Introduction

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load *e.g. when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid.*

On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required.

The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

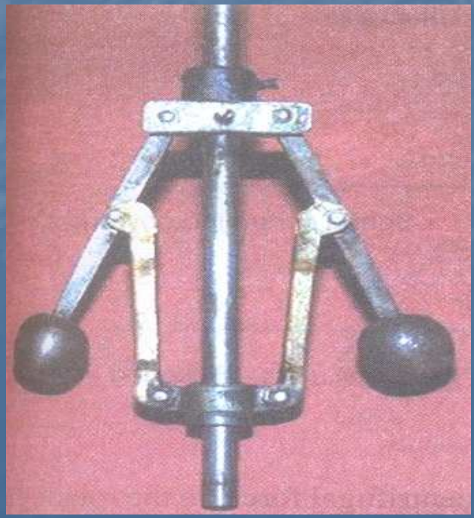
The function of a flywheel in an engine is entirely different from that of a governor. It controls the speed variation caused by the fluctuations of the engine turning moment during each cycle of operation.

It does not control the speed variations caused by a varying load.

Types of Governors

The governors may, broadly, be classified as

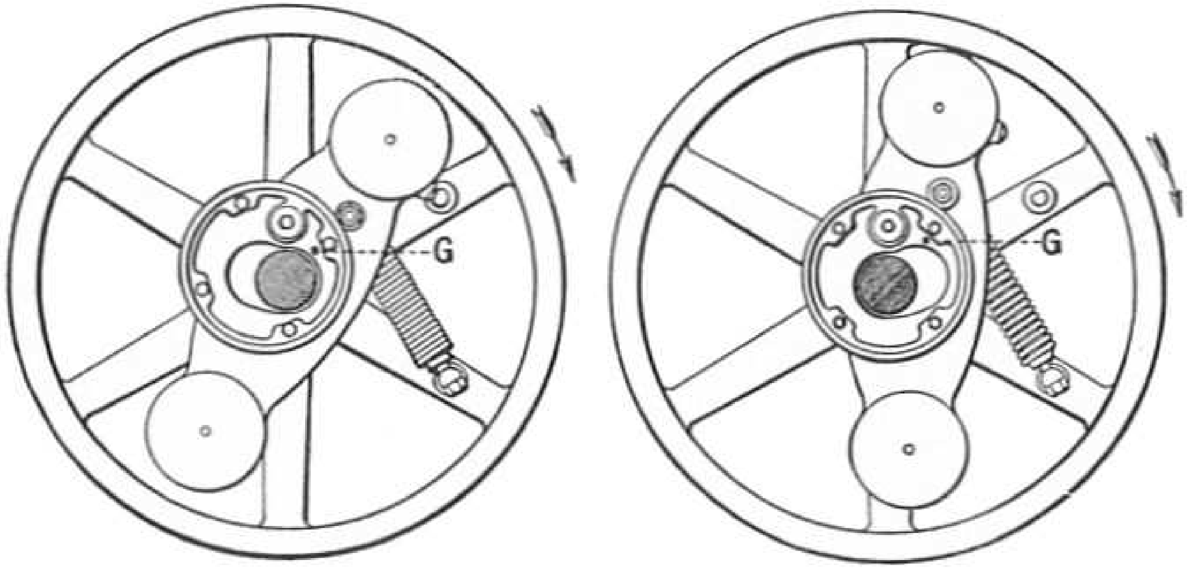
1. Centrifugal governors, and 2. Inertia governors.



Centrifugal governors

- Pendulum type
  - Watt governor
- Loaded type
  - Dead weight governors
    - Porter governor
    - Proell governor
  - Spring controlled governors
    - Hartnell governor
    - Hartung governor
    - Wilson-Hartnell governor
    - Pickering governor

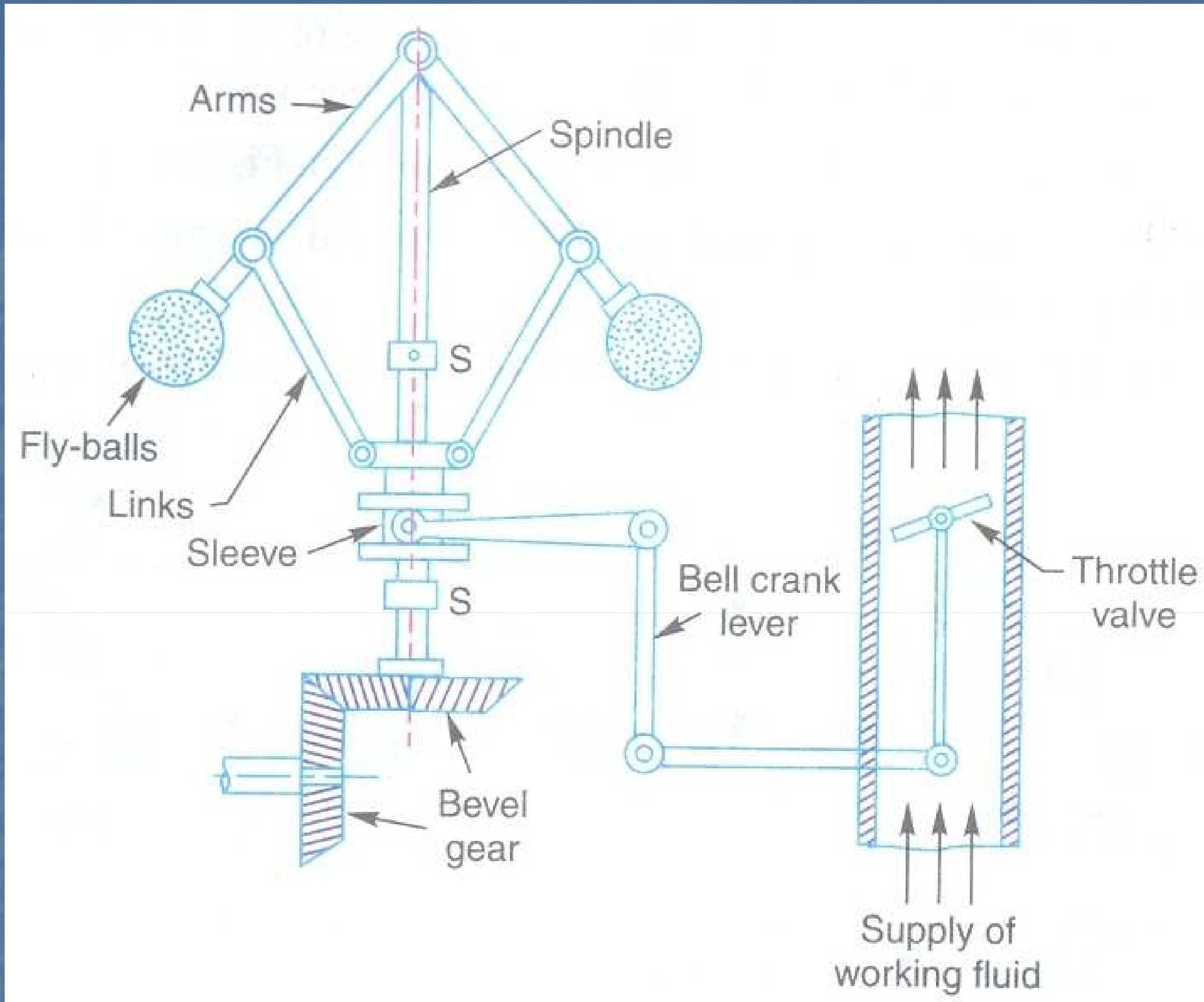
Inertia governor

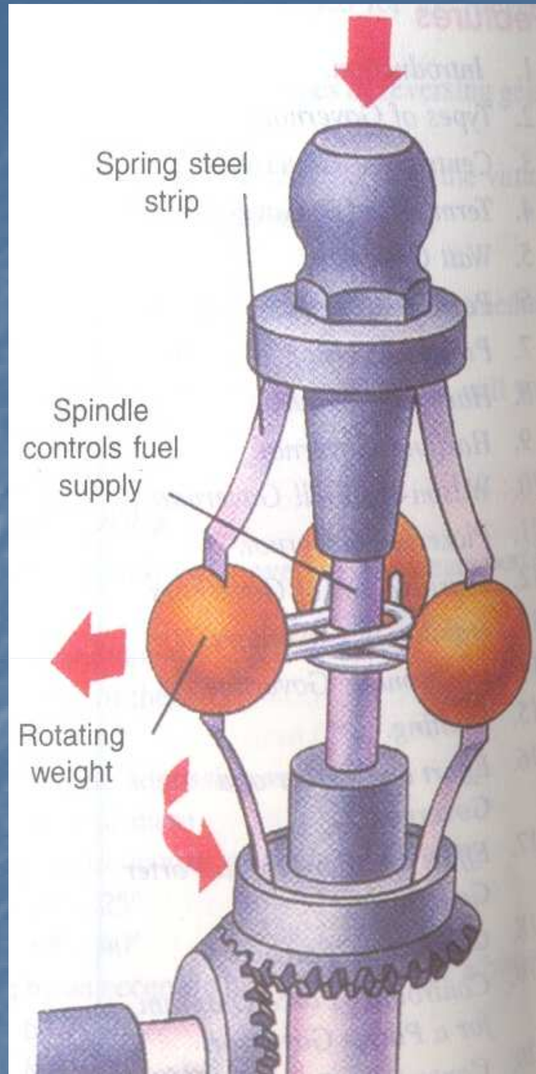


At Longest Cut-off   At Shortest Cut-off

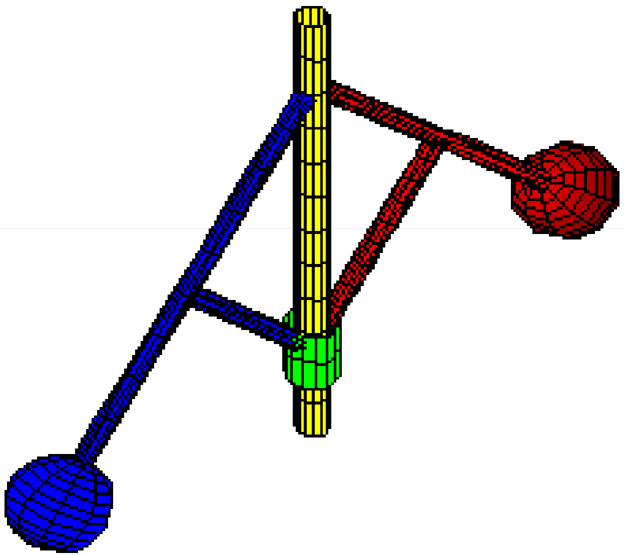
A speed-control device utilizing suspended masses that respond to speed changes by reason of their inertia

Inertia governor is more sensitive than the centrifugal, but it becomes difficult to completely balance the revolving parts. For this reason centrifugal governors are more frequently used.



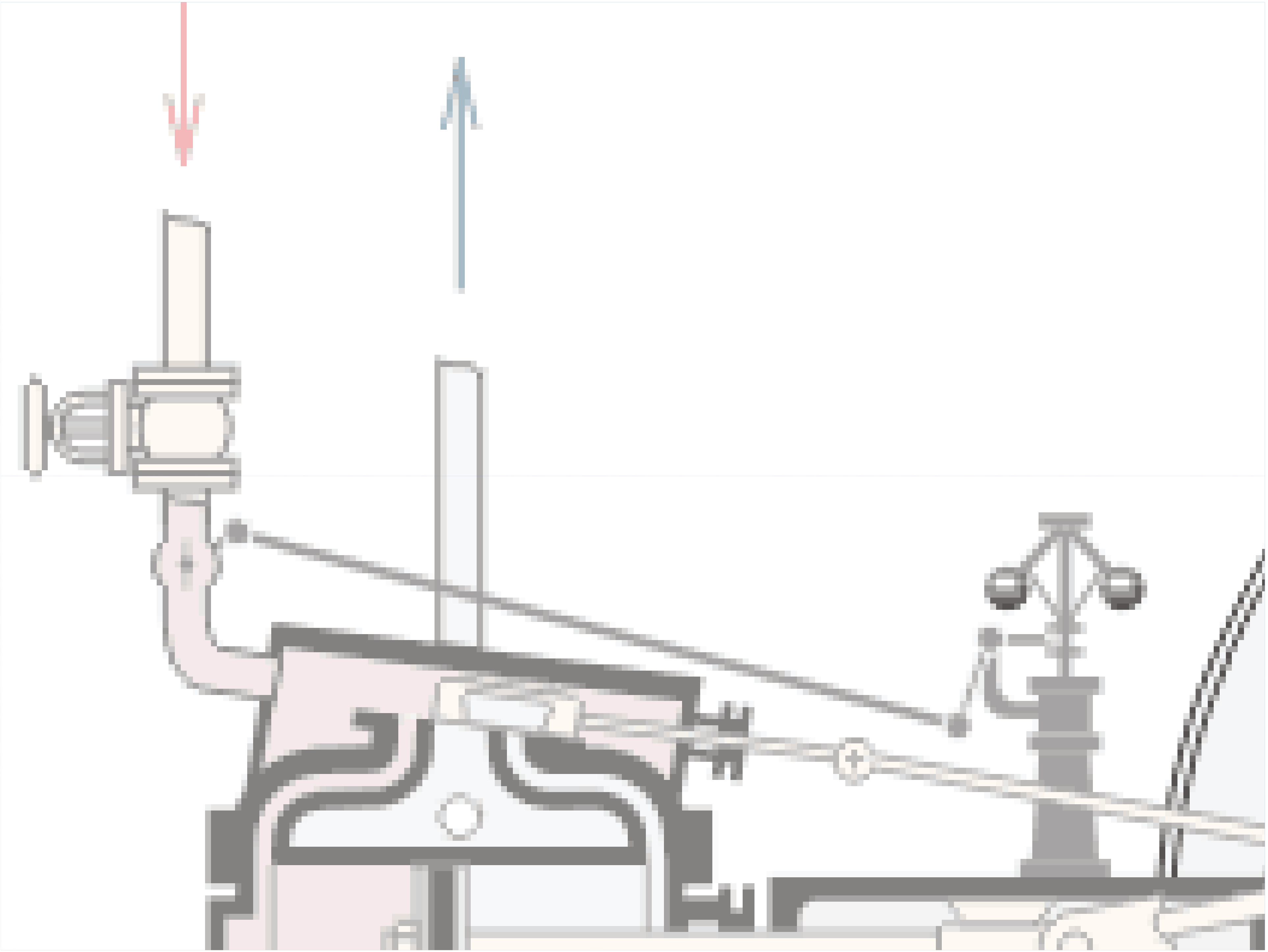


Centrifugal Governor



An 18th century governor

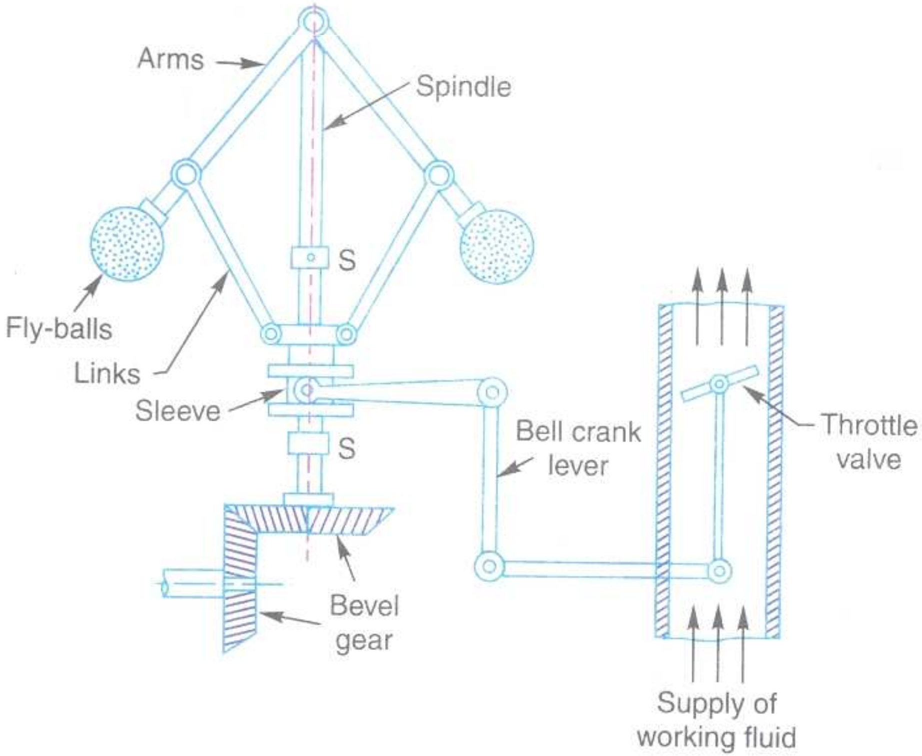
Boulton & Watt engine of 1788



Centrifugal Governors

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the *controlling force.*

It consists of two balls of equal mass, which are attached to the arms as shown in Figure These balls are known as governor balls or fly balls. The balls revolve with a spindle, which is driven by the engine through bevel gears.

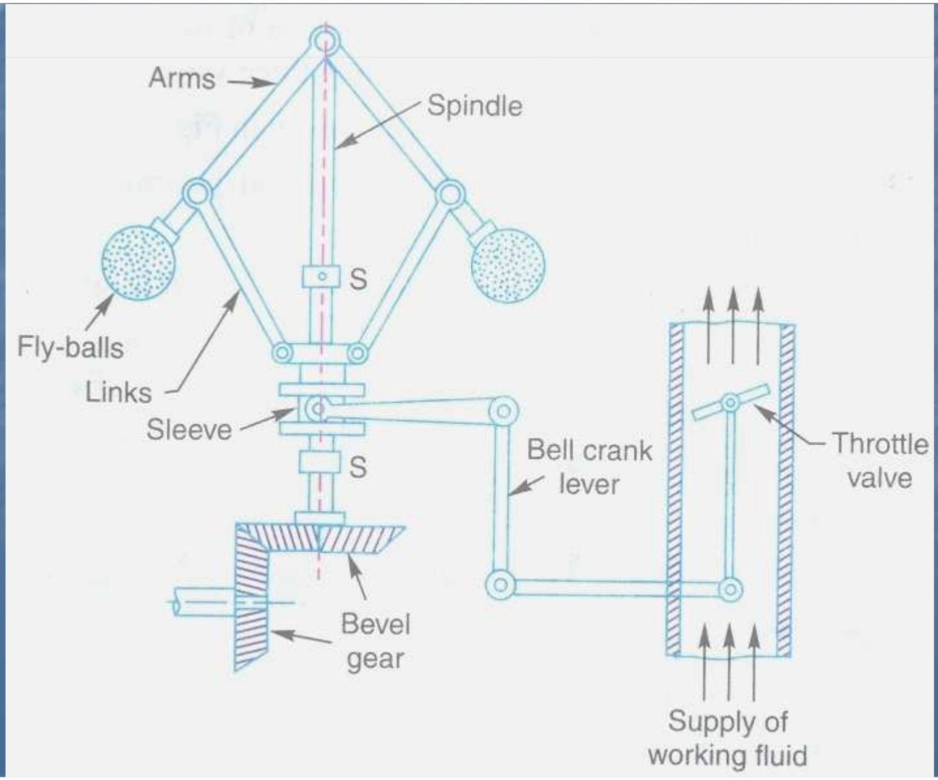


The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis.

The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle ; but can slide up and down.

The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops , *S are provided on the spindle.*

The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls.



Load on Engine
Increase

Speed
Decreases

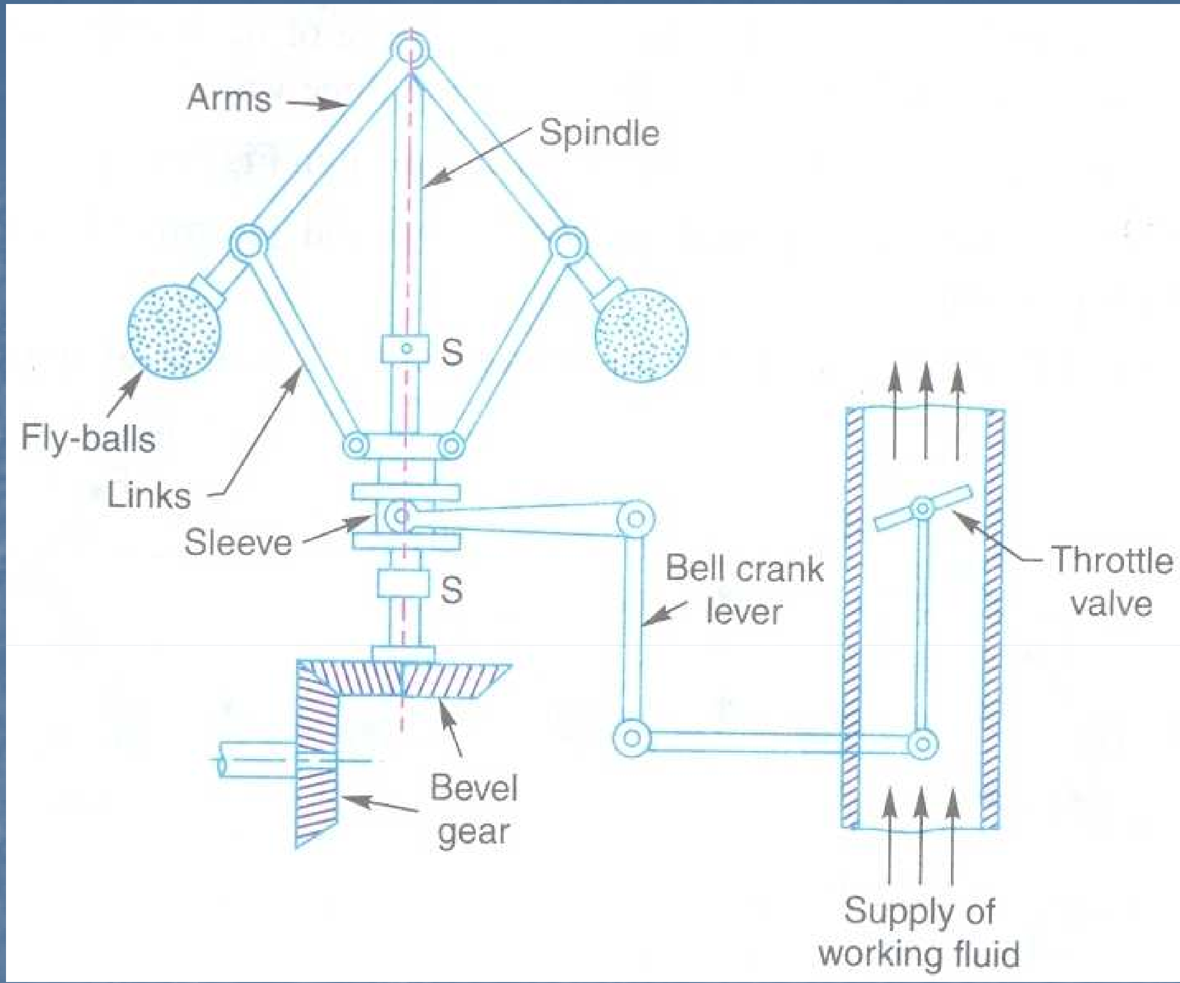
Centrifugal force
Decreases

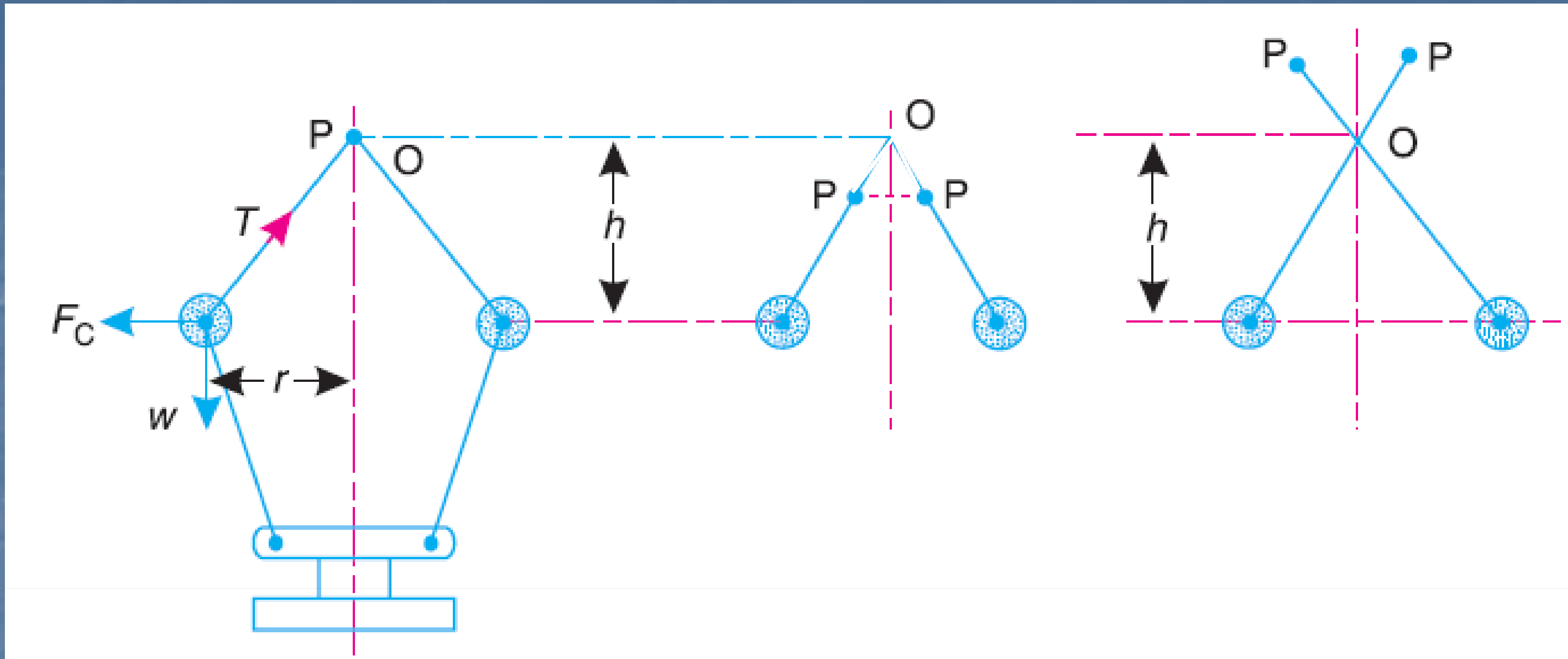
Sleeve moves
Down

That causes
throttle valve
open

Fuel supply
Increase

Speed
Will Increases

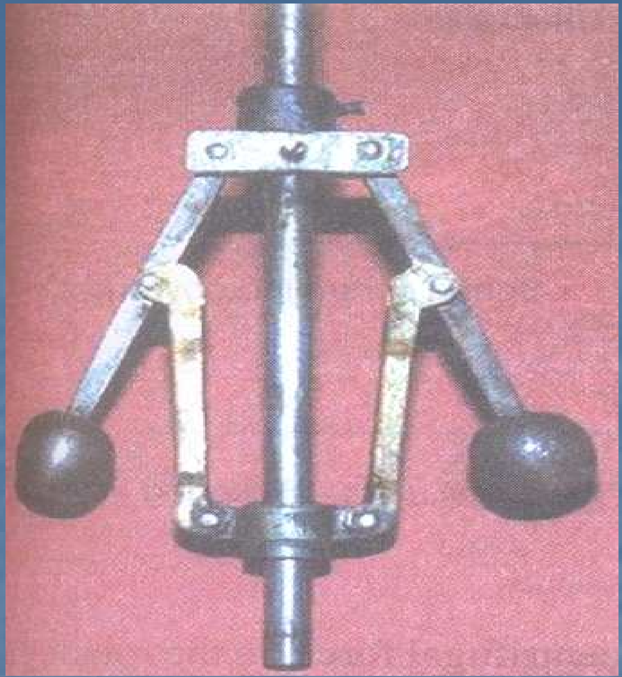


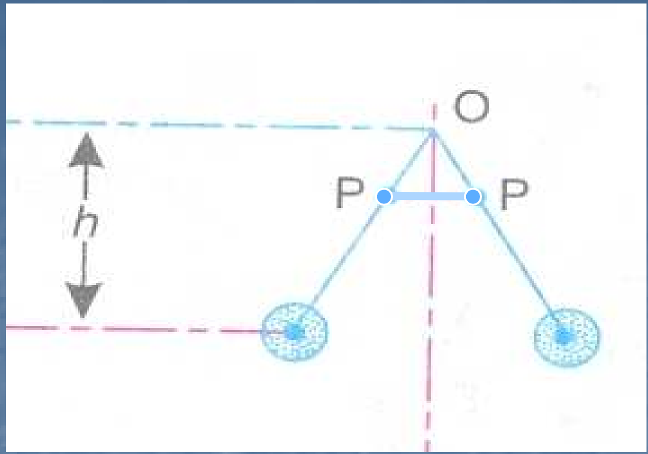


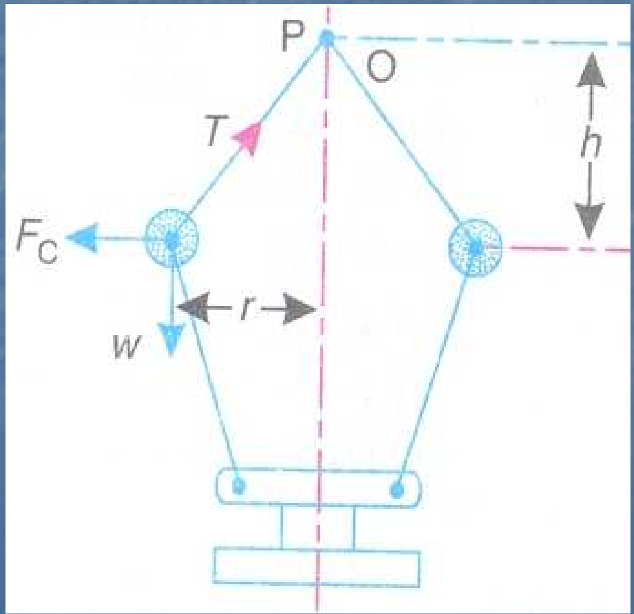
Watt Governor

The simplest form of a centrifugal governor is a Watt governor, as shown in Figure basically a conical pendulum with links attached to a sleeve of negligible mass. The is governor may be connected to the spindle in the following three ways :

1. The pivot P. may be on the spindle axis as shown in Fig. 18.2 (a).
2. The pivot P, may be offset from the spindle axis and the arms when produced intersect at, 0, as shown in Fig. (b).
3. The pivot P, may be offset, but the arms cross the axis at O, as shown in Figure is, (c).




O
P
P
h


P
O
T
h
F_C
r
w

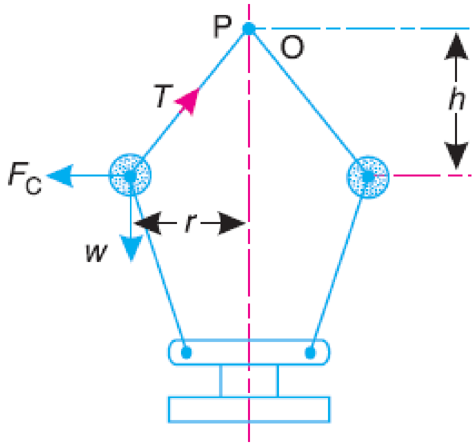
1. Height of a governor : It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by h.

2. Equilibrium speed: It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move up wards or downwards.

3. Mean equilibrium speed : it is the speed at the mean position of the balls or the sleeve.

4. Maximum and minimum equilibrium : It is the speed at the maximum and minimum radius of rotation of the ball without tending to move in either direction.

5. Sleeve lift: It is the vertical distance which sleeve travels due to change in equilibrium speed.



m = Mass of the ball in kg,

w = Weight of the ball in newtons = $m.g$,

T = Tension in the arm in newtons,

ω = Angular velocity of the arm and ball about the spindle axis in rad/s,

r = Radius of the path of rotation of the ball *i.e.* horizontal distance from the centre of the ball to the spindle axis in metres,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$, and

h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of

1. the centrifugal force (F_C) acting on the ball, **2.** the tension (T) in the arm, and **3.** the weight (w) of the ball.

Taking moments about point O, we have

$$F_C \times h = w \times r = m.g.r$$

or
$$m.\omega^2.r.h = m.g.r \quad \text{or} \quad h = g/\omega^2 \qquad \ldots (i)$$

When g is expressed in m/s² and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then

$$\omega = 2\pi N/60$$

$$\therefore \qquad h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres} \qquad \ldots (\because g = 9.81 \text{ m/s}^2) \ldots (ii)$$

Note

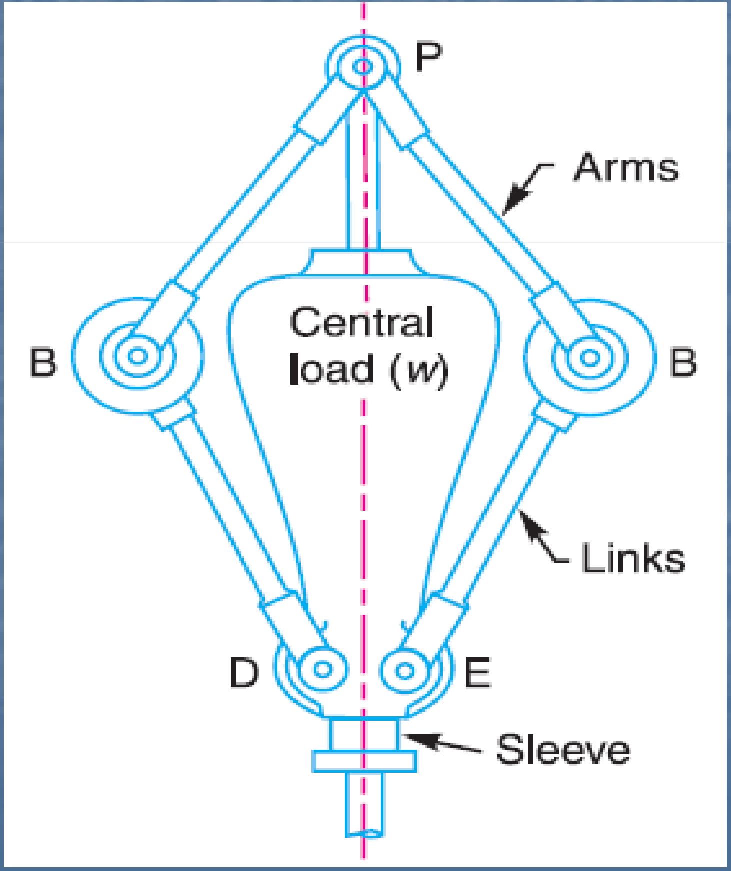
- One can see from the above expression that the height of a governor h is inversely proportional to N^2. Therefore at high speeds, the value of h is small.

- At such speeds, the change in the value of h corresponding to a small change in speed is insufficient to enable a governor of this type to operate the mechanism to give the necessary change in the fuel supply.

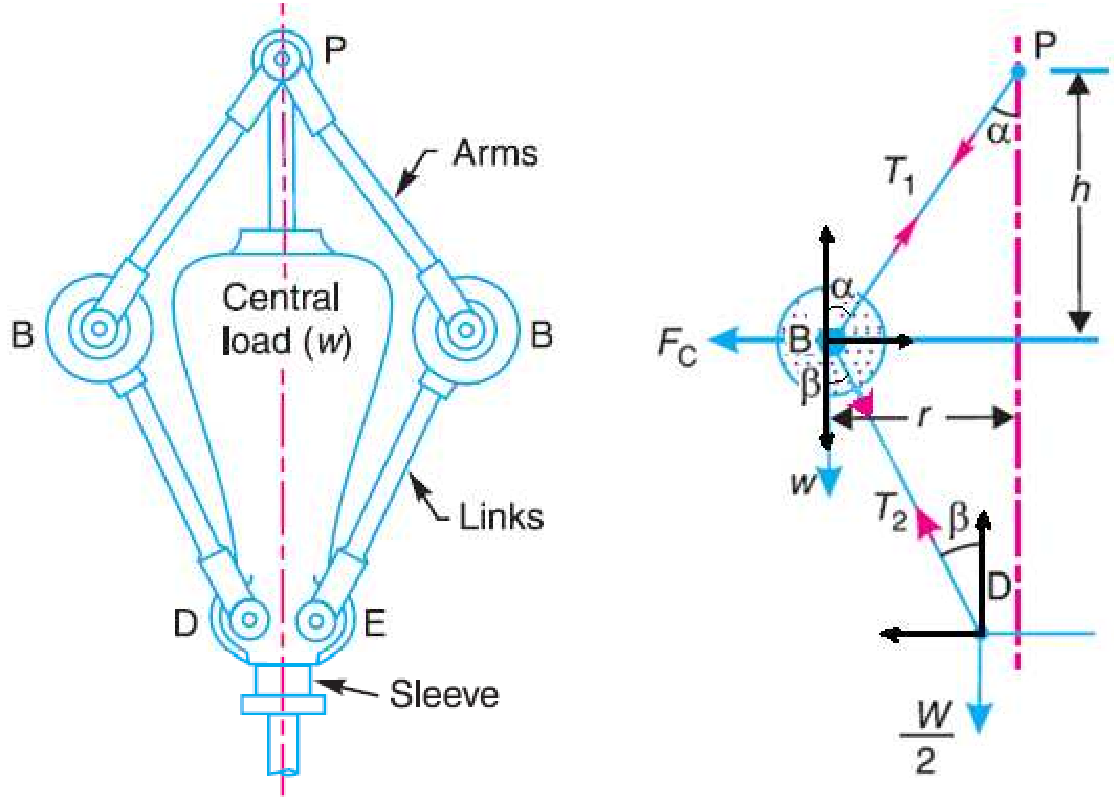
- This type governor may only work satisfactorily at relatively low speeds i.e. from 60 to 80 r.p.m.

Porter Governor

The Porter governor is a modification of a **Watt's governor**, with **central load** attached to the sleeve as shown in Fig.



The load moves up and down on the central spindle. This additional **downward force** increases the speed of revolution which is required to enable the balls to move to any pre determined level.



(a) (b)

Let m = Mass of each ball in kg,

w = Weight of each ball in newtons = $m.g$,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = $M.g$,

r = Radius of rotation in metres,

h = Height of governor in metres ,

N = Speed of the balls in r.p.m .,

ω = Angular speed of the balls in rad/s $= 2\,\pi N/60$ rad/s,

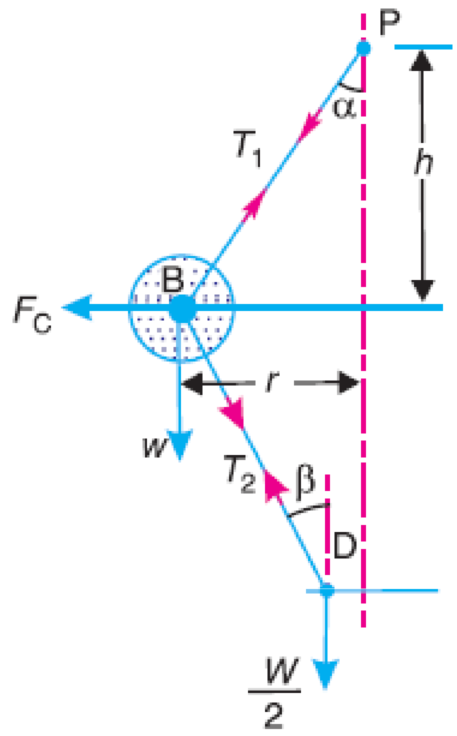
F_C = Centrifugal force acting on the ball in newtons $= m\,.\,\omega^2.r$,

T_1 = Force in the arm in newtons,

T_2 = Force in the link in newtons,

α = Angle of inclination of the arm (or upper link) to the vertical, and

β = Angle of inclination of the link (or lower link) to the vertical.



Though there are several ways of determining the relation between the height of the governor (h) and the angular speed of the balls (ω), yet the following two methods are important from the subject point of view :

1. Method of resolution of forces ; and
2. Instantaneous centre method.

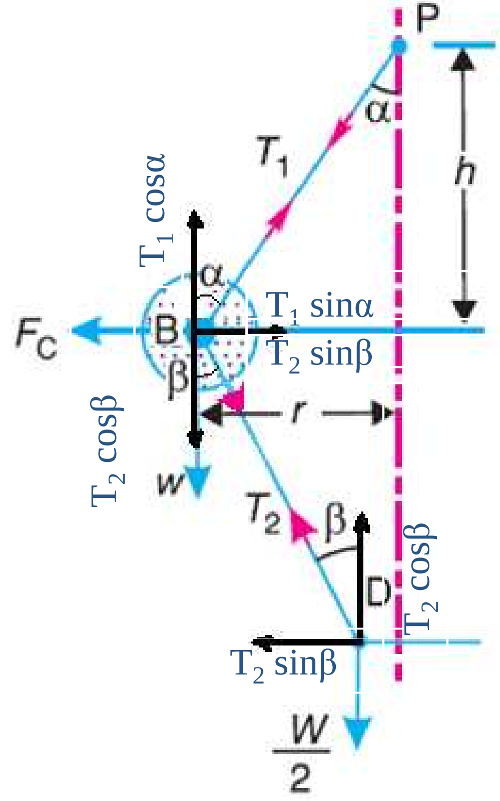
1. *Method of resolution of forces*

Considering the equilibrium of the forces acting at D, we have

$$T_2 \cos\beta = \frac{W}{2} = \frac{M.g}{2}$$

or

$$T_2 = \frac{M.g}{2\cos\beta} \qquad \ldots (i)$$



Again, considering the equilibrium of the forces acting on B. The point B is in equilibrium under the action of the following forces,

(*i*) The weight of ball ($w = m.g$),

(*ii*) The centrifugal force (F_C),

(*iii*) The tension in the arm (T_1), and

(*iv*) The tension in the link (T_2).

Resolving the forces vertically,

$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g \qquad \ldots (ii)$$

$$\ldots \left(\because\ T_2 \cos\beta = \frac{M.g}{2}\right)$$

Resolving the forces horizontally,

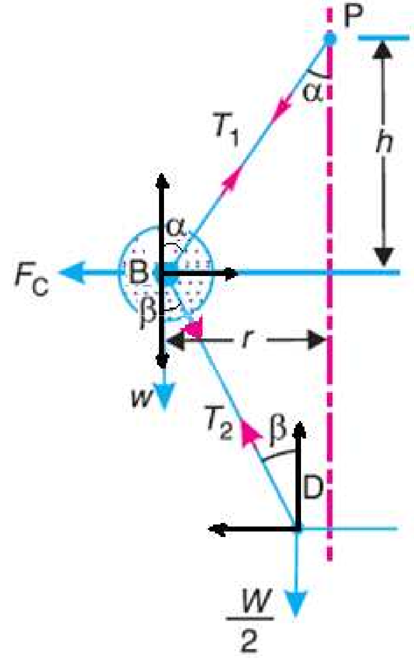
$$T_1 \sin\alpha + T_2 \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2\cos\beta} \times \sin\beta = F_C \qquad \ldots \left(\because\ T_2 = \frac{M.g}{2\cos\beta}\right)$$

$$T_1 \sin\alpha + \frac{M.g}{2} \times \tan\beta = F_C$$

$$T_1 \sin\alpha = F_C - \frac{M.g}{2} \times \tan\beta \qquad \ldots (iii)$$

Dividing equation *(iii)* by equation *(ii)*,



$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_C - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g}$$

or
$$\left(\frac{M.g}{2} + m.g\right)\tan\alpha = F_C - \frac{M.g}{2} \times \tan\beta$$

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

Substituting $\frac{\tan\beta}{\tan\alpha} = q$, and $\tan\alpha = \frac{r}{h}$, we have

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q \qquad \ldots (\because F_C = m.\omega^2.r)$$

$$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$$\therefore \quad h = \left[m.g + \frac{M.g}{2}(1+q) \right] \frac{1}{m.\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2} \quad \ldots (iv)$$

or
$$\omega^2 = \left[m.g + \frac{Mg}{2}(1+q) \right] \frac{1}{m.h} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

or
$$\left(\frac{2\pi N}{60} \right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$$\therefore \quad N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}\left(\frac{60}{2\pi} \right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h} \quad \ldots (v)$$

. . . (Taking $g = 9.81$ m/s^2)

Notes : 1. When the length of arms are equal to the length of links and the points P and D lie on the same vertical line, then

$$\tan\alpha = \tan\beta \quad \text{or} \quad q = \tan\alpha / \tan\beta = 1$$

Therefore, the equation (*v*) becomes

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h} \quad \ldots (vi)$$

2. When the loaded sleeve moves up and down the spindle, the frictional force acts on it in a direction opposite to that of the motion of sleeve.

If F = Frictional force acting on the sleeve in newtons, then the equations ***(v)*** and ***(vi)*** may be written as

$$N^2 = \frac{m.g + \left(\dfrac{M.g \pm F}{2}\right)(1+q)}{m.g} \times \frac{895}{h} \qquad \ldots \textbf{\textit{(vii)}}$$

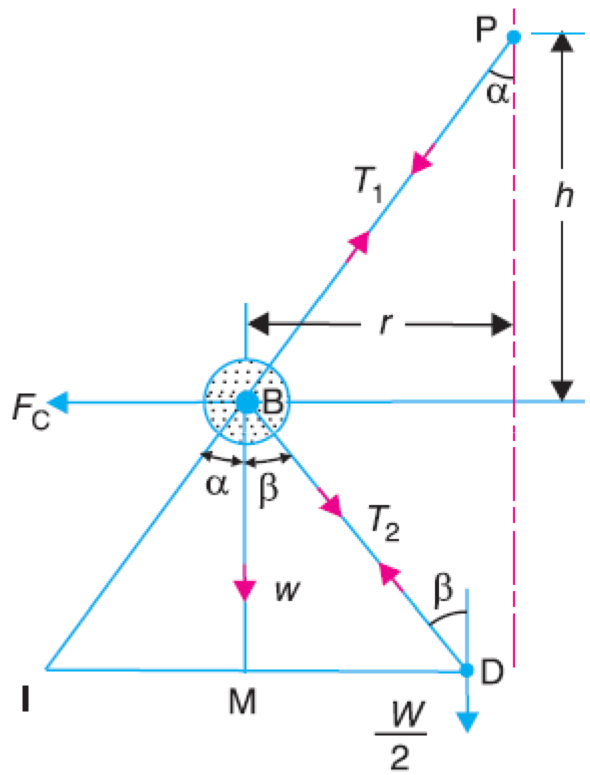
$$= \frac{m.g + (M.g \pm F)}{m.g} \times \frac{895}{h} \qquad \ldots \text{(When } q = 1) \ldots \textbf{\textit{(viii)}}$$

The + sign is used when the sleeve moves upwards or the governor speed increases and negative sign is used when the sleeve moves downwards or the governor speed decreases.

3. On comparing the equation ***(vi)*** with equation ***(ii)*** of Watt's governor (Art. 1), we find that the mass of the central load (M) increases the height of governor in the ratio $\dfrac{m+M}{m}$.

2. *Instantaneous centre method*

In this method, equilibrium of the forces acting on the link *BD* are considered. The instantaneous centre *I* lies at the point of intersection of *PB* produced and a line through *D* perpendicular to the spindle axis, as shown in Fig. Taking moments about the point *I*,



$$F_C \times BM = w \times IM + \frac{W}{2} \times ID$$

$$= m.g \times IM + \frac{M.g}{2} \times ID$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2} \times \frac{ID}{BM}$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right)$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM}{BM} + \frac{MD}{BM}\right)$$

$$= m.g \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta) \qquad \ldots\left(\because \frac{IM}{BM} = \tan\alpha, \text{ and } \frac{MD}{BM} = \tan\beta\right)$$

Dividing throughout by tan α,

$$\frac{F_C}{\tan\alpha} = m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right) = m.g + \frac{M.g}{2}(1+q) \qquad \ldots\left(\because q = \frac{\tan\beta}{\tan\alpha}\right)$$

We know that $F_C = m.\omega^2.r$, and $\tan\alpha = \frac{r}{h}$

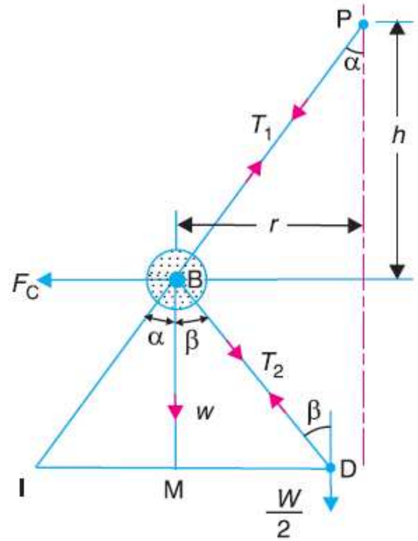
$$\therefore \quad m.\omega^2.r \times \frac{h}{r} = m.g + \frac{M.g}{2}(1+q)$$

or

$$h = \frac{m.g + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

When $\tan\alpha = \tan\beta$ or $q = 1$, then

$$h = \frac{m+M}{m} \times \frac{g}{\omega^2}$$

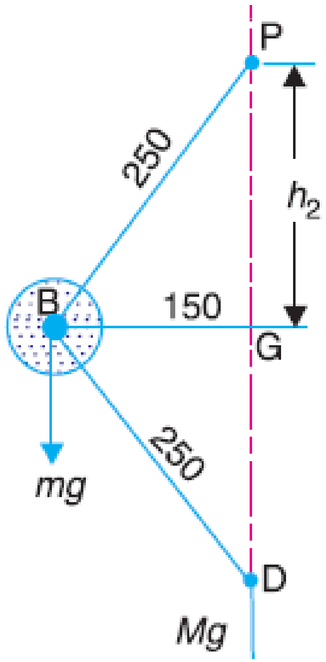


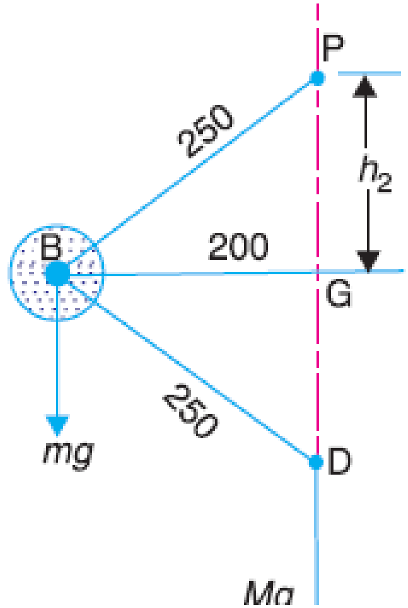
Example *The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.*

Solution. Given : $BP = BD = 250$ mm ; $m = 5$ kg ; $M = 30$ kg ; $r_1 = 150$ mm ; $r_2 = 200$ mm

Let N_1 = Minimum speed when $r_1 = BG = 150$ mm, and

N_2 = Maximum speed when $r_2 = BG = 200$ mm.

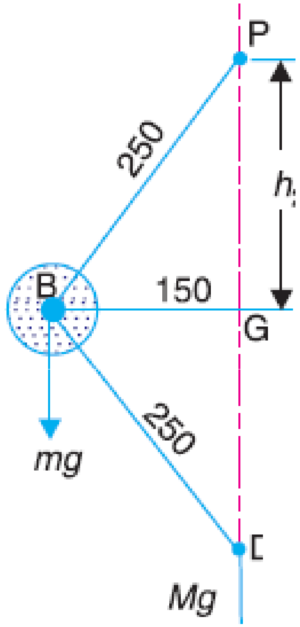




Speed range of the governor

From (a), height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$



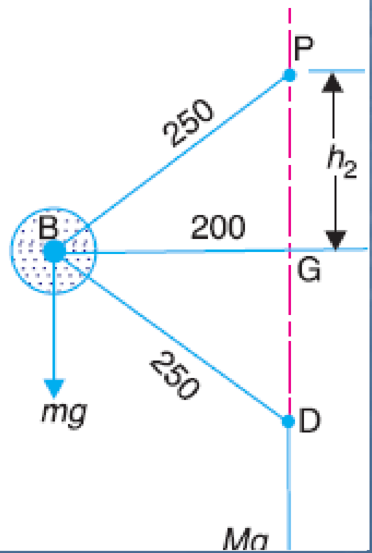
$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+30}{5} \times \frac{895}{0.2} = 31\,325$$

$N_1 = 177$ r.p.m.

From (b), height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2}$$

$$= 0.15 \text{ m}$$



$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+30}{5} \times \frac{895}{0.15} = 41\,767$$

$N_2 = 204.4$ r.p.m.

speed range of the governor

$= N_2 - N_1 = 204.4 - 177 = 27.4$ r.p.m. **Ans.**

Speed range when friction at the sleeve is equivalent of 20 N of load (i.e. when F = 20 N)

when the sleeve moves downwards, the friction force (F) acts upwards and the minimum speed is given by

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 - 20)}{5 \times 9.81} \times \frac{895}{0.2}$$

when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m\,.\,g} \times \frac{895}{h_2}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 + 20)}{5 \times 9.81} \times \frac{895}{0.15}$$

$$\therefore \qquad N_2 = 210 \text{ r.p.m.}$$

speed range of the governor

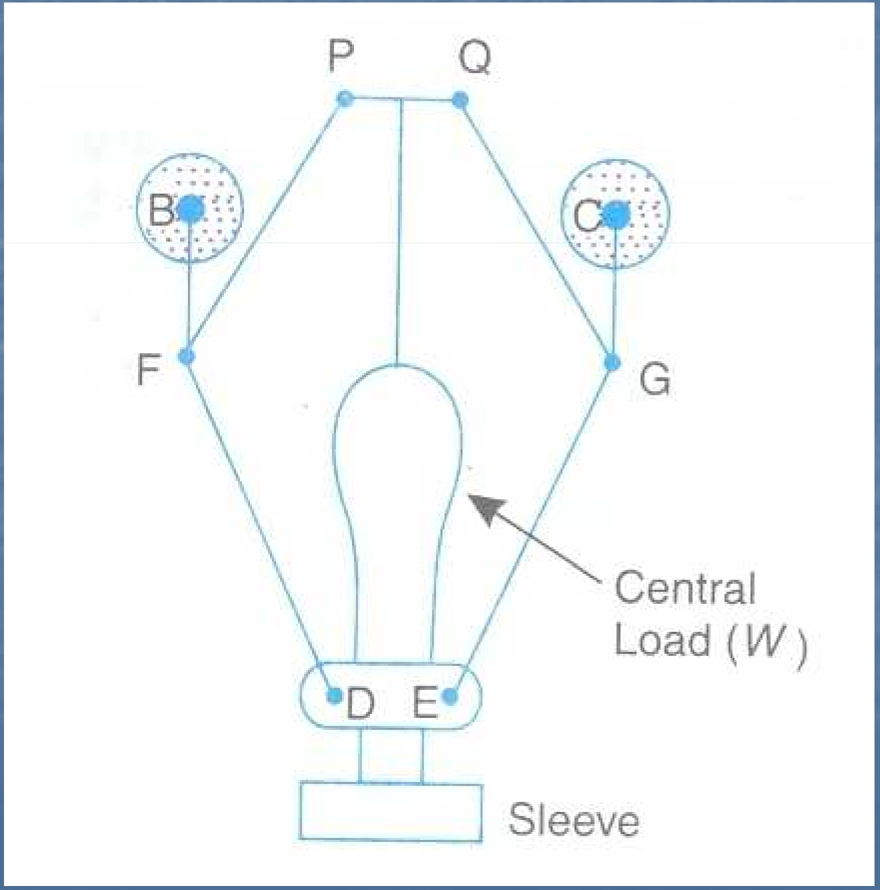
$= N_2 - N_1 = 210 - 172 = 38$ r.p.m. **Ans.**

Proell Governor

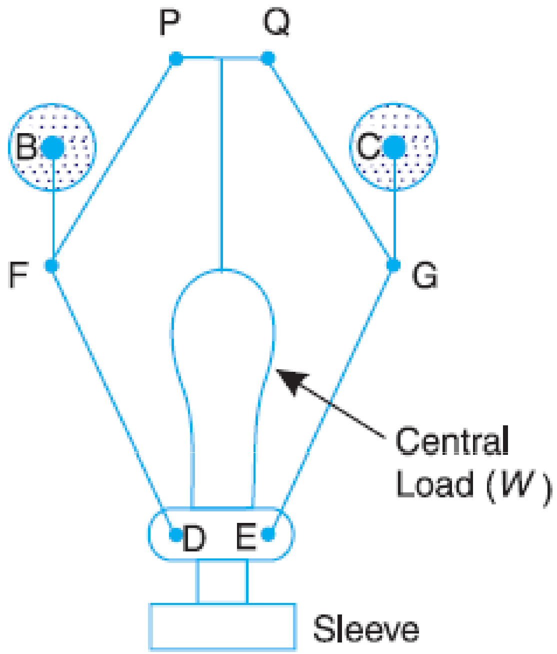
The Proell governor has the balls fixed at *B and C to the* ***extension of the links*** *DF and EG as shown in Fig. The arms FP and GQ are pivoted at P and Q respectively.*

By this change, it requires fly ball of **less mass** for same action.

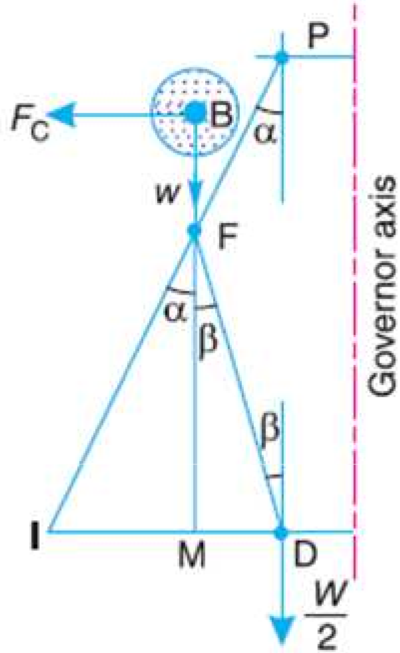
The central load moves up and down on the central spindle. This additional **downward force** increases the speed of revolution which is required to enable the balls to move to any pre determined level.



Consider the equilibrium of the forces on one-half of the governor as shown in Fig. The instantaneous centre (I) lies on the intersection of the line PF produced and the line from D drawn perpendicualr to the spindle axis. The prependicular BM is drawn on ID.



(a)



(b)

Taking moments about I,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \qquad \ldots (i)$$

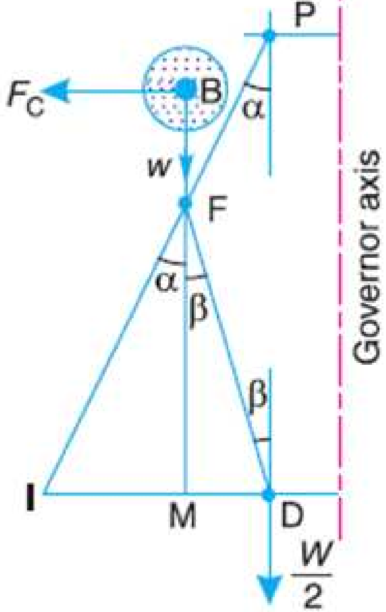
$$\therefore \qquad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \qquad \ldots (\because\ ID = IM + MD)$$

Multiplying and dividing by FM, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$



We know that $F_C = m.\omega^2 r$; $\tan\alpha = \dfrac{r}{h}$ and $q = \dfrac{\tan\beta}{\tan\alpha}$

$$\therefore \qquad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1 + q)\right]$$

and

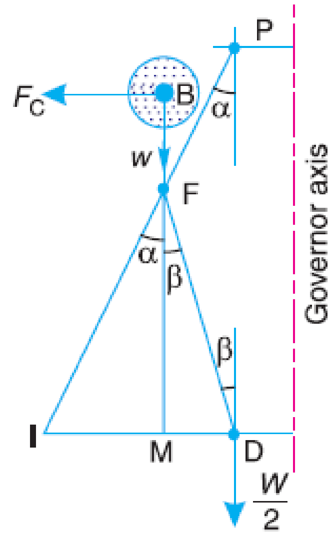
$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1 + q)}{m}\right]\frac{g}{h} \qquad \dots (ii)$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81\ \text{m/s}^2$, we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1 + q)}{m}\right]\frac{895}{h} \qquad \dots (iii)$$

Taking moments about I,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \quad \ldots (i)$$



1. The equation *(i)* may be applied to any given configuration of the governor.

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h} \quad \ldots (iii)$$

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h} \quad \ldots (v)$$

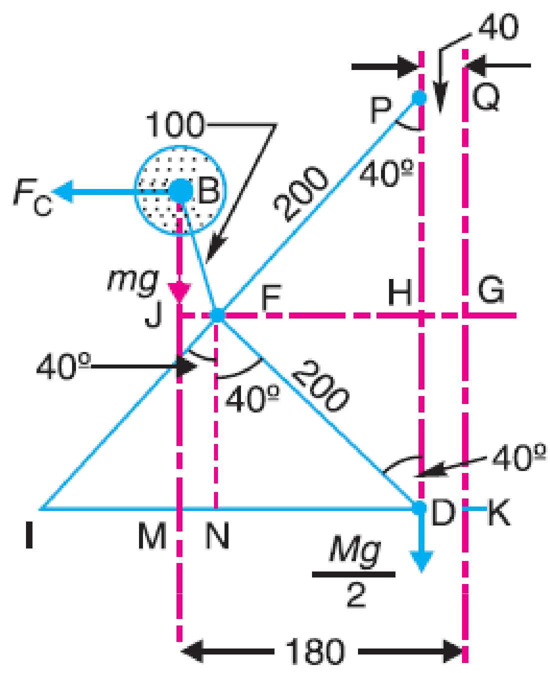
2. Comparing equation *(iii)* with the equation *(v)* of the Porter governor equilibrium speed reduces for the given values of m, M and h. Hence in order to have the same equilibrium speed for the given values of m, M and h, balls of smaller masses are used in the Proell governor than in the Porter governor.

3. When $\alpha = \beta$, then $q = 1$. Therefore equation *(iii)* may be written as

$$N^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h} \quad (h \text{ being in metres}) \ldots (iv)$$

Example *The following particulars refer to a Proell governor with open arms :*

Length of all arms = 200 mm ; distance of pivot of arms from the axis of rotation = 40 mm ; length of extension of lower arms to which each ball is attached = 100 mm ; mass of each ball = 6 kg and mass of the central load = 150 kg. If the radius of rotation of the balls is 180 mm when the arms are inclined at an angle of 40° to the axis of rotation, find the equilibrium speed for the above configuration.



Solution. Given : $PF = DF = 200$ mm ; $PQ = DK = HG = 40$ mm ; $BF = 100$ mm ; $m = 6$ kg; $M = 150$ kg ; $r = JG = 180$ mm $= 0.18$ m ; $\alpha = \beta = 40°$

Let N = Equilibrium speed.

$$F_C \times BM = m.g \times IM + \frac{M.g}{2} \times ID$$

From the equilibrium position of the governor, as shown in Fig.

$$PH = PF \times \cos 40°$$

$$= 200 \times 0.766 = 153.2 \text{ mm}$$

$$= 0.1532 \text{ m}$$

and $$FH = PF \times \sin 40° = 200 \times 0.643 = 128.6 \text{ mm}$$

$$JF = JG - HG - FH = 180 - 40 - 128.6 = 11.4 \text{ mm}$$

and $$BJ = \sqrt{(BF)^2 - (JF)^2} = \sqrt{(100)^2 - (11.4)^2} = 99.4 \text{ mm}$$

$$BM = BJ + JM = 99.4 + 153.2 = 252.6 \text{ mm} \quad \ldots (\because JM = HD = PH)$$

$$IM = IN - NM = FH - JF = 128.6 - 11.4 = 117.2 \text{ mm}$$

and $$ID = IN + ND = 2 \times IN = 2 \times FH = 2 \times 128.6 = 257.2 \text{ mm}$$

Now taking moments about the instantaneous centre I,

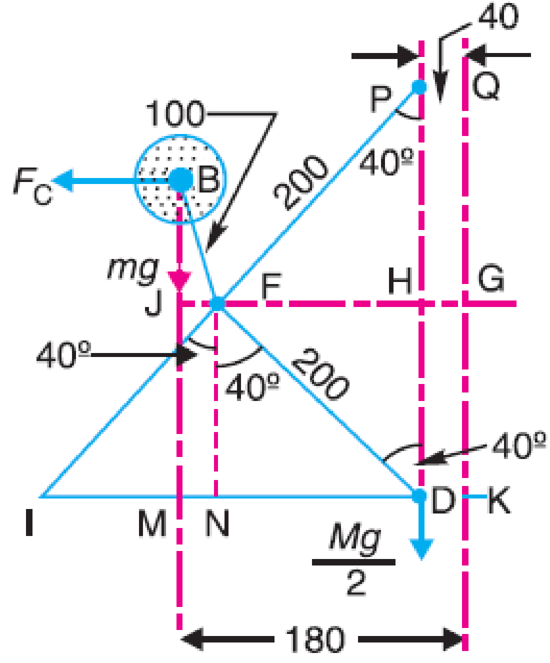
$$F_C \times BM = m.g \times IM + \frac{M.g}{2} \times ID$$

$$F_C \times 252.6 = 6 \times 9.81 \times 117.2 + \frac{150 \times 9.81}{2} \times 257.2 = 196\ 125$$

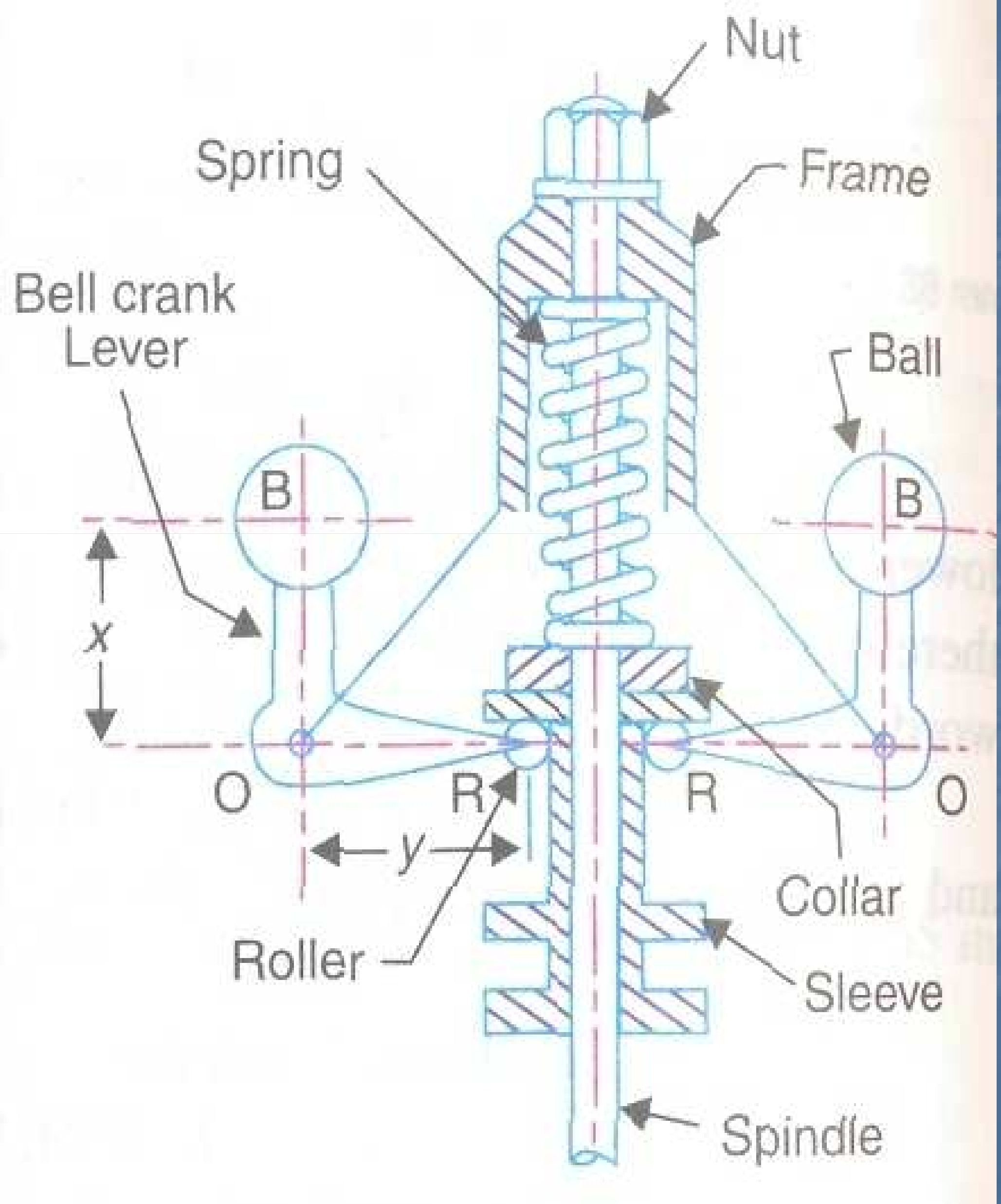
$$\therefore \quad F_C = \frac{196\ 125}{252.6} = 776.4 \text{ N}$$

$$F_C = 776.4 = m.\omega^2.r = 6\left(\frac{2\pi N}{60}\right)^2 0.18 = 0.012\ N^2$$

$$N^2 = \frac{776.4}{0.012} = 64\,700 \quad \text{or} \quad N = 254 \text{ r.p.m. } \textbf{Ans.}$$



Hartnell Governor



A Hartnell governor is a spring loaded governor as shown in Fig.

It consists of two bell crank levers pivoted at the points 0,0 to the frame.

The frame is attached to the governor spindle and therefore rotates with it.

Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR.

A helical compressive spring provides equal downward forces on the two rollers through a collar on the sleeve.

It serves the purpose like dead weight load. The spring force may be adjusted by screwing a nut up or down on the sleeve.


UNIVERSAL GOVERNOR APPARATUS

Let m = Mass of each ball in kg,

M = Mass of sleeve in kg,

r_1 = Minimum radius of rotation in metres,

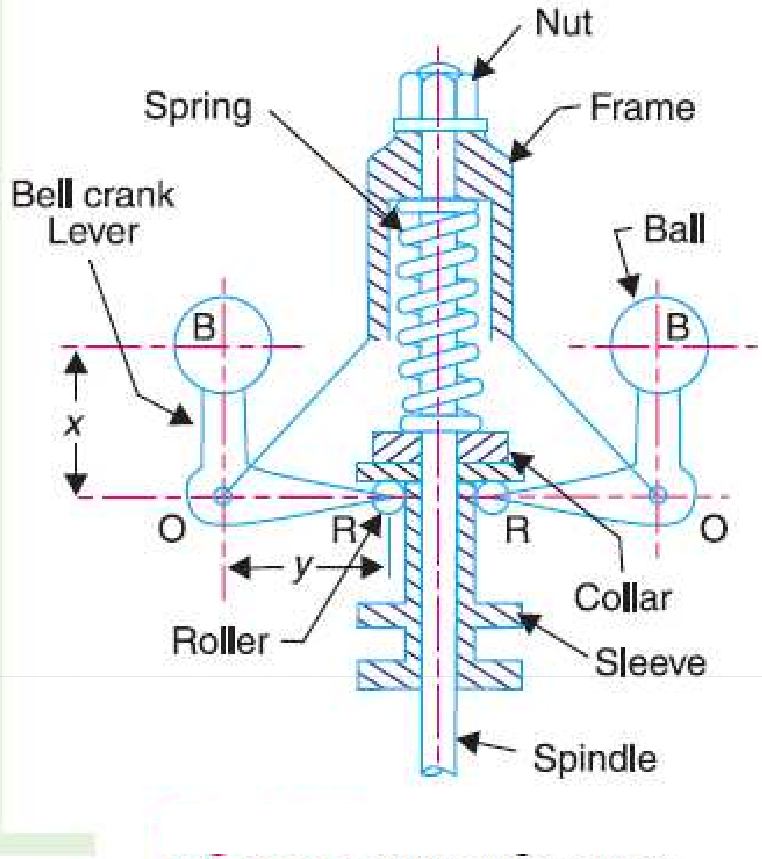
r_2 = Maximum radius of rotation in metres,

ω_1 = Angular speed of the governor at minimum radius in rad/s,

ω_2 = Angular speed of the governor at maximum radius in rad/s,

S_1 = Spring force exerted on the sleeve at ω_1 in newtons,

S_2 = Spring force exerted on the sleeve at ω_2 in newtons,



F_{C1} = Centrifugal force at ω_1 in newtons = $m(\omega_1)^2 r_1$,

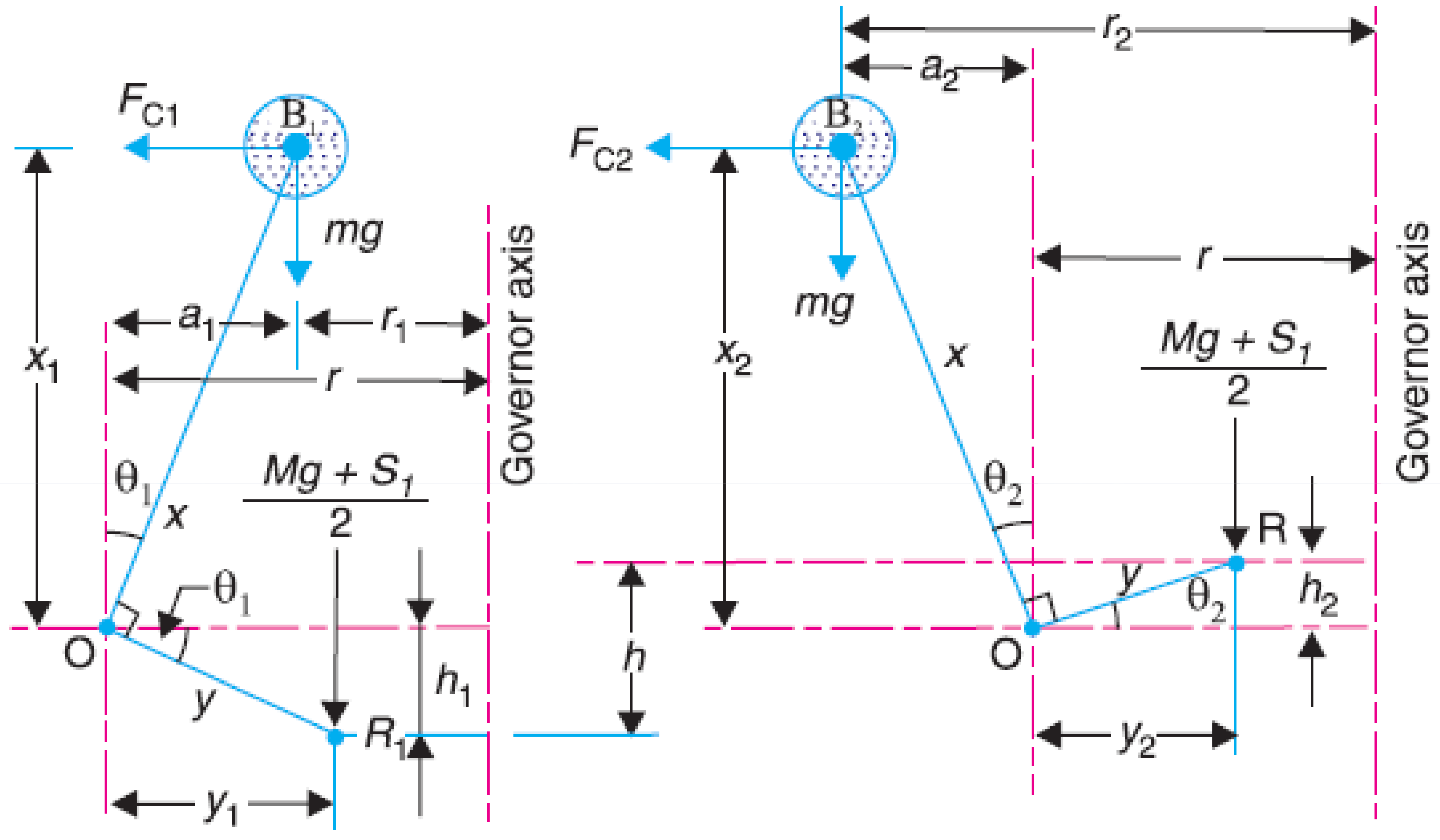
F_{C2} = Centrifugal force at ω_2 in newtons = $m(\omega_2)^2 r_2$,

s = Stiffness of the spring or the force required to compress the spring by one mm,

x = Length of the vertical or ball arm of the lever in metres,

y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid-position, in metres.

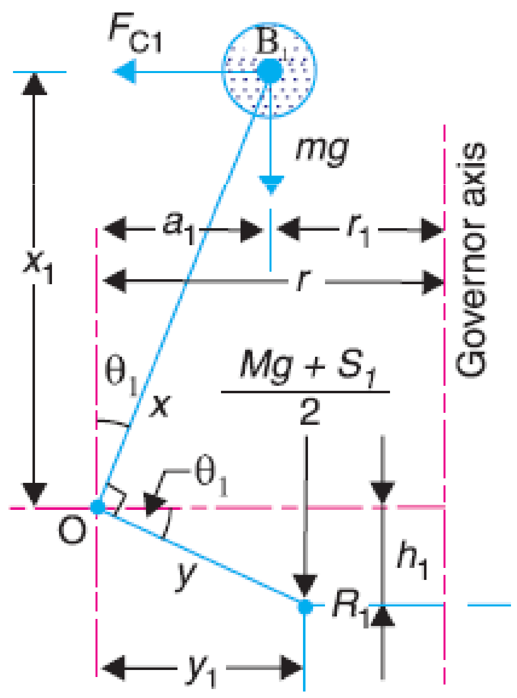


(*a*) Minimum position. (*b*) Maximum position.

Consider the forces acting at one bell crank lever. The minimum and maximum position is shown in Fig. Let h be the compression of the spring when the radius of rotation changes from r_1 to r_2.

For the minimum position *i.e.* when the radius of rotation changes from r to r_1, as shown in Fig. (*a*), the compression of the spring or the lift of sleeve h_1 is given by

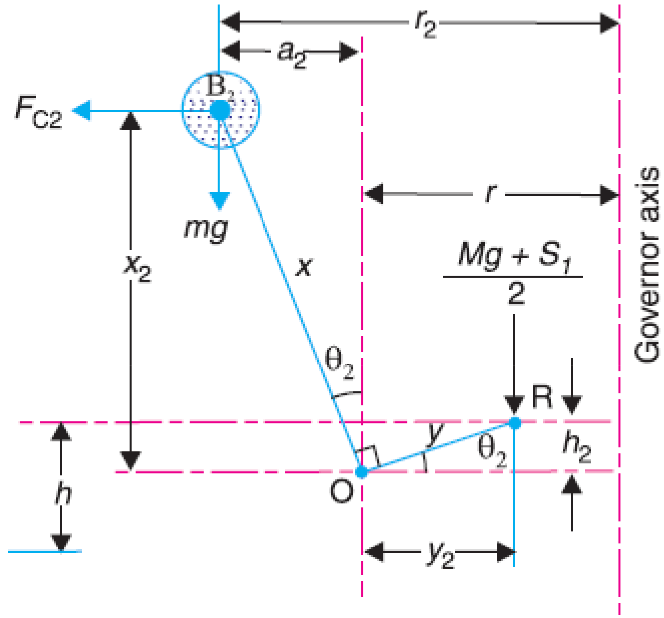
$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \qquad \ldots (i)$$



(*a*) Minimum position.

Similarly, for the maximum position *i.e.* when the radius of rotation changes from r to r_2, as shown in Fig. 18.19 (*b*), the compression of the spring or lift of sleeve h_2 is given by

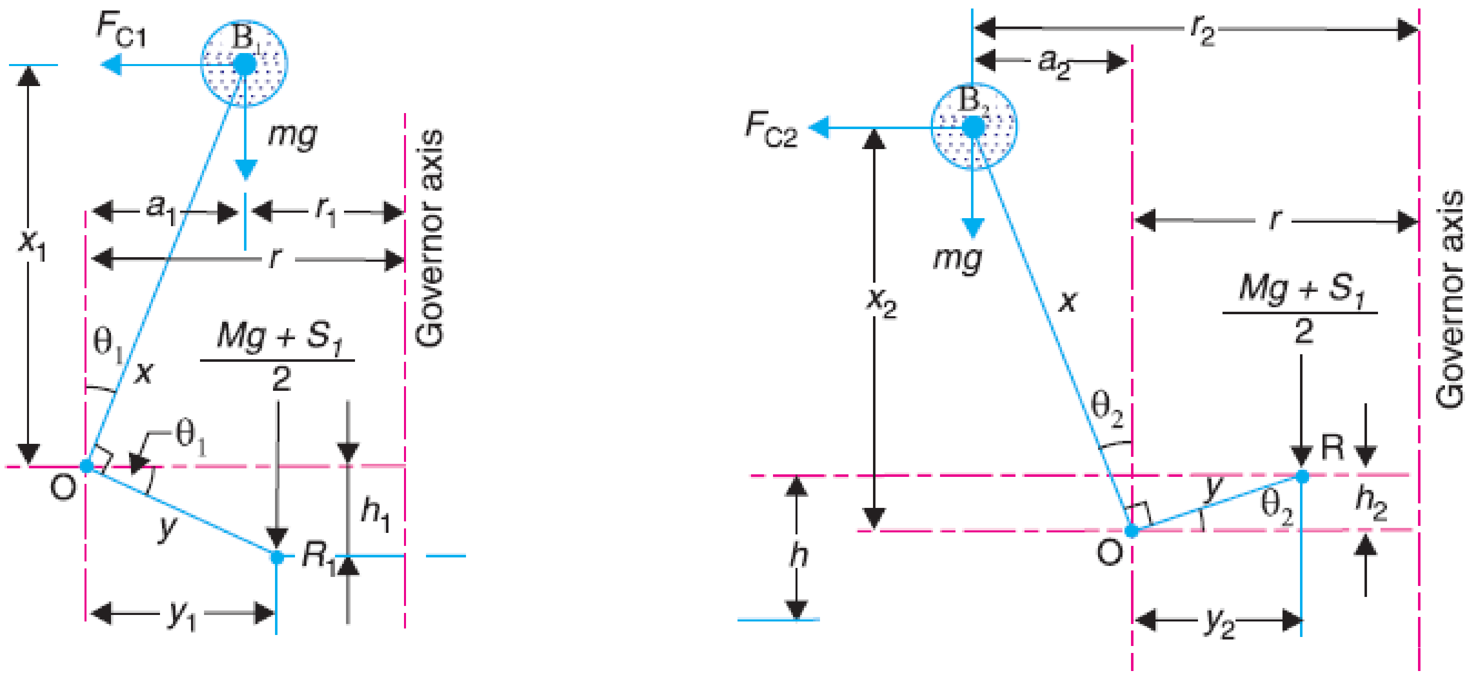
$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \qquad \ldots (ii)$$



Adding equations (*i*) and (*ii*),

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \qquad \frac{h}{y} = \frac{r_2 - r_1}{x} \qquad \ldots (\because\ h = h_1 + h_2)$$

$$\therefore \qquad h = (r_2 - r_1)\,\frac{y}{x} \qquad \ldots (iii)$$



Now for minimum position, taking moments about point O, we get

$$\frac{M.g + S_1}{2} \times y_1 + m.g \times a_1 = F_{C1} \times x_1$$

or $$M.g + S_1 = \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1) \qquad \ldots (iv)$$

Again for maximum position, taking moments about point O, we get

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

or $$M.g + S_2 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) \qquad \ldots (v)$$

Subtracting equation *(iv)* from equation *(v)*,

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1)$$

Neglecting the obliquity effect of the arms (*i.e.* $x_1 = x_2 = x$, and $y_1 = y_2 = y$) and the moment due to weight of the balls (*i.e.* $m.g$), we have for minimum position,

$$\frac{M.g + S_1}{2} \times y = F_{C1} \times x \quad \text{or} \quad M.g + S_1 = 2F_{C1} \times \frac{x}{y} \qquad \ldots (vi)$$

Similarly for maximum position,

$$\frac{M.g + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad M.g + S_2 = 2F_{C2} \times \frac{x}{y} \qquad \ldots (vii)$$

Subtracting equation *(vi)* from equation *(vii)*,

$$S_2 - S_1 = 2(F_{C2} - F_{C1})\frac{x}{y}$$

We know that

$$S_2 - S_1 = h.s, \quad \text{and} \quad h = (r_2 - r_1)\frac{y}{x}$$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{C2} - F_{C1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \qquad \ldots (ix)$$

Example *A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15 mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine : **1.** loads on the spring at the lowest and the highest equilibrium speeds, and **2.** stiffness of the spring.*

Solution. Given : $N_1 = 290$ r.p.m. or $\omega_1 = 2\pi \times 290/60 = 30.4$ rad/s ; $N_2 = 310$ r.p.m. or $\omega_2 = 2\pi \times 310/60 = 32.5$ rad/s ; $h = 15$ mm $= 0.015$ m ; $y = 80$ mm $= 0.08$ m ; $x = 120$ mm $= 0.12$ m ; $r = 120$ mm $= 0.12$ m ; $m = 2.5$ kg

1. *Loads on the spring at the lowest and highest equilibrium speeds*

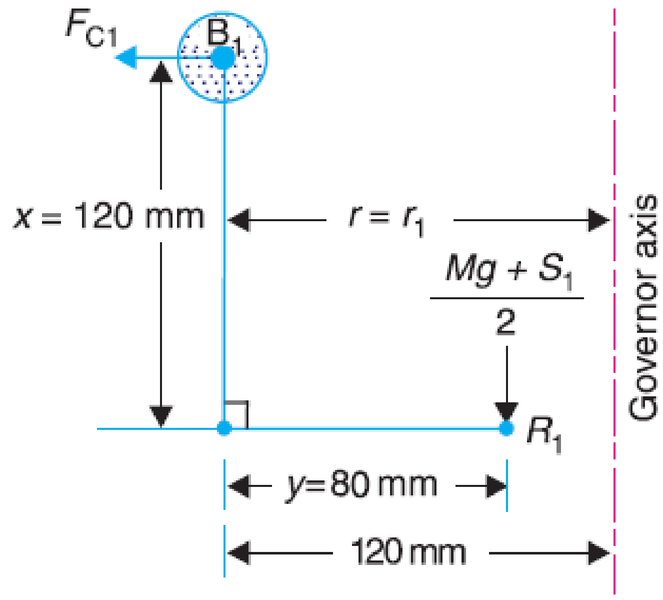
Let

S = Spring load at lowest equilibrium speed, and

S_2 = Spring load at highest equilibrium speed.

Since the ball arms are parallel to governor axis at the lowest equilibrium speed (*i.e.* at $N_1 = 290$ r.p.m.), as shown in Fig.

$$r = r_1 = 120 \text{ mm} = 0.12 \text{ m}$$



(*a*) Lowest position.

We know that centrifugal force at the minimum speed,

$$F_{C1} = m\,(\omega_1)^2\, r_1 = 2.5\,(30.4)^2\;0.12 = 277\text{ N}$$

Now let us find the radius of rotation at the highest equilibrium speed, *i.e.* at $N_2 = 310$ r.p.m.

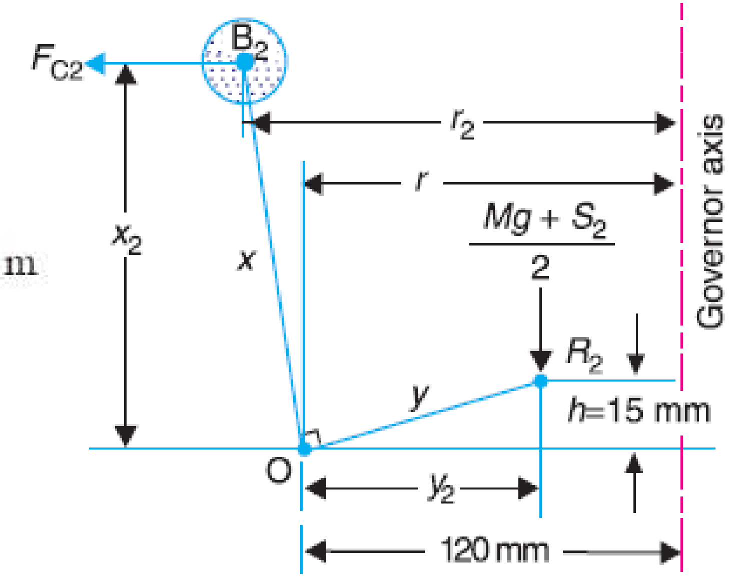
The position of ball arm and sleeve arm at the highest equilibrium speed is shown in Fig. 18.20 (*b*).

Let $\qquad r_2$ = Radius of rotation at $N_2 = 310$ r.p.m.

We know that $\qquad h = (r_2 - r_1)\,\dfrac{y}{x}$

$$r_2 = r_1 + h\left(\frac{x}{y}\right)$$

$$= 0.12 + 0.015\left(\frac{0.12}{0.08}\right) = 0.1425\text{ m}$$



(*b*) Highest position.

∴ Centrifugal force at the maximum speed,

$$F_{C2} = m\,(\omega_2)^2\, r_2$$

Neglecting the obliquity effect of arms and the moment due to the weight of the balls, we have for lowest position,

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 277 \times \frac{0.12}{0.08} = 831 \text{ N}$$

$\therefore$ $S_1 = 831$ N **Ans.** $(\because M = 0)$

and for highest position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 376 \times \frac{0.12}{0.08} = 1128 \text{ N}$$

$\therefore$ $S_2 = 1128$ N **Ans.** $(\because M = 0)$

2. ***Stiffness of the spring***

We know that stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = 19.8 \text{ N/mm}$$ **Ans.**

Example *In a spring loaded governor of the Hartnell type, the mass of each ball is 1kg, length of vertical arm of the bell crank lever is 100 mm and that of the horizontal arm is 50 mm. The distance of fulcrum of each bell crank lever is 80 mm from the axis of rotation of the governor. The extreme radii of rotation of the balls are 75 mm and 112.5 mm. The maximum equilibrium speed is 5 per cent greater than the minimum equilibrium speed which is 360 r.p.m. Find, neglecting obliquity of arms, initial compression of the spring and equilibrium speed corresponding to the radius of rotation of 100 mm.*

Solution. Given : $m = 1$ kg ; $x = 100$ mm $= 0.1$ m ; $y = 50$ mm $= 0.05$ m ; $r = 80$ mm $= 0.08$ m ; $r_1 = 75$ mm $= 0.075$ m ; $r_2 = 112.5$ mm $= 0.1125$ m ; $N_1 = 360$ r.p.m. or $\omega_1 = 2\pi \times 360/60 = 37.7$ rad/s

Since the maximum equilibrium speed is 5% greater than the minimum equilibrium speed (ω_1), therefore maximum equilibrium speed,

$$\omega_2 = 1.05 \times 37.7 = 39.6 \text{ rad/s}$$

We know that centrifugal force at the minimum equilibrium speed,

$$F_{C1} = m\,(\omega_1)^2\, r_1 = 1\,(37.7)^2\, 0.075 = 106.6 \text{ N}$$

and centrifugal force at the maximum equilibrium speed,

$$F_{C2} = m\,(\omega_2)^2\, r_2 = 1\,(39.6)^2\, 0.1125 = 176.4 \text{ N}$$

Initial compression of the spring

S_1 = Spring force corresponding to ω_1, and

S_2 = Spring force corresponding to ω_2.

Since the obliquity of arms is neglected, therefore for minimum equilibrium position,

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 106.6 \times \frac{0.1}{0.05} = 426.4 \text{ N}$$

$$S_1 = 426.4 \text{ N} \qquad ...(\because M = 0)$$

and for maximum equilibrium position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 176.4 \times \frac{0.1}{0.05} = 705.6 \text{ N}$$

$$\therefore \qquad S_2 = 705.6 \text{ N} \qquad ...(\because M = 0)$$

We know that lift of the sleeve,

$$h = (r_2 - r_1)\frac{y}{x} = (0.1125 - 0.075)\frac{0.05}{0.1} = 0.018\ 75 \text{ m}$$

and stiffness of the spring $s = \dfrac{S_2 - S_1}{h} = \dfrac{705.6 - 426.4}{0.018\ 75} = 14\ 890 \text{ N/m} = 14.89 \text{ N/mm}$

$\therefore$ Initial compression of the spring

$$= \frac{S_1}{s} = \frac{426.4}{14.89} = 28.6 \text{ mm} \quad \textbf{Ans.}$$

Equilibrium speed corresponding to radius of rotation r = 100 mm = 0.1 m

Let N = Equilibrium speed in r.p.m.

Since the obliquity of the arms is neglected, therefore the centrifugal force at any instant,

$$F_C = F_{C1} + (F_{C2} - F_{C1})\left(\frac{r - r_1}{r_2 - r_1}\right)$$

$$= 106.6 + (176.4 - 106.6)\left(\frac{0.1 - 0.075}{0.1125 - 0.075}\right) = 153 \text{ N}$$

We know that centrifugal force (F_C),

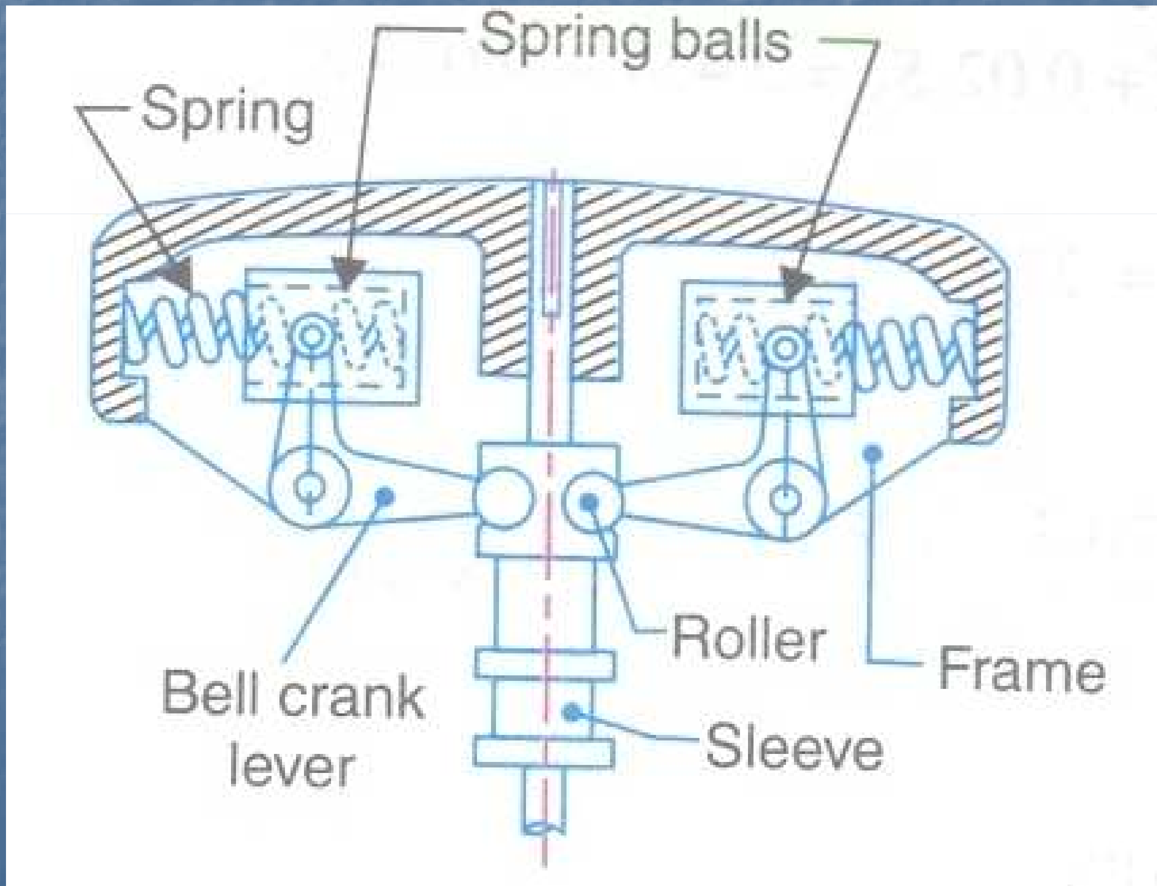
$$153 = m.\omega^2.r = 1\left(\frac{2\pi N}{60}\right)^2 0.1 = 0.0011\ N^2$$

$\therefore$ $N^2 = 153 / 0.0011 = 139\ 090$ $\quad N = 373$ r.p.m. **Ans.**

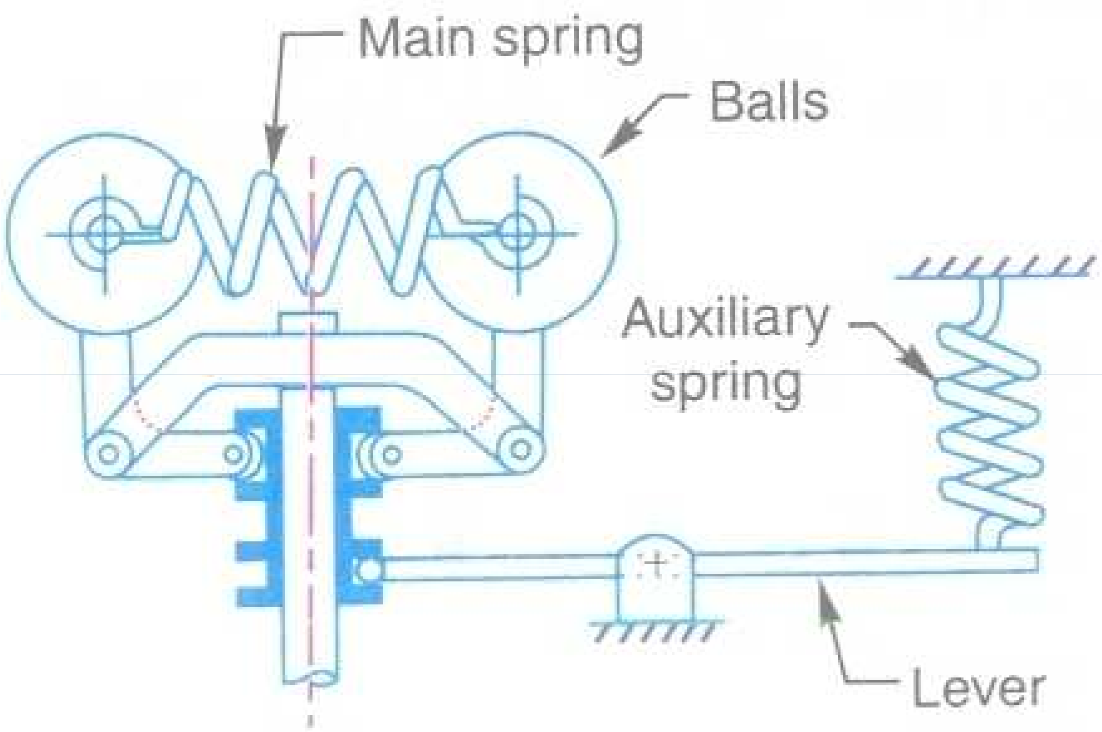
Hartung governor

Here vertical arms of bell crank lever is attached with spring ball which compress against the frame of governor.

This additional force against the centrifugal force increases the speed of revolution which is required to enable the balls to move to any pre determined level.



Wilson-Hartnell



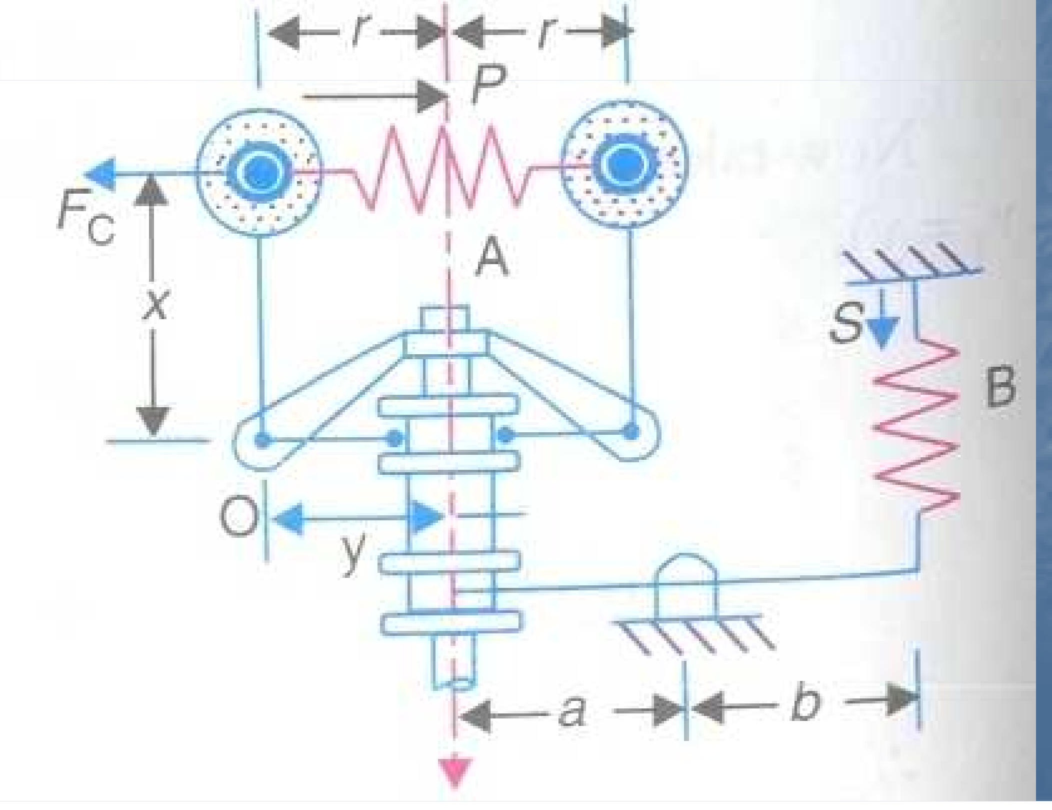
Wilson-Hartnell governor.

Wilson-Hartnell

In Wilson Hatnell governor balls are connected by a spring in tension.

An auxiliary spring is attached to the sleeve mechanism through a lever by means of which the equilibrium speed for a given radius may be adjusted.

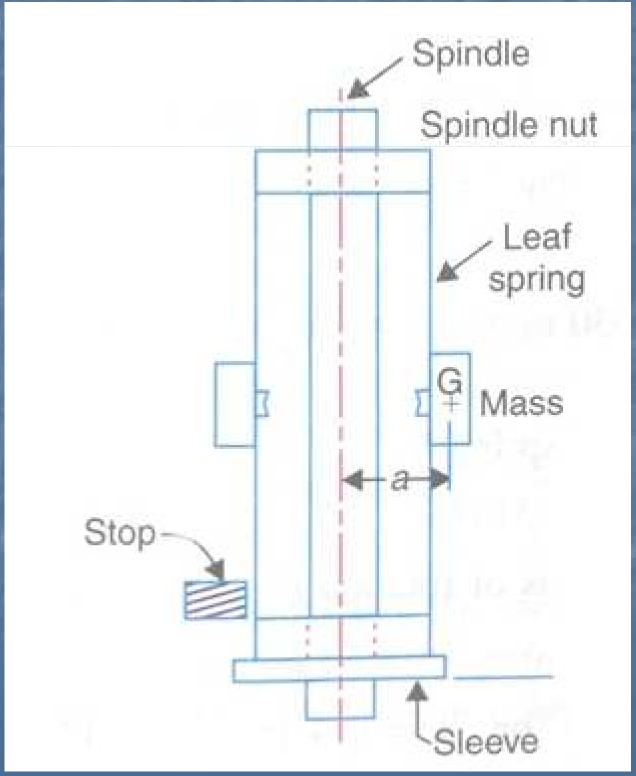
The main spring may be considered of two equal parts each belonging to both the balls. The line diagram of a Wilson Hartnell governor is shown in Fig.

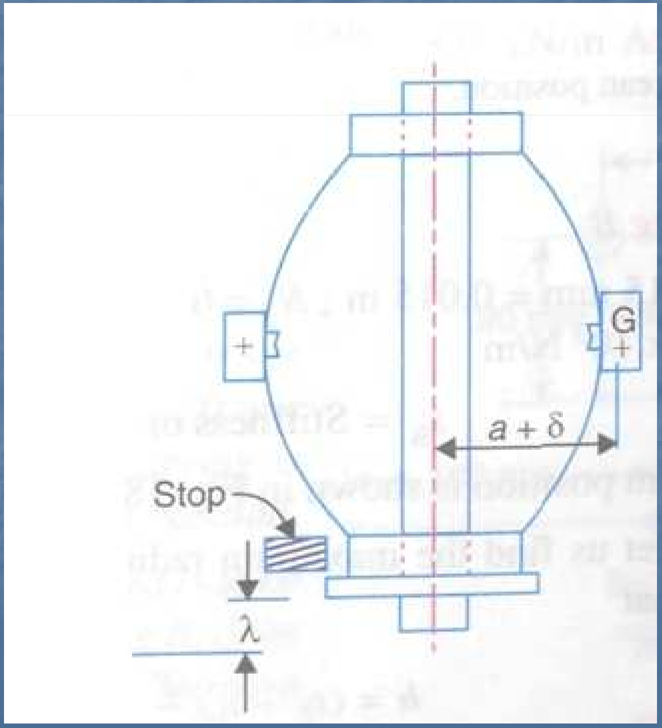


Pickering Governor

A Pickering governor is mostly used for driving gramophone. It consists of *three straight leaf springs arranged at equal angular intervals round the spindle.

Each spring carries a weight at the centre. The weights move outwards and the springs bend as they rotate about the spindle axis with increasing speed.





Sensitiveness

Consider two governors A and B running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor A is greater than the lift of the sleeve of governor B. It is then said that the governor is more sensitive than the governor B.

In general, the greater the lift of the sleeve corresponding to a given fractional change in speed, the greater is the sensitiveness of the governor.

It may also be stated in another way that for a given lift of the sleeve, the sensitiveness of the governor increases as the speed range decreases.

This definition of sensitiveness may be quite satisfactory when the governor is considered as an independent mechanism. But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load, position of the sleeve should be as small a fraction as possible of the mean equilibrium speed.

The actual displacement of the sleeve is immaterial, provided that it is sufficient to change the energy supplied to the engine by the required amount.

For this reason, the sensitiveness is defined as the ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.

Let N_1 = mini. Equilibrium

N_2 = max. Equilibrium

N = Mean Equilibrium

$$\text{Sensitiveness of governor} = (N_2 - N_1) / N$$

$$\text{Where } N = (N_1 + N_2) / 2$$

Stability

A governor is said to be stable when for every speed with in the working range there is a definite configuration i.e. there is only one radius of rotation of the fly balls.

Isochronous Governor

a governor is said to be Isochronous when equilibrium speed with in the working range is constant for all radii of rotation of the fly balls by neglecting friction.

Hunting

a governor is said to be hunt if speed of the engine fluctuates continuously above and below mean speed.

This is caused by too sanative governor which changes the fuel supply by large amount when a small changes in speed of rotation.

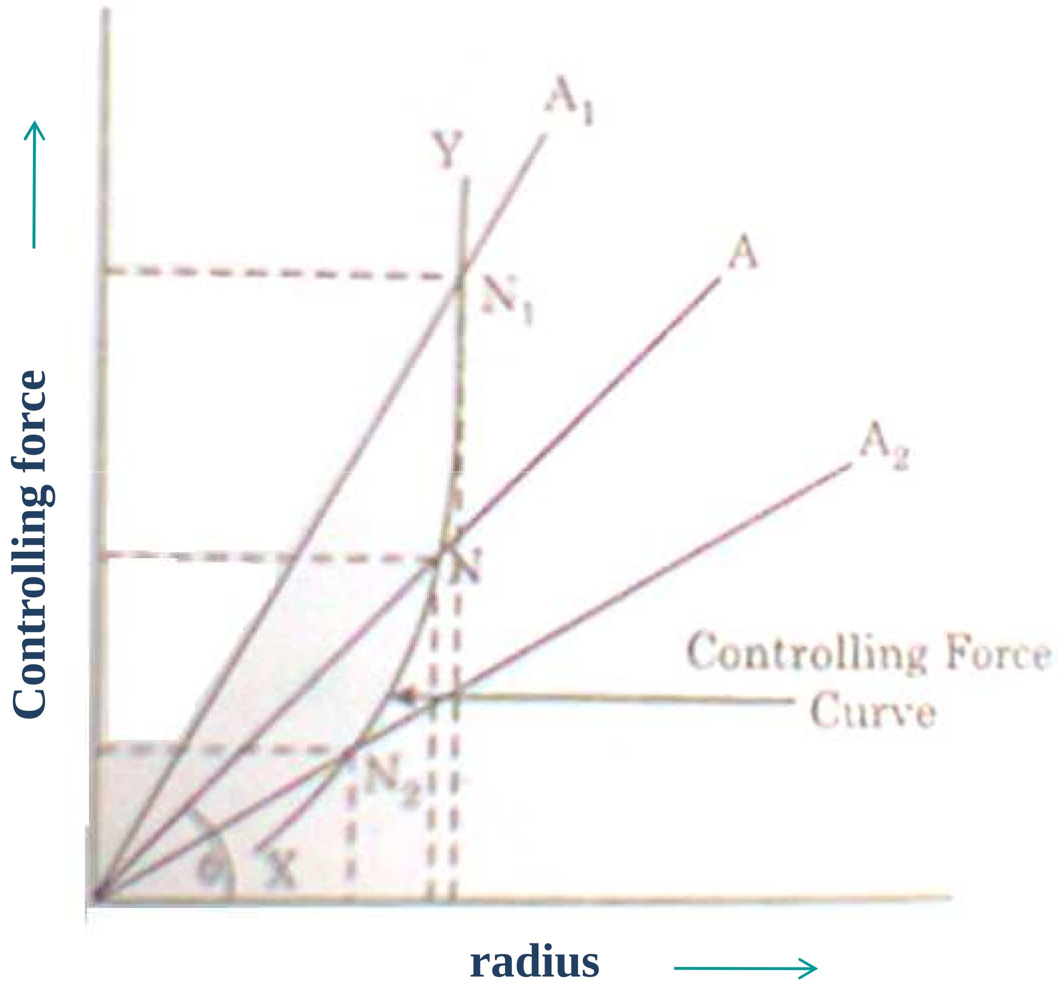
Effort of a governor

It is a mean force extended at the sleeve for given percentage change of speed

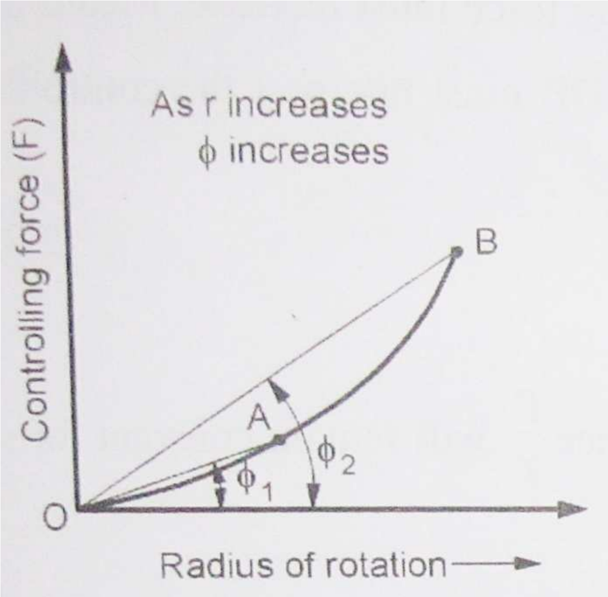
Power of a governor

It is the work done at sleeve for given percentage change of speed

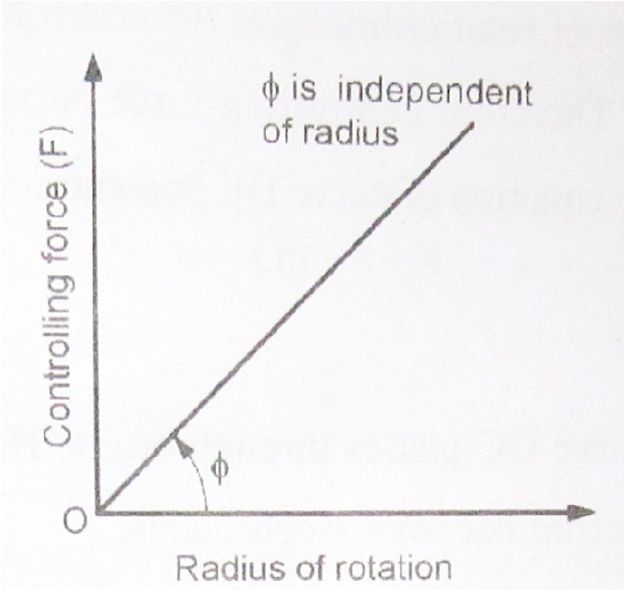
= mean effort X distance moved by sleeve



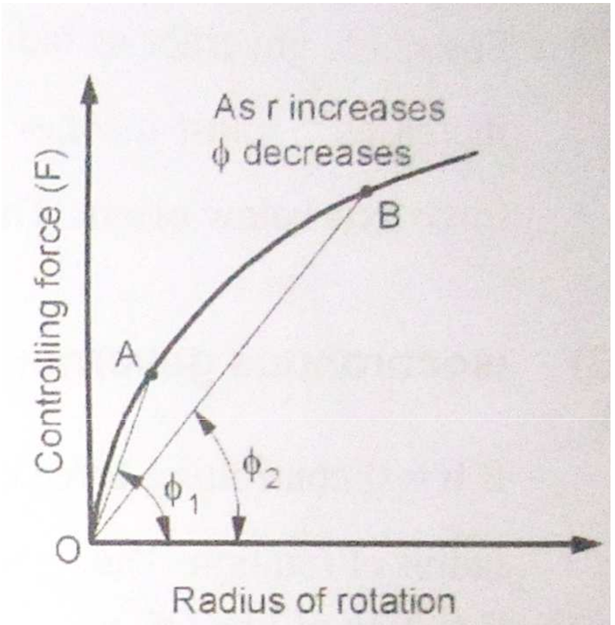
Controlling force
A1
Y
A
N1
A2
N
Controlling Force
Curve
N2
X
radius



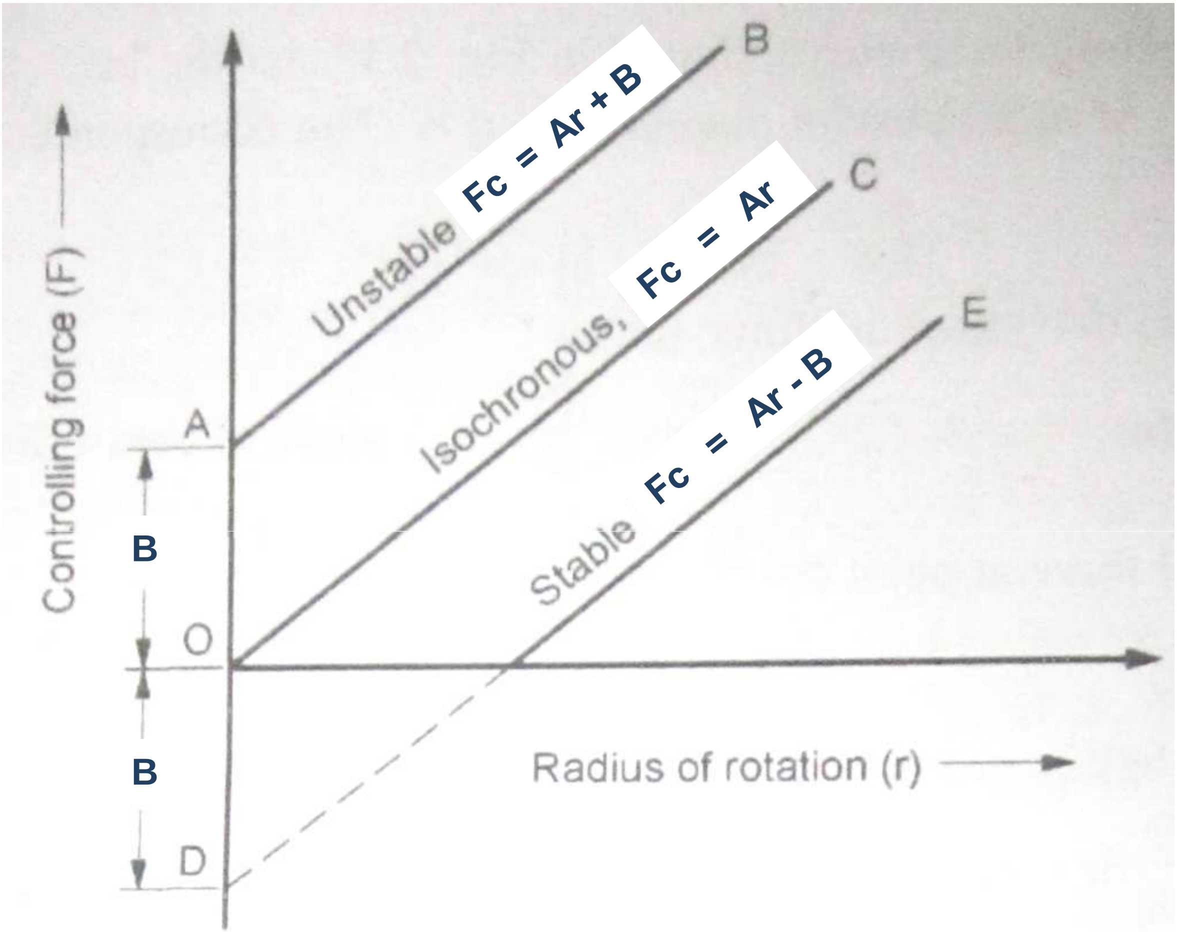
Stable governor:
As 'r' increase 'angle ' must increase.



Isochronous governor:
'angle' is constant and is independent of radius



Unstable governor:
As 'r' increases ‘angle ' will decrease.



**Controlling force diagram
for spring loaded governors**

(A) Stable governor :

For stable governor as radius of rotation increase the controlling force must increase. Means as F/r increases - must increase. Therefore controlling force curve DE must intersect the controlling force axis below origin. The equation of curve DE becomes **Fc = Ar - B**

(B) Isochronous governor :

It b = 0 controlling force curve OC passes through origin. Hence Fc / r will remain constant for radius of rotation. Thus governor becomes isochronous. Equation of line OC is Fc = Ar

(C) Unstable governor :

If B *is positive, then* Fc / r *decrease as r increases. It means that as the radius increases* controlling force decreases. Thus governor becomes unstable Equation of line AB is F = Ar+ B