Electrostatics-II

Work, Energy and Potential

- **Relating Electric Field E & Potential V**

Energy to move a point charge through a Field

- Force on charge Q due to an electric field

$$F_E = QE$$

- Differential work done by an external source moving Q

$$= -Q\mathbf{E}\cdot\mathbf{a}_L dL = -Q\mathbf{E}\cdot d\mathbf{L}$$

$$dW = -Q\mathbf{E}\cdot d\mathbf{L}$$

- Work required to move a charge Q by a finite distance in the presenace of E field is given by

$$W = -Q\int_{\text{init}}^{\text{final}} \mathbf{E}\cdot d\mathbf{L}$$

Line Integral

- differential length elemts are given by

$$d\mathbf{L} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z \quad \text{(cartesian)}$$

$$d\mathbf{L} = d\rho\,\mathbf{a}_\rho + \rho\,d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z \quad \text{(cylindrical)}$$

$$d\mathbf{L} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi \quad \text{(spherical)}$$

Potential Difference

- The work done by an external source in moving a charge Q from one point to another in an electric field E,

$$W = -Q \int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L}$$

- Potential difference V is defined as the **work done (by an external source) in moving a unit positive charge from one point to another in an electric field,**

$$\text{Potential difference } = V = -\int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L}$$

Numerical on potential difference

D4.4. An electric field is expressed in cartesian coordinates by $\mathbf{E} = 6x^2\mathbf{a}_x + 6y\mathbf{a}_y + 4\mathbf{a}_z$ V/m. Find: (*a*) V_{MN} if points M and N are specified by $M(2, 6, -1)$ and $N(-3, -3, 2)$; (*b*) V_M if $V = 0$ at $Q(4, -2, -35)$; (*c*) V_N if $V = 2$ at $P(1, 2, -4)$.

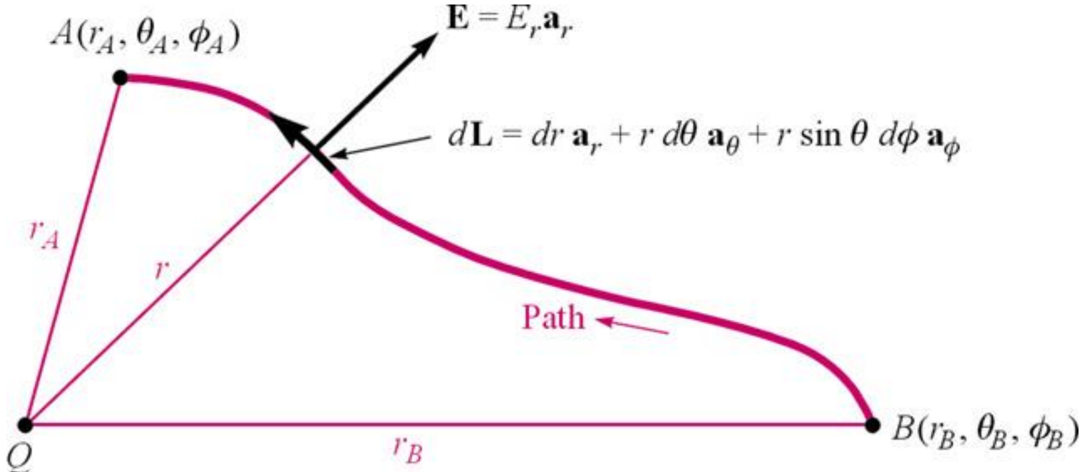
Potential Difference

- Potential Difference

$$V = -\int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L}$$

- Using radial distances from the point charge

$$V_{AB} = -\int_{B}^{A} \mathbf{E} \cdot d\mathbf{L}$$



$$V_{AB} = -\int_{r_B}^{r_A} \frac{Q}{4\pi\varepsilon_0 \cdot r^2}\,dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

Absolute Potential

- Potential difference is measured in ***joules per coulomb(volts)***.
- Hence the potential difference between points A and B is

$$V_{AB} = -\int_B^A \mathbf{E} \cdot d\mathbf{L} \quad \mathrm{V}$$

- We can try out this definition by finding the potential difference between points A and B at radial distances r_A and r_B from a point charge Q. Choosing an origin at Q;

$$\mathbf{E} = E_r \mathbf{a}_r = \frac{Q}{4\pi\epsilon_0 r^2}\mathbf{a}_r$$

and

$$d\mathbf{L} = dr\, \mathbf{a}_r$$

we have

$$V_{AB} = -\int_B^A \mathbf{E} \cdot d\mathbf{L} = -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon_0 r^2}\, dr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

Absolute Potential

- It is often convenient to define ***absolute potential, of a point***, rather than the potential difference between two points, but this means only that we agree to measure every potential difference with respect to a specified reference point which we consider to have zero potential.

- The most universal zero reference point in experimental or physical potential measurements is **"*ground*",** by which we mean the potential of the surface region of the earth itself.

- **The zero reference is more conveniently selected at infinity.** Let V_B=0 at Infinity, the absolute potential at point A is

$$V = \frac{Q}{4 \pi \varepsilon_0 \cdot r}$$

- If the Absolute potential at point A is V_A and that at B is V_B , then

$$V_{AB} = V_A - V_B$$

Definitions

- ***Equipotential Surface***: It is a surface composed of all those points having the same value of potential.

- No work is involved in moving a unit charge around on an equipotential surface, for, by definition, there is no potential difference between any two points on this surface.

- ***Conservative Field:***

- No work is done in carrying the unit charge around any closed path in the presance of conseravtive field.

Facts on Potential

1. The potential due to a single point charge is the work done in carrying a unit positive charge from infinity to the point at which we desire the potential, and the work is independent of the path chosen between those two points.

2. The potential field in the presence of a number of point charges is the sum of the individual potential fields arising from each charge.

3. The potential due to a number of point charges or any continuous charge distribution may therefore be found by carrying a unit charge from infinity to the point in question along any path we choose.

Which means that no work is done in carrying the unit charge around any closed path.

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0$$

This is true only for static fi .

Relationship between potential and electric field intensity

- given **E**, find V: What about reverse???

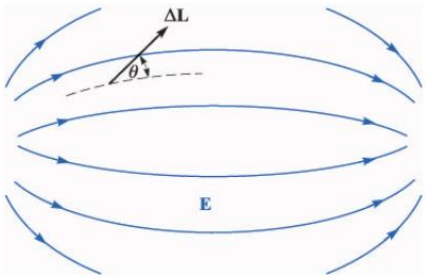
$$V = -\int \mathbf{E} \cdot d\mathbf{L}$$

- For a very short element of length ▲L along which E is essentially constant, leading to an incremental potential difference

$$\Delta V \doteq -\mathbf{E} \cdot \Delta \mathbf{L}$$

$$\Delta V \doteq -E \Delta L \cos\theta$$

$$\frac{dV}{dL} = -E\cos\theta$$



- In which direction should ▲L be placed to obtain a maximum value of ▲v??

- It is obvious that the maximum positive increment of potential, ▲V_{max} , will occur when $\cos\theta = -1$ (i.e. when ▲L points in the direction opposite to **E**)

$$\left.\frac{dV}{dL}\right|_{\max} = E$$

Relationship between E & V

- Two characteristics of relationship between E and V:
- 1. The magnitude of the electric field intensity is given by the maximum value of the rate of change of potential with distance.
- 2. This maximum value is obtained when the direction of the distance increment is opposite to E or, in other words, the direction of E is opposite to the direction in which the potential is increasing the most rapidly.
- Let a_N be a unit vector normal to the surface and directed toward the higher potentials

$$\mathbf{E} = -\frac{dV}{dN}\mathbf{a}_N$$

- which shows that the magnitude of E is given by the maximum space rate of change of V and the direction of E is normal to the surface (in the direction of decreasing potential).

• The gradient of a scalar is a vector.

• The gradient shows the maximum space rate of change of a scalar quantity and the *direction* in which the maximum occurs.

• The operation on V by which **-E** is obtained

$$\mathbf{E} = -\text{ grad } V = -\nabla . V$$

$$\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z$$

$$\text{grad } V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\mathbf{E} = -\left(\frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z\right)$$

Gradients in different coordinate systems

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z \qquad \text{(cartesian)}$$

Cartesian

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z \qquad \text{(cylindrical)}$$

Cylindrical

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi \qquad \text{(spherical)}$$

Spherical

Example 4.3

Given the potential field, $V = 2x^2y - 5z$, and a point P(-4, 3, 6), find the following: potential V, electric field intensity **E**

potential $V_P = 2(-4)^2(3) - 5(6) = 66$ V

electric field intensity - use gradient operation

$$\mathbf{E} = -4xy\mathbf{a_x} - 2x^2\mathbf{a_y} + 5\mathbf{a_z}$$

$$\mathbf{E}_P = 48\mathbf{a_x} - 32\mathbf{a_y} + 5\mathbf{a_z}$$

Where is energy stored?

- The location of potential energy cannot be precisely pinned down in terms of physical location - in the molecules of the pencil, the gravitational field, etc?

- So where is the energy in a capacitor stored?

- Electromagnetic theory makes it easy to believe that the energy is stored in the field itself.