

EXAMPLE 5.29 Find the complements of the following expressions.

(a) $AB + A(B + C) + \bar{B}(B + D)$

(b) $A + B + \bar{A}\bar{B}C$

(c) $\bar{A}B + A\bar{B}C + \bar{A}BCD + \bar{A}\bar{B}C\bar{D}E$

(d) $ABEF + AB\bar{E}\bar{F} + \bar{A}\bar{B}EF$

(e) $\bar{B}\bar{C}D + \overline{(B + C + D)} + \bar{B}\bar{C}\bar{D}E$

(f) $AB + \bar{A}\bar{C} + A\bar{B}C$

(g) $(\bar{A}\bar{B} + A\bar{C})(BC + B\bar{C})(ABC)$

(h) $A\bar{B}C + \bar{A}BC + ABC$

(i) $(\bar{A}BC)(\overline{A + B + C})$

(j) $A + \bar{B}C(A + B + \bar{C})$

(k) $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

Solution

To obtain the complement of an expression change the ANDs to ORs, ORs to ANDs, 0s to 1s, 1s to 0s and complement each variable. Based on this rule the complements of the above functions are as follows.

(a) $(\bar{A} + \bar{B}) \cdot (\bar{A} + \bar{B} \cdot \bar{C}) \cdot (\bar{B} + \bar{B}\bar{D}) = (\bar{A} + \bar{B}) \cdot (\bar{A} + \bar{B} \cdot \bar{C}) \cdot (B + \bar{B}\bar{D})$

(b) $\bar{A} \cdot \bar{B} \cdot (\bar{A} + \bar{B} + \bar{C}) = (\bar{A}) \cdot (\bar{B}) \cdot (A + B + \bar{C})$

(c) $(\bar{A} + \bar{B}) \cdot (\bar{A} + BC) \cdot (\bar{A} + \bar{B} + \bar{C} + \bar{D}) \cdot (\bar{A} + \bar{B}\bar{C} + \bar{D} + \bar{E})$
 $= (A + \bar{B}) \cdot (\bar{A} + BC) \cdot (A + \bar{B} + \bar{C} + \bar{D}) \cdot (A + BC + D + \bar{E})$

(d) $(\bar{A} + \bar{B} + \bar{E} + \bar{F}) \cdot (\bar{A} + \bar{B} + \bar{E} + \bar{F}) \cdot (\bar{A} + \bar{B} + \bar{E} + \bar{F})$
 $= (\bar{A} + \bar{B} + \bar{E} + \bar{F}) \cdot (\bar{A} + \bar{B} + E + F) \cdot (A + B + \bar{E} + \bar{F})$

(e) $(\bar{B} + \bar{C} + \bar{D}) \cdot \overline{(B + C + D)} \cdot (\bar{B} + \bar{C} + \bar{D} + \bar{E})$
 $= (B + C + \bar{D}) \cdot (B + C + D) \cdot (B + C + D + \bar{E})$

(f) $(\bar{A} + \bar{B}) \cdot (AC) \cdot (\bar{A} + B + \bar{C})$

(g) $(\bar{A} + \bar{B}) \cdot (\bar{A} + \bar{C}) + (\bar{B} + \bar{C}) \cdot (\bar{B} + \bar{C}) + (\bar{A} + \bar{B} + \bar{C})$
 $= (\bar{A} + B) \cdot (\bar{A} + C) + (\bar{B} + \bar{C}) \cdot (\bar{B} + C) + (\bar{A} + \bar{B} + \bar{C})$

(h) $(\bar{A} + \bar{B} + \bar{C}) \cdot (\bar{A} + \bar{B} + \bar{C}) \cdot (\bar{A} + \bar{B} + \bar{C})$

(i) $(\bar{A}BC) + \overline{(A + B + C)} = (ABC) + (A + B + C)$

(j) $\bar{A} \cdot (\bar{B} + \bar{C}) + (\bar{A}\bar{B}\bar{C}) = \bar{A} \cdot (B + \bar{C}) + (\bar{A}\bar{B}C)$