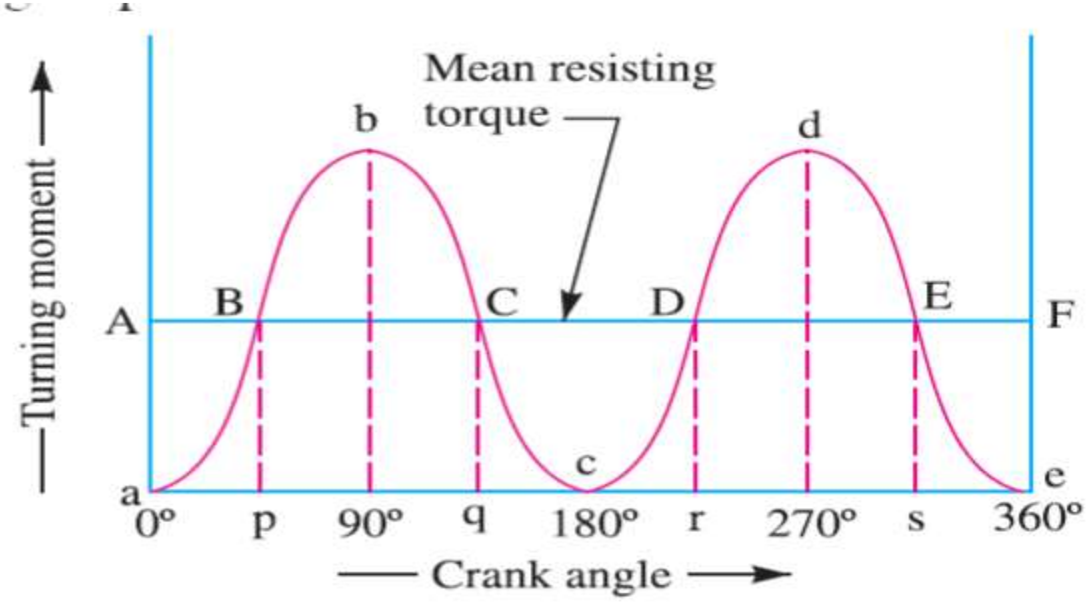
Introduction

- A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply.
- In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in I.C. engines, the energy is developed only during power stroke which is much more than the engine load, and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed.
- A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.
- **A flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.**

Fluctuation of energy

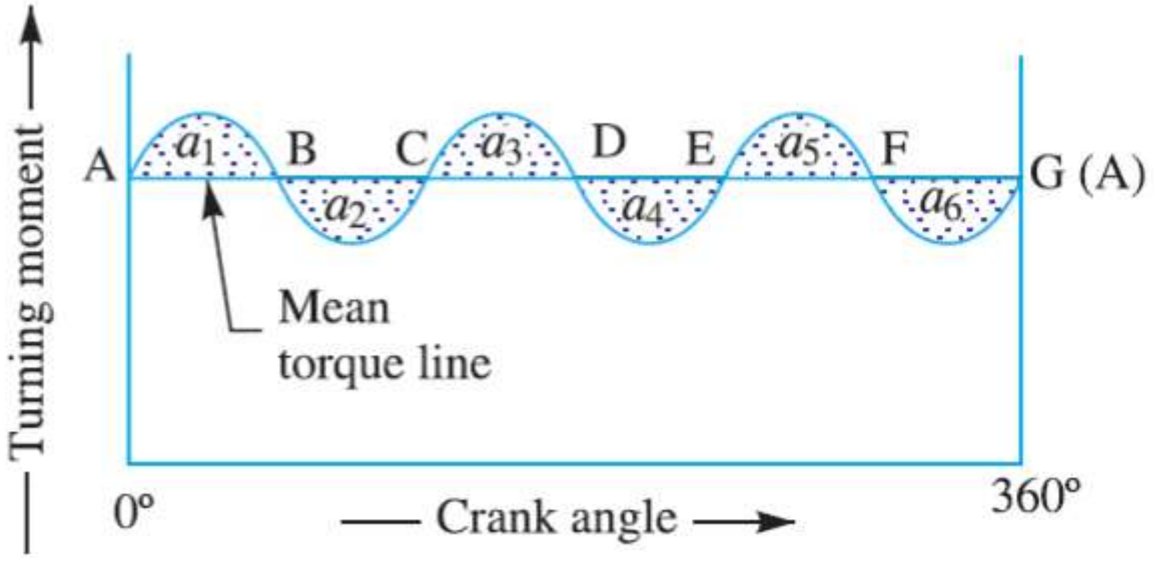
- The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider a turning moment diagram for a single cylinder double acting steam engine as shown in Fig. The vertical ordinate represents the turning moment and the horizontal ordinate (abscissa) represents the crank angle.
- A little consideration will show that the turning moment is zero when the crank angle is zero. It rises to a maximum value when crank angle reaches 90º and it is again zero when crank angle is 180º. This is shown by the curve abc in Fig. and it represents the turning moment diagram for outstroke. The curve cde is the turning moment diagram for instroke and is somewhat similar to the curve abc.
- The variations of energy above and below the mean resisting torque line are called **fluctuation of energy**.



Turning moment diagram for a single cylinder double acting steam engine.

Maximum Fluctuation of energy

- The difference between the maximum and the minimum energies is known as **maximum fluctuation of energy**.
- A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. The horizontal line AG represents the mean torque line. Let a1, a3, a5 be the areas above the mean torque line and a2, a4 and a6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.



Turning moment diagram for a multi-cylinder engine.

Let the energy in the flywheel at $A = E$, then from Fig

Energy at $B = E + a_1$

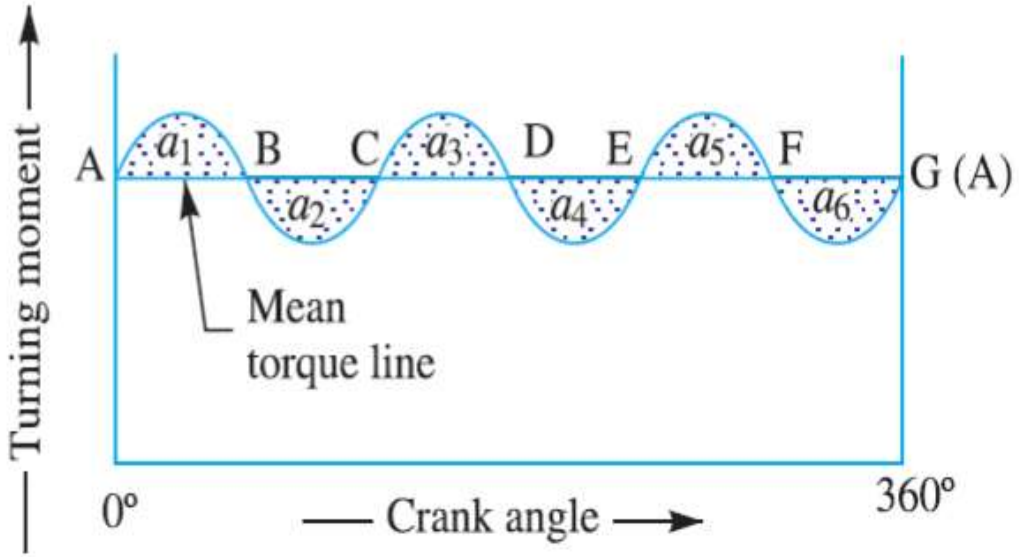
Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6 =$ Energy at A



Let us now suppose that the maximum of these energies is at B and minimum at E.

$\therefore$ Maximum energy in the flywheel

$$= E + a_1$$

and minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

$\therefore$ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

Coefficient of Fluctuation of Energy

- It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It is usually denoted by C_E. Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The workdone per cycle may be obtained by using the following relations:

1. Workdone / cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned in radians per revolution

$= 2\pi$, in case of steam engines and two stroke internal combustion engines.

$= 4\pi$, in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation *i.e.*

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = $2\pi N / 60$

2. The workdone per cycle may also be obtained by using the following relation:

$$\text{Workdone / cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute.

= N, in case of steam engines and two stroke internal combustion engines.

= $N/2$, in case of four stroke internal combustion engines.

Table 22.2. Coefficient of fluctuation of energy (C_E) for steam and internal combustion engines.

S.No.	*Type of engine*	*Coefficient of fluctuation of energy* (C_E)
1.	Single cylinder, double acting steam engine	0.21
2.	Cross-compound steam engine	0.096
3.	Single cylinder, single acting, four stroke gas engine	1.93
4.	Four cylinder, single acting, four stroke gas engine	0.066
5.	Six cylinder, single acting, four stroke gas engine	0.031

Coefficient of Fluctuation of Speed

- The difference between the maximum and minimum speeds during a cycle is called the **maximum fluctuation of speed.**
- The ratio of the maximum fluctuation of speed to the mean speed is called **coefficient of fluctuation of speed.**

Let N_1 = Maximum speed in r.p.m. during the cycle,

N_2 = Minimum speed in r.p.m. during the cycle, and

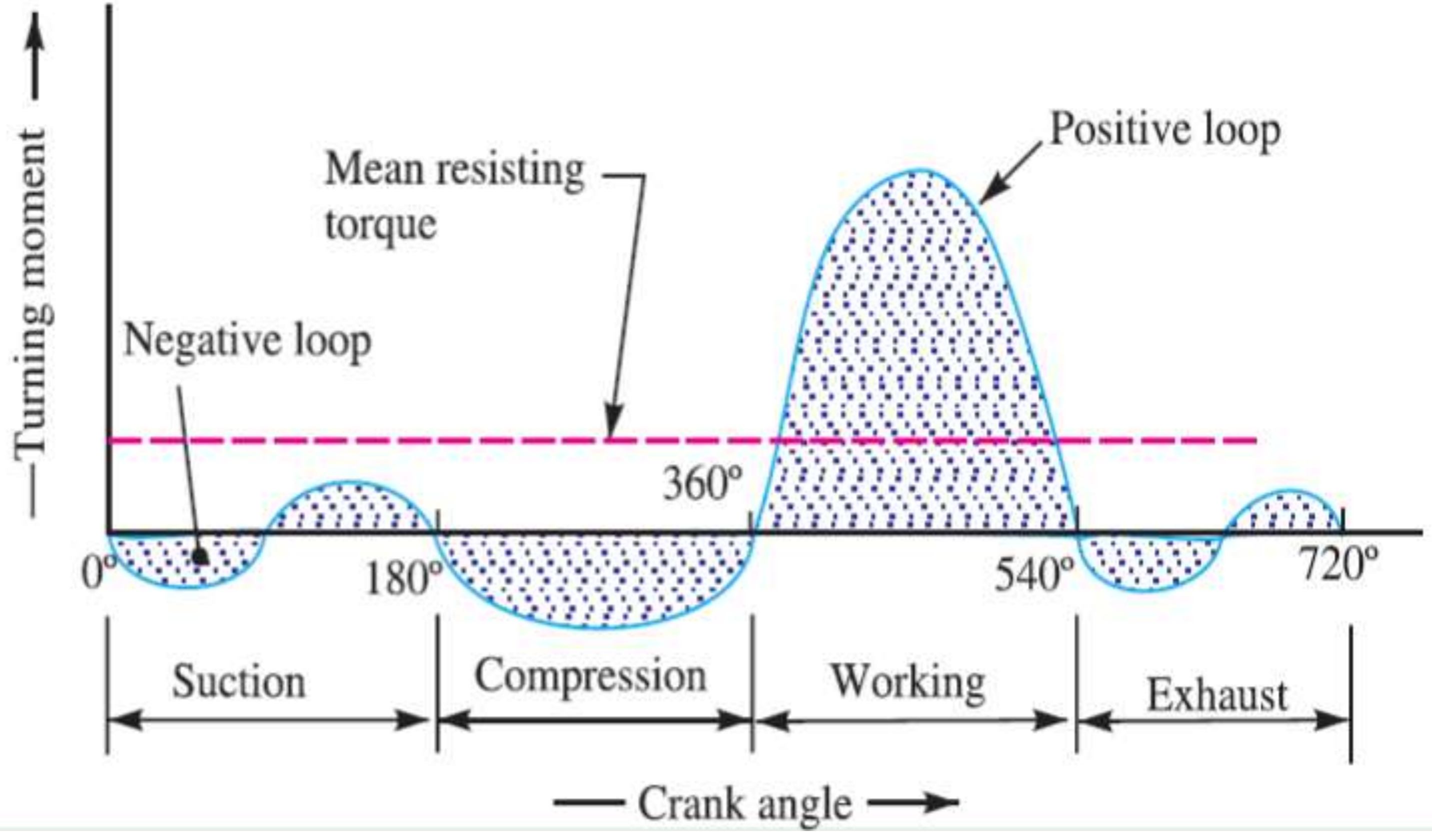
N = Mean speed in r.p.m. $= \dfrac{N_1 + N_2}{2}$

$\therefore$ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$



Tunring moment diagram for a four stroke internal combustion engine.

Energy Stored in a Flywheel

Let m = Mass of the flywheel in kg,

k = Radius of gyration of the flywheel in metres,

I = Mass moment of inertia of the flywheel about the axis of rotation in kg-m^2

$= m.k^2$,

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad / s,

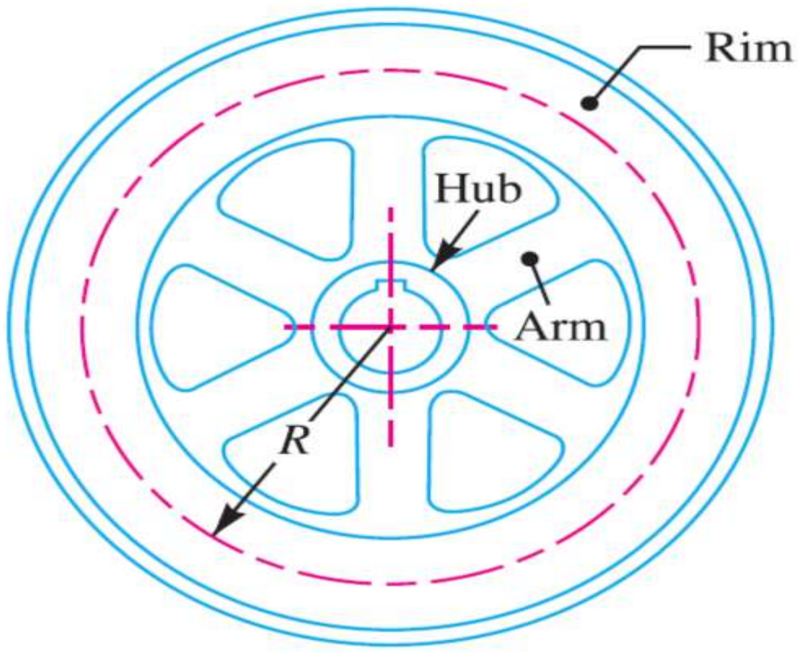


Fig. 22.5. Flywheel.

N = Mean speed during the cycle in r.p.m. $= \dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad / s $= \dfrac{\omega_1 + \omega_2}{2}$

C_S = Coefficient of fluctuation of speed $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$

We know that mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \text{ (in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.} = \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2$$

$$= \frac{1}{2} \times I\left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times I(\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= I.\omega(\omega_1 - \omega_2) \qquad \ldots\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right) \ldots(i)$$

$$= I.\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right) \qquad \ldots[\text{Multiplying and dividing by } \omega]$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \qquad \ldots(\because I = m.k^2) \ldots(ii)$$

$$= 2\,E.C_S \qquad \ldots\left(\because E = \frac{1}{2} \times I.\omega^2\right) \ldots(iii)$$

The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore substituting $k = R$ in equation (*ii*), we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S \qquad \ldots(\because v = \omega.R)$$

From this expression, the mass of the flywheel rim may be determined.