Static Force Analysis

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STATIC EQUILIBRIUM

- A body is in static equilibrium if it remains in its state of rest or motion.
- If the body is at rest, it tends to remain at rest and if in motion, it tends to keep the motion.
- In static equilibrium,
 - the vector sum of all the forces acting on the body is zero and
 - the vector sum of all the moments about any arbitrary point is zero.
- Mathematically,

$$\Sigma \mathbf{F} = 0$$
$$\Sigma \mathbf{T} = 0$$

• In a planer system, forces can be described by two-dimensional vectors and therefore,

$$\Sigma \mathbf{F}_x = 0$$

$$\Sigma \mathbf{F}_y = 0$$

$$\Sigma \mathbf{T}_z = 0$$

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EQUILIBRIUM OF TWO AND THREE-FORCE MEMBERS

- A member under the action of two forces will be in equilibrium if
 - the forces are of the same magnitude,
 - the forces act along the same line, and
 - the forces are in opposite directions.
- Figure shows such a member.
- A member under the action of three forces will be in equilibrium if
 - the resultant of the forces is zero, and
 - the lines of action of the forces intersect at a point (known as point of concurrency).



EQUILIBRIUM OF TWO AND THREE-FORCE MEMBERS (Contd....)

- Figure (a) shows a member acted upon by three forces F₁, F₂ and F₃ and is in equilibrium as the lines of action of forces intersect at one point *O* and the resultant is zero.
- This is verified by adding the forces vectorially [Fig.(b)].
- As the head of the last vector F_3 meets the tail of the first vector F_1 , the resultant is zero.
- Figure (d) shows a case where the magnitudes and directions of the forces are the same as before, but the lines of action of the forces do not intersect at one point.
- Thus, the member is not in equilibrium.



- Consider a member in equilibrium in which force *F*₁ is completely known, *F*₂ known in direction only and *F*₃ completely unknown.
- The point of applications of F_1 , F_2 and F_3 are A, B and C respectively.
- To solve such a problem, first find the point of concurrency O from the two forces with known directions, i.e. from F₁, and F₂.
- Joining O with C gives the line of action of the third force F_3 .
- To know the magnitudes of the forces F_2 and F_3 , take a vector of proper magnitude and direction to represent the force F_1 .
- From its two ends, draw lines parallel to lines of action of the forces F₂ and F₃ forming a force triangle [Fig.].
- Mark arrowheads on F_2 and F_3 so that F_1 , F_2 and F_3 are in the same order.



MEMBER WITH TWO FORCES AND A TORQUE

- A member under the action of two forces and an applied torque will be in equilibrium if
 - the forces are equal in magnitude, parallel in direction and opposite in sense and
 - the forces form a couple which is equal and opposite to the applied torque.



 Figure shows a member acted upon by two equal forces F₁, and F₂ and an applied torque T for equilibrium,

$$T = F_1 \times h = F_2 \times h$$

where T, F_1 and F_2 are the magnitudes of T, F_1 and F_2 respectively.

 T is clockwise whereas the couple formed by F₁, and F₂ is counterclockwise.

FORCE CONVENTION

- The force exerted by member $i\ on\ member\ j\ is\ represented\ by\ F_{ij}$



FREE BODY DIAGRAMS

- A free body diagram is a sketch or diagram of a part isolated from the mechanism in order to determine the nature of forces acting on it.
- Figure (a) shows a four-link mechanism.
- The free-body diagrams of its members 2, 3 and 4 are shown in Figs.
 (b), (c) and (d) respectively.
- Various forces acting on each member are also shown.
- As the mechanism is in static equilibrium, each of its members must be in equilibrium individually.



- Member 4 is acted upon by three forces F, F₃₄ and F₁₄.
- Member 3 is acted upon by two forces F₂₃ and F₄₃
- Member 2 is acted upon by two forces F_{32} and F_{12} and a torque T.
- Initially, the direction and the sense of some of the forces may not be known.
- Link 3 is a two-force member and for its equilibrium F₂₃ and F₄₃ must act along *BC*.
- Thus, F_{34} being equal and opposite to F_{43} also acts along BC.



- Assume that the force F on member 4 is known completely.
- To know the other two forces acting on this member completely, the direction of one more force must be known.
- For member 4 to be in equilibrium, F_{14} passes through the intersection of F and F_{34} .
- By drawing a force triangle (F is completely known), magnitudes of F₁₄ and F₃₄ can be known [Fig.(e)].

Now $F_{34} = F_{43} = F_{23} = F_{32}$

• Member 2 will be in equilibrium if F_{12} is equal, parallel and opposite to F_{32} and $C = F_{43}$ (c)



A four-link mechanism with the following dimensions is acted upon by a force 80N at angle 150° on link *DC* [Fig.(a)): AD = 50 mm, AB = 40 mm, BC = 100 mm, DC = 75 mm, DE = 35 mm. Determine the input torque T on the link AB for the static equilibrium of the mechanism for the given configuration.

As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually. Member 4 is acted upon by three forces F F_{34} and F_{14} . Member 3 is acted upon by two forces F_{23} and F_{43} Member 2 is acted upon by two forces F_{32} and F_{12} and a torque T. Initially, the direction and the sense of some of the forces are not known.

Force F on member 4 is known completely. To know the other two forces acting on this member completely, the direction of one more force must be known. To know that, link 3 will have to be considered first which is a two-force member.

(4)

150°

F = 80 N

(1)

(3)

(a)

120°

- As link 3 is a two-force member [Fig.(b)], for its equilibrium, F_{23} and F_{43} must act along *BC* (at this stage, the sense of direction of forces F_{23} and F_{43} is not known). Thus, the line of action of F_{34} is also along *BC*.
- As force F₃₄ acts through point C on link 4, draw a line parallel to BC through C by taking a free body of link 4 to represent the same. Now, as link 4 is three force member, the third force F₁₄ passes through the intersection of F and F₃₄ [Fig (c)].
- By drawing a force triangle (F is completely known), magnitudes of F₁₄ and F₃₄ are known [Fig(d)].



- Now, $F_{34} = -F_{43} = F_{23} = -F_{32}$
- Member 2 will be in equilibrium [Fig. (e)] if F_{12} is equal, parallel and opposite to F_{32} and

 $T=-F_{32} \times h = 47.8 \times 39.3 = -1878.54 \text{ N.mm}$ The input torque has to be equal and opposite to this couple i.e. T= 1878.5 N/mm (clockwise)

h=Perpendicular distance between two equal and opposite forces.



Figure shows a slider crank mechanism in which the resultant gas pressure 8×10^4 N/m² acts on the piston of cross sectional area $0.1m^2$ The system is kept in equilibrium as a result of the couple applied to the crank 2, through the shaft at O₂. Determine forces acting on all the links (including the pins) and the couple on 2. OA=100mm, AB=450mm.



Force triangle for the forces acting on(4)is drawn to some suitable scale. Magnitude and direction of P known and lines of action of $F_{34} \& F_{14}$ known.



Measure the lengths of vectors and multiply by the scale factor to get the magnitudes of $F_{14} \& F_{34}$. Directions are also fixed.

Since link 3 is acted upon by only two forces, F_{43} and F_{23} are collinear, equal in magnitude and opposite in direction *i.e.*, $F_{43} = -F_{23} = 8.8 \times 10^3 \text{ N}$

Also, $F_{23} = -F_{32}$ (equal in magnitude and opposite in direction).



Link 2 is acted upon by 2 forces and a torque, for equilibrium the two forces must be equal, parallel and opposite and their sense must be opposite to T_2 .

There fore,

 $F_{32} = -F_{12} = 8.8 \text{x} 10^3 \text{ N}$



 F_{32} & F_{12} form a counter clock wise couple of magnitude, $F_{23} * h = F_{12} * h$ =(8.8x10³)x0.125 = 1100 Nm.

To keep 2 in equilibrium, T: should act clockwise and magnitude is 1100 Nm.

Note: h is measured perpendicular to F_{32} & F_{12}

For the static equilibrium of the quick return mechanism shown in Fig (a), determine the input torque T_2 : to be applied on link AB Tor a force of 300N on the slider D. The dimensions of the various links arc OA=400mm, AB=200mm, OC=800mm. CD=300mm







• For the mechanism shown in Fig., determine the torque on the link AB for the static equilibrium of the mechanism.

 F_2

R

 \mathbf{F}_{23}

(d)











• Answers from Composite Graphical Solution



Graphical Solution by Superposition method



Graphical Solution by Superposition method



 $T_2 = F_{32} \times h_2 = 33.2 \times 13.2 = 438$ N.mm lockwise Total torque = 262 + 438 = 700 N.mm







In a four-link mechanism shown in Fig., torque T_3 and T_4 have magnitudes of 30N.m and 20N.m respectively. The link lengths are AD = 800 mm, AB = 300 mm, BC = 700 mm and CD = 400 mm. For the static equilibrium of the mechanism, determine the required input torque T_2



In a four-link mechanism shown in Fig., torque T_3 and T_4 have magnitudes of 30N.m and 20N.m respectively. The link lengths are AD = 800 mm, AB = 300 mm, BC = 700 mm and CD = 400 mm. For the static equilibrium of the mechanism, determine the required input torque T_2







Neglecting torque T3

Torque \mathbf{T}_4 on the link 4 is balanced by a couple having two equal, parallel and opposite forces at *C* and *D*. As the link 3 is a two-force member, \mathbf{F}_{43} and therefore, \mathbf{F}_{34} and \mathbf{F}_{14} will be parallel to *BC*.

$$F_{34} = F_{14} = \frac{T_4}{h_{4a}} = \frac{20}{0.383} = 52.2 \text{ N}$$

and $F_{34} = F_{43} = F_{23} = F_{32} = F_{12} = 52.2 \text{ N}$ $T_{2a} = F_{32} \times h_{2a} = 52.2 \times 0.274 = 14.3 \text{ N.m}$ counter-clockwise.

Neglecting torque T4

 \mathbf{F}_{43} is along *CD*. The diagram is self-explanatory.

$$F_{43} = F_{23} = \frac{T_3}{h_{3b}} = \frac{30}{0.67} = 44.8 \text{ N}$$

$$F_{23} = F_{32} = F_{12} = 44.8 \text{ N}$$

$$T_{2b} = F_{32} \times h_{2b} = 44.8 \times 0.042 = 1.88 \text{ N.m}$$

counter-clockwise.

$$T_2 = T_{2a} + T_{2b} = 14.3 + 1.88 = \underline{16.18 \text{ N}}$$

counter-clockwise

PRINCIPLE OF VIRTUAL WORK

- The principle of virtual (imaginary) work can be stated as the work done during a virtual displacement from the equilibrium is equal to zero.
- Virtual displacement may be defined as an imaginary infinitesimal displacement of the system.
- By applying this principle, an entire mechanism is examined as a whole and there is no need of dividing it into free bodies.



- Consider a slider-crank mechanism shown in Fig.
- It is acted upon by the external piston force F, the external crankshaft torque T and the force at the bearings.
- As the crank rotates through a small angular displacement $\delta \theta$, the corresponding displacement of the piston is δx , the various forces acting on the system are
 - Bearing reaction at O (performs no work)
 - Force of cylinder on piston, perpendicular to piston displacement (produces no work)
 - Bearing forces at A and B, being equal and opposite (AB is a two-force member), no work is done
 - Work done by torque T= T $\delta \theta$
 - Work done by force $F = F \delta x$
- Work done is positive if a force acts in the direction of the displacement and negative if it acts in the opposite direction.

According to the principle of virtual work,

$$W = T \,\delta\theta + F \,\delta x = 0$$

As virtual displacement must take place during the same interval δt ,

$$T\frac{d\theta}{dt} + F\frac{dx}{dt} = 0$$
or
$$T\omega + Fv = 0$$

where ω is the angular velocity of the crank and v, the linear velocity of the piston.

$$T = -\frac{F}{\omega}v$$

The negative sign indicates that for equilibrium, T must be applied in the opposite direction to the angular displacement.