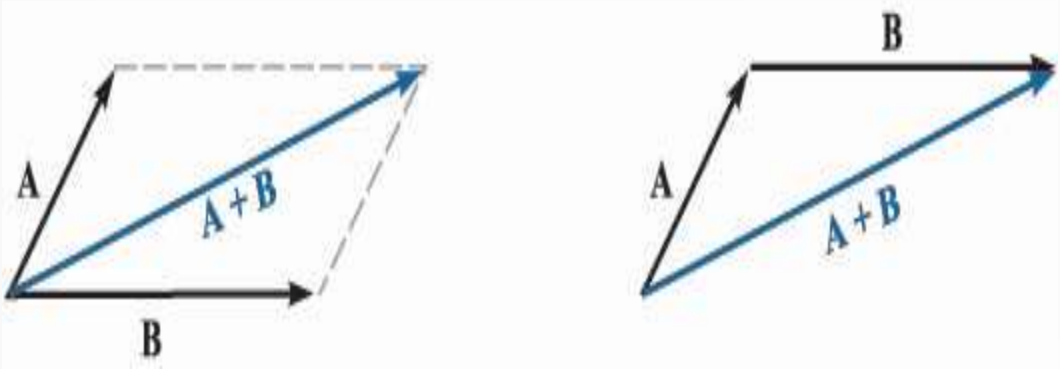
Introduction
Electromagnetics

E&C Department

Scalars and Vectors

- The scalars are the quantity whose values may be represented by a real number.
- The mass ,density, pressure and volume are the scalar quantity.

- On the other hand, the vectors are defined by both magnitude and direction in space.
- Force, velocity and acceleration are example of the vectors.

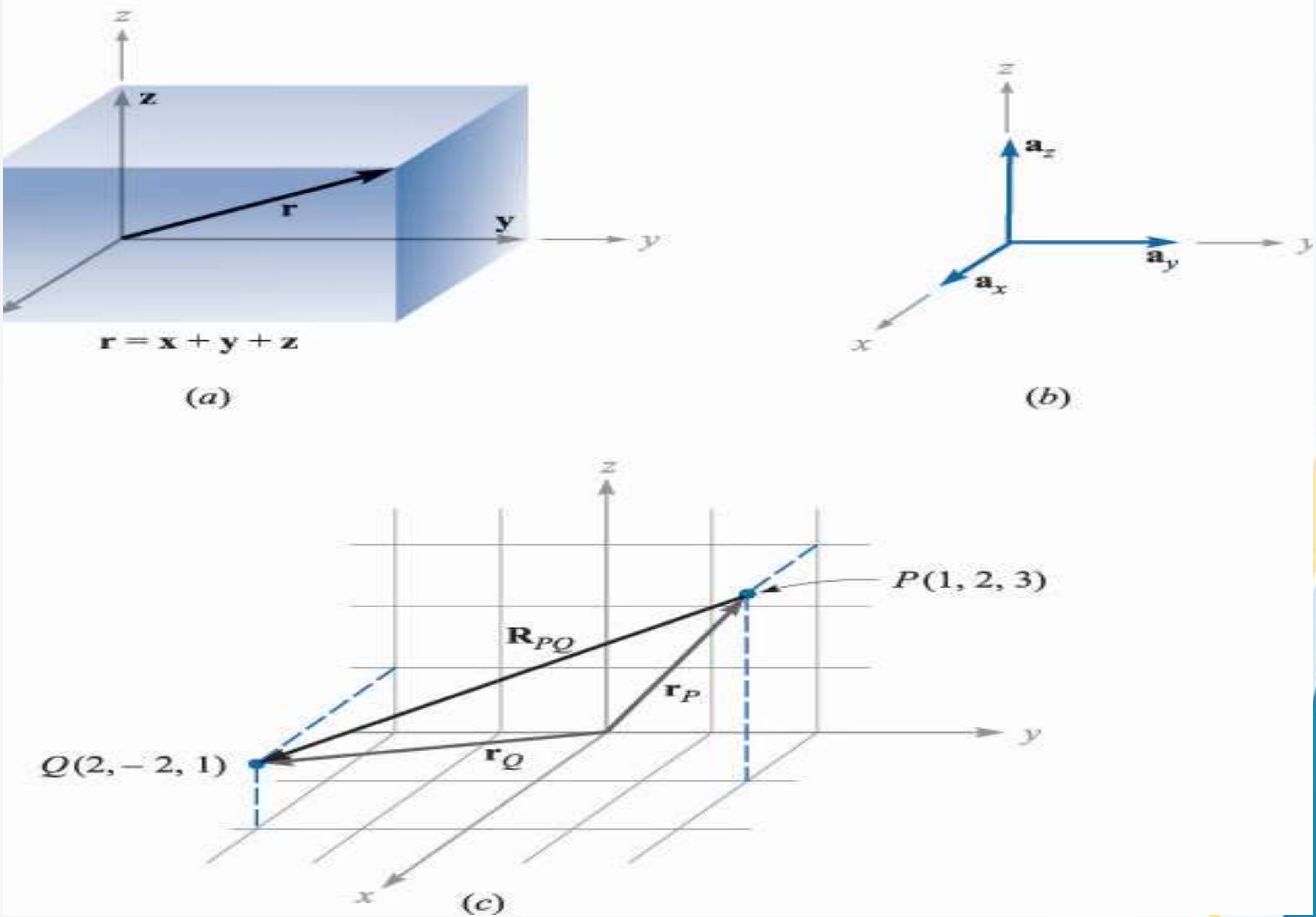


- In the figure, the two vectors are shown, A and B.
- The vector addition follows the parallelogram law.
- The vector addition follows, the commutative law
- A+B=B+A
- The vector addition follows, the associative law,
- A(B+C)=(A+B)+C
- For vector subtraction, reversed the sign or direction of the second vector,
- A-B=A+(-B)
- The vector, when multiplied by a scalar, the magnitude of the vector is changed but the direction remains same if the scalar is positive real number but if scalar is negative real number, the direction is reversed.
- r(A+B)= rA+rB

- The division of the vector by a scalar is nothing but the multiplication of the vector by the scalar's reciprocal.

- Two vectors are said to be equal when their difference is zero.
- A-B=0

Cartesian Co-ordinate system

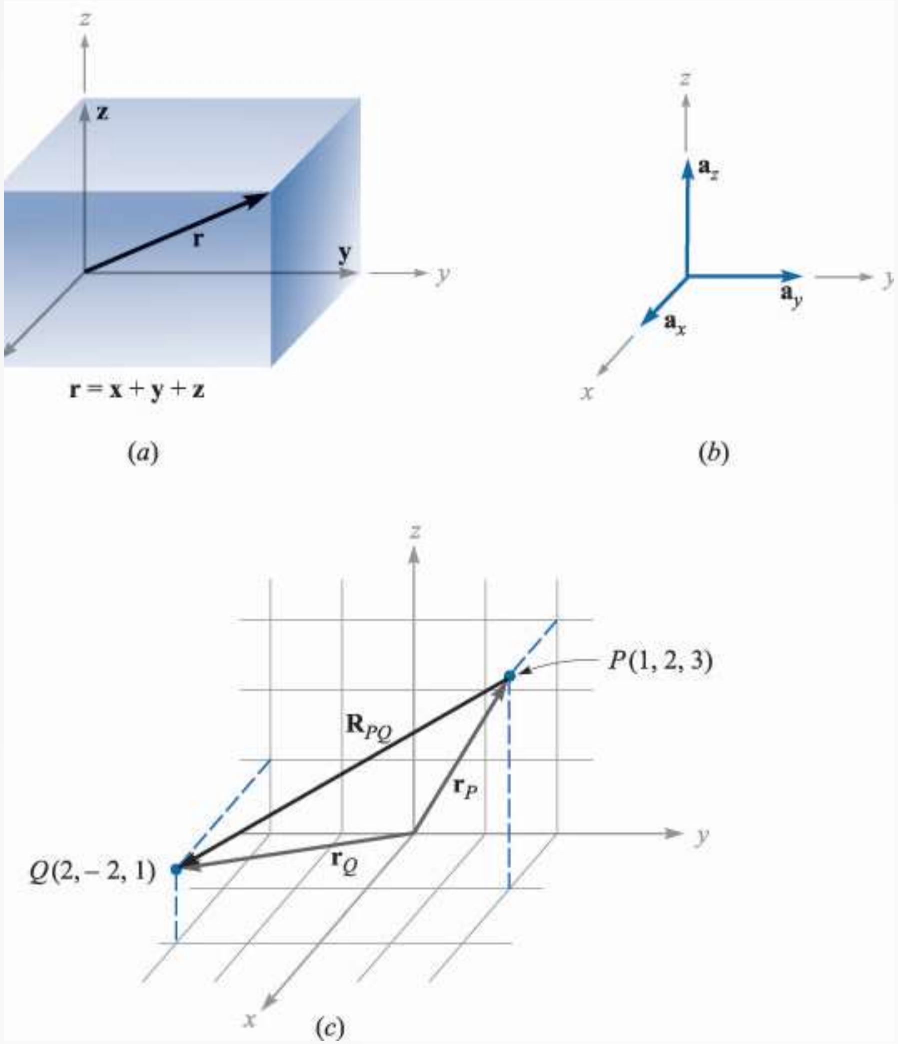


- As shown in figure, Point 'P'(1,2,3) and Point 'Q'(2,-2,1).
- So the desired vector from point P to Q is,
- $R_{PQ} = r_Q - r_P = (2-1)a_x + (-2-2)a_y + (1-3)a_z = a_x - 4a_y - 2a_z$
- Any vector B in the Cartesian co-ordinate system is given by,
- $B = B_x a_x + B_y a_y + B_z a_z$
- It's magnitude is given by,

$$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

- The unit vector in the direction of B is,

$$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}} = \frac{\mathbf{B}}{|\mathbf{B}|}$$


z
z
r
y
y
r = x + y + z
(a)
z
$\mathbf{a}_z$
$\mathbf{a}_y$
$\mathbf{a}_x$
y
x
(b)
z
P(1, 2, 3)
$\mathbf{R}_{PQ}$
$\mathbf{r}_P$
$\mathbf{r}_Q$
Q(2, − 2, 1)
y
x
(c)

The Dot Product

- The dot product between two vectors A and B is given by the product of their magnitude and a smaller cosine angle between them,

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos\theta_{AB}$$

- The dot product is a scalar.
- The dot product obeys the commutative law,

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

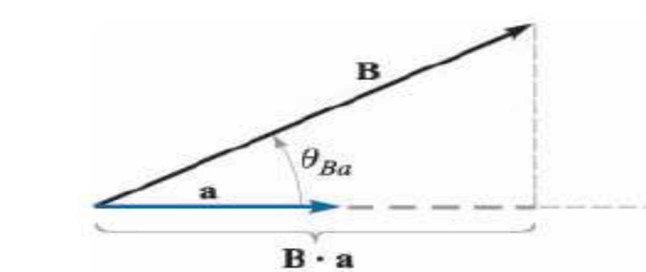
- Suppose $\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$ and $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$

$$\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\mathbf{a}_x \cdot \mathbf{a}_y = \mathbf{a}_y \cdot \mathbf{a}_x = \mathbf{a}_x \cdot \mathbf{a}_z = \mathbf{a}_z \cdot \mathbf{a}_x = \mathbf{a}_y \cdot \mathbf{a}_z = \mathbf{a}_z \cdot \mathbf{a}_y = 0$$

$$\mathbf{A} \cdot \mathbf{A} = A^2 = |\mathbf{A}|^2$$

- One of the important applications of the dot product is to finding the component of the vector in a given direction.



- As shown in figure, we can obtain the component of B in the direction specified by the unit vector 'a',

$$\mathbf{B} \cdot \mathbf{a} = |\mathbf{B}|\,|\mathbf{a}| \cos\theta_{Ba} = |\mathbf{B}| \cos\theta_{Ba}$$

- The sign of the component is positive for $0 < \theta_{Ba} < 90$ deg, and sign is negative when $90 < \theta_{Ba} < 180$ deg.

The Cross Product

- The cross product of two vector A and B is given by,

$$\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\, |\mathbf{B}| \sin \theta_{AB}$$

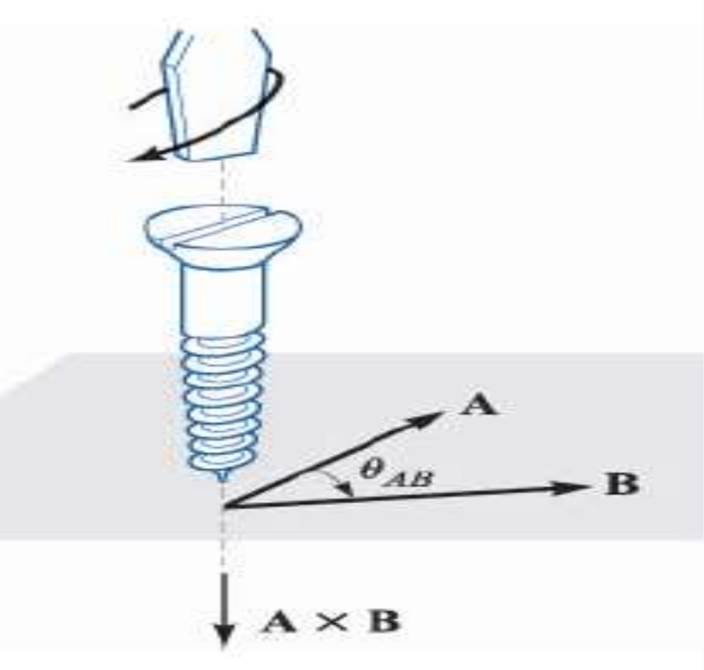
- The cross product is vector.

- The direction of AxB is perpendicular to the plane containing vectors A and B and it is along that perpedicular which is in the direction of advanced of the right-handed screw as A is turned into B.

- The magnitude of AxB is the product of magnitude of A, B and sine of angle between A and B.

- The cross product is not commutative,

$$\mathbf{B} \times \mathbf{A} = -(\mathbf{A} \times \mathbf{B})$$

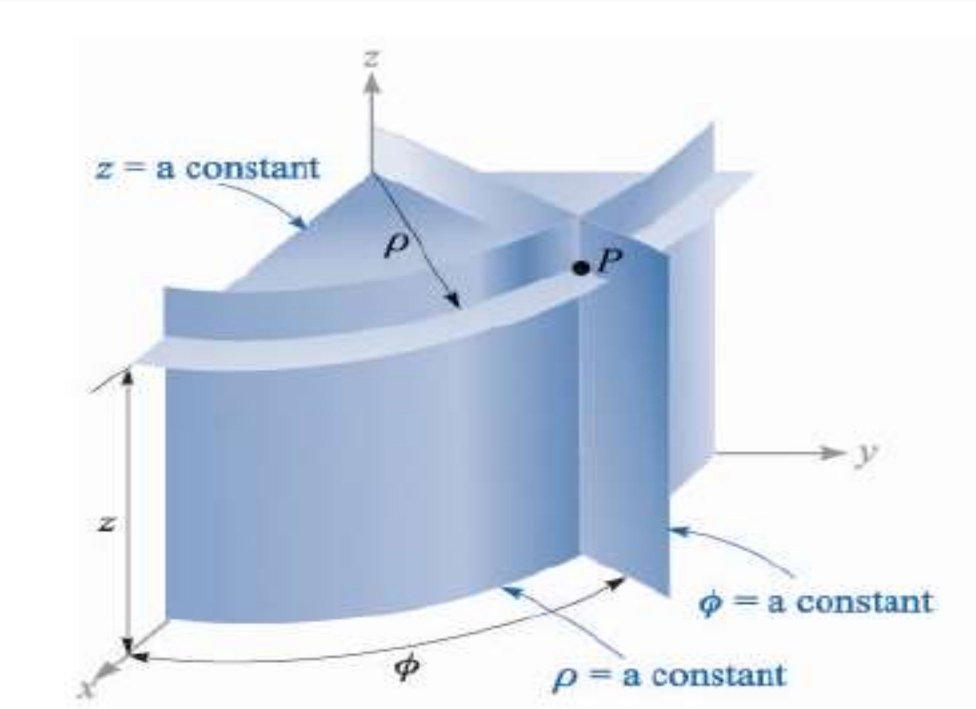


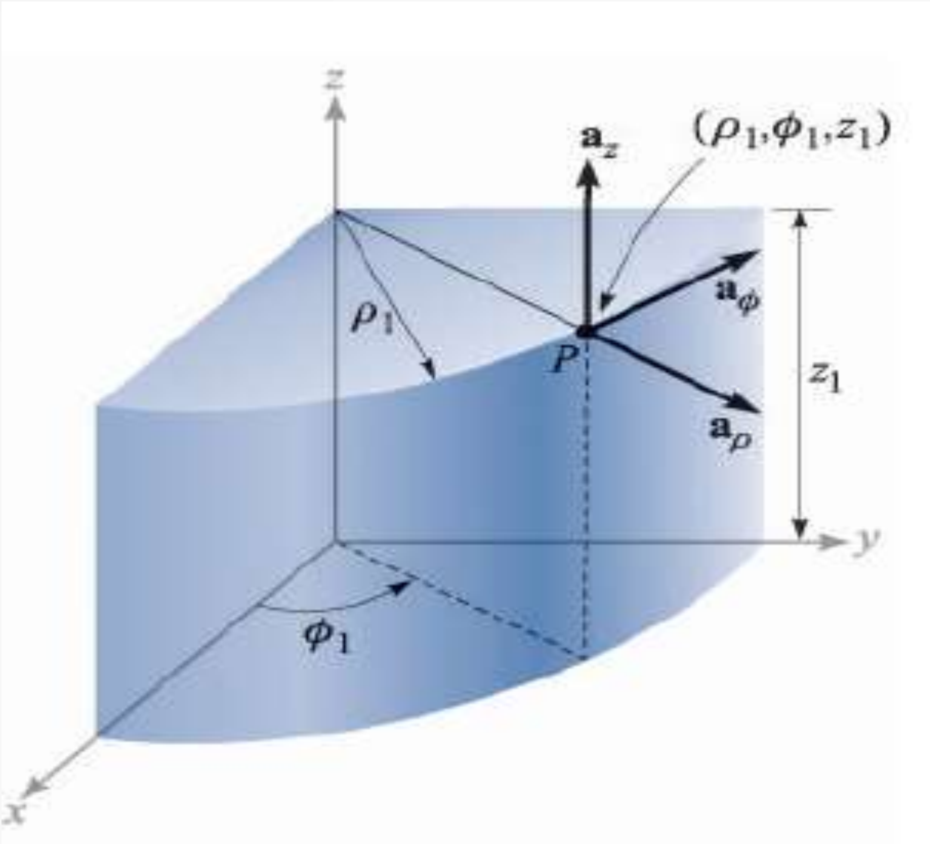
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y)\mathbf{a}_x + (A_z B_x - A_x B_z)\mathbf{a}_y + (A_x B_y - A_y B_x)\mathbf{a}_z$$

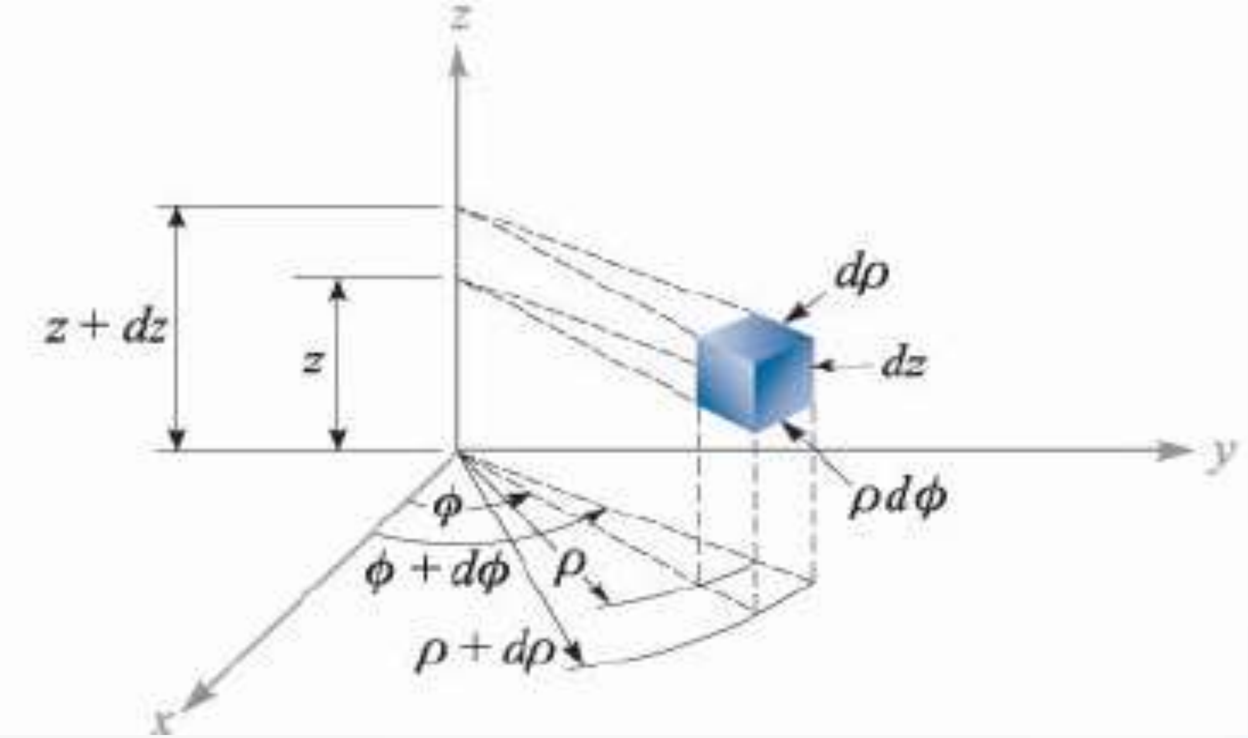
Cylindrical Coordinate System

- Consider any point as a intersection of three mutually perpendicular surfaces.
- These surfaces are, circular cylinder (ρ= constant), a plane (Φ= constant), and a plane (z= constant).





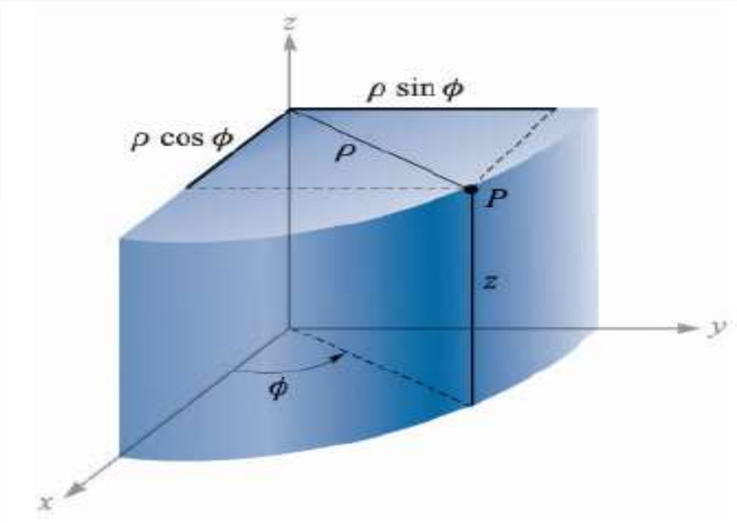
- The unit vector a_ρ at a point P is directed radially outward, normal to the cylindrical surface $\rho=\rho_1$. The unit vector a_Φ is lies normal to the plane $\Phi=\Phi_1$, points in the direction of increasing Φ, lies in the plane $z=z_1$ and tangent to the cylindrical surface $\rho=\rho_1$.



- A differential volume element in the cylindrical coordinate system is as shown in figure.

- Here dz and dρ are dimensionally a length, but dΦ is not a length, the length element is ρ dΦ.
- The surfaces have area of ρdρ dΦ, dρ dz and ρ dΦ dz.
- The volume becomes ρdρ dΦ dz.

- The variables of Cartesian coordinate system and cylindrical coordinate are related to each other as,



- $X = \rho \cos\Phi$
- $Y = \rho \sin\Phi$
- $Z = Z$.

$$\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$

$$\mathbf{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$$

$$A_\rho = \mathbf{A}\cdot\mathbf{a}_\rho \qquad \text{and} \qquad A_\phi = \mathbf{A}\cdot\mathbf{a}_\phi$$

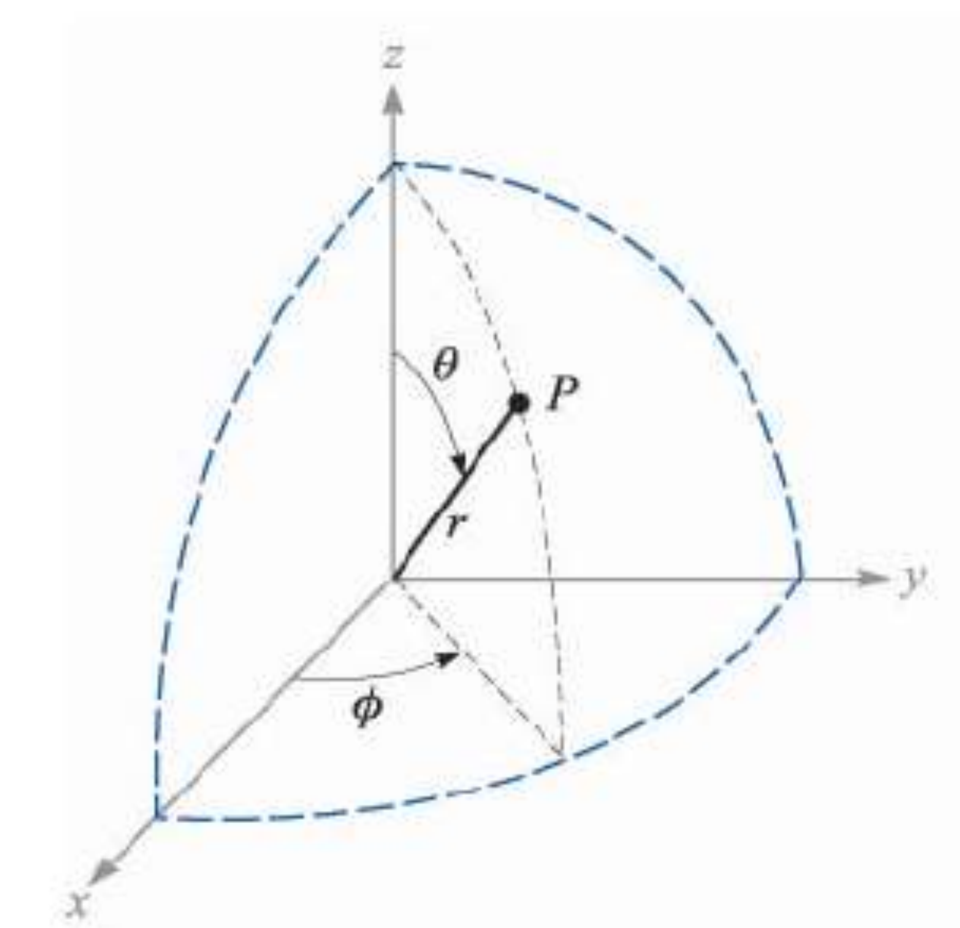
$$A_\rho = (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z)\cdot\mathbf{a}_\rho = A_x\mathbf{a}_x\cdot\mathbf{a}_\rho + A_y\mathbf{a}_y\cdot\mathbf{a}_\rho$$

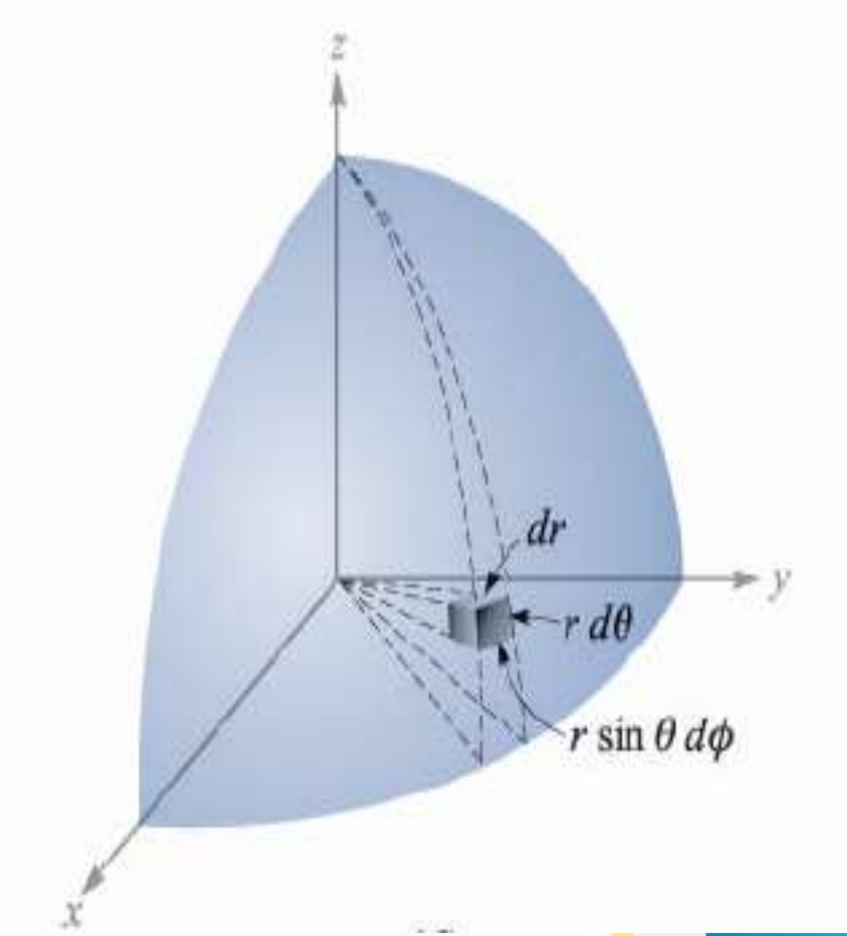
$$A_\phi = (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z)\cdot\mathbf{a}_\phi = A_x\mathbf{a}_x\cdot\mathbf{a}_\phi + A_y\mathbf{a}_y\cdot\mathbf{a}_\phi$$

$$A_z = (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z)\cdot\mathbf{a}_z = A_z\mathbf{a}_z\cdot\mathbf{a}_z = A_z$$

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x\cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y\cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z\cdot$	0	0	1

Spherical Coordinate System





- There are three variables in the spherical coordinate system,
  r, Θ, and Φ.
- The 'r' defined, the distance from the origin to the point, the surface r= constant is a sphere.

- Θ is the angle between, the Z-axis and the line drawn to point from the origin. The surface Θ= constant is a cone, which is a circle of radius r sin Θ. The coordinate Θ corresponds to the angle in elevation plane (latitude).

- The angle Φ is the angle between the X-axis and the line drawn in the Z=0 plane. The coordinate Φ corresponds to the angle in azimuthpalne (longitude). The surface Φ=constant is a plane passing through Θ=0 line.

- Any point is the intersection of 3 mutually perpendicular surfaces - a sphere, a cone and a plane.

- A differential volume element in the spherical coordinate system is as shown in figure.
- Here dr is dimensionally a length, but dθ and dΦ are not a length, the length element are r dθ and r sin θ dΦ.
- The surfaces have area of r dr dθ , r sin θ dr dΦ and r^2 sin θ dθ dΦ.
- The volume becomes r^2 sin θ dr dθ dΦ.

- X= r sin θ cos Φ, Y= r sin θ sin Φ, Z= r cos θ

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z\cdot$	$\cos\theta$	$-\sin\theta$	0

Thank You