EXAMPLE 5.31 Simplify the following Boolean expressions to a minimum number of literals.

(a) $\bar{x}\,\bar{y} + xy + \bar{x}y$

(b) $x\bar{y} + \bar{y}\,\bar{z} + \bar{x}\,\bar{z}$

(c) $(x + y)(x + \bar{y})$

(d) $\bar{x}y + xy + x\bar{z} + x\bar{y}\,\bar{z}$

(e) $(A + B)(\bar{A} + C)(\bar{B} + D)(C\bar{D})$

(f) $\bar{A}\bar{C} + ABC + A\bar{C}$ to three literals

(g) $\overline{(\bar{x}\,\bar{y} + z)} + z + xy + wz$ to three literals

(h) $\bar{A}B(\bar{D} + \bar{C}D) + B(A + \bar{A}CD)$ to one literal.

(i) $(\bar{A} + C)(\bar{A} + \bar{C})(\bar{A} + B + \bar{C}D)$ to one literal

(j) $AB + A(B + C) + \bar{B}(B + D)$

(k) $A + B + \bar{A}\bar{B}C$

(l) $\bar{A}B + \bar{A}B\bar{C} + \bar{A}BCD + \bar{A}B\bar{C}\bar{D}E$

(m) $ABEF + AB\overline{EF} + \overline{AB}EF$

(n) $ABC\bar{D} + A + AB\bar{D} + (\bar{D})(\bar{A}\bar{B}\bar{C})$

(o) $x[y + z(\overline{xy + xz})]$

(p) $\bar{x}\,\bar{z} + \bar{y}\,\bar{z} + y\bar{z} + xyz$

Solution

The simplication of the above Boolean expressions is as follows:

(a) $\bar{x}\,\bar{y} + xy + \bar{x}y = \bar{x}\,\bar{y} + y(x + \bar{x}) = \bar{x}\,\bar{y} + y = (y + \bar{y})(y + \bar{x}) = \bar{x} + y$

(b) $x\bar{y} + \bar{y}\,\bar{z} + \bar{x}\,\bar{z} = x\bar{y} + \bar{x}\,\bar{z} + \bar{y}\,\bar{z}(x + \bar{x}) = x\bar{y} + \bar{x}\,\bar{z} + \bar{y}\,\bar{z}x + \bar{y}\,\bar{z}\,\bar{x}$

$= x\bar{y}(1 + \bar{z}) + \bar{x}\,\bar{z}(1 + \bar{y}) = x\bar{y} + \bar{x}\,\bar{z}$

(c) $(x + y)(x + \bar{y}) = xx + xy + x\bar{y} + y\bar{y} = x + xy + x\bar{y} + 0 = x(1 + y + \bar{y}) + 0 = x$

(d) $\bar{x}y + xy + x\bar{z} + x\bar{y}\,\bar{z} = y(x + \bar{x}) + x\bar{z}(1 + \bar{y}) = y + x\bar{z}$

(e) $(A + B)(\bar{A} + C)(\bar{B} + D)(C\bar{D}) = (A\bar{A} + AC + \bar{A}B + BC)(\bar{B}C\bar{D} + DC\bar{D})$

$= (AC + \bar{A}B + BC)\bar{B}C\bar{D} = A\bar{B}C\bar{D}$

(f) $\bar{A}\bar{C} + ABC + A\bar{C} = \bar{C}(\bar{A} + A) + ABC = \bar{C} + ABC = (\bar{C} + C)(\bar{C} + AB) = \bar{C} + AB$

(g) $\overline{(\bar{x}\,\bar{y} + z)} + z + xy + wz = \overline{\bar{x}\,\bar{y}} \cdot \bar{z} + z(1 + w) + xy = (x + y)\bar{z} + z + xy$

$= (z + \bar{z})(z + x + y) + xy$

$= x + y + z + xy = x + y + z$

(h) $\bar{A}B(\bar{D}+\bar{C}D)+B(A+\bar{A}CD)=\bar{A}B\bar{D}+\bar{A}B\bar{C}D+AB+\bar{A}BCD$

$$=\bar{A}BD(C+\bar{C})+\bar{A}B\bar{D}+AB=\bar{A}BD+\bar{A}B\bar{D}+AB$$

$$=\bar{A}B(D+\bar{D})+AB=AB+\bar{A}B=B(A+\bar{A})=B$$

(i) $(\bar{A}+C)(\bar{A}+\bar{C})(\bar{A}+B+\bar{C}D)=(\bar{A}+\bar{A}C+\bar{A}\bar{C}+C\bar{C})(\bar{A}+B+\bar{C}D)$

$$=\bar{A}(1+C+\bar{C})(\bar{A}+B+\bar{C}D)=\bar{A}(\bar{A}+B+\bar{C}D)$$

$$=\bar{A}+\bar{A}B+\bar{A}\bar{C}D=\bar{A}(1+B+\bar{C}\bar{D})=\bar{A}$$

(j) $AB+A(B+C)+\bar{B}(B+D)=AB+AB+AC+\bar{B}B+\bar{B}D$

$$=AB+AC+\bar{B}D=A(B+C)+\bar{B}D$$

(k) $A+B+\bar{A}\bar{B}C=(A+\bar{A})(A+\bar{B}C)+B=A+B+\bar{B}C=A+(B+\bar{B})(B+C)=A+B+C$

(l) $\bar{A}B+\bar{A}B\bar{C}+\bar{A}BCD+\bar{A}B\bar{C}\bar{D}E=\bar{A}B(1+\bar{C}+CD+\bar{C}\bar{D}E)=\bar{A}B$

(m) $ABEF+AB\overline{EF}+\overline{AB}EF=AB(EF+\overline{EF})+\overline{AB}EF=AB+\overline{AB}EF$

$$=(AB+\overline{AB})(AB+EF)=AB+EF$$

(n) $ABC\bar{D}+A+AB\bar{D}+(\bar{D})(\bar{A}\bar{B}\bar{C})=ABC\bar{D}+A+AB\bar{D}+(\bar{A}\bar{B}\bar{C}\bar{D})$

$$=A(1+B\bar{D}+BC\bar{D})+\bar{A}\bar{B}\bar{C}\bar{D}=A+\bar{A}\bar{B}\bar{C}\bar{D}$$

$$=(A+\bar{A})(A+\bar{B}\bar{C}\bar{D})=A+\bar{B}\bar{C}\bar{D}$$

(o) $x[y+z(\overline{xy+xz})]=x[y+z(\overline{xy}\cdot\overline{xz})]=xy+xz\cdot\overline{xy}\cdot\overline{xz}=xy+0$

$$=xy$$

(p) $\bar{x}\bar{z}+\bar{y}\bar{z}+y\bar{z}+xyz=\bar{x}\bar{z}+\bar{z}(y+\bar{y})+xyz=\bar{x}\bar{z}+\bar{z}+xyz$

$$=\bar{z}(1+\bar{x})+xyz=\bar{z}+xyz=(\bar{z}+z)(\bar{z}+xy)=\bar{z}+xy$$