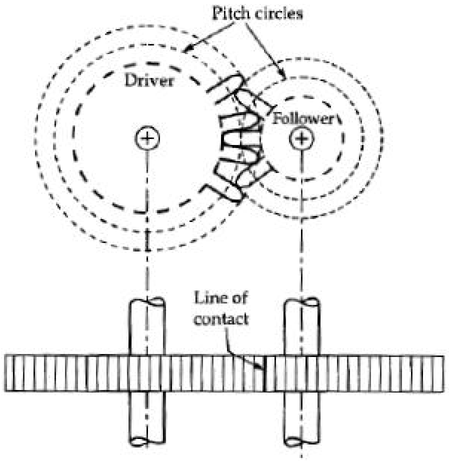
Design of Spur Gear

Introduction

- Any toothed member designed to transmit motion to another one, or receive motion from it, by means of successively engaging tooth is called a (toothed) gear.
- A gear is a toothed wheel with teeth cut on the periphery of a cylinder or a cone.
- Teeth of one gear meshes with teeth of the other gear, hence it is called a mesh drive or positive drive.
- It is preferred when medium or larger power is to be transmitted.
- Rotation of one gear will cause rotation of the other in the opposite direction the bigger one the gear wheel.



ADVANTAGES OF GEARS

- Compact drive on account of small centre distance.
- High efficiency, reliable service and simple operation.
- Positive drive due to negligible slip between contacting surfaces.
- Give higher speed ratio and can transmit higher power.
- It is possible to transmit power between parallel, non-parallel, intersecting, and nonintersecting shafts.
- Power can be transmitted at higher speeds.

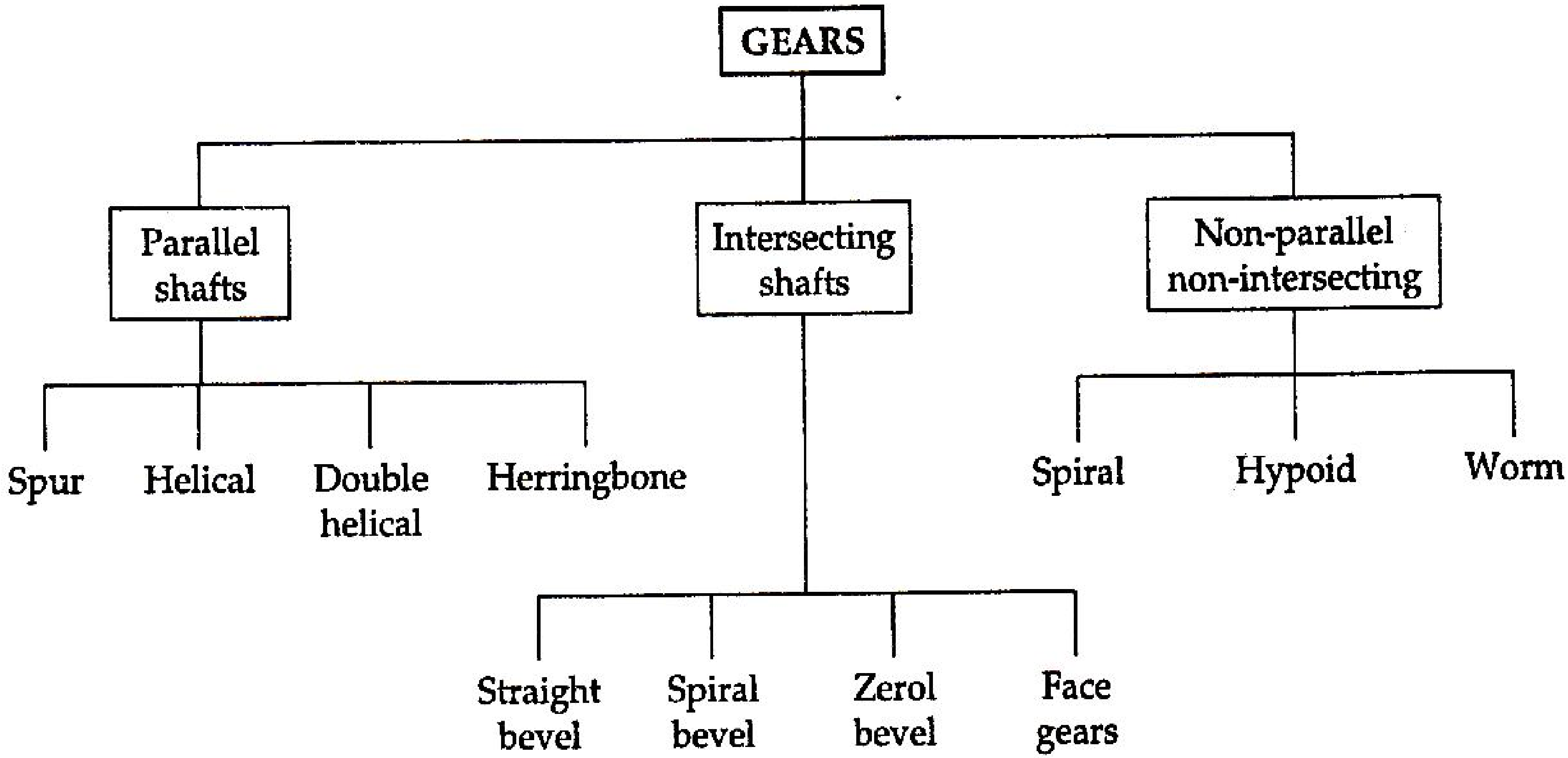
LIMITATIONS OF GEARS

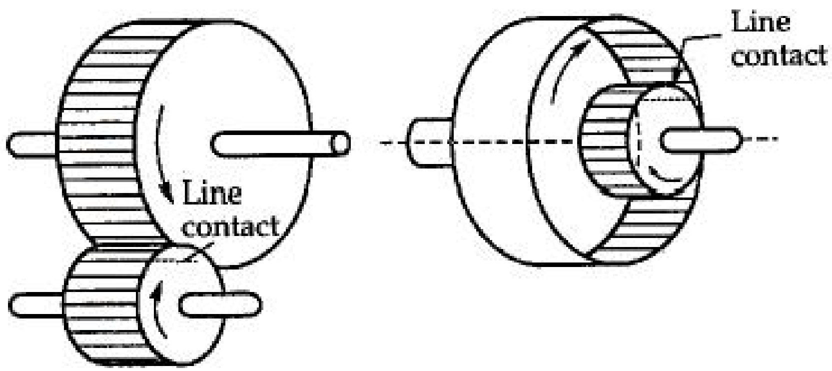
- The error in tooth meshing may cause undesirable vibrations and noise during operation.
- Costlier than belts and chain drives.
- Power cannot be transmitted over long distances.
- Precise alignment of shafts is required.
- Require continuous lubrication.

APPLICATIONS OF GEARS

- Metal cutting machine tools
- Automobiles
- Tractors
- Hoisting and transporting machinery
- Rolling mills
- Marine engines, etc.

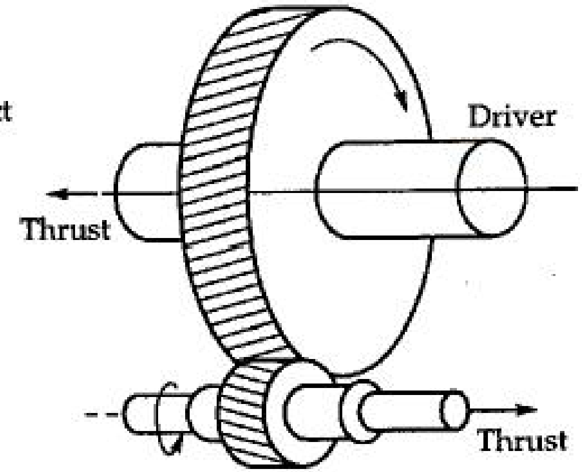
CLASSIFICATION OF GEARS



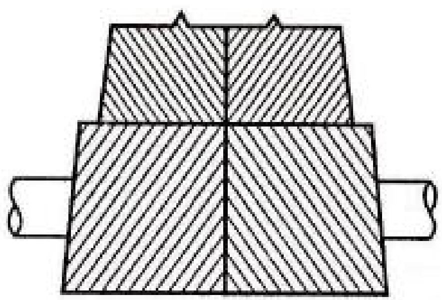


(a) External (b) Internal

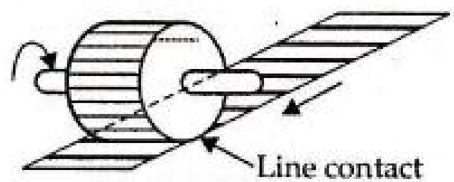
(i) Spur Gears



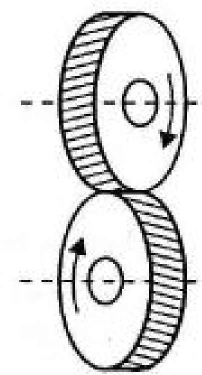
(ii) Helical Gears



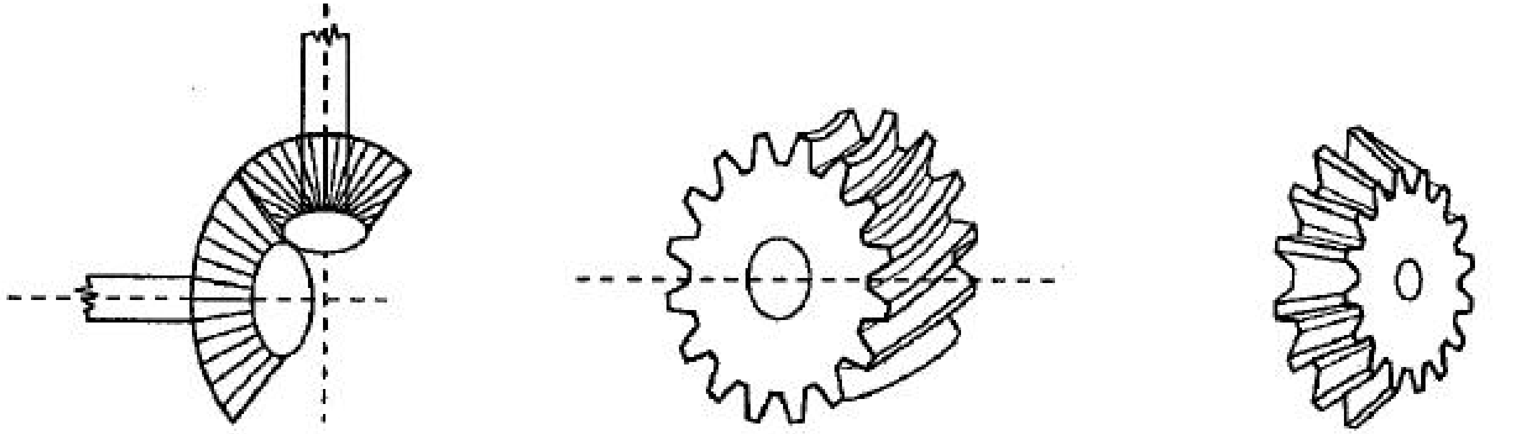
(iii) Double Helical (Herringbone) Gears



(iv) Spur Rack and Pinion

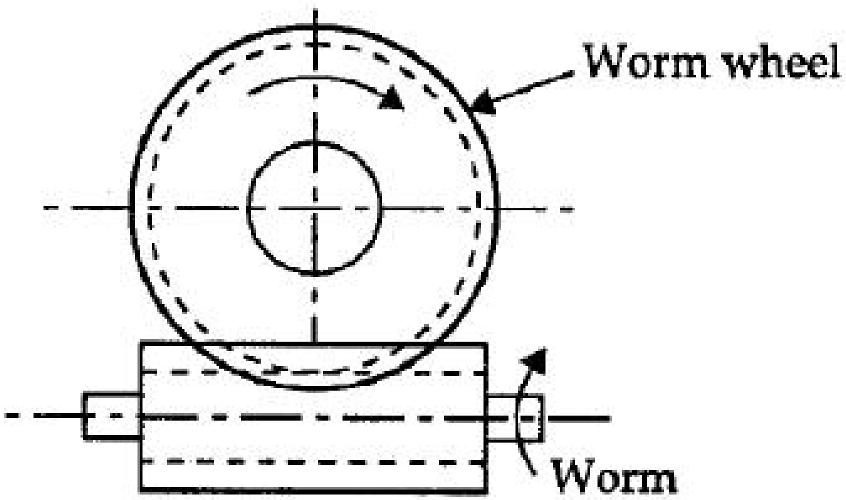


(v) Crossed Helical Gears

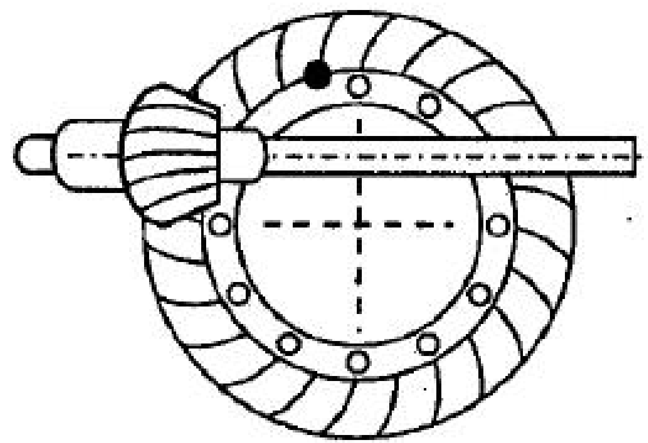


(a) Bevel Gears (b) Spiral Bevel Gears (c) Zerol Bevel Gears

(vi) Bevel Gears



(vi) Worm Gears

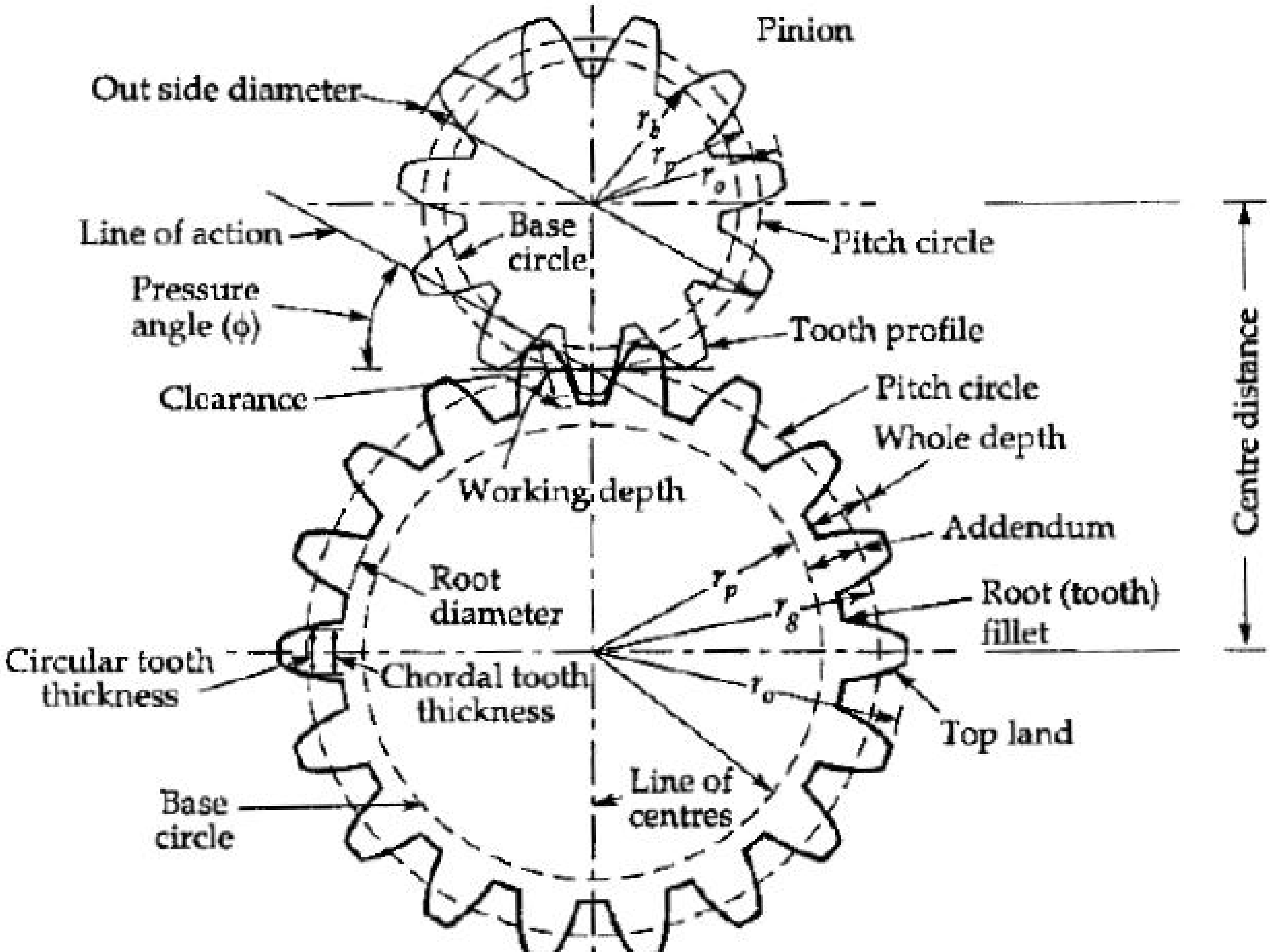


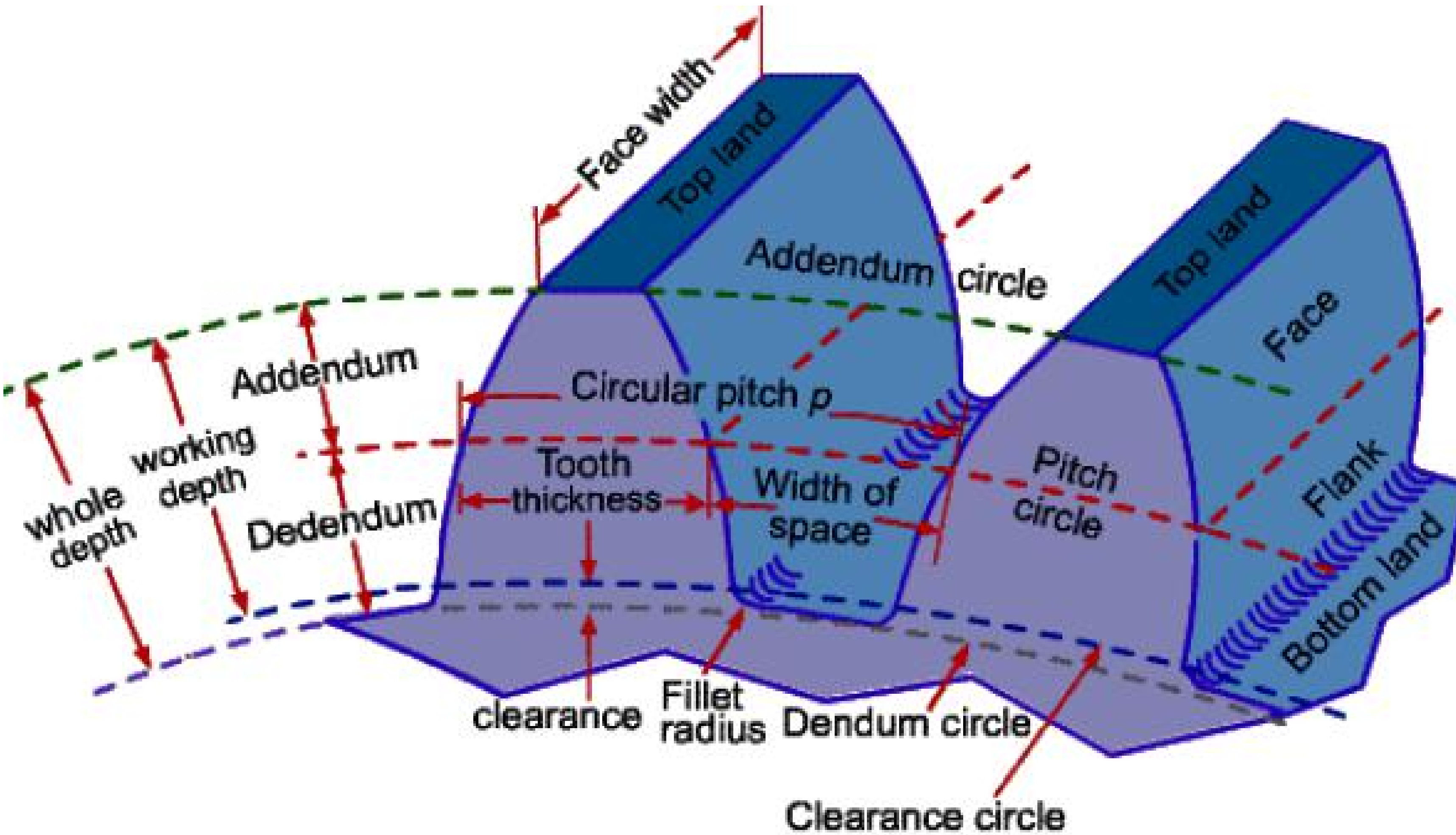
(viii) Hypoid Gears

GEAR MATERIALS

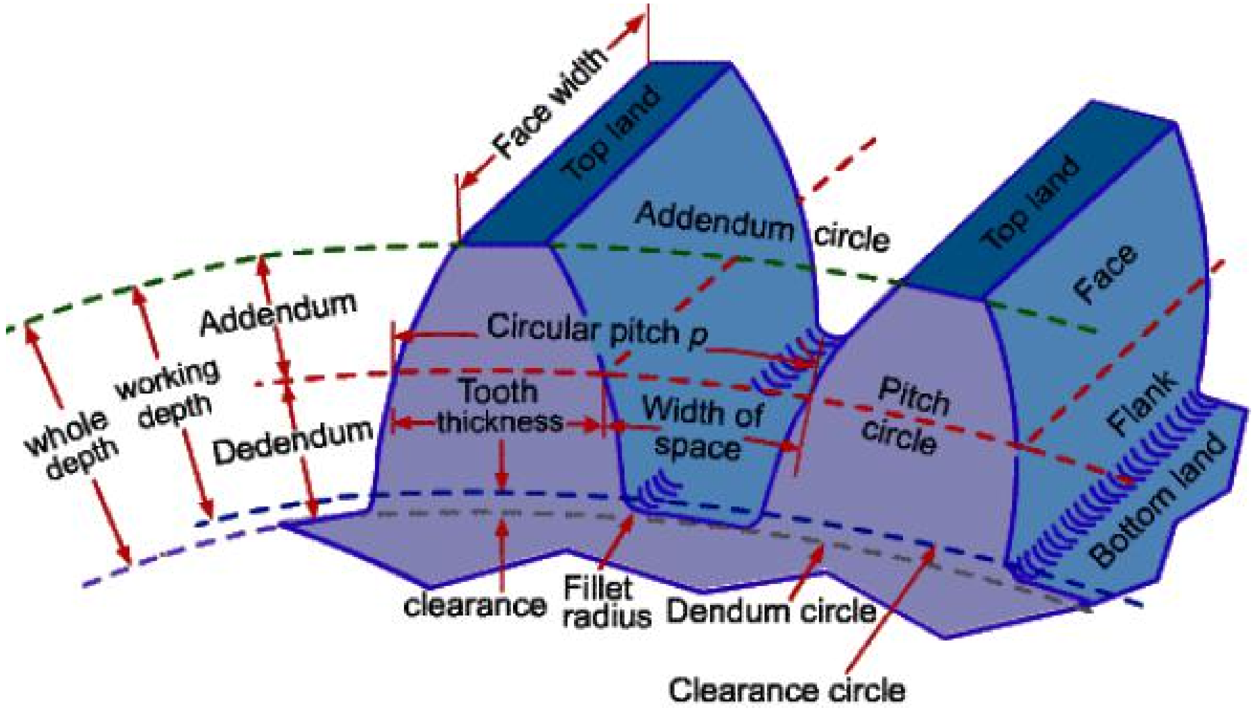
- The gear material should have sufficient static strength (ultimate or yield) and endurance strength against fluctuating loads to resist failure due to the breakage of tooth.
- It should have sufficient surface endurance strength to avoid failure due to destructive pitting caused by excessive wear.
- The material should have a low coefficient of friction to avoid failure due to scoring caused by high sliding velocities during high speed power transmission.
- The coefficient of thermal expansion of material should be low to limit the thermal stresses causing distortion and warping.

SPUR GEAR TERMINOLOGY

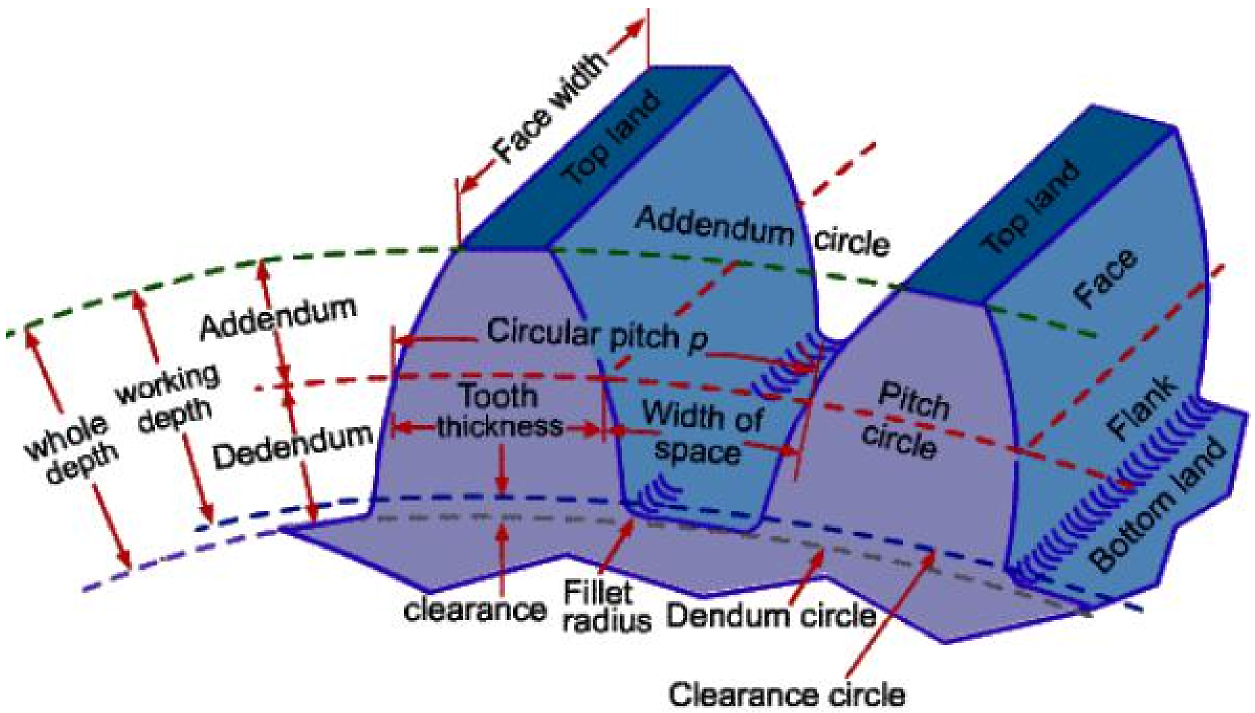



Face width
Top land
Addendum circle
Top land
Face
Addendum
Circular pitch p
working depth
whole depth
Tooth thickness
Width of space
Pitch circle
Flank
Dedendum
Bottom land
clearance
Fillet radius
Dendum circle
Clearance circle

- **Circular pitch (p):** The distance on the pitch circle from a point on a tooth to the corresponding point of the adjacent tooth.
- **Diametral pitch (P):** It is defined as the number of teeth of the gear divided by the pitch circle diameter.
- **Module (m):** It is defined as the pitch circle diameter per unit number of teeth
- **Pressure angle (ϕ):** The angle between the line of action (a line through the pitch point and tangential to the base circles) and a line perpendicular to the line of centers at the pitch point is known as pressure angle.



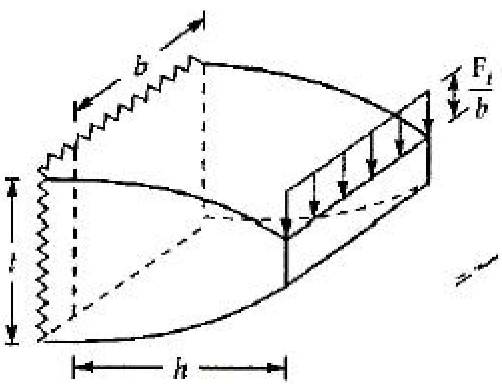
- **Centre distance (C):** The distance between the centers of the two gears in mesh is known as centre distance.
- **Backlash:** The difference between tooth space and tooth width is known as backlash.
- **Clearance:** The difference between the dedendum of one gear and the addendum of the mating gear is known as clearance.
- **Fundamental Law of Gearing:** This law may be stated as "The shape of the teeth of a gear must be such that the common normal at the point of contact between two teeth must always pass through a fixed point on the line of centers”.

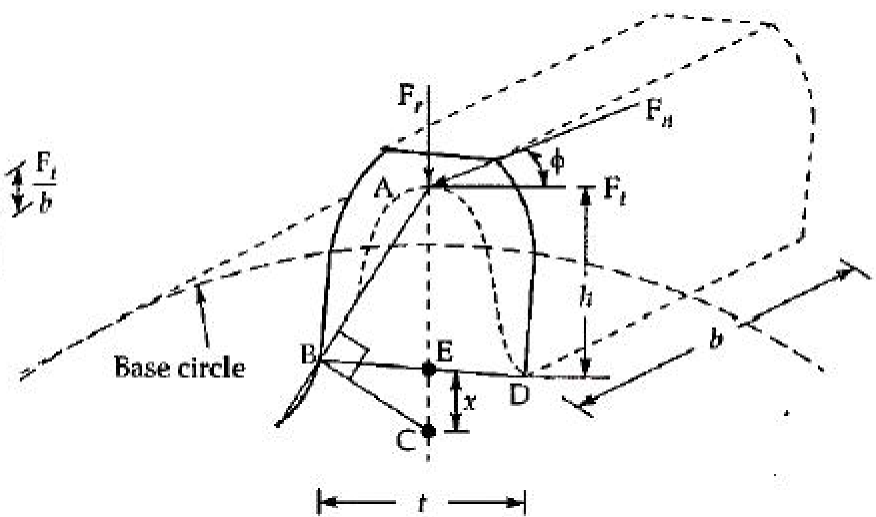


- **Gear ratio (i):** The ratio of the number of teeth of the wheel (gear) to that of the pinion is called gear ratio.
- **Transmission ratio (i):** The ratio of the angular speed of the first driving gear of a train of gears to that of the last driven gear is called transmission ratio.
- **Cycloid:** A plane curve described by a point on a circle (generating circle), which rolls without slip on a fixed line (base line) is known as cycloid.
- **Involute:** A plane curve described by a point on a straight line which rolls without slip on a fixed circle is known as involute.
- **Face width:** The width over the toothed part of a gear, measured along a straight line generator of the reference cylinder is known as face width.
- **Base circle:** In an involute cylindrical gear, the base circle of the involutes of the tooth profiles is known as base circle.

BEAM STRENGTH OF SPUR GEAR TEETH
(Design for Static Load)

- First, the gears must operate together without tooth interference, with a proper length of contact and without undue noise.
- Second, the gear teeth must have the ability to transmit the applied loads without failure and with a certain margin of safety. This involves the ability of the teeth to resist not only the load resulting from the power transmitted but also the increases in load due to impact and shock caused by inaccuracy of tooth contour, tooth deflection tooth acceleration and stress-concentration at the root of the tooth or fatigue strength. The total resulting load is commonly referred to as the dynamic load.
- Third, the wearing qualities of teeth must be considered. This is known as the wear load.





Determination of Lewis equation

- The static strength of the tooth is determined by assuming the tooth to be a cantilever beam (Fig. (a)) acted upon by the moment resulting from the transmitted load obtained from the power transmitted.
- The design stress is based upon the ultimate strength of the material with a factor of safety of about 3.
- This analysis was given by Wilfred Lewis in 1892 and the design equation is known as the Lewis Equation.
- In order to take into account the effects of tooth fabrication and additional loads due to impact, the design is further modified by a velocity factor.
- These modifications in design were presented by Earle Buckingham in 1932, after which the gear design has been based upon the dynamic load and the endurance limit of the material and the wear load.

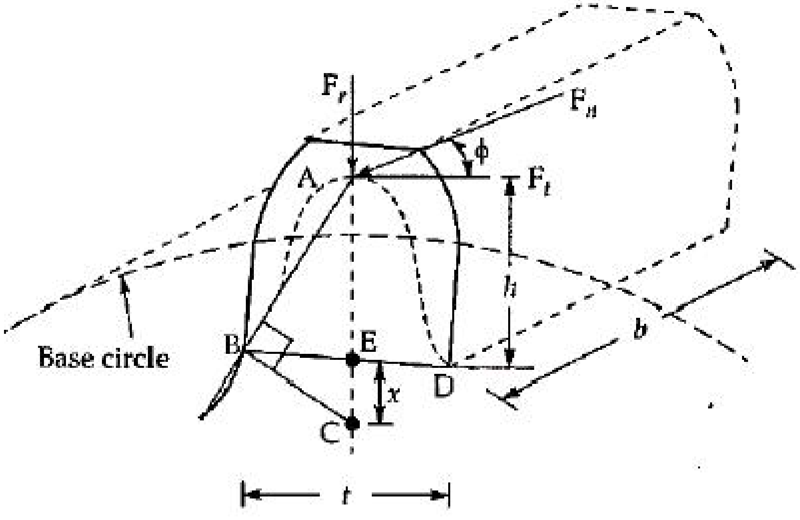
The Lewis equation is based on the following assumptions

1. The gear tooth is treated as a cantilever beam.
2. The effect of the radial component, which induces compressive stresses, is neglected.
3. It is assumed that the tangential component is uniformly distributed over the face width of the gear. This is possible when the gears are rigid and accurately machined.
4. The effect of stress concentration is neglected.
5. It is assumed that at any time, only one pair of teeth is in contact and takes the total load.

- Fig. (b) shows a gear tooth with the force acting at the tip of the tooth. The normal force Fn, is resolved into its components Fr and Ft acting at point A, the intersection of the line of action of the normal tooth load and the centre of the tooth.
- The radial component Fr produces compressive stress in the tooth and the tangential component Ft causes bending stresses. The direct compressive stress is small enough as compared to the bending stress and is ignored in determining the strength of the tooth.
- The maximum bending stress may be located and computed as follows:
- Through the point A in Fig. (b), draw a parabola (shown in dash line) tangent to the tooth curves at B and D. This parabola represents the outline of a beam of uniform strength, and therefore the maximum stress in the actual tooth will be the point of tangency B or D. This stress is:

$$\sigma_b = \frac{M\,c}{I} = \frac{6\,F_t\,h}{bt^2}$$

$$\therefore F_t = \frac{\sigma_b b t^2}{6\,h}$$



Both t and h are based upon the size of the tooth and its profile; hence the equation.

$$h = \left(\frac{\sigma_b\,b}{6\,F_t}\right) t^2 = \text{constant} \times t^2$$

Clearly this is the equation of a parabola. Triangles ABE and BCE are similar, thus

$$\frac{x}{(t/2)} = \frac{(t/2)}{h} \quad or \quad h = \frac{t^2}{4x}$$

$$\therefore F_t = \frac{\sigma_b b t^2}{6t^2} \cdot 4x = \sigma_b b \frac{4x}{6}$$

If we define a factor $y = 2x/3p$, called the Lewis form factor, based on circular pitch then, we get

$$F_t = \sigma_b\, b\, y\, p = \frac{\sigma_b\, b\, Y}{P} = \sigma_b\, b\, Y\, m = \sigma_b\, b\, \pi\, y\, m$$

Where, y = Lewis form factor based upon circular pitch

Y = Lewis form factor based upon diametral pitch

b =face width, mm = 3 π m to 4 π m

p = circular pitch, mm

σ_b = Permissible bending stress, N/mm^2

The permissible bending stress in the Lewis equation is taken as ⅓ of the ultimate tensile strength

$$\therefore \sigma_b = \frac{1}{3}\sigma_{ut}$$

The values of y may be obtained from the following relations:

$$y = 0.124 - \frac{0.684}{z}$$, for 14.5° involute

$$y = 0.154 - \frac{0.912}{z}$$, for 20° involute full depth (FD)

$$y = 0.170 - \frac{0.95}{z}$$, for 20° involute stub

Velocity Factor

- Slight inaccuracies in profile and tooth spacing both, teeth being not absolutely rigid, variations in the applied load and repetitions of the loading cause impact and fatigue stresses that become more severe as the pitch line velocity increase. To allow for these additional stresses, a velocity factor C*v is introduced into* the Lewis equation. This factor is given by:

$$C_v = \frac{3.05}{3.05 + v_m}$$, for ordinary industrial gears operating at velocity upto 10 m/s

$$C_v = \frac{6.1}{6.1 + v_m}$$, for accurately cut gears operating at velocity upto 20 m/s

$$C_v = \frac{5.56}{5.56 + v_m}$$, for precision gears cut with a high degree of accuracy and operating at velocity of 20 m/s and over

where v_m is the mean speed in m/s.

Tangential Load on Gear Tooth

- The tangential load acting on the gear tooth is the load perpendicular to the pitch circle radius.

$$F_t = 10^3 \times \frac{P}{v}, \quad N$$

Service Factor, Cs

The service factor accounts for increase in the tangential force due to fluctuation of the torque developed by the prime mover and the torque required to run the machine. It depends upon the prime mover and the driven machine

$$C_s = \frac{\text{maximum torque}}{\text{rated torque}}$$

DESIGN FOR DYNAMIC LOAD

- The dynamic force is introduced in the gear teeth due to the following factors:

1. Inaccuracies of the tooth profile,
2. Errors in tooth spacing,
3. Misalignment between bearings
4. Elasticity of parts, and
5. Inertia of rotating masses

Buckingham's Dynamic Load Equation

$$F_d = (\mathbf{F}_t) + \frac{21v(b \times C + F_t)}{21v + \sqrt{b \times C + F_t}}$$

where F_d = Total load on gear including load due to dynamic action,

C = Load stress factor, $= \dfrac{k \times e}{\left(\dfrac{1}{E_p} + \dfrac{1}{E_g}\right)}$ in N/mm,

k = 0.107 for 14.5^{o} involute Full Depth

= 0.111 for 20^{o} involute Full depth

= 0.115 for 20^{o} involute Stub,

E_p & E_g = modulus of elasticity of pinion and gear materials respectively,

e = Sum of errors between two meshing teeth, mm

= e_p+ e_g

e_p = error for pinion

e_g =error for gear

DESIGN FOR WEAR

- The failure of the gear tooth due to pitting occurs when the Hertz's contact stresses between two meshing teeth exceed the surface endurance strength of the material.
- Pitting is a surface fatigue failure which is characterized by small pits on the surface of the gear tooth. In order to avoid this type of failure, the proportions of gear tooth and surface hardness should be selected in such a way that the wear strength of the gear tooth is more than the effective load between the meshing teeth.
- The wear load is determined by the surface endurance limit of the material, curvature of the surface, and relative hardness of the surfaces.

$$F_w = \frac{\sigma_{es}{}^2 \times b \sin\phi}{1.4}\left[\frac{2\,d_p\,d_g}{d_p + d_g}\right]\left[\frac{1}{E_p} + \frac{1}{E_g}\right]$$

σes = Surface endurance limit,
b = Face width
ϕ = Pressure angle
Ep & Eg = Modulus of elasticity of pinion and gear materials respectively,
dp & dg = Pitch circle diameter of pinion and gear respectively,

For a safe design, *Fw* ≥ *Fd*

If pinion and gear are of steel, $\sigma_{es} = \left(2.76 \times BHN - 70\right), \quad N/mm^2$

GEAR TOOTH FAILURES

1. Breakage of the tooth due to static and dynamic loads,
2. Surface destruction.

- **Breakage of Tooth**
- The complete breakage of the tooth can be avoided by adjusting module and face width so that the beam strength of the gear tooth is more than the sum of static and dynamic loads.
- The static beam strength of a gear tooth was suggested by Wilfred Lewis. The dynamic load is caused due to small machining errors resulting into inertia and impact loads on the gear tooth.

2. Surface Destruction

- The wear of gear tooth takes place due to the combined action of rolling and sliding.
- Rolling causes contact stresses and sliding causes rubbing action.
- Pinion is subjected to more rubbing action as it rotates faster than the gear.

The principal types of gear tooth wear are:

1. Abrasive wear
2. corrosive wear
3. Pitting
4. Scoring

- **Abrasive wear:**

  The tooth surface is scratched by foreign particles in the lubricant, such as dirt, rust and weld spatter of metallic debris. This can be reduced by oil filter, using high viscosity lubricants, and surface hardness.

- **Corrosive wear**:

  The corrosion of the tooth surface is caused by corrosive elements, such as extreme pressure (EP) additives present in the lubricating oils and foreign materials due to external contamination.

  These elements attack the tooth surface, resulting in fine wear uniformly distributed over the entire surface.

  The corrosive wear can be controlled by complete enclosure of the gears, selecting proper additives and replacing the lubricant at regular intervals of use.

- **Pitting:**

  It is a type of fatigue failure caused by repeated applications of stress cycles. Pitting phenomenon is of two types: initial pitting and destructive pitting.

- **Scoring:**

  The oil film between the gear teeth may breakdown under excessive surface pressure, high sliding velocity and inadequate supply of lubricant. This results in generation of excessive frictional heat and overheating of the contacting surfaces of gear teeth. This may lead to metal -to-metal contact.

  Scoring is a stick-slip phenomenon, in which alternate welding and shearing takes place rapidly at the high spots. This increases the wear rate faster.

  Scoring can be controlled by selecting proper surface speed, surface pressure and flow rate of lubricant to keep the temperature of contacting surfaces within permissible limits. The bulk temperature of lubricant can be reduced by providing fins on the gear box, air cooling by a fan or circulating cold water.

Design Procedure for Spur Gears

1. Design tangential tooth load $$W_T = \frac{P}{v} \times C_S$$

Pitch line velocity in m / s $$v = \frac{\pi D.N}{60}$$

CS = Service factor

Type of load	*Type of service*		
	Intermittent or 3 hours per day	*8-10 hours per day*	*Continuous 24 hours per day*
Steady	0.8	1.00	1.25
Light shock	1.00	1.25	1.54
Medium shock	1.25	1.54	1.80
Heavy shock	1.54	1.80	2.00

2. Apply the Lewis equation as follows

$$W_T = \sigma_w.b.p_c.y = \sigma_w.b.\pi\, m.y$$
$$= (\sigma_o.C_v)\ b.\pi\, m.y$$

Notes :
(i) The Lewis equation is applied only to the weaker of the two wheels (i.e. pinion or gear).
(ii) When both the pinion and the gear are made of the same material, then pinion is the weaker.
(iii) When the pinion and the gear are made of different materials, then the product of ($\sigma w \times y$) or ($\sigma o \times y$) is the *deciding factor. The Lewis equation is used to that wheel for which ($\sigma w \times y$) or ($\sigma o \times y$) is less.
(iv) The product ($\sigma w \times y$) is called strength factor of the gear.
(v) The face width (b) may be taken as 3 pc to 4 pc (or 9.5 m to 12.5 m) for cut teeth and 2 pc to 3 pc (or 6.5 m to 9.5 m) for cast teeth.

3. Calculate the dynamic load (WD) on the tooth by using Buckingham equation

$$W_D = W_T + W_I$$

$$= W_T + \frac{21v\,(b.C + W_T)}{21v + \sqrt{b.C + W_T}}$$

In calculating the dynamic load (WD), the value of tangential load (WT) may be calculated by neglecting the service factor (CS)

$$W_T = P / v,$$

4. Find the static tooth load (i.e. beam strength or the endurance strength of the tooth) by using the relation,

$$W_S = \sigma_e . b . p_c . y = \sigma_e . b . \pi \, m . y$$

For safety against breakage, WS should be greater than WD.

5. Finally, find the wear tooth load by using the relation

$$W_w = D_P . b . Q . K$$

The wear load (Ww) should not be less than the dynamic load (WD).

HELICAL GEAR

- A helical gear has teeth in the form of a helix around the gear. The helix may be right handed on onegear and left handed on the other gear.
- The pitch surfaces are cylindrical like spur gears but the teethwind around the cylinder helically like screw threads. Helical gears are used to transmit powerbetween parallel shafts.
- Helical is the most commonly used gear in transmissions. They also generate large amounts of thrust and use bearings to help support the thrust load. Helical gears can be used to adjust the rotation angle by 90 deg. when mounted on perpendicular shafts.


Single Helical
Gear Set
Double Helical
Gear Set
Herringbone
Gear Set
Crossed
Helical Gear
Set
Helical Rack
& Pinion

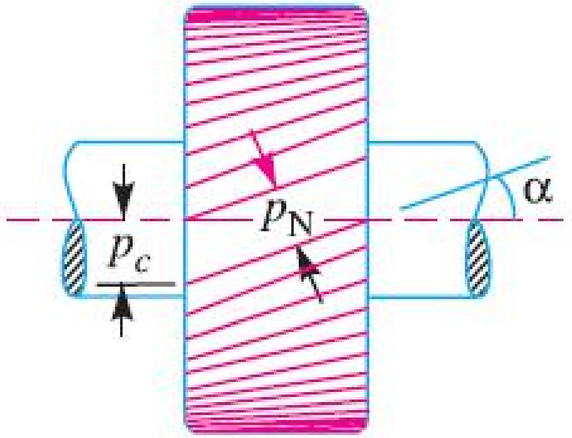
COMPARISON BETWEEN SPUR AND HELICAL GEARS

Spur Gears	Helical Gears
1. Teeth are cut parallel to the axis of the shaft.	1. Teeth are cut in the form of a helix on the pitch cylinder between meshing gears.
2. Contact between meshing teeth occurs along the entire face width of the tooth.	2. Contact between meshing gears begins with a point on the leading edge of the tooth and gradually extends along the diagonal line across the tooth.
3. Load application is sudden resulting into impact conditions and generating noise in high speed applications.	3. Pick up of load by the tooth is gradual, resulting in smooth engagement and quiet operation even at high speeds.
4. Used for parallel shafts only.	4. Crossed helical ears are used on shafts with crossed axes.
5. Speed is limited to about 20 m/s.	5. Used in automobiles, turbines and high speed applications upto 50 m/s.
6. Imposes radial load only.	6. Imposes radial and axial thrust loads.
7. Contact ratio is low.	7. Contact ratio is high.

HELICAL GEARS TERMINOLOGY

- **Helix angle :** It is a constant angle made by the helices with the axis of rotation.
- **Axial pitch :** It is the distance, parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and pc. The axial pitch may also be defined as the circular pitch in the plane of rotation or the diametral plane.
- **Normal pitch**. It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by pN. The normal pitch may also be defined as the circular pitch in the normal plane which is a plane perpendicular to the teeth.

Normal pitch, $p_N = p_c \cos \alpha$

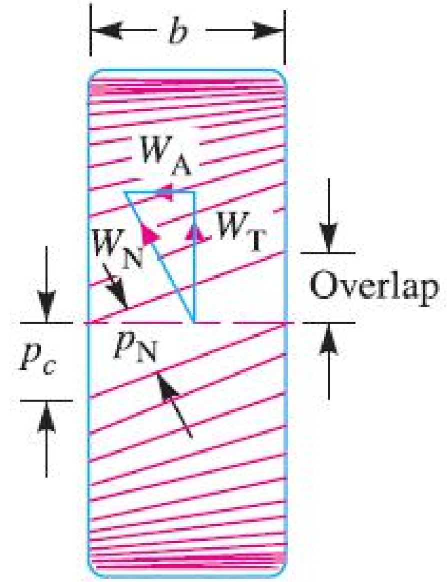


Face Width of Helical Gears

- In order to have more than one pair of teeth in contact, the tooth displacement (i.e. the advancement of one end of tooth over the other end) or overlap should be atleast equal to the axial pitch, such that , Overlap = pc = b tan α
- The normal tooth load (WN) has two components : tangential component (WT) and axial component (WA).
- The axial or end thrust is given by WA = WN sin α = WT tan α
- As the helix angle increases, then the tooth overlap increases. But at the same time, the end thrust also increases, which is undesirable.
- So, overlap should be 15 percent of the circular pitch.

$$\text{Overlap} = b \tan \alpha = 1.15\, p_c$$

$$b = \frac{1.15\ p_c}{\tan\ \alpha} = \frac{1.15 \times \pi m}{\tan\ \alpha} \quad \ldots (\because p_c = \pi m)$$



Formative or Equivalent Number of Teeth for Helical Gears

- The formative or equivalent number of teeth for a helical gear may be defined as the number of teeth that can be generated on the surface of a cylinder having a radius equal to the radius of curvature at a point at the tip of the minor axis of an ellipse obtained by taking a section of the gear in the normal plane.

$$T_E = T / \cos^3 \alpha$$

T = Actual number of teeth on a helical gear

α = Helix angle.

Strength of Helical Gears

- In helical gears, the contact between mating teeth is gradual, starting at one end and moving along the teeth so that at any instant the line of contact runs diagonally across the teeth. Therefore in order to find the strength of helical gears, a modified Lewis equation is used

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y'$$

W_T = Tangential tooth load,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y' = Tooth form factor or Lewis factor corresponding to the formative or virtual or equivalent number of teeth.

The dynamic tooth load on the helical gears

$$W_D = W_T + \frac{21\,v\,(b.C\cos^2\alpha + W_T)\cos\alpha}{21\,v + \sqrt{b.C\cos^2\alpha + W_T}}$$

The static tooth load or endurance strength of the tooth

$$W_S = \sigma_e.b.\pi\,m.y'$$

The maximum or limiting wear tooth load for helical gears

$$W_w = \frac{D_P.b.Q.K}{\cos^2\alpha}$$

$$K = \frac{(\sigma_{es})^2\sin\phi_N}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

ϕ_N = Normal pressure angle.