Minterm:-

→ Binary variable may appear in normal form or (x) its complement form (x')

→ Suppose 2 variable are there so 2^n combination
$2^2 = 4$. xy, $x'y'$, $x'y$, xy'

→ So AND product in SOP (sum of product) is called minterm.

x	y		
0	0	→ $x'y'$	m_0
0	1	→ $x'y$	m_1
1	0	→ xy'	m_2 ← for minterm
1	1	→ xy	m_3

0 Depends ← complement variable (x')
1 represent ← simple variable (x)

$f_1 = x'y'z + xy'z' + xyz$
$: 001 + 100 + 111$
$= m_1 + m_4 + m_7$.

Any Boolean fn can be expressed as sum of minterm.

$f_1 = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$. maxterm.
$= (0\,0\,0)$

maxterm ← OR term in POS.

Any Boolean fn can be expressed as Product of maxterm

$= (\overset{4}{0}\overset{2}{0}\overset{1}{0})\ (\overset{4}{0}\overset{2}{1}\overset{1}{0})\ (\overset{4}{0}\overset{2}{1}\overset{1}{1})\ (\overset{4}{1}\overset{2}{0}\overset{1}{1})\ (\overset{4}{1}\overset{2}{1}\overset{1}{0})$
$= M_0\ \ M_2\ \ M_3\ \ M_5\ \ M_6$.

* minterms and maxterms are related.

$(x'y'z')'$ $\quad m_0 \quad x+y+z \quad M_0$

$\boxed{m_0' = M_0.}$ ← complement with each other.

(2) $A + B'C = A(B + \bar{B})$ Express the Boolean f $F = A + B'C$ in a minterms.

$= AB + A\bar{B} \quad (C + \bar{C})$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$

$\bar{B}C = \bar{B}C(A + \bar{A})$

$= \bar{B}CA + \bar{A}\bar{B}C$

$A + B'C = \overset{421}{ABC} + \overset{421}{AB\bar{C}} + \overset{421}{A\bar{B}C} + \overset{421}{A\bar{B}\bar{C}} + \overset{421}{\bar{A}\bar{B}C}$.

$= 111 + 110 + 101 + 100 + 001$

$= m_7 + m_6 + m_5 + m_4 + m_1$

$= m_1 + m_4 + m_5 + m_6 + m_7$

$\Sigma m(1, 4, 5, 6, 7)$

Graphical & space

→ Boolean f^n may be expressed algebraically from a given truth Table.

find the minterms and maxterms for F described by following truth Table.

→ The canonical sum of products form of a logic f^n can be obtained by using following procedure.

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Chapter-6
The Karnaugh map (K-map).

2

K map ← Systematic method of simplifying Boolean expression

→ graphical representation of Boolean expression.

2^n cells → $2^2 = 4$. cell ← Two variable K map

$2^3 = 6$. Three variable Kmap.

$2^4 : 8$ four variable Kmap.

→ Product Term in SOP format ← minterm

sum term in POS format ← maxterm.

→ Each minterm is → AND operation of all variable in it (variable may be normal or complemented)

→ min term → complemented variable represent by 0

non complemented variable represnt by 1

→ for maxterm opposite from minterm.

* Boolean expression to SOP form [expansion].

multiply missing variable with sum of its complement.

(min term in SOP form).

for ~~max~~ min term.

(1)

$\bar{A}+\bar{B} = \bar{A}(B+\bar{B}) + \bar{B}(A+\bar{A})$ ← (mutiplying missing variable with sum of its complement) 1st step.

$= \bar{A}B + \bar{A}\bar{B} + \bar{B}A + \bar{A}\bar{B}$ ←

$= \bar{A}B + \bar{A}\bar{B} + A\bar{B}$.

$= 01 + 00 + 10$. (← (4 2 1))

$= m_1 + m_0 + m_2$

$\boxed{= \Sigma m(0,1,2)}$ ← in SOP
min term

This

Two variable kmap ← $2^n \leftarrow 2^2 = 4$ cells.
minterm

So m_3 ← is missing in SOP form.

maxterm M_3 will be present in POS form.

$\boxed{\text{max term } = \Pi M_3}$ ← POS.

$\bar{A}+\bar{B} = X\bar{A} + X\bar{B}$ [Put x in each variable.

$= X0 + \bar{B}X$

$= X0 + 0X$ } Put complemented variable=0.

$= X0 + 0X$

$= 10 + 00 + 01 + 00$ make all possible combination

$= 10 + 01 + 00$

$= \Sigma m(0,1,2)$

Ex→2 $A + B\bar{C} + AB\bar{D} + ABCD$

$A(B+\bar{B}) = AB + A\bar{B}$

$= A(B+\bar{B})(C+\bar{C})(D+\bar{D})$

$= ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D}$

→ $B\bar{C} = B\bar{C}(A+\bar{A})(D+\bar{D})$

$= AB\bar{C}D + AB\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}$

→ $AB\bar{D} = AB\bar{D}(C+\bar{C})$

$= ABC\bar{D} + AB\bar{C}\bar{D}$

$A+B\bar{C}+AB\bar{D}+ABCD = ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}$

$2^4 = 16$.

$= \Sigma m(4,5,8,9,10,11,12,13,14,15)$ ← min term in SOP form

$= \Pi M(0,1,2,3,6,7)$. ← max term in POS form

$A+B\bar{C}+AB\bar{D}+ABCD$

$= AXXX$

$= \cancel{A}000 + 1001 + 1010 + 1100$
$+ 1101 + 1110 + 111$

$= m8 + m9 + m10 + m11 + m12 + m13 + m14 + m15$

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$B\bar{C} = XB\bar{C}X = X10X$.

$= 0100 + 0101 + 1100 + 1101$

$= m4 + m5 + m12 + m13$

$AB\bar{D} = ABX\bar{D} = 11X0 = 1100 + 1110$

$= m12 + m14$.

$\Sigma m = (4,5,8,9,10,11,12,13,14,15)$

sion of a Boolean Expression to POS form

1) $A(\bar{B}+A)B$ to maxterm and minterm.

$A = A + B\bar{B} = (A+B)(A+\bar{B})$

$B: B + A\bar{A} = (A+B)(\bar{A}+B)$

adding product of each terms and its complement

$\rightarrow A(\bar{B}+A)B = \underline{(A+B)}(A+\bar{B})(\bar{B}+A)\underline{(A+B)}(\bar{A}+B)$

$= (A+B)(\bar{A}+B)(A+\bar{B})$

$= 00 \quad (01)(10) \quad = \Pi M(0,1,2) \leftarrow$ max term in pos

$A(\bar{B}+A)B: A \times (\bar{B}+A) \cancel{B} \times B$

$= 0 \times (\bar{B}+A) \cancel{0B} \times 0$

$= 00 \quad (01) \quad (10)$

$\rightarrow$ Expand $A(\bar{A}+B)(\bar{A}+B+\bar{C})$ to max term and min term

$A = \underline{A + B\bar{B}} + C\bar{C} = (A+B)(A+\bar{B}) + C\bar{C}$

$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)$

$(A+\bar{B}+\bar{C}) \cancel{(\bar{A}+B+\bar{C})}$

$\bar{A}+B: (\bar{A}+B) + C\bar{C} = (\bar{A}+B+C)(\bar{A}+B+\bar{C})$

$A(\bar{A}+B)(\bar{A}+B+\bar{C}) = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)$

$(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

$A = 0XX = (000)(001)$
$(010)(011)$
$= m_0 m_1 m_2 m_3$

$\bar{A}+B = 10X = 100, 101$
$= m_4, m_5$

$\bar{A}+B+\bar{C} = 101 = m_5$

$\Pi m(0,1,2,3,4,5)$

$= (000)(001)(010)(011)$

$(100)(101)$

$= \Pi M(0,1,2,3,4,5) \leftarrow$ max term in pos

min term $\Sigma m(6,7) \leftarrow$ min term in sop

→ Write down the algebric form for minterm

m_0	m_5	m_9	m_{14}
	8421	8421	8421
	0101	1001	1110
$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$A\bar{B}\bar{C}D$	$ABC\bar{D}$

M_3	M_9	M_{11}	M_{14}
8421	8421	8421	8421
0011	1001	1011	1110
$(A+B+\bar{C}+\bar{D})$	$(\bar{A}+B+C+\bar{D})$	$(\bar{A}+B+\bar{C}+\bar{D})$	$(\bar{A}+\bar{B}+\bar{C}+D)$

max term :- suppose there are two binary variable a & b and associate with OR operation. then it is called max term

$a+b$, $\bar{a}+\bar{b}$, $\bar{a}+b$, $\bar{b}+a$

$F = xy + x'z$

$= (xy + zz') + (x'z + yy')$

$xy \quad (z\bar{z})$

$(x' + y + zz') \quad (x + z + yy') \quad (y + z + xx')$

$(x' + y) \quad (x + 1) \quad (y + 1)$

$(100) \quad ($

$A\bar{C} + B$

$= A\bar{C}(B+\bar{B}) + B\,(A+\bar{A})\,(C+\bar{C})$

$A\bar{C}(B+\bar{B}) = AB\bar{C} + A\bar{B}\bar{C}$

$B(A+\bar{A})(C+\bar{C}) = (AC + A\bar{C} + \bar{A}C + \bar{A}\bar{C})\,B$

$= ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$.

$A\bar{C}+B = ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C}$.

$= \overset{4\,2\,1}{111} + \overset{4\,2\,1}{110} + \overset{4\,2\,1}{011} + \overset{4\,2\,1}{010} + \overset{4\,2\,1}{100}$

$= m_7 + m_6 + m_3 + m_2 + m_4$

4) $AD + ABC$.

$AD(B+\bar{B})\,(C+\bar{C})$

$= AD\,(BC + B\bar{C} + \bar{B}C + \bar{B}\bar{C})$

$= \underline{ABCD} + AB\bar{C}D + \underline{A\bar{B}CD} + A\bar{B}\bar{C}D$.

$ABC\,(D+\bar{D}) = A\underline{B}CD + ABC\bar{D}$

$AD+ABC = ABCD + AB\bar{C}D + A\bar{B}CD + A\bar{B}\bar{C}D + ABC\bar{D}$

$= \underset{8\,4\,2\,1}{1111} + \underset{8\,4\,2\,1}{1101} + \underset{8\,4\,2\,1}{1011} + \underset{8\,4\,2\,1}{1001} + \underset{8\,4\,2\,1}{1110}$

$= m_{15} + m_{13} + m_{11} + m_9 + m_{14}$

maxterm.

5

$F(A,B,C) = (A+B')(B+C)(A+C)$ obtain the product of sum / max-term form following the

$= (A+B'+CC')(AA'+B+C)(A+C+BB')$

$= (A+B'+C)(A+B+C')(B+C+A)(B+C+A')$
$(A+C+B)(A+C+B')$

$= (A+B'+C)(A+B+C')(A+B+C)(A'+B+C)$

$= \overset{421}{(010)}\ \overset{421}{(001)}\ \overset{421}{(000)}\ \overset{421}{(100)}$

$= \Pi(2,1,0,4)$

$= \Pi(0,1,2,4)$

$\boxed{\text{mostly used distributive property}}$

$F(A,B,C) = (A+B'C)$

$= (A+B')(A+C)$

$= (A+B'+CC')(A+C+BB')$

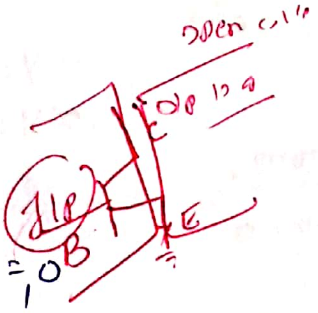
$= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C)$

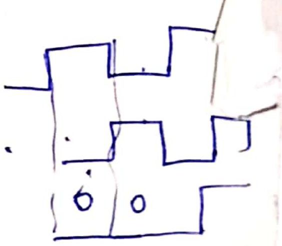
$= (A+B'+C)(A+B'+C')(A+B+C)$

$= \overset{421}{(010)}\ \overset{421}{(011)}\ \overset{421}{(000)}$

$= \Pi(2,3,0)$

- n variable





→ if complement = 0 / 1

minterm:

→ A product term which contain variables of the function either complemented or uncomplemented for is called minterm.

Product of sum = maxterm.

Sum of Product = minterm.

$m_0 = \bar{A}\bar{B}\bar{C}$
$m_1 = \bar{A}\bar{B}C$
$m_2 = \bar{A}B\bar{C}$

→ n variable to have 2^n minterm.

→ It is denoted as m_0, m_1, m_2.

→ Express the following f^n as sum of minterm and product of max-term.

$F(A,B,C,D) = \bar{B}D + \bar{A}D + BD$

$= \bar{B}D + BD + \bar{A}D$

$= (\bar{B}+B)D + \bar{A}D$

$= \bar{A}D + D.$

$= D(1+\bar{A})$

$= D.$

$D(A+\bar{A})(B+\bar{B})(C+\bar{C})$

$\bar{B}D + \bar{A}D + BD = x\bar{B}xD + \bar{A}xxD + xBxD$

$= x0x1 + 0xx1 + x1x1$

$= 0001,$
$0011,$
$1001,$
$1011,$

(b) $(xy+z)(xz+y) = (x+z)(y+z)(y+x)(y+z)$

$= (x+y)(y+z)(x+z).$

$= (x+y+zz')(y+z+xx')(x+z+yy')$

$= (x+y+z)(x+y+z')(x+y+z)(x'+y+z)(x+z+y)(x+z+y')$

$= (x+y+z)(x+y+z')(x'+y+z)(x+z+y')$

$= (\overset{4\,2\,1}{0\,0\,0})(\overset{4\,2\,1}{0\,0\,1})(\overset{4\,2\,1}{1\,0\,0})(\overset{4\,2\,1}{0\,1\,0})$

$= \Pi m(0,1,4,2)$

$= \Pi m(0,1,2,4)$ ← max term.

$= \Sigma m(3,5,6,7)$ ← minterm.

$xyz = 111$

$(xyz)' = x'+y'+z' = 000$

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$AB + C$

$AB + \bar{A}C$ find product of maxterm

$= (AB + \bar{A})(AB + C)$

$= (A + \bar{A})(\bar{A} + B)(A + C)(B + C)$

$= (\bar{A} + B)(B + C)(A + C)$

$= (\bar{A} + B + CC')(B + C + AA')(A + C + BB')$

$= (A' + B + C)(A' + B + C')(A' + B + C)(A + B + C)(A + B + C)(A + B' + C).$

$F = \underset{000}{(A + B + C)}\ \underset{010}{(A + B' + C)}\ \underset{100}{(A' + B + C)}\ \underset{101}{(A' + B + C')}$

$= \Pi m(0, 2, 4, 5)$

→ <u>Kmap (Karnaugh map)</u>

→ used to simplify Boolean expression
1) algebric 2) graphical. method ← Kmap

→ K map is f^n of nvariable. ← kmap ← 2^2 terms or 2^2 cells are there
2 variables

→ two 2 variable kmap ← $2^n = 2^2 = 4$ cells, 4 minterms.

→ Each cells represent unique minterm

minterm	I/P	O/P
0	A B 0 0	1
1	$\bar{A}B = 01$	0
2	$A\bar{B} = 10$	1
3	$AB = 11$	1

← presence or absence of minterm in perticular expression s.

$= \bar{A}\bar{B} + A\bar{B} + AB$ This is only two SOP.

A \ B	0	1
0	$\bar{A}\bar{B}$ (0)	$\bar{A}B$ (1)
1	$A\bar{B}$ (2)	AB (3)

A \ B	0	1
0	1	0
1	1	1.

$\Sigma m(0, 2, 3).$

$f = \bar{A}B + A\bar{B}$

A \ B	0	1
0	0	1
1	1	0

$x'y + xy' + xy$

x \ y	0	1
0		1
1	1	1

Expand $A(\bar{B}+A)B$ to maxterm and minterm

$- A(\bar{B}+A)B$

$A + B\bar{B} = (A+B)(A+\bar{B})$

$(\bar{B}+A) = A+\bar{B}$

$B = \cancel{B+(A\bar{A})} = B + A\bar{A}$

$= (\bar{A}+B)(A+B)$

$(A+B)(A+\bar{B})(\bar{A}+B)$

$(\overset{0}{0}\overset{1}{0})(\overset{0}{0}\overset{1}{1})(\overset{0}{1}\overset{1}{0})$

$\Pi m(0,1,2)$

$\Sigma m(3)$

$A - Ax - 0x = (00)(01)$

$(\bar{B}+A) = (10)$

$B - xB - x0 = (10)(00)$

$(00)(01)(10) \quad \Pi m(0,1,2)$

Scale :

Expand $A(\bar{A}+B)(\bar{A}+B+\bar{C})$.

$A \rightarrow \quad A\,x\,x \rightarrow 0\,x\,x$

$\rightarrow 0\,0\,0$

$0\,1\,0$

$0\,1\,1$

$0\,0\,1$

$\bar{A}+B+X \rightarrow \cancel{0}\,1\,0\,X = (1\,0\,1)\ (1\,0\,0)$

$\bar{A}+B+\bar{C} \rightarrow 1\,0\,1$

$$\overset{4\,2\,1}{(0\,0\,1)}\quad \overset{4\,2\,1}{(0\,1\,0)}\quad \overset{4\,2\,1}{(0\,1\,1)}\quad \overset{4\,2\,1}{(0\,0\,1)}\quad \overset{4\,2\,1}{(1\,0\,1)}\quad \overset{4\,2\,1}{(1\,0\,0)}$$

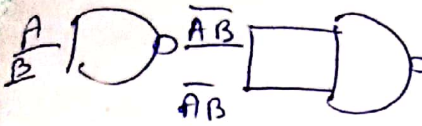
$\Pi m\,(1, 2, 3, 1, 5, 4)$

$\Pi m\ 0, 1, 2, 3, 4, 5\ )$

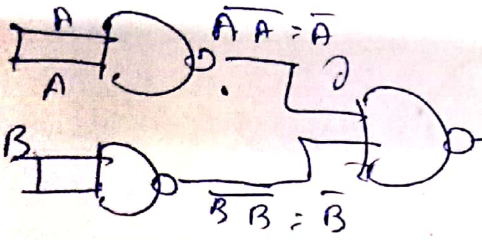
$\Sigma m\,(6, 7, 8)$

NAND Gate & NOR gate as Universal gate..

$\overline{A \cdot A} = \bar{A}$ NOT gate

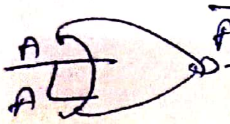


$\overline{\overline{AB} \cdot \overline{AB}} = \overline{\overline{AB}} = AB$ AND gate.

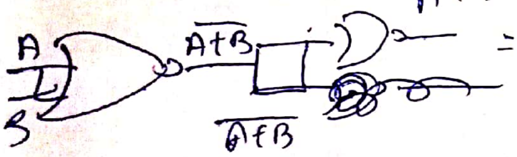


$\overline{\bar{A} \cdot \bar{B}} = A + B$: OR gate

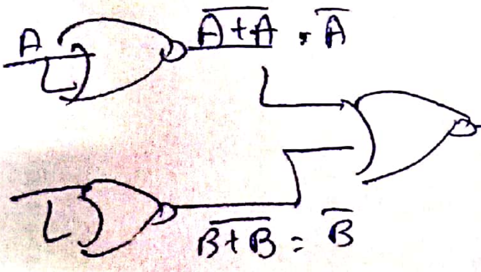
NOR



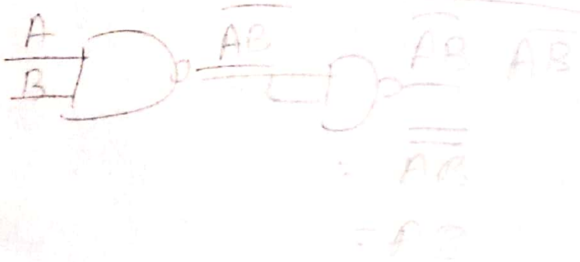
$\overline{A + A} = \bar{A}$ NOT gate



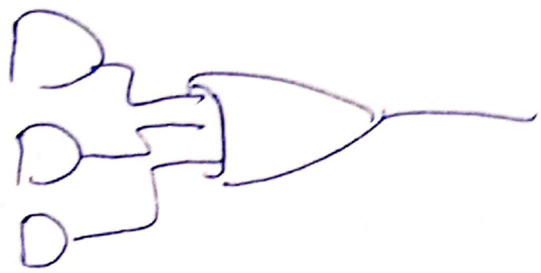
$\overline{\overline{A+B} + \overline{A+B}} = \overline{\overline{A+B}} = A + B$ → OR gate



$\overline{\bar{A} + \bar{B}} = \bar{\bar{A}} \cdot \bar{\bar{B}} = AB$ → AND gate.



SOP ← Sum of product



POS →