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Current & Conductors:

- Electrical charges in a motion constitutes an electric current. (Ampere)

$$I = \frac{dQ}{dt} \quad \text{(rate of change of charges)}$$

time

- Current density: $\bar{J}$ (A/m^2)

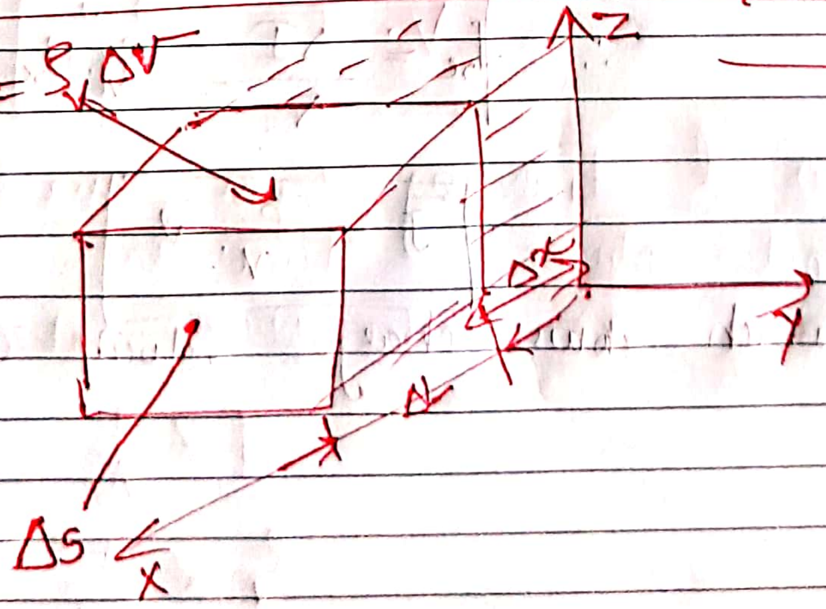
incremental current ΔI crossing inc. surface Δ

$$\Delta I = J_N \cdot \Delta S$$

$$\therefore \boxed{I = \int_S \bar{J} \cdot \overline{ds}}$$

- Relation between current density & vol. charge density

$\Delta Q = \rho_v \Delta v$



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Element of charge

$$\Delta Q = \rho_v \cdot \Delta V = \rho_v \cdot \Delta S \cdot \Delta L$$

Let charge element is oriented with its edges parallel to x-axes & posses only x comp. of velocity

In time Δt, element of charge has moved dist Δx

$\therefore$ moved charge $\Delta Q = \rho_v \cdot \Delta S \cdot \Delta x$ in time interval of Δt

$$\therefore \Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \cdot \Delta S \cdot \frac{\Delta x}{\Delta t}$$

$$\therefore \Delta I = \rho_v \cdot \Delta S \cdot v_x$$

where $v_x \rightarrow$ x component of velocity

$$J_x = \frac{\Delta I}{\Delta S} = \rho_v \cdot v_x$$

$$\therefore \boxed{\bar{J} = \rho_v \cdot \bar{v}}$$

$\rightarrow$ which shows charge velocity constitutes a current.

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Continuity of current :

Principle of conservation of charges shows that "charges can neither be created nor be destroyed, although equal amount of +ve & -ve charges may be simultaneously created (obtained by seperation) or destroyed (lost by recombination)

→ Current through a closed surface is

$$I = \oint_S \vec{J}.\vec{ds}$$

→ outward flow of +ve charges must be balanced by a decrease of ~~+ve~~ charge within closed surface.

If the charge inside close surface $= Q_i$ then rate of decrease $= -\frac{dQ_i}{dt}$

& from principle of conservation of charges

$$\boxed{I = \oint \vec{J}.\vec{ds} = -\frac{dQ_i}{dt}}$$ ← integral form of ct. eqn

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⇒ differential (point) form of ct. eqⁿ:

$$I = \oint_S \bar{J} \cdot \bar{ds} = \int_{Vol} (\bar{\nabla} \cdot \bar{J})\, dv \quad \left[\begin{array}{l}\text{divergence} \\ \text{theorem}\end{array}\right]$$

$$\Rightarrow \int_{Vol} (\bar{\nabla} \cdot \bar{J})\, dv = -\frac{\partial Q_i}{\partial t}$$

$$\Rightarrow \; \& \; Q_i = \int_{Vol} \rho_v \, dv$$

$$\therefore \int_{Vol} (\bar{\nabla} \cdot \bar{J})\, dv = -\frac{\partial}{\partial t} \int_{Vol} \rho_v \cdot dv$$

$$(\bar{\nabla} \cdot \bar{J})\, \Delta v = -\frac{\partial \rho_v}{\partial t} \cdot \Delta v$$

$$\therefore \boxed{\bar{\nabla} \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}}$$

⇒ "Current diverging from small volume per unit volume = time rate of decrease of charge per unit volume at ~~every~~ point."

$$J \propto \frac{1}{r}$$

$$\rho_v \propto \frac{1}{r^2}$$

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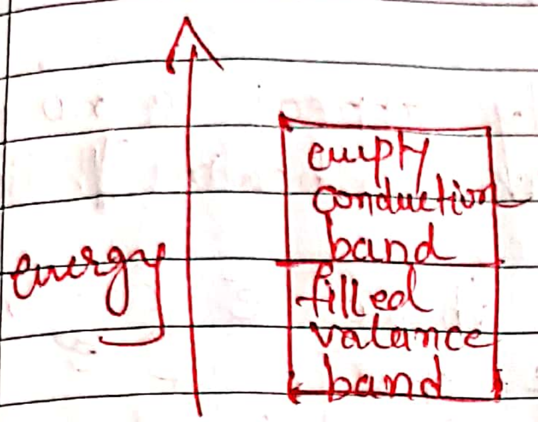
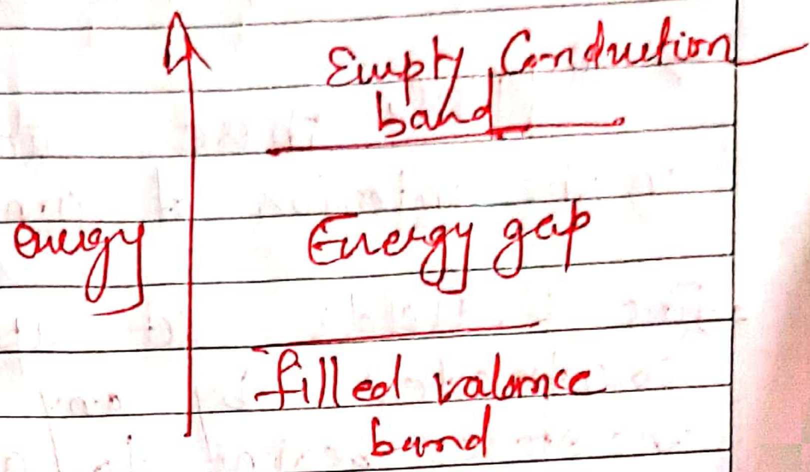
Mettalic Conductor



Insulator



⇒ free $\bar{e}$ move under the influence of $\bar{E}$ field.

With field $\bar{E}$, electron having charge $q = -e$ exp a force

$$\bar{F} = +q\bar{E} = -e\bar{E}$$

⇒ drift velocity of $\bar{e}$

$$v_d = -\mu_e \bar{E}$$ where μ_e – mobility of $\bar{e}$

$\therefore J = \rho_v \cdot (-\mu_e \bar{E})$ & ρ_e = charge density of $\bar{e}$

$$\therefore \bar{J} = -\rho_e \mu_e \bar{E}$$

$\therefore \boxed{\bar{J} = \sigma \bar{E}}$ → σ : conductivity $= -\rho_e \mu_e$

Conductor Properties:

→ Suppose, there suddenly appear a no of $\bar{e}$ in the interior of conductor. (heating / battery)

→ The E. fields set up by these $\bar{e}$ are not counteracted by any +ve charge & $\bar{e}$ therefore begins to accelerate from each other

→ This continues until the $\bar{e}$ reach the surface of conductor ~~or until an~~ (as the surrounding material is an insulator)

Properties

(1) The volume charge density within the conductor is zero. ($\rho_v = 0$)

(2) The surface charge density ρ_s resides on the exterior surface of the conductor. ($\rho_s \neq 0$)

(3) For electrostatics, no charge & no E-field may exist within a conducting material ($E_i = 0$)

(4) The conductor is an equipotential body.

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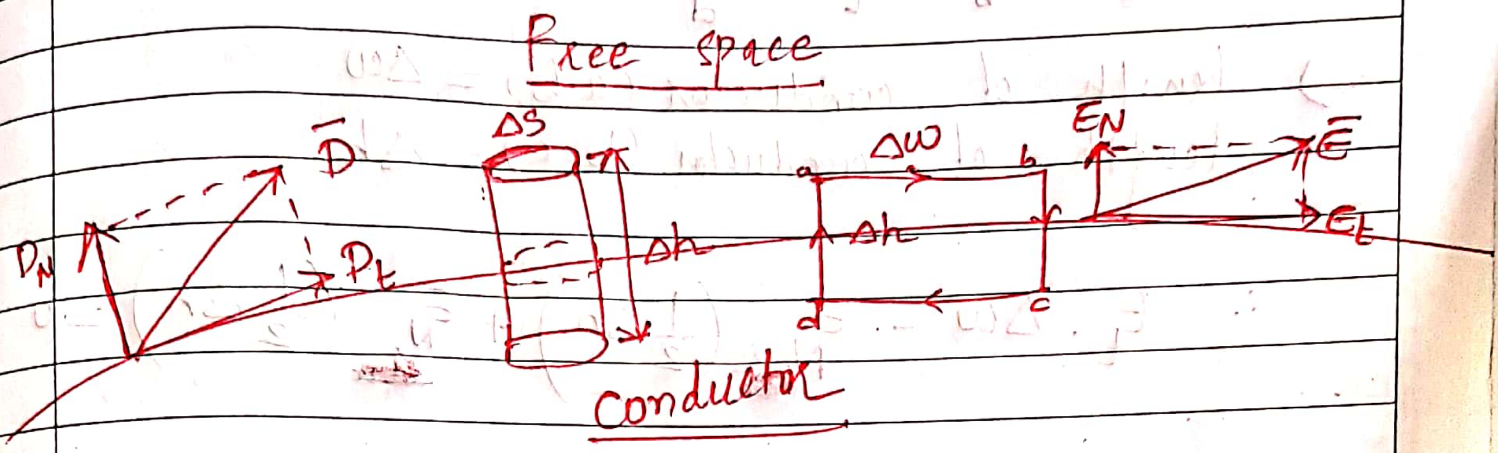
* Boundary condition at the interface of conductors & free space:



⇒ External E-field can be decomposed into 2 comp: tangential comp.
normal comp.

* Apply Gauss's law to find normal comp. of E-field
* Apply definition of pot. diff. to find tangential comp.

⇒ $\bar{D}_i$ & $\bar{E}_i = 0$

① To find tangential field;

$$\oint \bar{E}\cdot d\bar{L} = 0 \quad (\text{KVL})$$

around small closed path a-b-c-d.

$$\therefore \int_a^b + \int_b^c + \int_c^d + \int_d^a = 0$$

$\Rightarrow$ length of conductor (a-b) = $\Delta\omega$
length of conductor (b-c) = Δh

$$\therefore E_t \cdot \Delta\omega - E_N \left(\frac{1}{2}\Delta h\right) + E_N \left(\frac{1}{2}\Delta h\right) = 0$$

$$\therefore \boxed{E_t = 0}$$

$\therefore$ Tangential comp. of external E. field is always zero. $\Rightarrow$ conductor is an equipotential surface.

$$\therefore \boxed{D_t = 0}$$

② To find normal comp. of field (E_N & D_N);
Apply Gauss's law

$$\oint \bar{D} \cdot \bar{ds} = Q$$

on a small cylinder (height = Δh & area of top & bottom faces = Δs)

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$$\therefore \int_{top} + \int_{bottom} + \int_{sides} = Q$$

$$\therefore \int_{top} \bar{D} \cdot \bar{ds} + \int_{bottom} \bar{D} \cdot \bar{ds} + \int_{sides} \bar{D} \cdot \bar{ds} = Q$$

$$\therefore \int_{top} \bar{D} \cdot \bar{ds} + 0 + D_t \cdot \left(\frac{\Delta S}{2}\right) + D_t \left(\frac{\Delta S}{2}\right) = Q$$

$$(\because D_t = 0)$$

$$\therefore D_N \cdot \Delta S = Q = \rho_s \cdot \Delta S$$

$$\therefore \boxed{D_N = \rho_s}$$

$$\therefore \boxed{E_N = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}}$$

"Electric flux density (C/m^2) leaving surface normally is equal to the surface charge density (C/m^2) i.e. $D_N = \rho_s$."

$\Rightarrow$ "Electric field must approach a conducting surface normally."

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* Dielectric Materials:

→ It does not contain free charges & so it does not constitute conduction current.

→ It consists of bound charges which are bound in place by atomic & moleculer forces and can only shift positions slightly in response to external fi

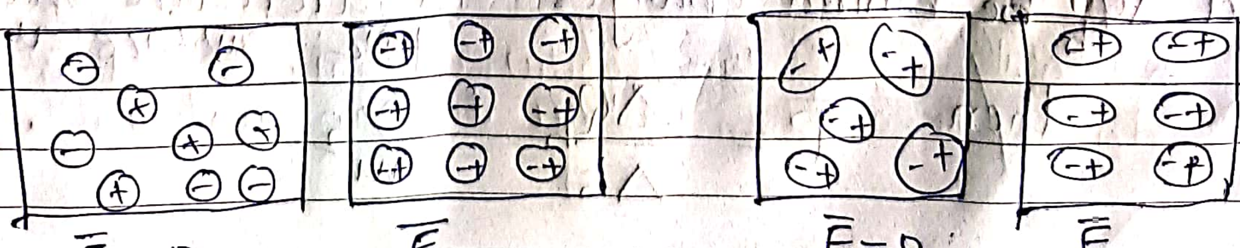
→ The bound charges are also the source of electrostatic field.

→ The charecteristic of dielectric material is to store electric energy. This storage takes place by means of shift in relative positions of internal bound +ve & −ve charges against normal atomic forces.

2 types of dielectric materials:

(1) Non-polar: does not posses permenant dipole moments.

(2) Polar: posses permanent dipole moments.



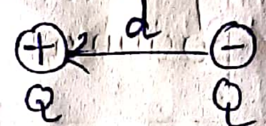
$\bar{E} = 0$ $\bar{E}$ $\bar{E} = 0$ $\bar{E}$

(Non-polar) (Polar)

⇒ A dielectric material in an Electric field can be viewed as a free space arrangement of microscopic electric dipoles which are composed of +ve & −ve bound charges.

⇒ Either type of dipole is described by its dipole moment

$$p = Qd$$



where Q = +ve bound charge composing dipole
& d = vector from −ve to +ve charge

⇒ If there are n dipoles per unit volume & if we consider volume Δv, total dipole moment

$$p_{total} = \sum_{i=1}^{n\Delta v} p_i$$

⇒ Polarization is defined as dipole moment per unit volume

$$\therefore \boxed{\bar{P} = \lim_{\Delta v \to 0} \frac{\sum_{i=1}^{n\Delta v} p_i}{\Delta v}} \quad C/m^2$$

Let ΔQ_b be bound charges in dielectric material per surface area of ΔS

then $\bar{P} = \dfrac{\Delta Q_b}{\Delta S}$ C/m^2

⇒ Due to applied electric field $\bar{E}$, electric ~~polar~~ dipole moment $p = Qd$ occurs in each molecule of dielectric material.

Due to total dipole moment per unit volume, dipole gets polarized by $\bar{P}$ in C/m^2.

⇒ No of bound charges within closed surface is equivalent to,

$$\boxed{Q_b = -\oint \bar{P} \cdot \bar{ds}} \quad \text{——— (1)}$$

⇒ Let consider other than free space lake one dielectric material:

which may have Total charge consisting of free & bound charges.

$$\boxed{Q_T = Q + Q_b} \quad \text{——— (2)}$$

where Q = free charges enclosed by surface
Q_b = bound charges enclosed by surface

& from Gauss's law ; $\psi = Q_T$

$$\therefore \boxed{Q_T = \oint_S (\varepsilon_0 \bar{E}) \cdot \bar{ds}} \quad \text{——— (3)}$$

Using eq^n ①, ② & ③

$$Q = Q_T - Q_b$$

$$\therefore Q = \oint (\varepsilon_0 \bar{E}) \cdot \bar{ds} - \oint (-\bar{P}) \cdot \bar{ds}$$

$$\therefore \oint \bar{D} \cdot \bar{ds} = \oint (\varepsilon_0 \bar{E} + \bar{P}) \cdot \bar{ds} \quad \text{(Gauss law)}$$

$$\therefore \boxed{\bar{D} = \varepsilon_0 \bar{E} + \bar{P}}$$ for a polarizable material.

⇒ Relating charges with charge densities,

using divergence theorem

$$Q_T = \int_v \rho_T \, dv \Rightarrow \boxed{\bar{\nabla} \cdot (\varepsilon_0 \bar{E}) = \rho_T}$$

$$Q_b = \int_v \rho_b \, dv \Rightarrow \boxed{\bar{\nabla} \cdot \bar{P} = -\rho_b} \quad — ④$$

$$\& \; Q = \int_v \rho_v \, dv \Rightarrow \boxed{\bar{\nabla} \cdot \bar{D} = \rho_v} \quad — ⑤$$

⇒ Eq^n ⑤ shows free volume charge density is the source of Electric fields / Elect flux densitie. Simillarly eq^n ④ shows bound volume charge density causes polarization in material.

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Imp ques → For isotropic material, there is a linear relationship between $\overline{P}$ & $\overline{E}$

$$\overline{P} = \chi_e \varepsilon_0 \overline{E}$$

where χ_e = electrical susceptibility of material.

$$\therefore \overline{D} = \varepsilon_0 \overline{E} + \chi_e \varepsilon_0 \overline{E}$$

$$= (1+\chi_e) \varepsilon_0 \overline{E}$$

$D = \varepsilon_0 \varepsilon_r \overline{E}$ where $\varepsilon_r = 1+\chi_e$

$\therefore \boxed{\overline{D} = \varepsilon \overline{E}}$ where $\varepsilon = \varepsilon_0 \varepsilon_r$

where ε_r = relative permittivity or dielectric constant of material.

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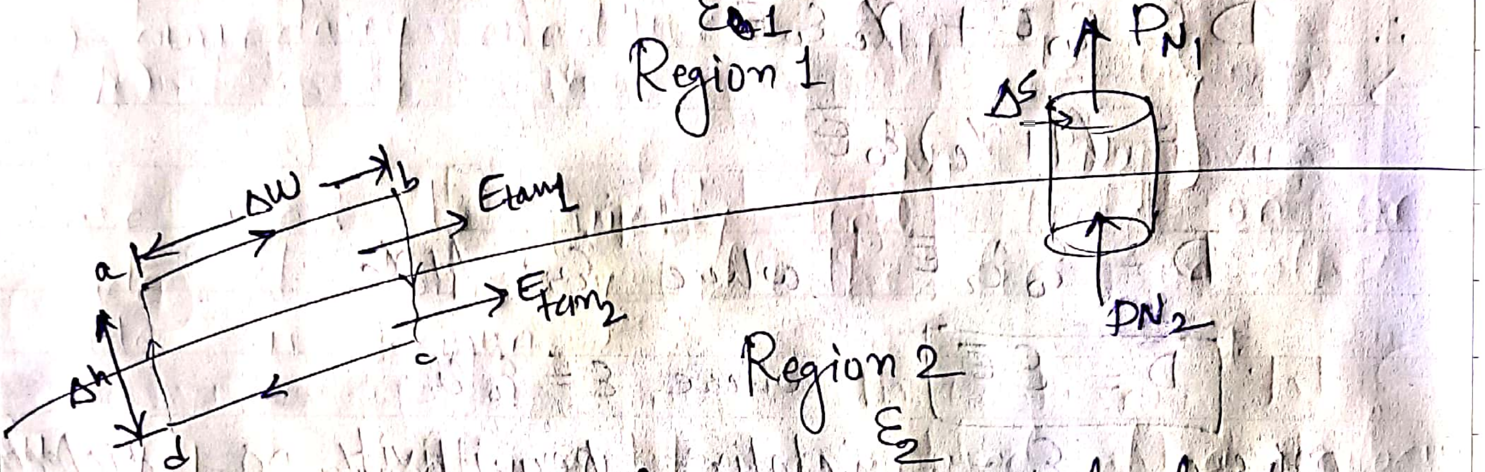
* Boundary conditions for perfect dielectric material

let consider the interface between 2 dielectrics having permittivities ε_1 & ε_2 & occupying regions 1 & 2 respectively.



→ To find tangential components of fields:

$$\oint \bar{E} \cdot \bar{dl} = 0$$ (KVL around small closed path a-b-c-d-a)

$$\therefore \int_a^b + \int_b^c + \int_c^d + \int_d^a = 0$$

$$\therefore E_{tan_1} \cdot \Delta w - E_{tan_2} \Delta w + E_{N_1} \Delta h - E_{N_2} \Delta h = 0$$

$$\therefore \boxed{E_{tan_1} = E_{tan_2}} \quad (\because \Delta h \to 0) \text{ at interface}$$

$\therefore$ Tangential Electric field intensity is continuous across the boundary.

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$$\therefore \frac{D_{tan_1}}{\varepsilon_1} = \frac{D_{tan_2}}{\varepsilon_2}$$

$$\therefore \boxed{\frac{D_{tan_1}}{D_{tan_2}} = \frac{\varepsilon_1}{\varepsilon_2}}$$ ⇒ tangential comp. of Electric flux density is discontinuous.

⇒ To find Normal components of field

Apply Gauss law for small cylinder at i/f;

$$\oint \bar{D} \cdot \bar{ds} = Q$$

$$\therefore \int_{top} + \int_{bottom} + \int_{sides} = Q$$

$$\therefore D_{N_1} \cdot \Delta s - D_{N_2} \cdot \Delta s = \Delta Q \quad \left(\because \text{sides are short} \; \Delta h \to 0 \right)$$

$$\therefore (D_{N_1} - D_{N_2}) \cdot \Delta s = \rho_s \cdot \Delta s$$

$$\therefore \boxed{D_{N_1} - D_{N_2} = \rho_s}$$

but in dielectric materials, no free charges are available so $\rho_s = 0$

$$\therefore \boxed{D_{N_1} = D_{N_2}}$$

⇒ Normal comp. of $\bar{D}$ is continuous.

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$$\because \quad \epsilon_1 E_{N_1} = \epsilon_2 E_{N_2}$$

$$\boxed{\frac{E_{N_1}}{E_{N_2}} = \frac{\epsilon_2}{\epsilon_1}}$$ & normal $\bar{E}$ is discontinuous.

$\Rightarrow$ boundary conditions: $\left.\begin{array}{l} E_{t_1} = E_{t_2} \\ \& \ D_{N_1} = D_{N_2} \end{array}\right\}$

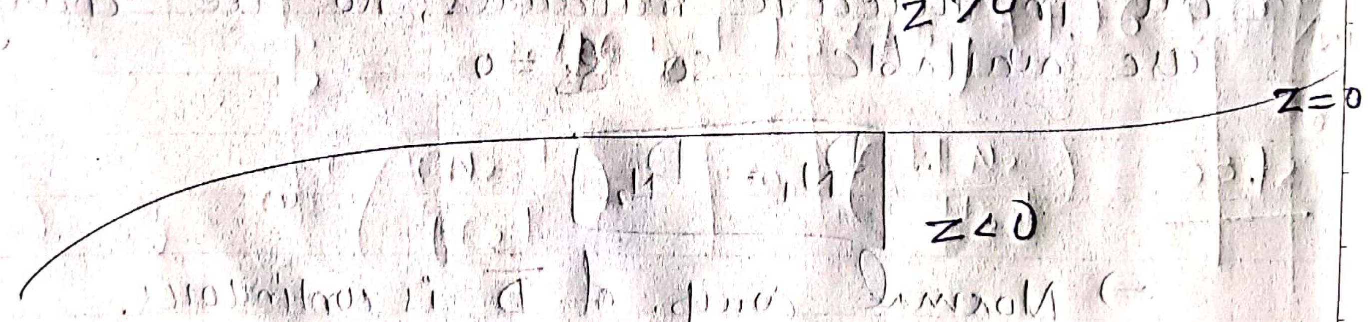
are used to quickly find field on one side of boundary if the field on other side is known.

Que. Let region $z<0$ be composed of a uniform dielectric material ($\epsilon_r = 2.5$) while region $z>0$ is char. by $\epsilon_r = 4$ & $\chi_{e_2} = 3$.

Let $D_1 = -30\,a_x + 50\,a_y + 70\,a_z \ nC/m^2$ find out

(i) D_{N_1} (ii) $\bar{D}_{t_1}$ (iii) D_{t_1} (iv) D_1 (v) θ_1 & (vi) $\bar{P}_1$

(vii) $\bar{D}_2$ (ix) $\bar{P}_2$ (x) θ_2



* Derivation of Poisson's & Laplace's Equations:

→ From point form of Gauss's law

$$\bar{\nabla}\cdot\bar{D} = \rho_v$$

& definition of $\bar{D} = \varepsilon\bar{E}$

& gradient relationship $\bar{E} = -\bar{\nabla}V$

$$\Rightarrow \bar{\nabla}\cdot(\varepsilon\bar{E}) = \rho_v$$

$$\therefore \bar{\nabla}\cdot(\varepsilon\, -\bar{\nabla}V) = \rho_v$$

$$\boxed{\therefore \bar{\nabla}\cdot\bar{\nabla}V = -\frac{\rho_v}{\varepsilon}} \rightarrow \text{Poisson's eq}^n$$

$$\Rightarrow \nabla\cdot\bar{\nabla}V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$\therefore \bar{\nabla}^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\therefore \boxed{\bar{\nabla}^2 V = -\frac{\rho_v}{\varepsilon}}$$

→ If $\rho_v = 0$ → zero volume charge density but allowing pt, line & sheet charge density at singular loc^n as source of field then:

$$\boxed{\bar{\nabla}^2 V = 0} \rightarrow \text{Laplacian's eq}^n$$

$$\boxed{\therefore \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0} \rightarrow \text{cartesian}$$

→ Laplace's & poisson's eq^{n}s are useful for solving many practical electrostatic field problems, where only electrostatic condition (potential / charge) at some boundaries are known. Solution of E-field & potential can be found throughout volume.

& $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$ → cylindrical

& $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right)$

$+ \frac{1}{r^2 \sin^2\theta} \cdot \frac{\partial^2 V}{\partial \phi^2}$ → spherical

Uniqueness Theorem.

⇒ let there be 2 solutions of Laplace's eq^ns (V_1 & V_2).

$\therefore \nabla^2 V_1 = 0$

& $\nabla^2 V_2 = 0$

$\therefore \nabla^2 (V_1 - V_2) = 0$

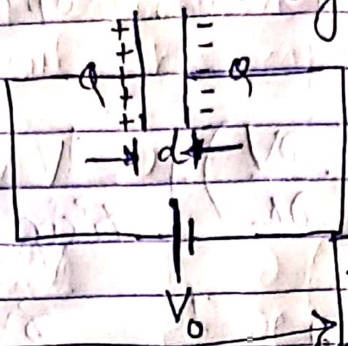
$\Rightarrow (V_1 - V_2) = \text{constant}$

$\therefore V_1 = V_2$

"For a given boundary condition, there can be only one solution of Laplace's eq^n"

⇒ There can not be 2 different solutions of Laplace's eq^ns & satisfy same boundary condition. sol^n of laplace's eq^n is determined uniquely if its value is known on all boundaries of the region.

Capacitance = $\frac{\text{total charge on either conductor}}{\text{magnitude of pot. diff.}}$



$$\therefore \boxed{C = \frac{Q}{V_0}}$$

Solve laplace's eq^n in 1-D. Let V_0 is pot. diff. bet^n 2 plates of capacitor & Determine capacitance.

Que 1-D potential field: $\frac{\partial V}{\partial x^2} = 0$

$$\therefore \frac{dV}{dx} = A$$

$$\therefore \boxed{V = Ax + B}$$

Where A & B can be calculated from boundary condition.

$\Rightarrow$ for a capacitor, $x = d \rightarrow V = V_0$
$x = 0 \rightarrow V = 0$

$$\therefore V_0 = A(d) + B \Rightarrow A = V_0/d$$

$$\& \; 0 = A(0) + B \Rightarrow B = 0$$

$$\therefore \boxed{V = \frac{V_0 x}{d}}$$ sol^n to laplace's eq^n

$$\Rightarrow \bar{E} = -\bar{\nabla} \cdot V = -\frac{\partial}{\partial x}\left(\frac{V_0 x}{d}\right)\bar{a_x} = -\frac{V_0}{d}\bar{a_x}$$

$$\therefore \bar{D} = \varepsilon \bar{E} = -\frac{\varepsilon V_0}{d}\bar{a_x}$$

$$\& \quad Q = \oint \bar{D} \cdot \bar{ds} = \int_S \left(-\frac{\varepsilon V_0}{d}\right) ds$$

$$\therefore Q = \frac{-\varepsilon_0 V_0 \cdot S}{d}$$

$$\therefore \boxed{C = \frac{|Q|}{V_0} = \frac{\varepsilon_0 S}{d}}$$

Q. Solve laplace's eqn in cylindrical coordinate system. Let potential varies w.r.t. to ρ only. Find out the capacitance of co-axial cable.

Ans

$$\nabla^2 V = 0 \quad \therefore \nabla^2 V_\rho = 0$$

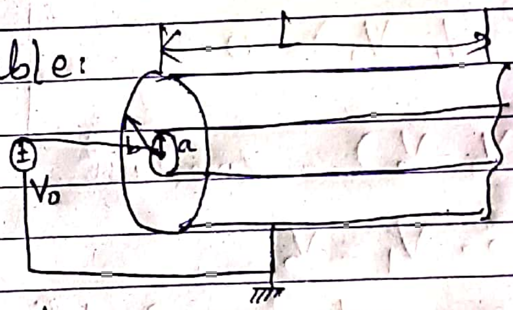
$$\therefore \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$$

$$\therefore \rho \frac{\partial V}{\partial \rho} = A$$

$$\therefore \frac{\partial V}{\partial \rho} = \frac{A}{\rho}$$

$$\therefore \boxed{V = A \ln \rho + B}$$

⇒ Co-axial cable:



Let $V = V_0$ at $\rho = a$
$V = 0$ at $\rho = b$ $\quad (b > a)$

$$\therefore V = \frac{V_0 \ln (b/\rho)}{\ln (b/a)}$$

$$\Rightarrow \bar{E} = -\nabla \cdot V = \frac{V_0}{\rho} \cdot \frac{1}{\ln (b/a)} \bar{a}_\rho$$

$$\Rightarrow D_N \Big|_{\rho = a} = \frac{\varepsilon V_0}{a} \cdot \frac{1}{\ln (b/a)}$$

$$\therefore \quad Q = \oint_S \bar{D} \cdot \bar{ds} = \frac{\varepsilon V_0}{a} \cdot \frac{2\pi a L}{\ln(b/a)} \quad \left(\because ds = 2\pi a \times L \right)$$

$$\boxed{C = \frac{Q}{V_0} = \frac{2\pi \varepsilon L}{\ln(b/a)}}$$