Basics of AC Circuit

UNIT-III

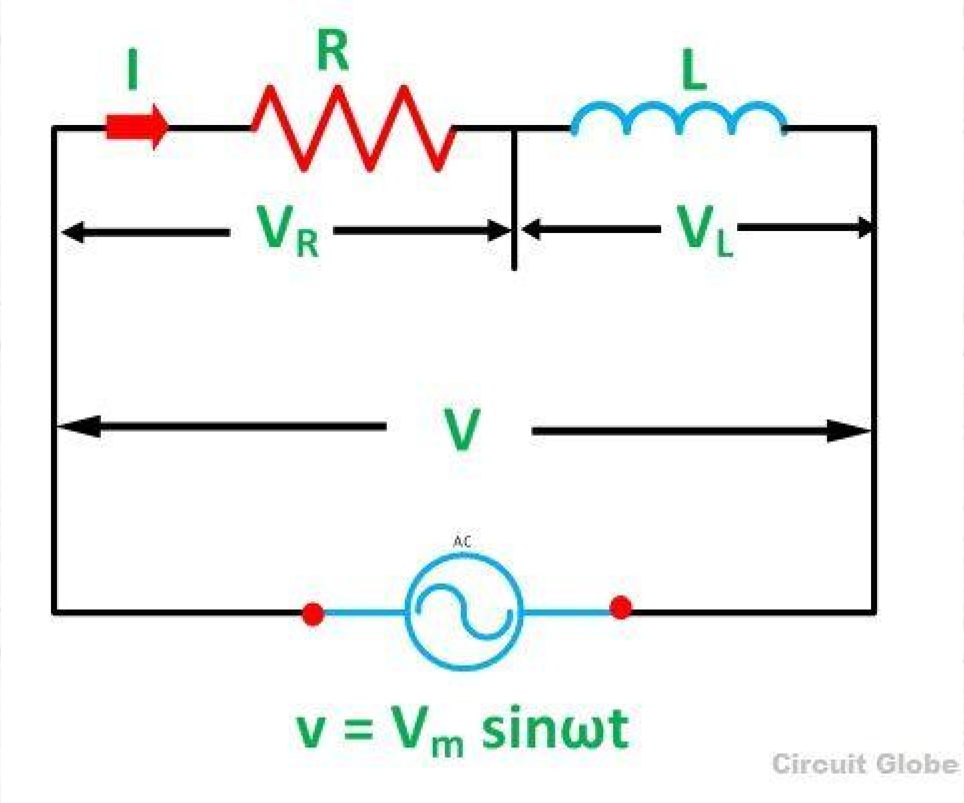
UNIT- III

AC series circuit RL, RC, RLC series circuit, active or real power, power factor in ac circuit, resonance in RLC series circuit, graphical representation of resonance. Resonance curve, Q factor.

Parallel AC circuit

Introduction, methods of solving parallel ac circuit, equivalent impedance method, admittance, admittance method, Series- parallel circuit, resonance in parallel circuit, comparison of series and parallel resonance.

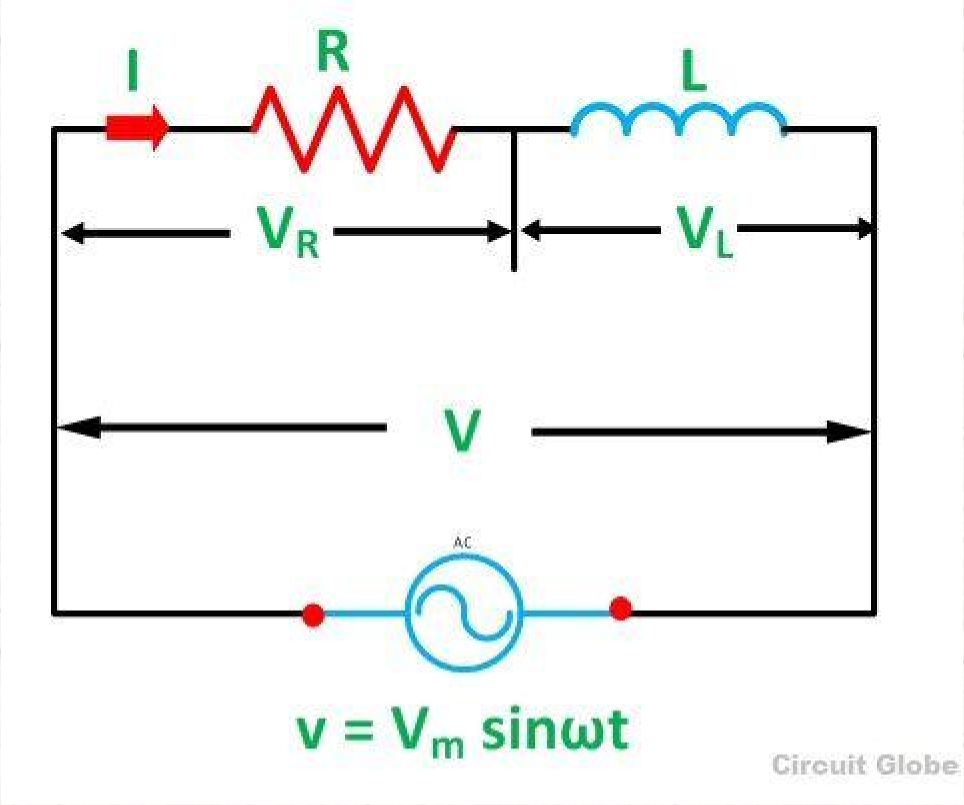
SERIES RL CIRCUIT



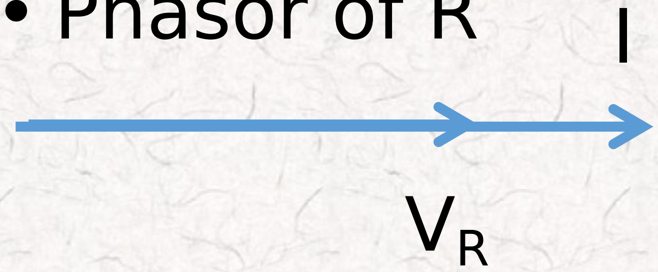
- Here R and L are connected in series across an AC cource of Vm sin ωt.
- Voltage drop across R is V_R
- Voltage drop across L in V_L
- Total voltage V is the phasor sum of V_R and V_L. That is

$V = V_R + V_L$

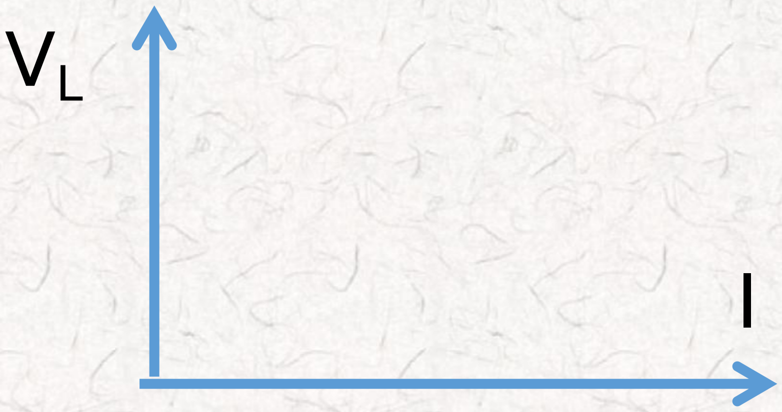
SERIES RL CIRCUIT



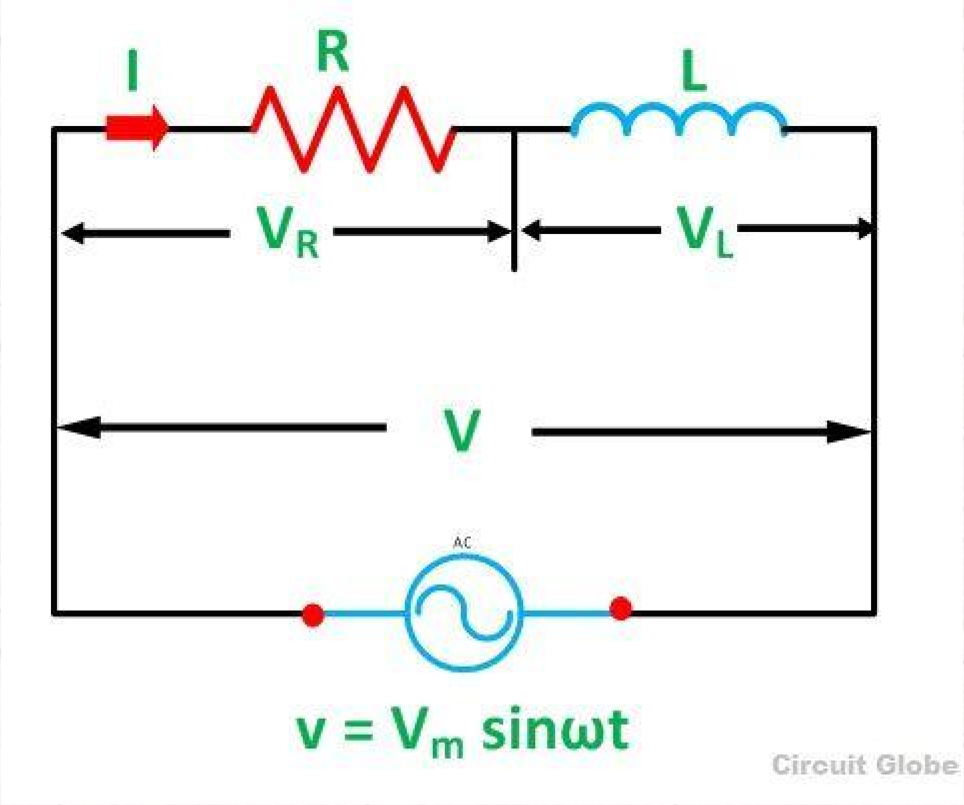
- Phasor of R



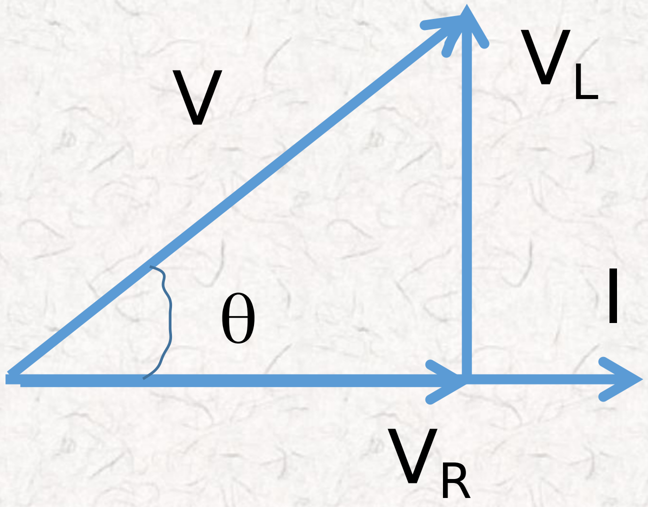
- Phasor of L



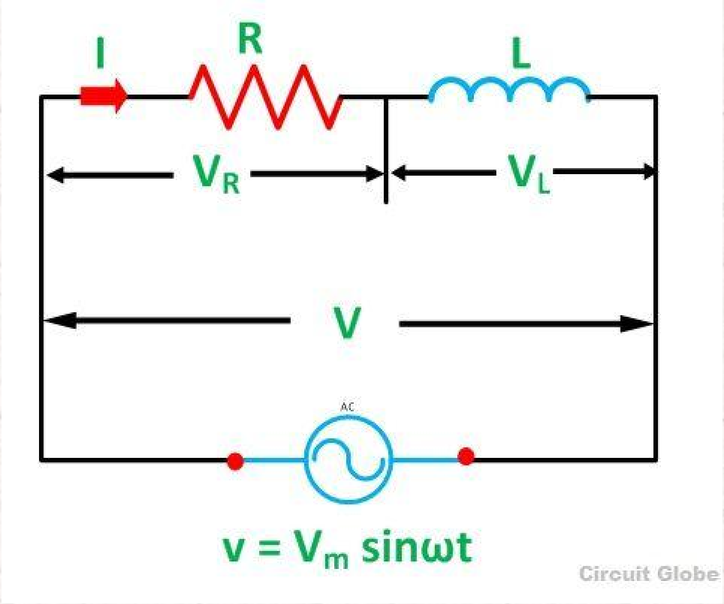
SERIES RL CIRCUIT



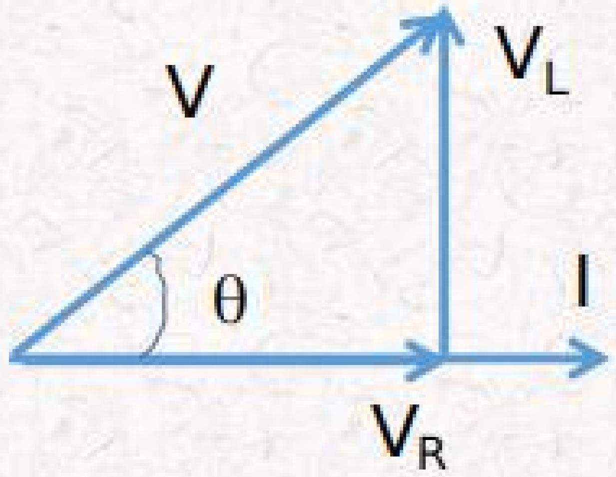
- Combining the two phasors, with current I as the reference we get the phasor of series RL circuit.
- It is called the voltage phasor

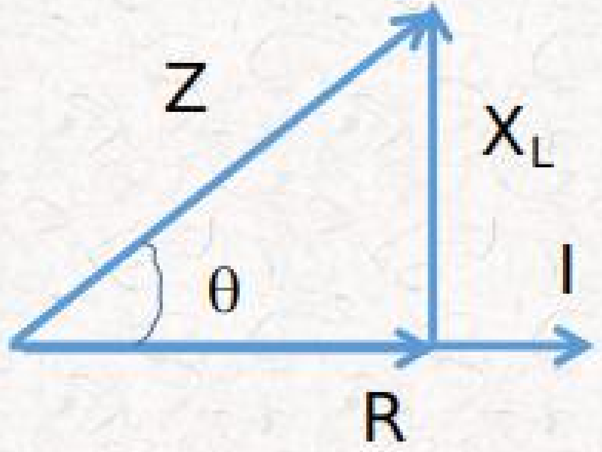


SERIES RL CIRCUIT

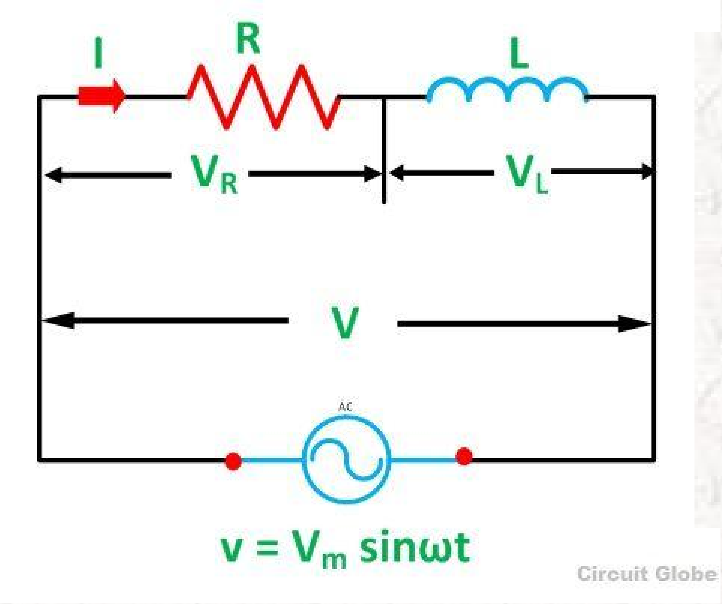


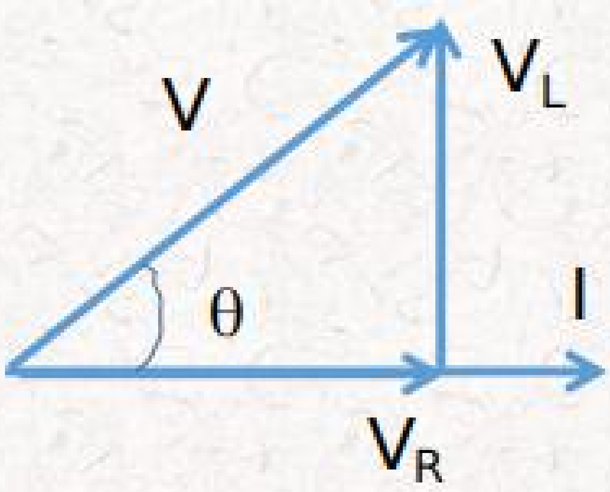
- V_R = IR : V_L = I X_L : V = IZ
- Where X_L =ωL
- Where Z is the total impedance.
- Dividing the sides of the phasor by I will give us the impedance phasor.

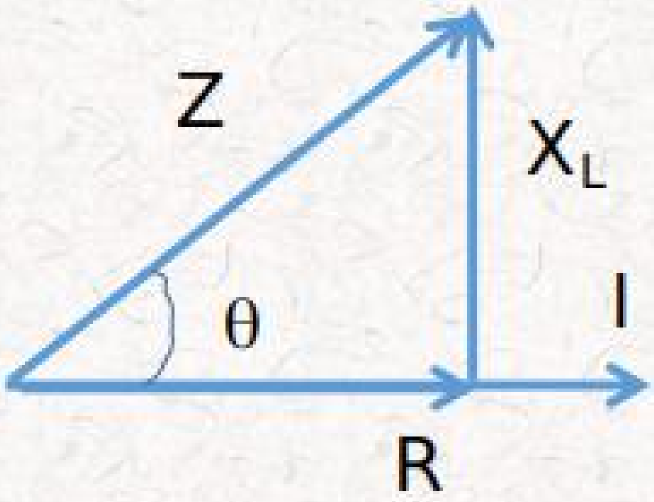




SERIES RL CIRCUIT



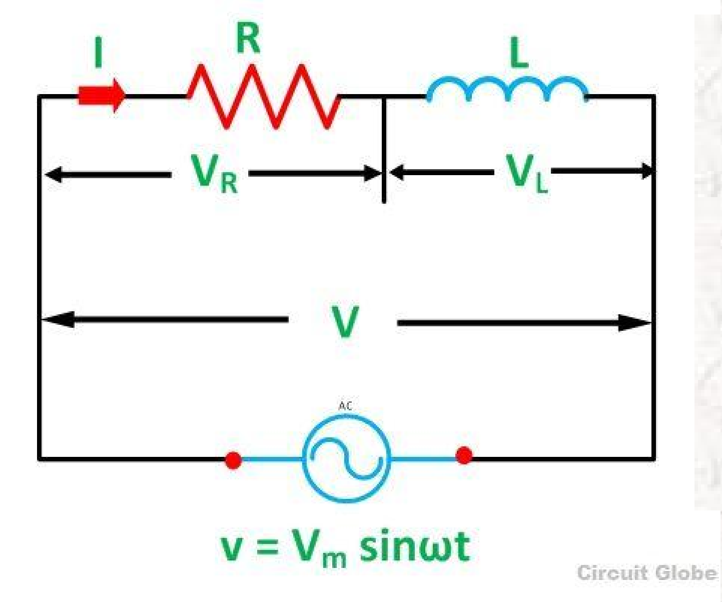


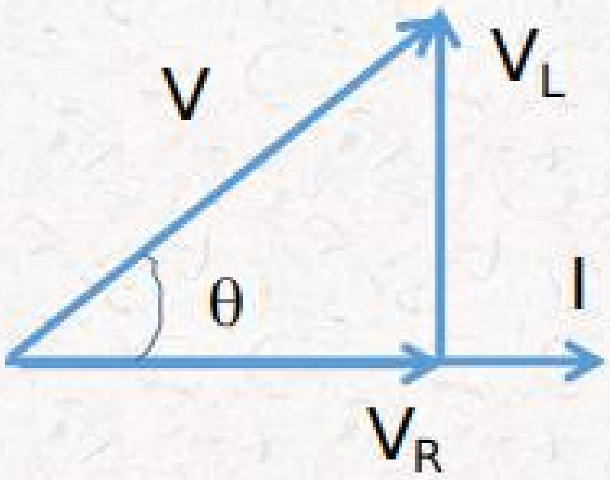


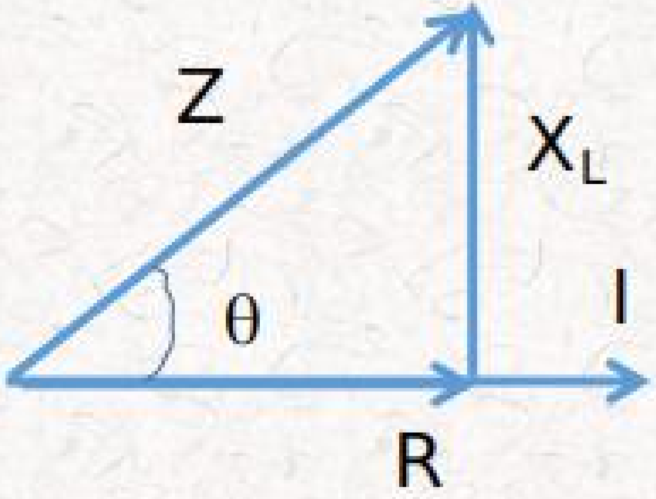
From impedance phasor we can write

$Z=R+jX_L$ (X_L has been rorated in anticlock wise direction by 90^0 from origin)

SERIES RL CIRCUIT



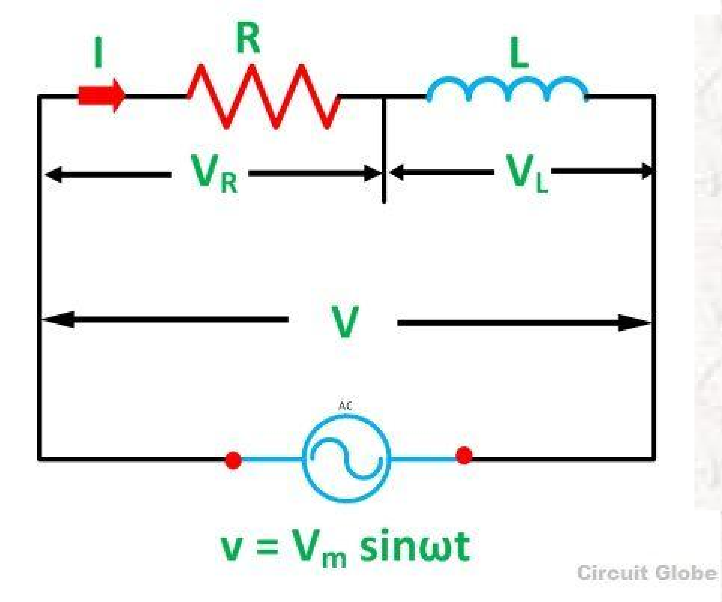


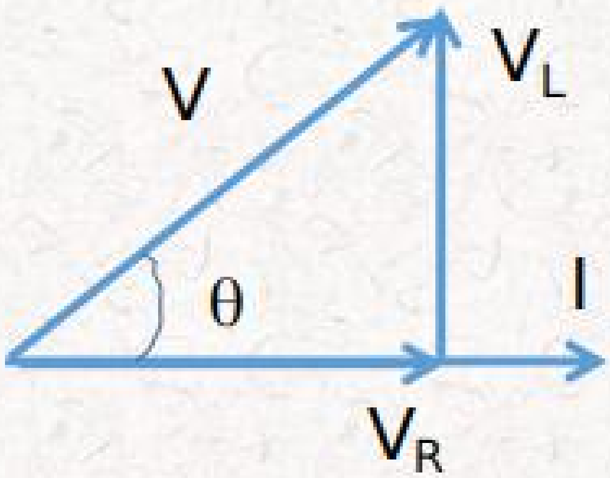


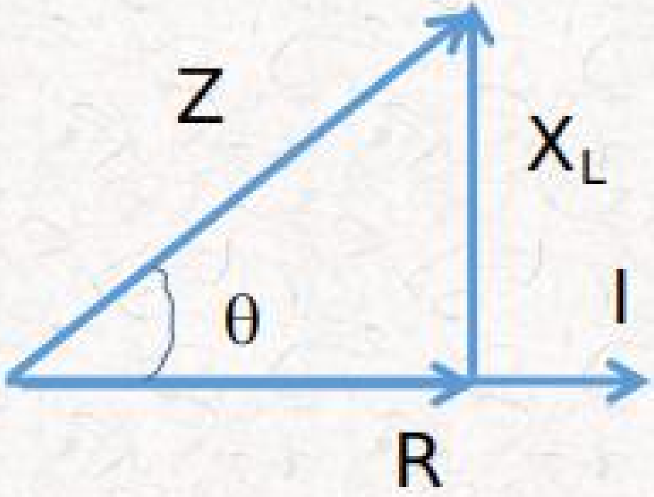
We know V=IZ. So $current\ I = \frac{V}{Z} = \frac{V}{R + jX_L}$

Once we obtained the expressdion for current I now we can calculate $V_R = IR$ and $V_L = I\,X_L$

SERIES RL CIRCUIT







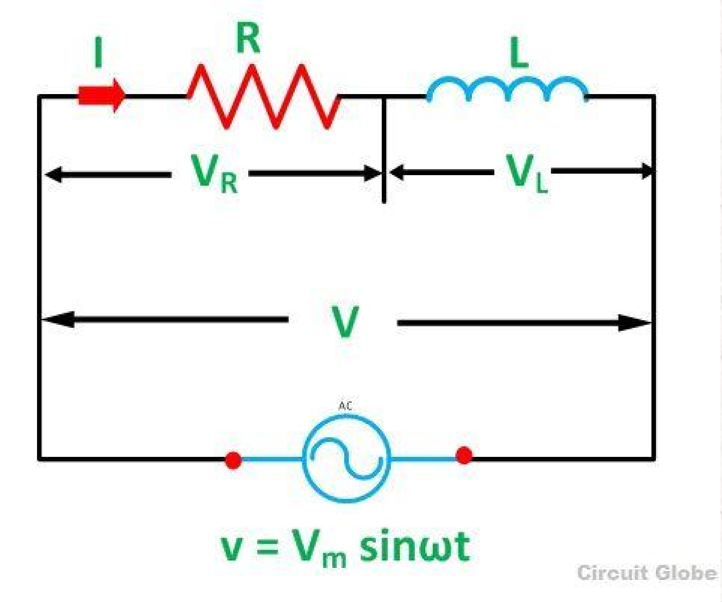
From the phasor:

$$Phase\ angle\ \theta = tan^{-1}\frac{V_L}{V_R} = tan^{-1}\frac{IX_L}{IR} = tan^{-1}\frac{X_L}{R}$$

Power factor = Cos θ

Power = VI Cos θ

SERIES RL CIRCUIT



For the series RL circuit shown in the Figure, R= 3Ω and L = 0.0127 and the supply voltage is 141 V. Calculate the following:

i) Inductive reactance. ii) Impedance of the circuit

iii) current I iv) Phase angle v) Power factor

vi) Real Power vii) Draw the phasor

SERIES RL CIRCUIT

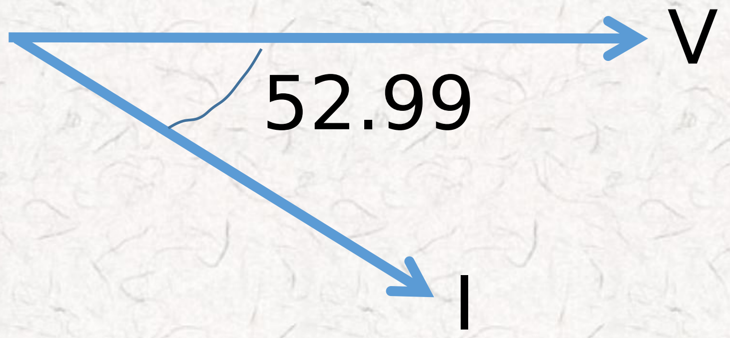
Inductive reactance (X_L) $= \omega L = 2\Pi f L = 2\Pi \times 50 \times 0.0127 = 3.98\ \Omega$

Impedance (Z)$= R + jX_L = 3 + j\,3.98\ \ \Omega$

$$\boldsymbol{Current}\ (\boldsymbol{I}) = \frac{V}{Z} = \frac{141}{3+j3.98} = 17.028 - j22.59\ A$$

$$= 28.29\angle -52.99\ A$$

Phasor diagram



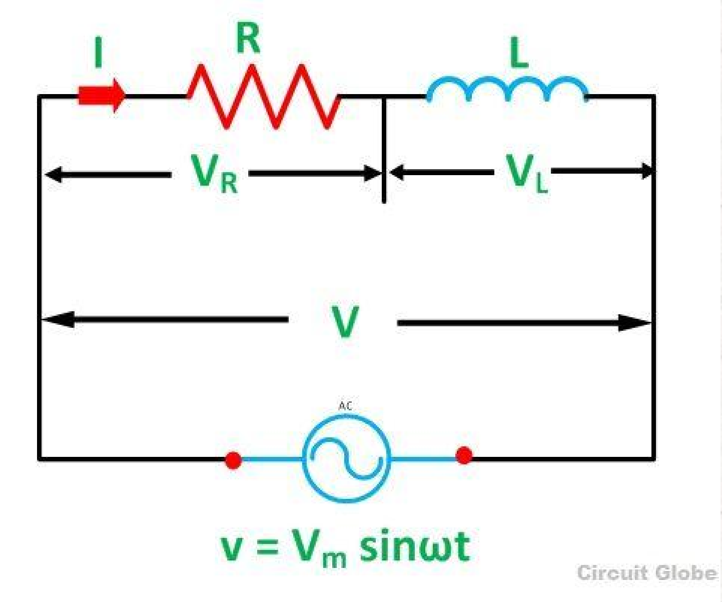
SERIES RL CIRCUIT

From the phasor **phase angle** = 52.99^0

Power factor= cos 52.99 = 0.602

Real Power = VI Cos 52.99 = 141 x 28.29 x 0.602 =2401.311 W

Problem-1



For the series RL circuit shown in the Figure, R= 30 Ω and L = 0.1 H and the supply voltage is $230 \angle 45$ V. Calculate the following:

i) Inductive reactance.   ii) Impedance of the circuit

iii) current I   iv) Phase angle   v) Power factor

vi) Real Power   vii) Draw the phasor

Problem-2

Example 22. *A coil has a resistance of 5 Ω and an inductance of 31.8 mH. Calculate the current taken by the coil and power factor when connected to 200 V, 50 Hz supply.*

Draw the vector diagram.

If a non-inductive resistance of 10 Ω is then connected in series with coil, calculate the new value of current and its power factor.

Problem-3

Example 20. *A coil takes 2.5 amps. when connected across 200 volt 50 Hz mains. The power consumed by the coil is found to be 400 watts. Find the inductance and the power factor of the coil.*

Problem-4

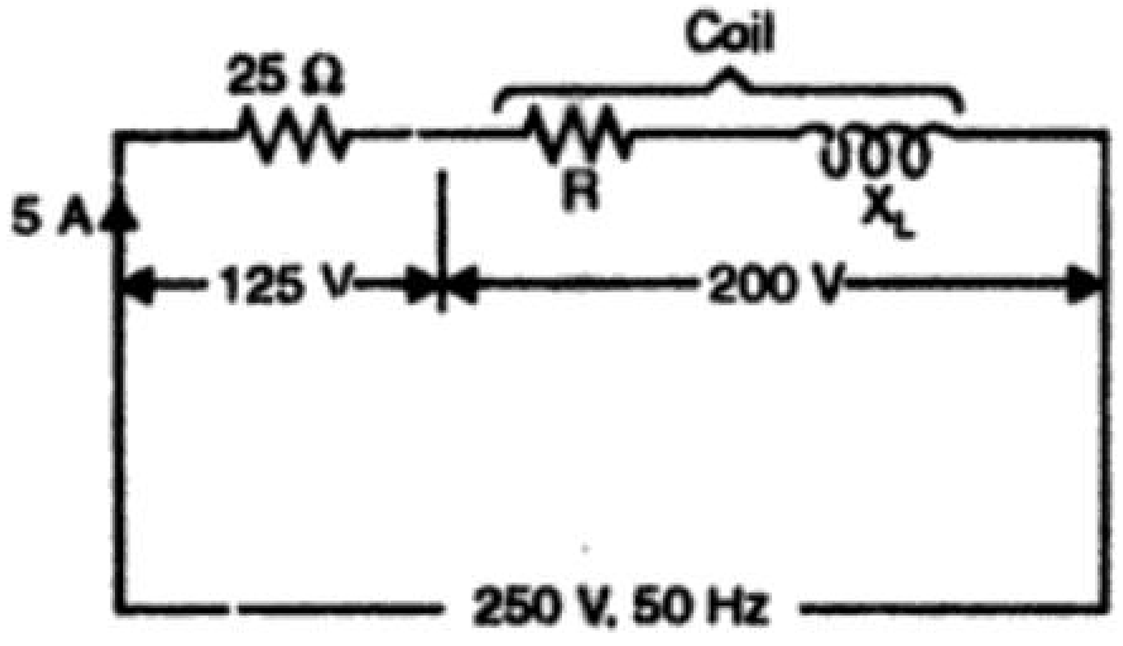
Example 23. *A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V, 50 Hz. If the voltage across the resistance is 125 V and across the coil 200 V, calculate :*

(i) Impedance, reactance and resistance of the coil,

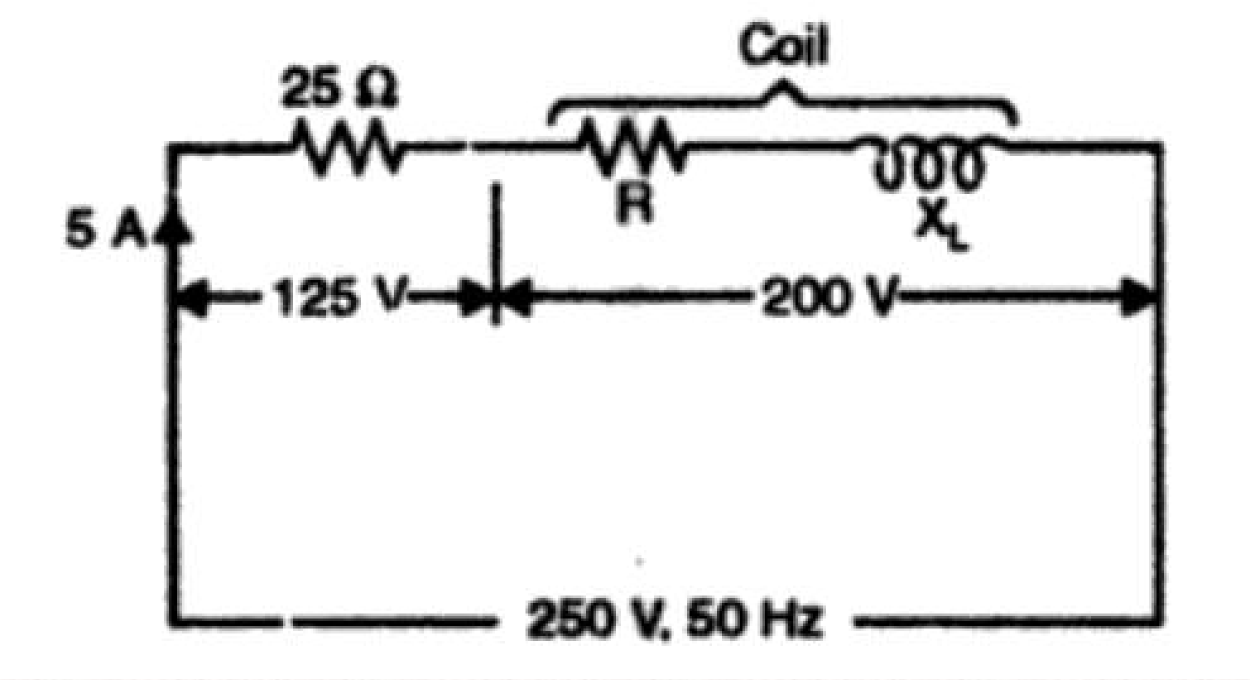
(ii) The power absorbed by the coil,

(iii) The total power.

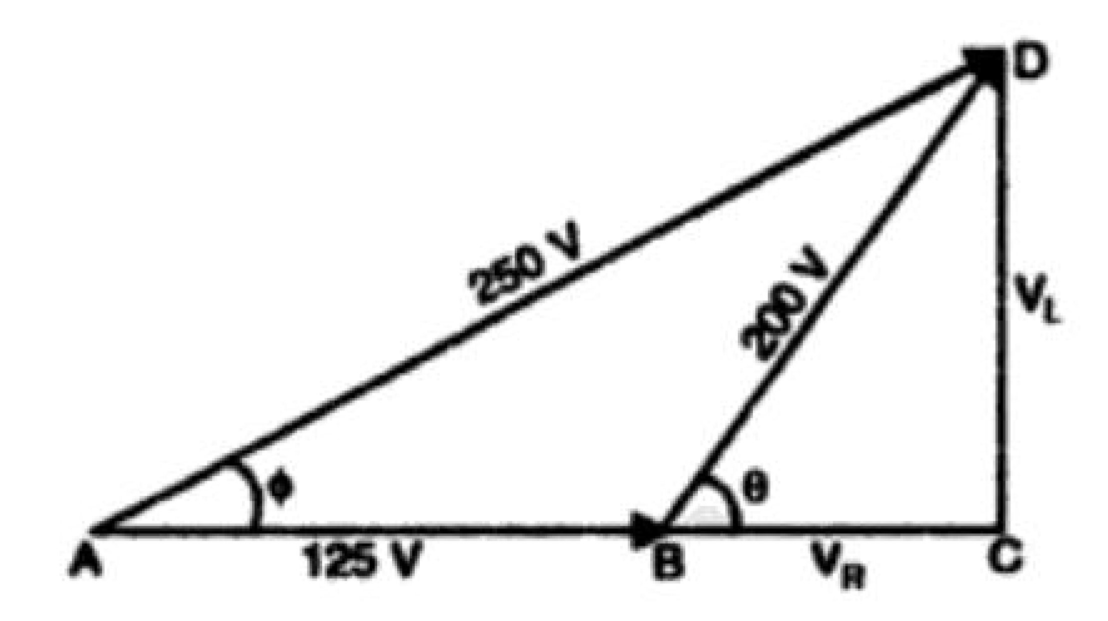
Draw the vector diagram. **(Elect. Engg. Madras Univ.)**



Problem-4

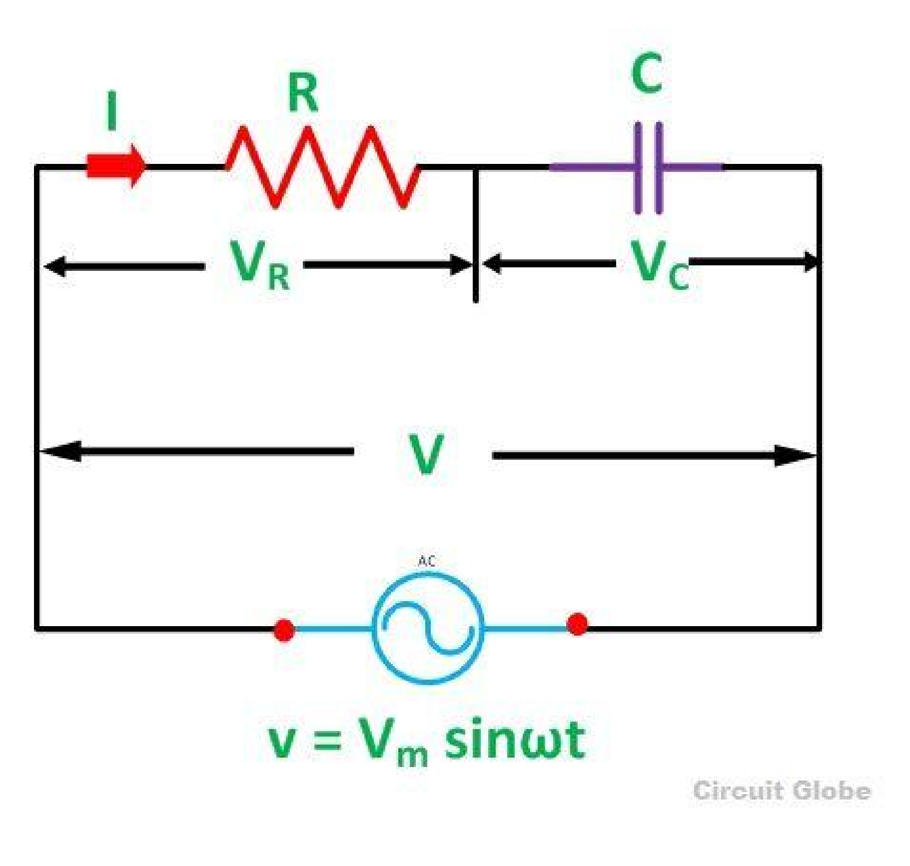


Coil
25 Ω
R
X_L
5 A
125 V
200 V
250 V, 50 Hz



D
250 V
200 V
V_L
ϕ
θ
A
125 V
B
V_R
C

SERIES RC CIRCUIT



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- Voltage drop across R is V_R
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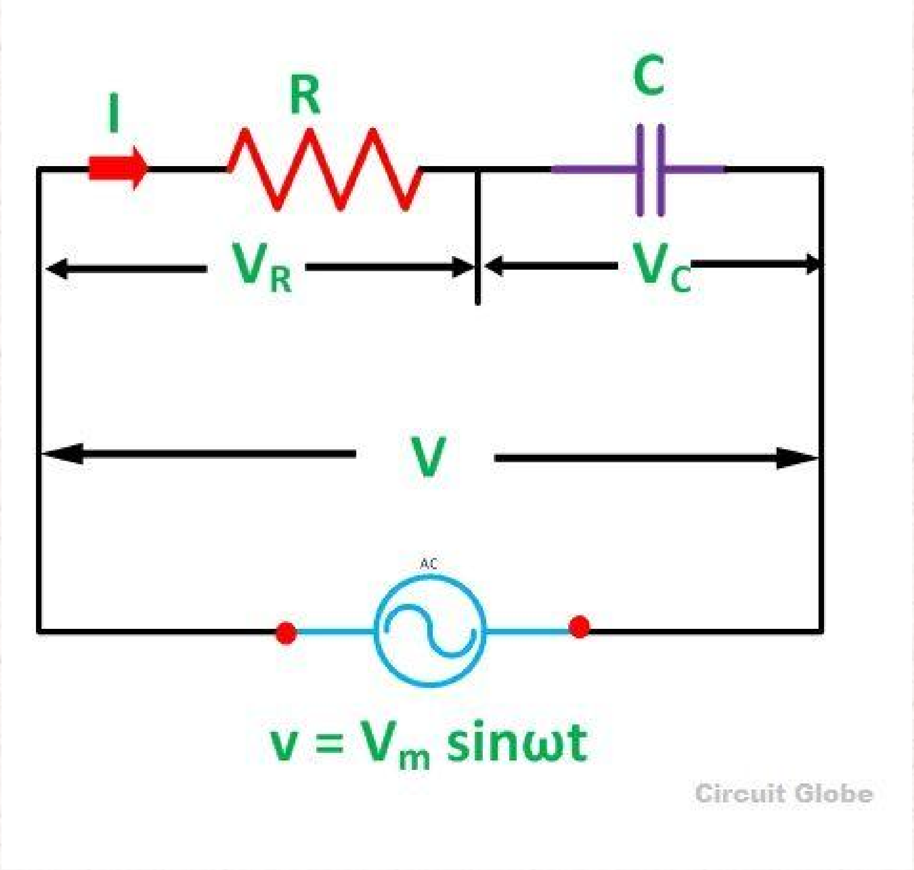
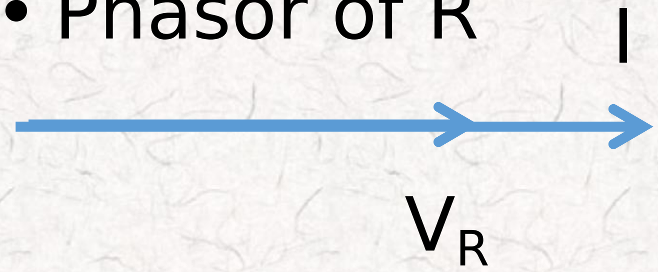
$V = V_R + V_C$

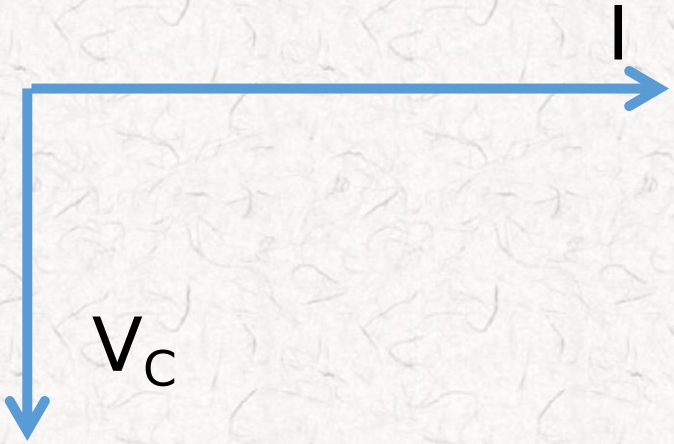
SERIES RC CIRCUIT



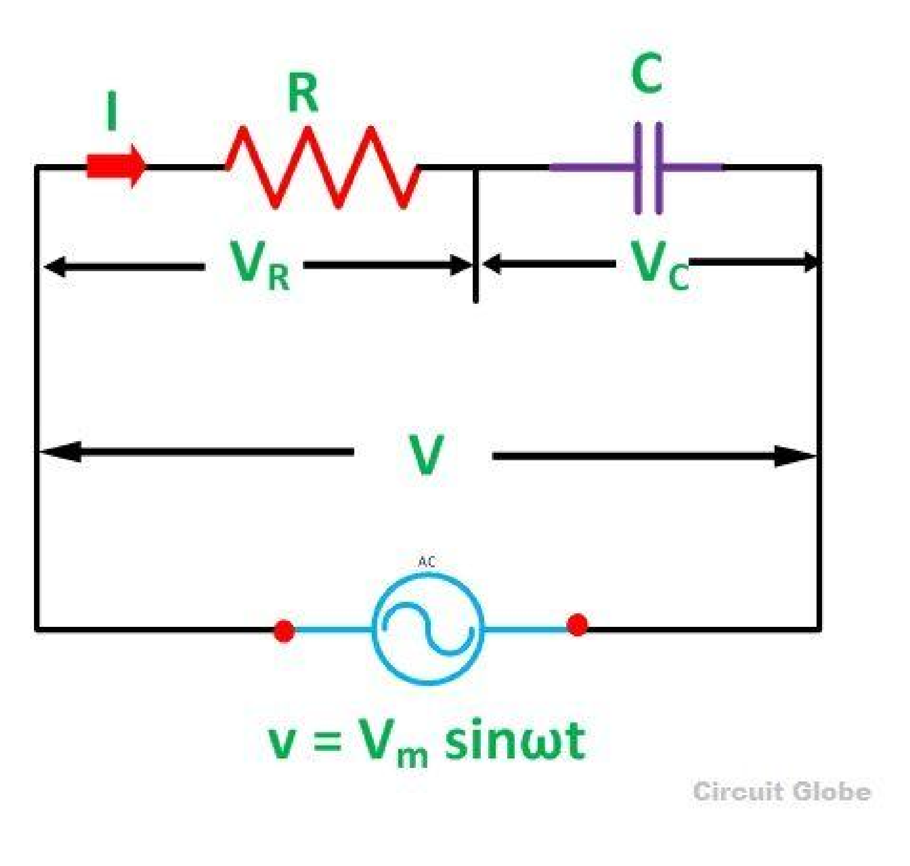
- Phasor of R



- Phasor of C



SERIES RC CIRCUIT



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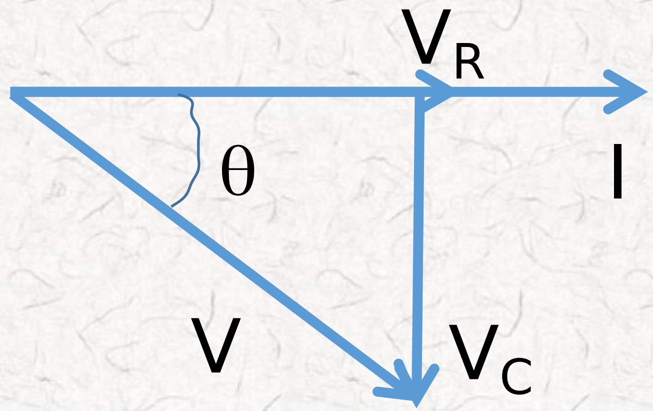
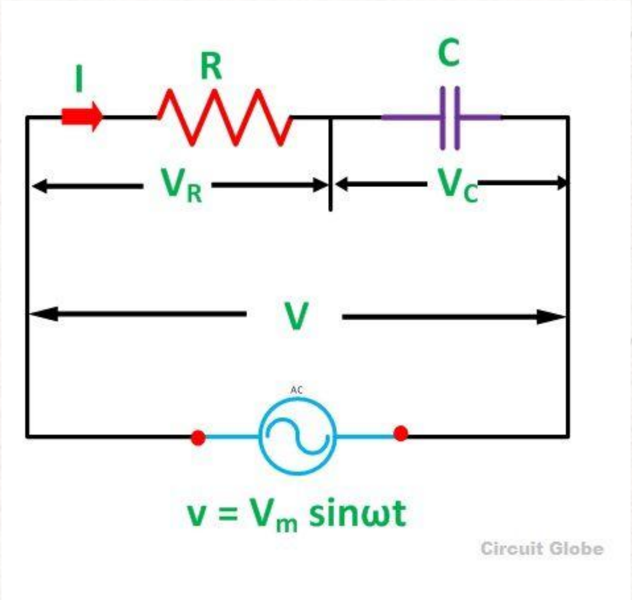


SERIES RC CIRCUIT



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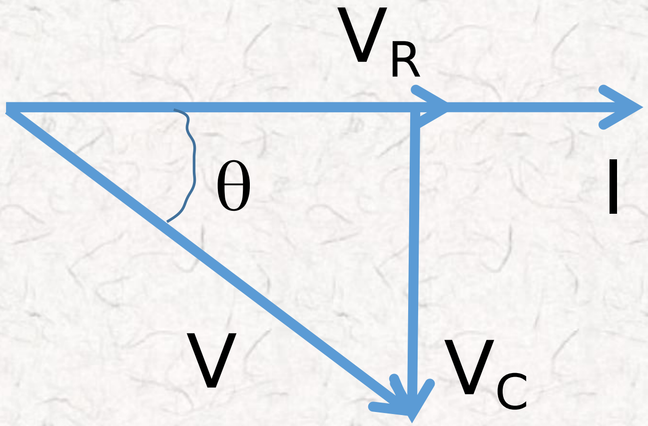
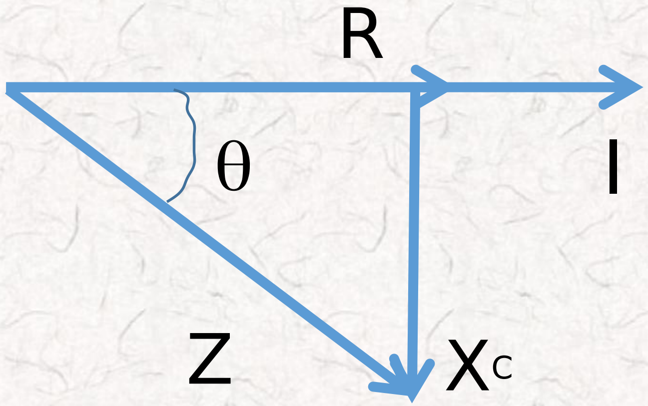
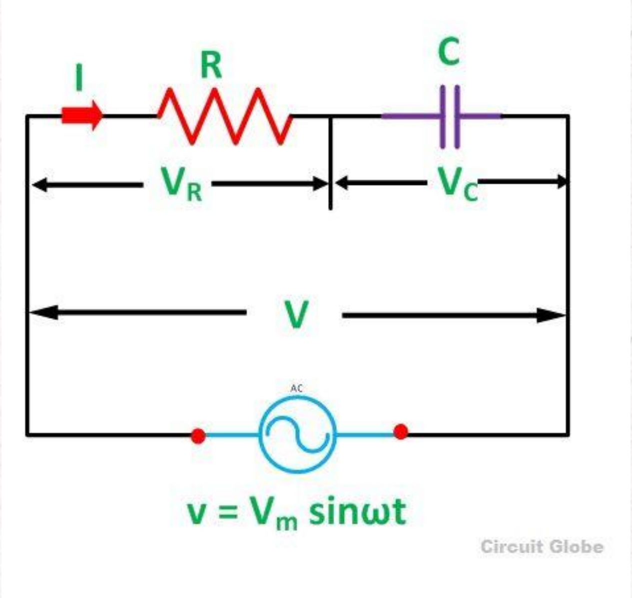
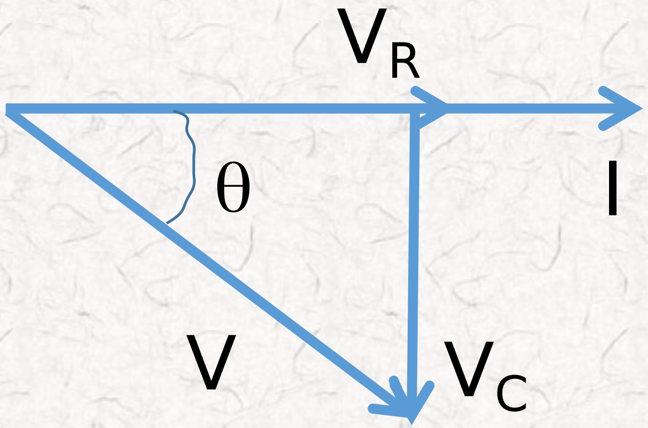
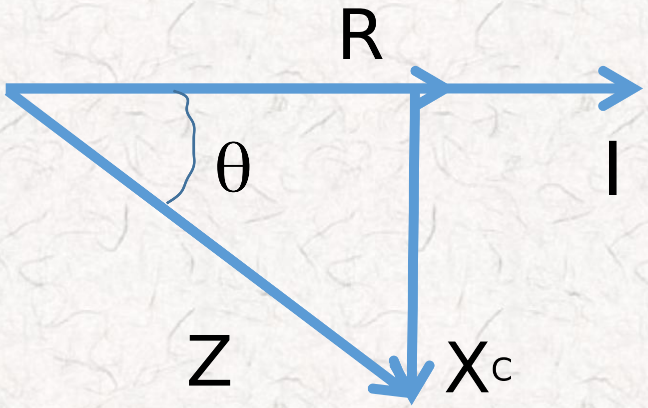




SERIES RC CIRCUIT







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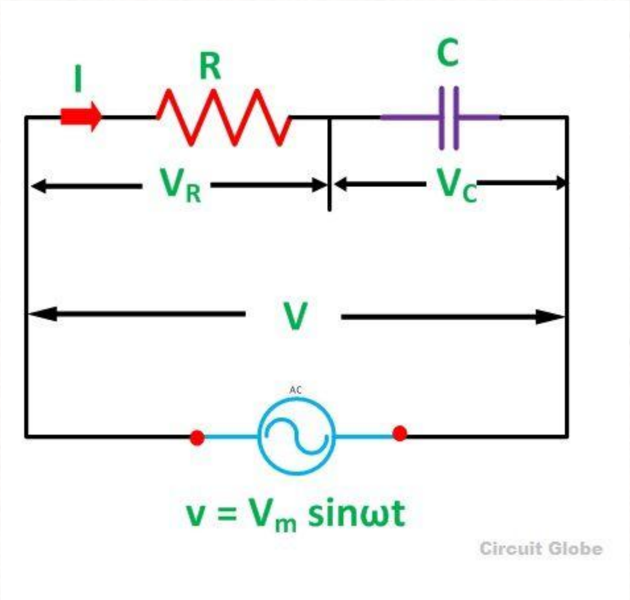
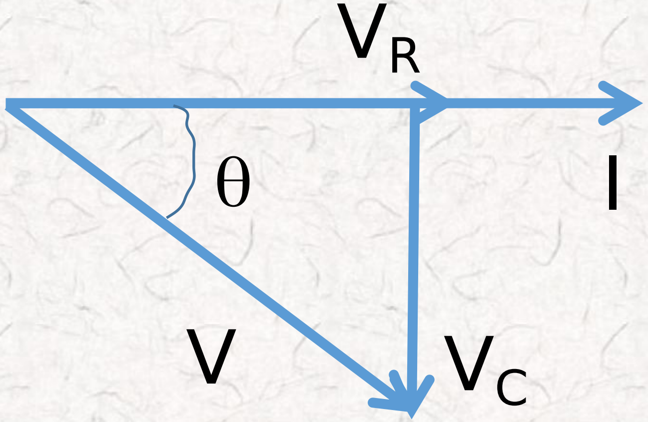
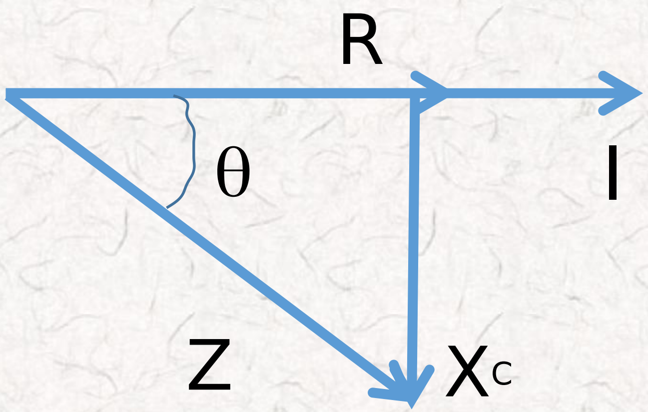
SERIES RC CIRCUIT







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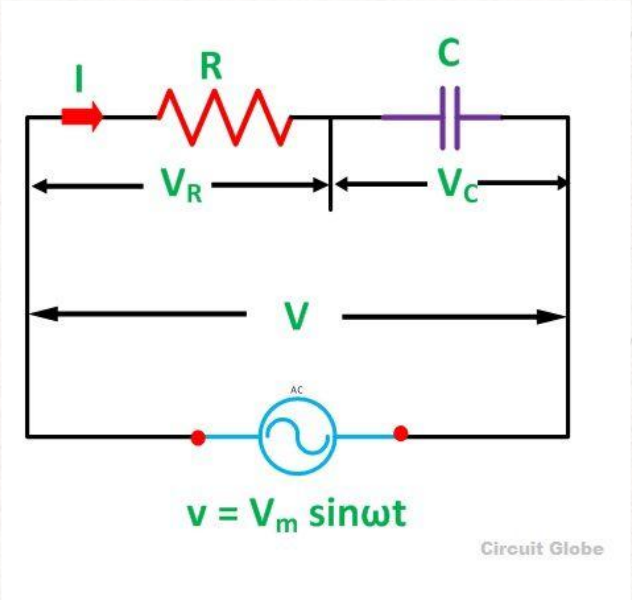
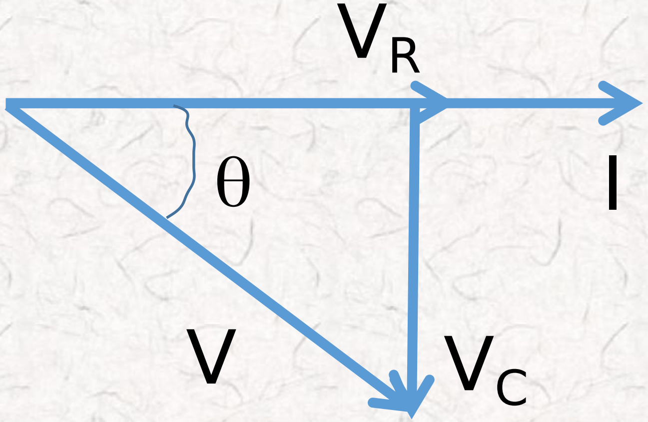
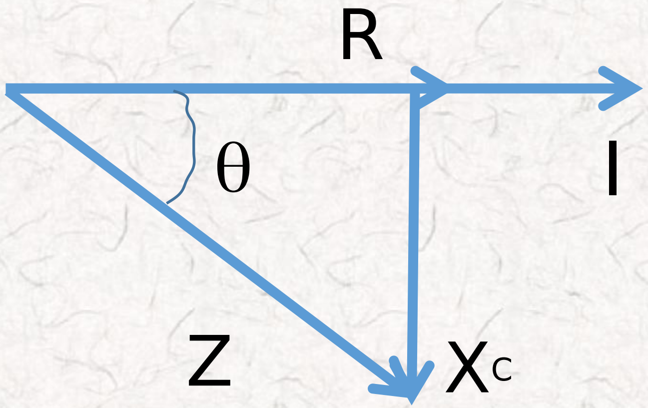
SERIES RC CIRCUIT







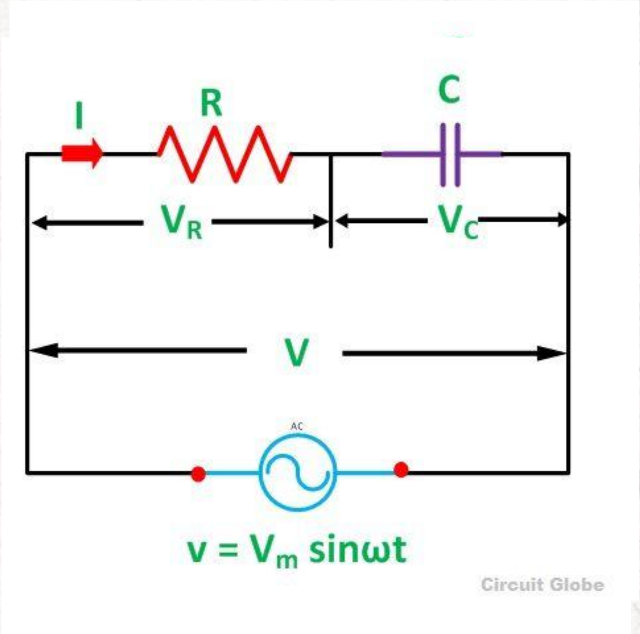
From the phasor:

$$Phase\ angle\ \theta = tan^{-1}\frac{V_C}{V_R} = tan^{-1}\frac{IX_C}{IR} = tan^{-1}\frac{X_C}{R}$$

Power factor = Cos θ

Power = VI Cos θ

SERIES RC CIRCUIT



For the series RC circuit shown in the Figure, R= 10 Ω and C = 10μF and the supply voltage is 141 V. Calculate the following:

i) Capacitative reactance. ii) Impedance of the circuit

iii) current I iv) Phase angle v) Power factor

vi) Real Power vii) Draw the phasor

SERIES RC CIRCUIT

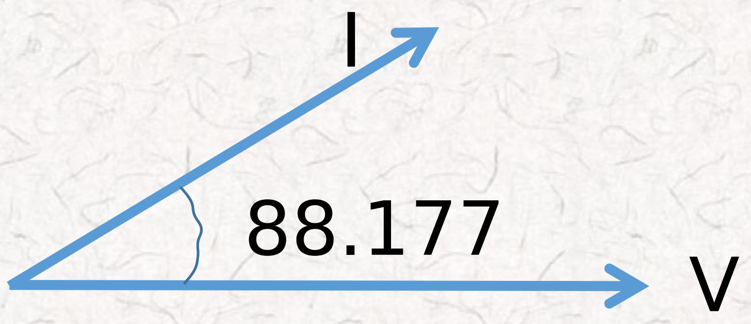
Capacitative reactance (X_C) =1/ωC = 1/ (2ΠfC) = 1/ (2Π x 50 x 10x10^{-6} = 318.31 Ω

Impedance (Z)= R-jX_C =10-j 318.31 Ω

$$\boldsymbol{Current\ (I)} = \frac{V}{Z} = \frac{141}{10 - j318.31} = 0.014 + j0.44\ A$$

$$= 0.44\angle 88.177\ A$$

Phasor diagram



SERIES RC CIRCUIT

From the phasor **phase angle** = 88.177^0

Power factor= cos 88.177 = 0.0318

Real Power = VI Cos 88.177 = 141 x 0.44 x 0.0318 =1.972 W

SERIES RC CIRCUIT

Example 26. *An alternating voltage of (176 + j132) is applied to a circuit and the current in the circuit is given by (6.6 + j 8.8) A. Determine :*

(*i*) *Values of elements of the circuit.*

(*ii*) *Power factor of the circuit.*

(*iii*) *Power consumed.*

IV) Reactive Power

V) Apparent Power

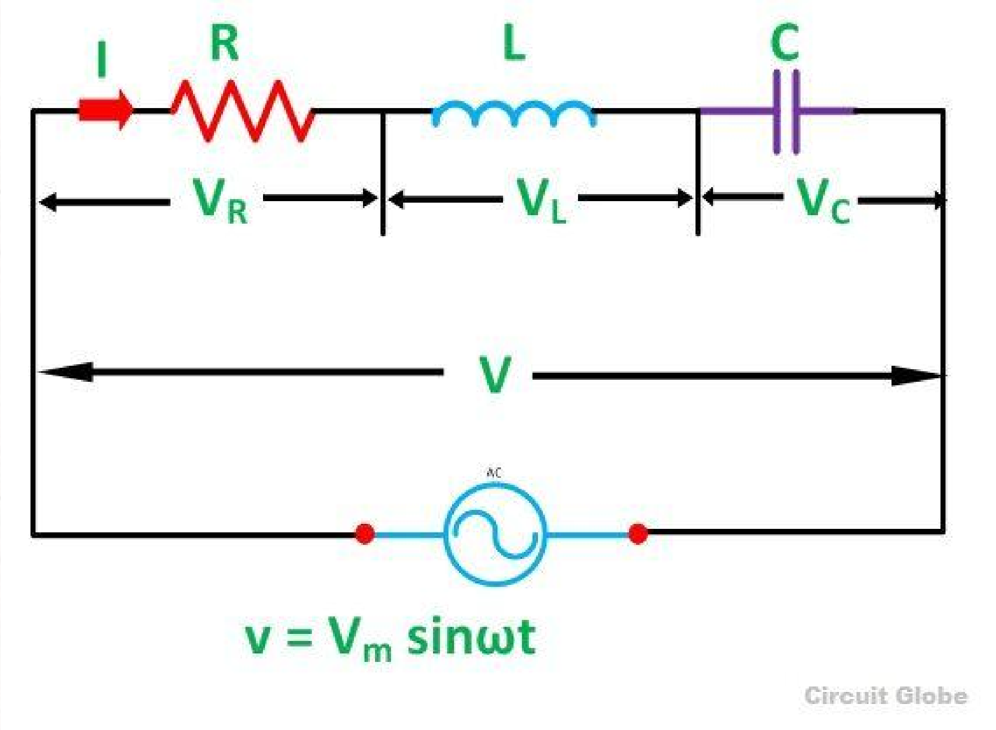
SERIES RC CIRCUIT

Example 29. *A capacitance of 20 μF and a resistance of 100 ohms are connected in series across 120 V, 60 Hz mains. Determine the average power expended in the circuit.*

SERIES RC CIRCUIT

Example 31. *A two element series circuit is connected across an A.C. source $e = 200\sqrt{2}\ \sin(\omega t + 20°)$ V. The current in the circuit then is found to be $i = 10\sqrt{2}\ \cos(314\ t - 25°)$ A. Determine parameters of the circuit.* **(Allahabad University)**

SERIES RLC CIRCUIT

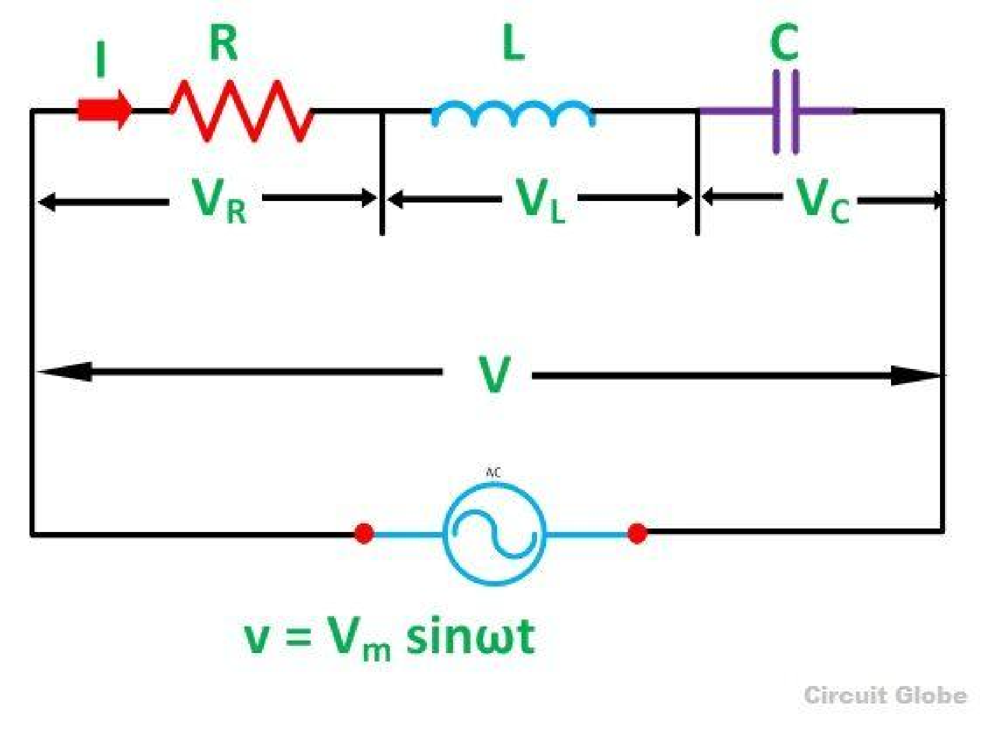


- Here R L and C are connected in series across an AC cource of Vm sin ωt.
- Voltage drop across R is V_R
- Voltage drop across L in V_L
- Voltage drop across C in V_C
- Total voltage V is the phasor sum of V_R V_L and V_C .

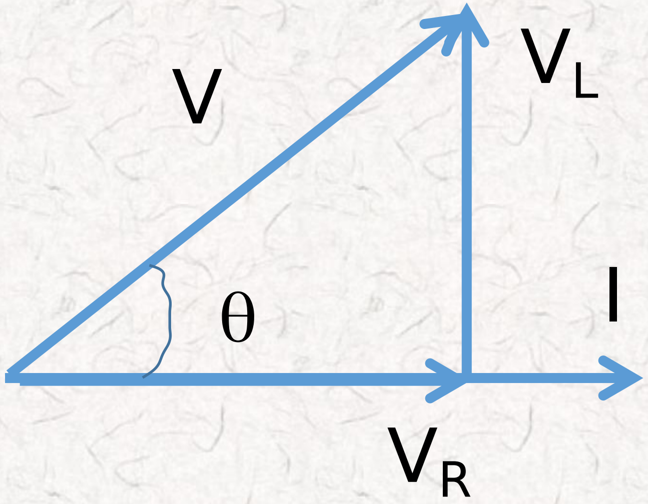
That is

$V = V_R + V_L + V_C$

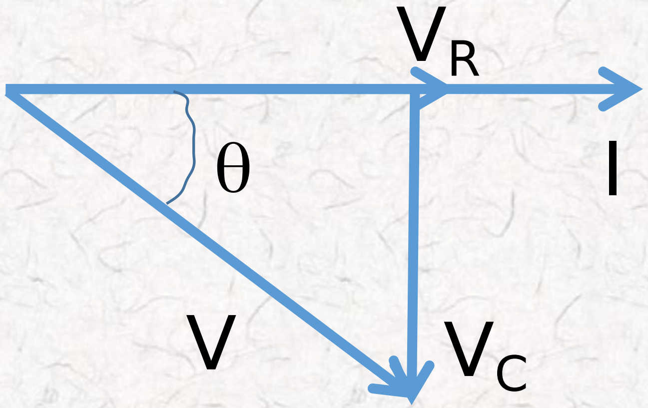
SERIES RLC CIRCUIT



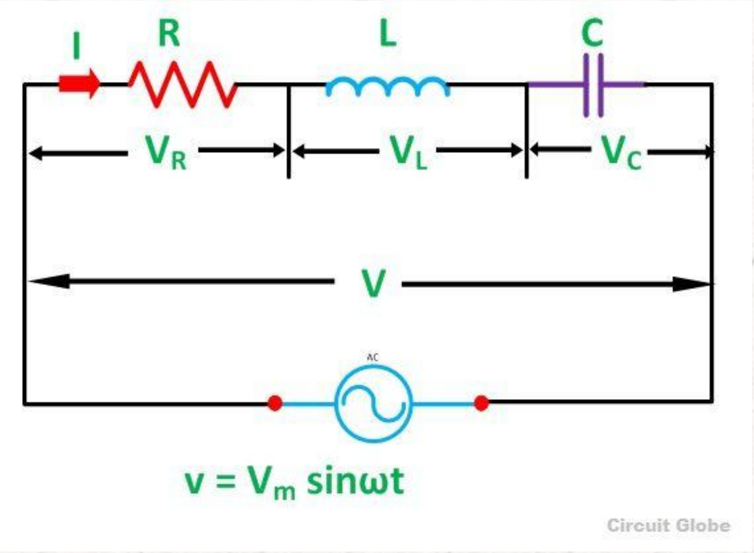
- Phasor of RL Circuit



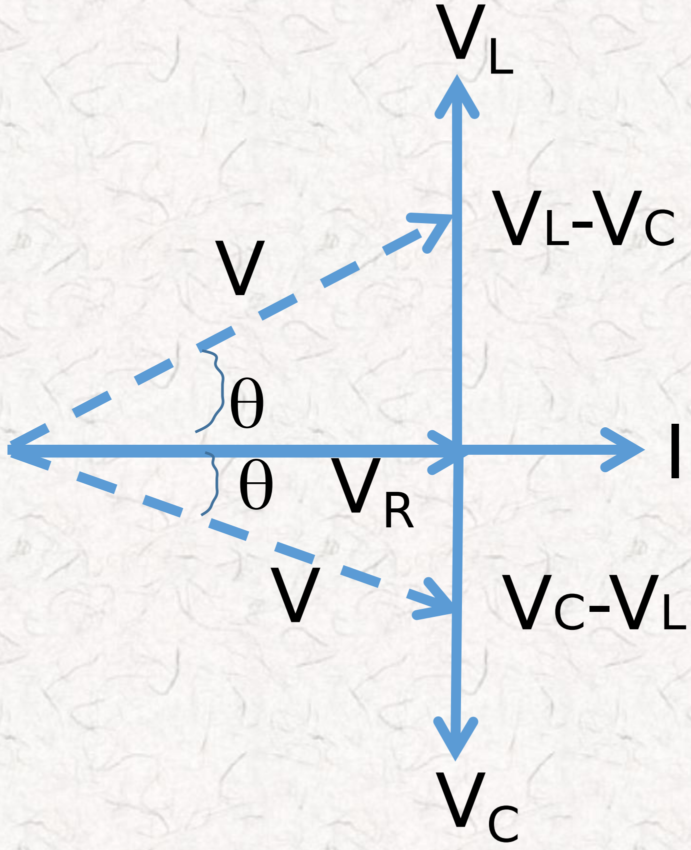
- Phasor of RC Circuit



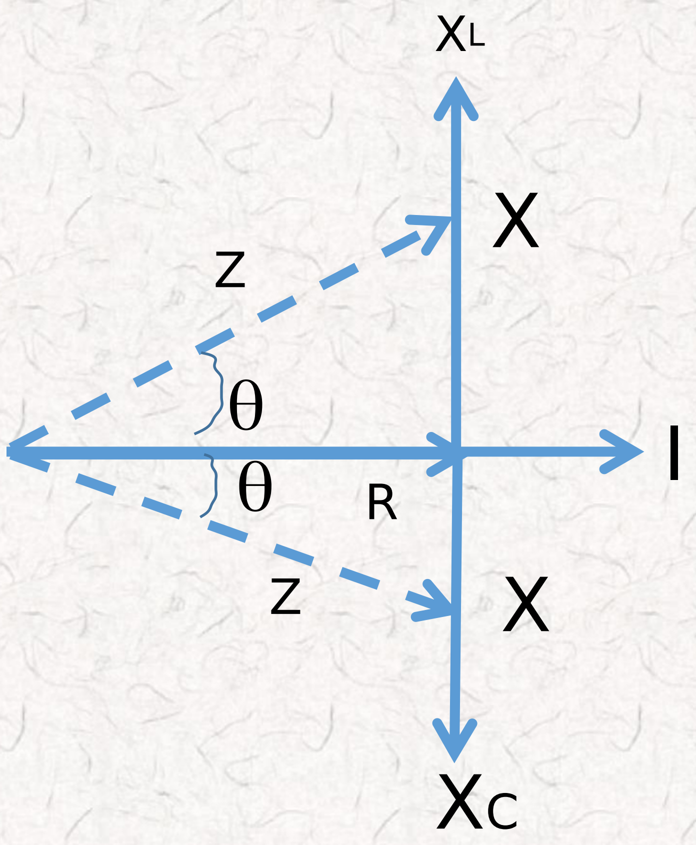
SERIES RLC CIRCUIT



- Combining the two phasors, with current I as the reference we get the phasor of series RLC circuit.
- It is called the voltage phasor

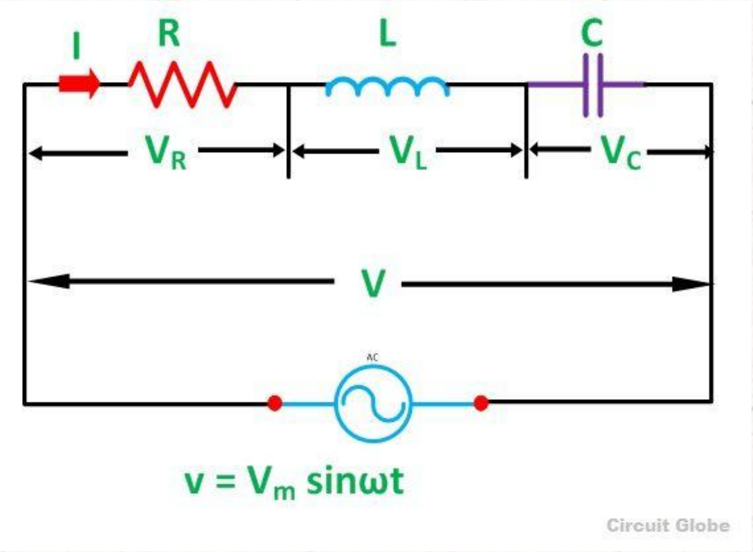


SERIES RLC CIRCUIT



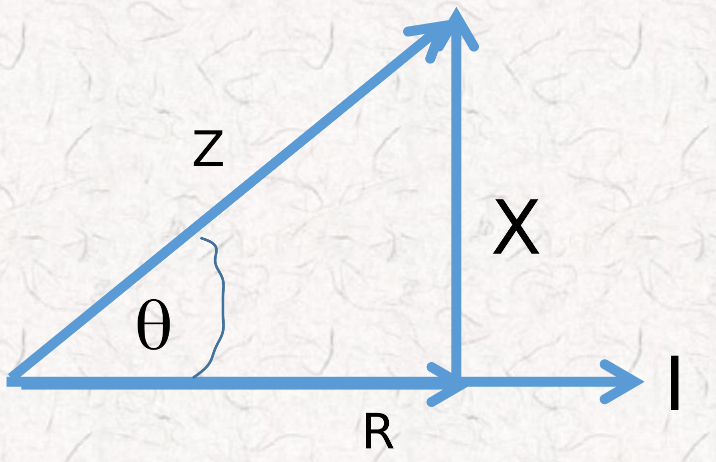
$$X = X_L \sim X_C$$

$$Z = R \pm jX$$

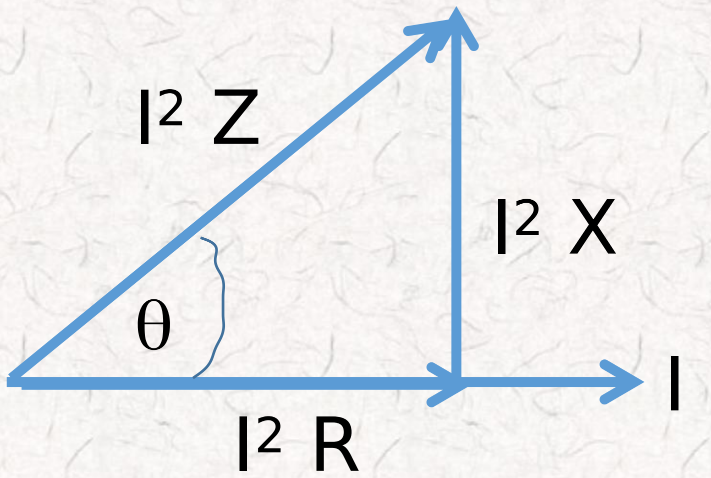


- $V_R = IR$: $V_L = I\,X_L$: $V_C = I\,X_C$: $V = IZ$
- Where $X_L = \omega L$ and $X_C = 1/\omega C$
- Where Z is the total impedance.
- Dividing the sides of the phasor by I will give us the impedance phasor.

POWER TRIANGLE



Impedance Phasor



Impedance Phasor

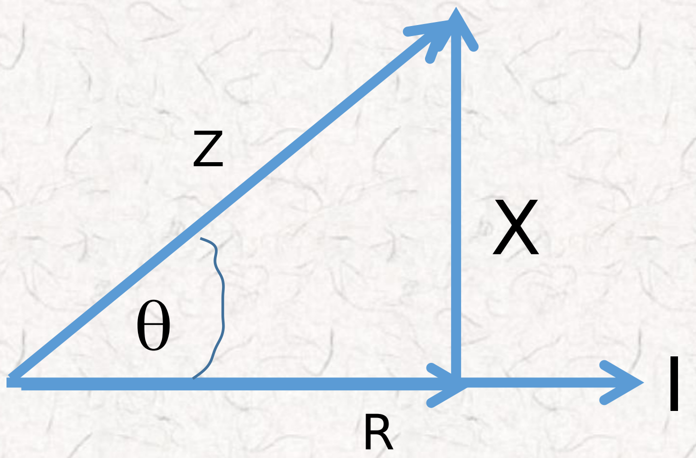
I^2 R: Active Power (P), unit is watts (W)

I^2 X: Reactive Power (Q), Unit is Volt-amperes Reactive, (VAr)

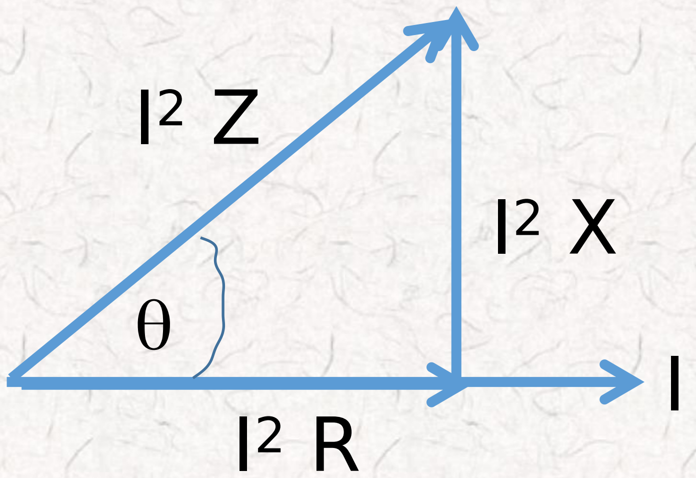
I^2 Z: Apparent Power (S), Unit is Volt-amperes, (VA)

S=P+jQ

POWER TRIANGLE



Impedance Phasor



Impedance Phasor

$P = VI \cos\theta$ (W)

$Q = VI \sin\theta$ (VAr)

$S = VI$ (VA)

$S=P+jQ$

Example 32. *A resistance 12 Ω, an inductance of 0.15 H and a capacitance of 100 μF are connected in series across a 100 V, 50 Hz supply. Calculate :*

(i) *The current.*

(ii) *The phase difference between current and the supply voltage.*

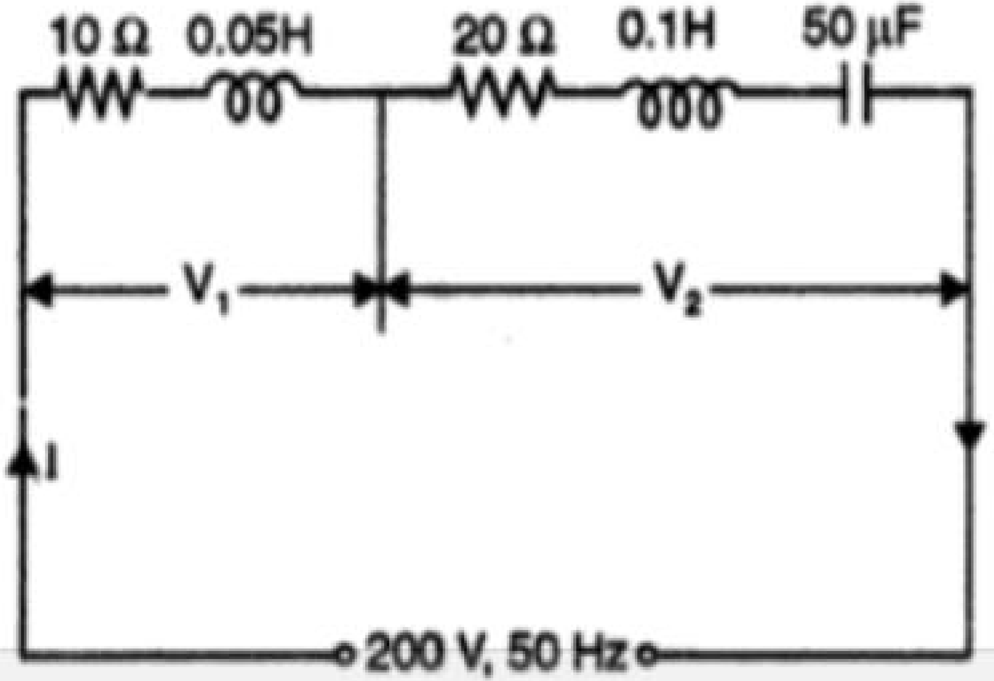
(iii) *Power consumed.*

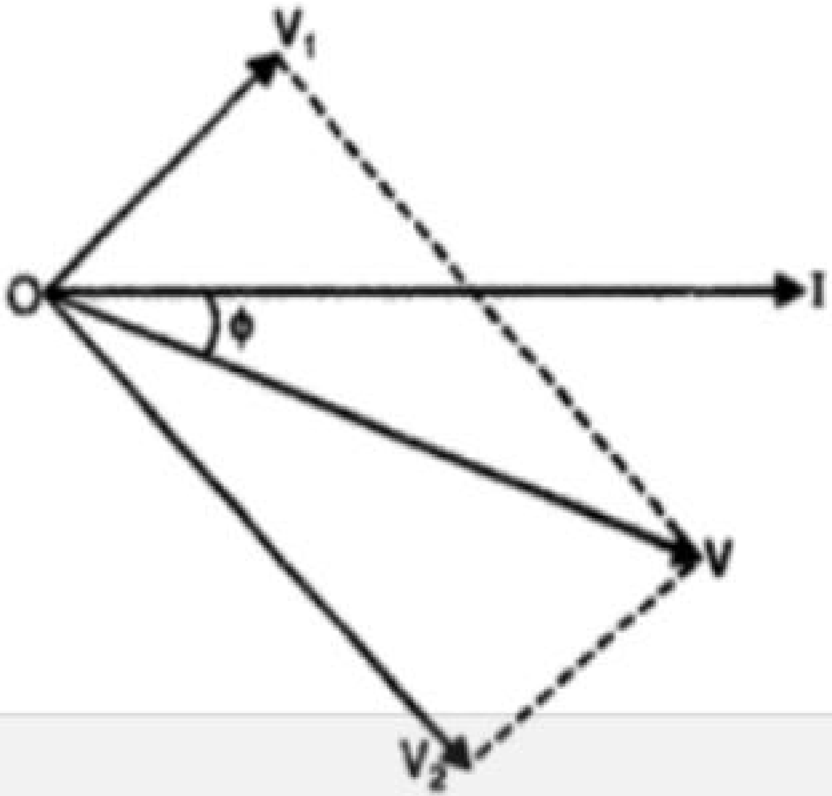
Draw the vector diagram of supply voltage and the line current.

Example 33. *For the circuit shown in Fig. 50 find the values of (i) current I, (ii) V_1 and V_2 and (iii) p.f.*

Draw the vector diagram. **(Bangalore University)**

Solution. Refer Fig. 50



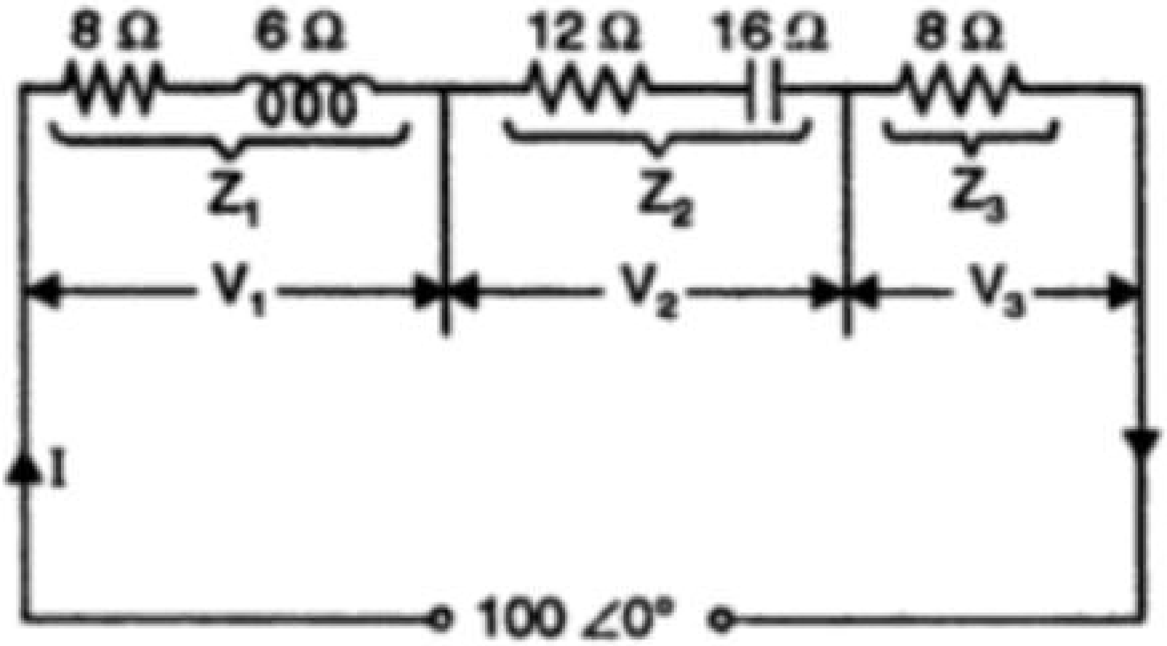


Example 34. *For the circuit shown in Fig. 51. Calculate :*

(*i*) *Current ;* (*ii*) *Voltage drops* V_1, V_2 *and* V_3 *;*

(*iii*) *Power absorbed by each importance ;* (*iv*) *Total power absorbed by the circuit.*

Take voltage vector along the reference axis.



Example 35. *Fig. 52 shows a circuit connected to a 230 V, 50 Hz supply. Determine the following :*

(i) Current drawn *(ii) Voltages V_1 and V_2*

(iii) Power factor.

Draw also the phasor diagram.

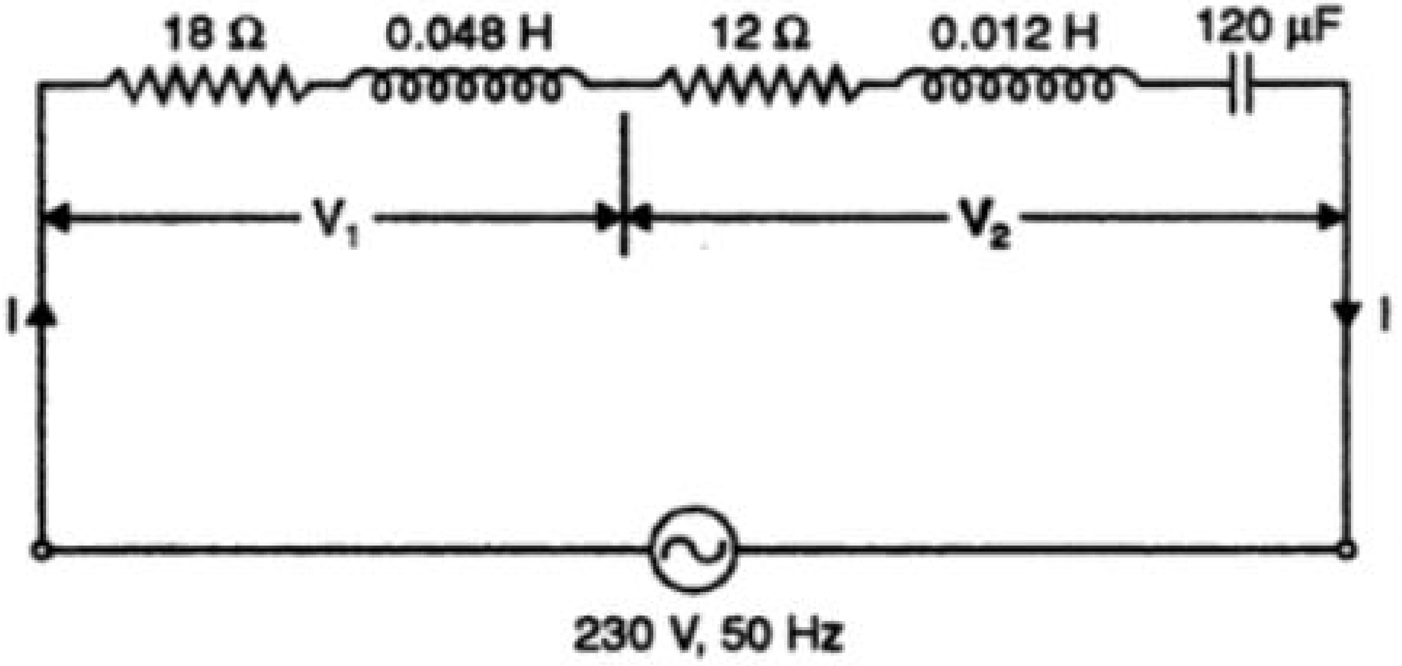
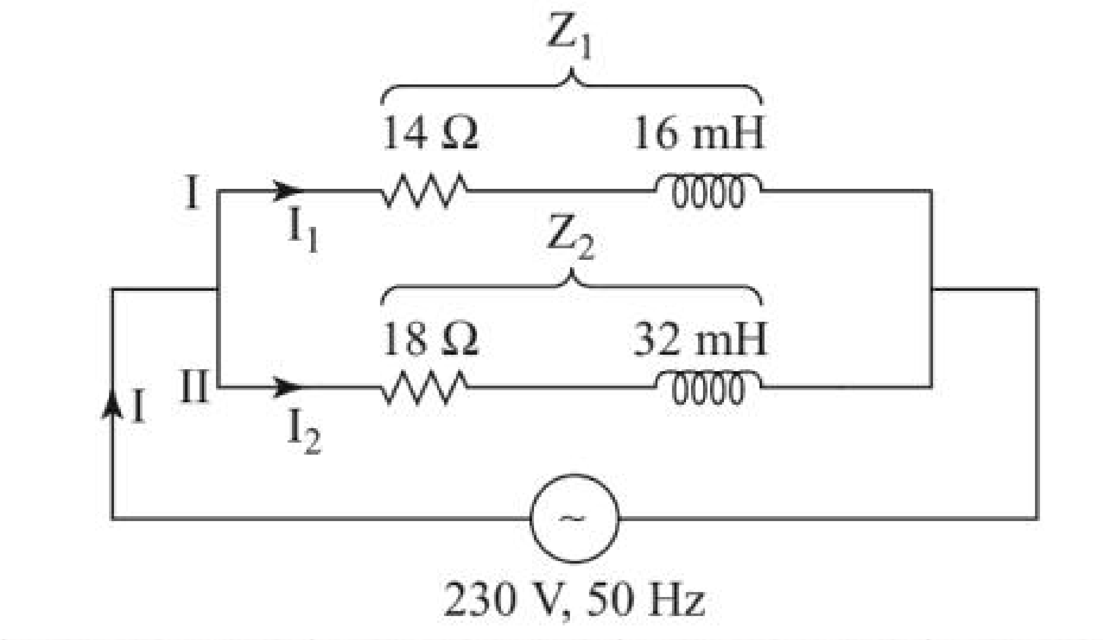


Fig. 52

(Pune University)

PARALLEL AC CIRCUITS



Calculate the following:

i) Z_1 and Z_2

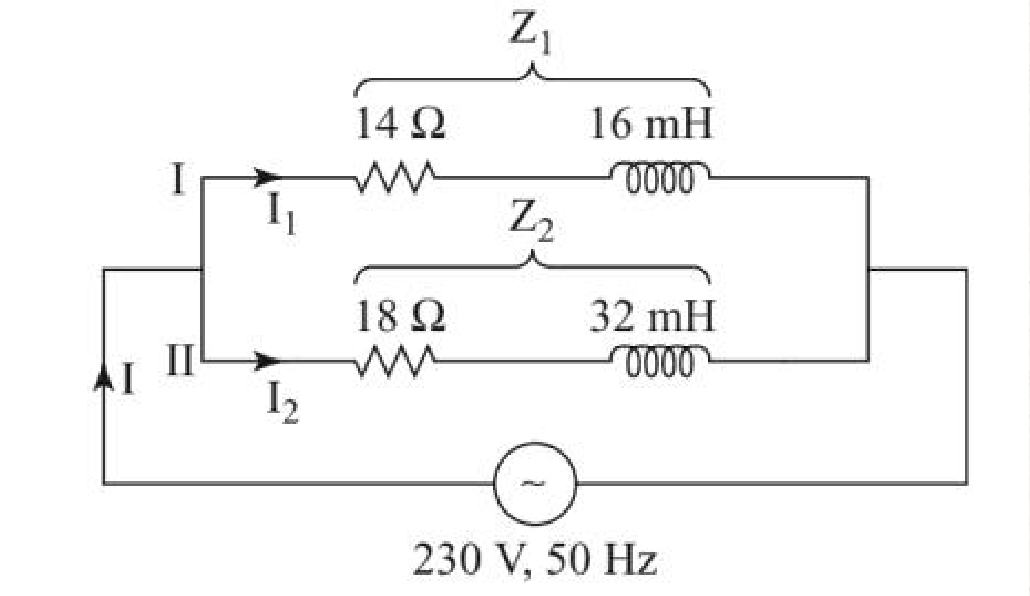
ii) Voltage drop across Z_1 and Z_2

iii) Currents I_1 and I_2

iv) Real Power, Reactive Power and Apparent Power of the circuit

v) Draw the complete phasor

PARALLEL AC CIRCUITS



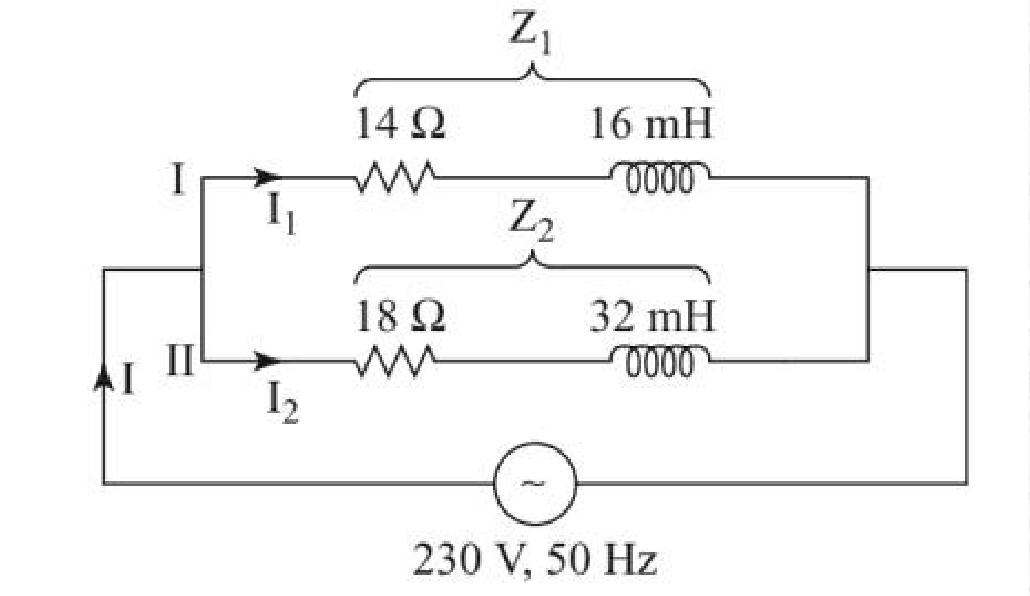
$$X_1 = 2\pi f L_1 = 2\times\pi\times 50\times 16\times 10^{-3} = 5.02\,\Omega$$

$$X_2 = 2\pi f L_2 = 2\times\pi\times 50\times 32\times 10^{-3} = 10.05\,\Omega$$

$$Z_1 = R_1 + jX_1 = 14 + j5.02\,\Omega = 14.87\angle 19.73\,\Omega$$

$$Z_2 = R_2 + jX_2 = 18 + j10.05\,\Omega = 20.62\angle 29.18\,\Omega$$

PARALLEL AC CIRCUITS

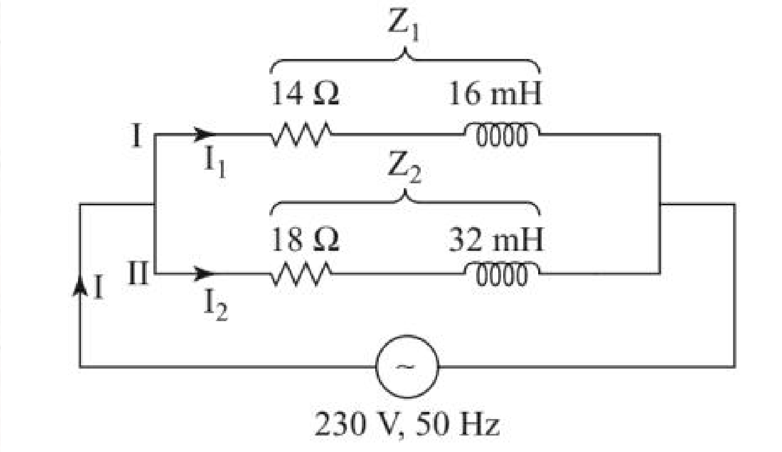


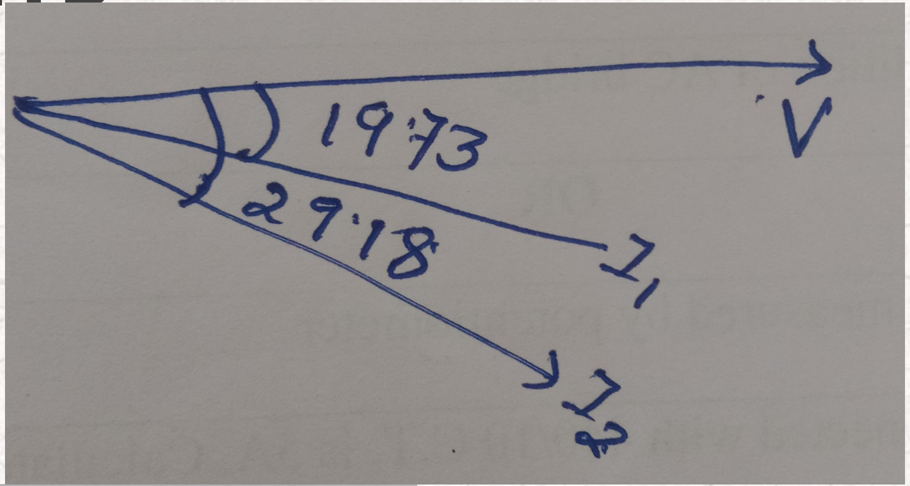
$$V_1 = V_2 = 230V$$

$$I_1 = \frac{V_1}{Z_1} = \frac{230\angle 0}{14.87\angle 19.73} = 15.47\angle -19.73\ A = 14.56 - j5.22\ A$$

$$I_2 = \frac{V_2}{Z_2} = \frac{230\angle 0}{20.62\angle 29.18} = 7.88\angle -29.18\ A = 6.88 - j3.84\ A$$

PARALLEL AC CIRCUITS





$I = I_1 + I_2 = 21.44 - j9.06 \text{ A} = 23.28 \angle -22.91 \text{ A}$

Real Power $= VI\cos\theta = 230 \times 23.28 \times \cos(22.91)$

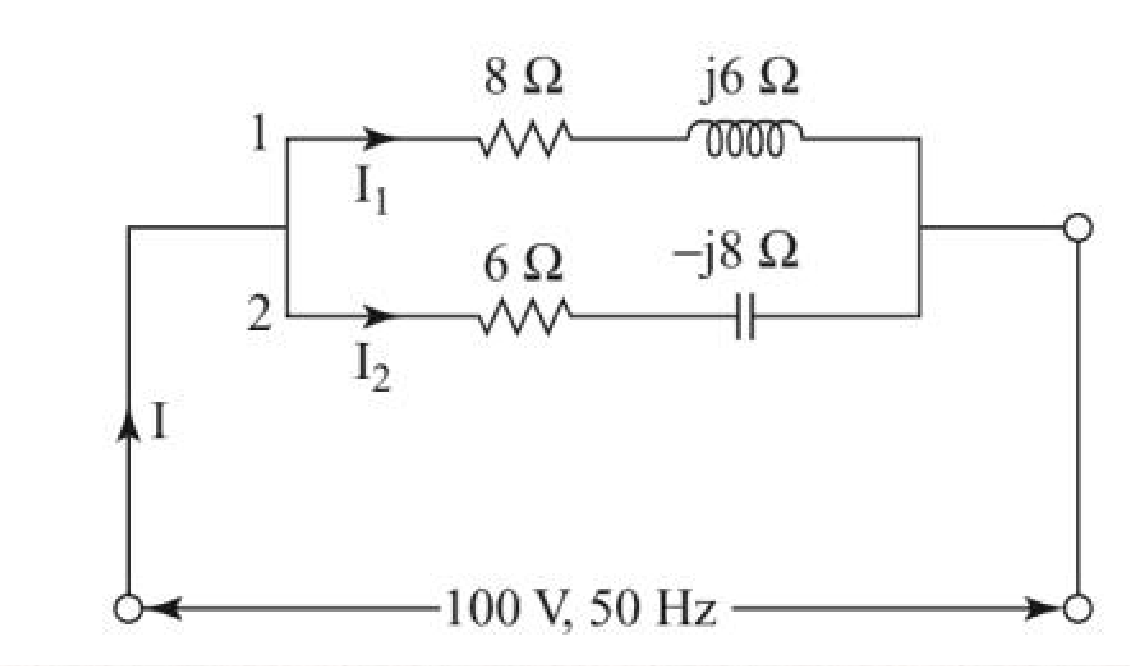
$= 4932.03$ W

Reactive Power $= VI\sin\theta = 230 \times 23.28 \times \sin(22.91)$

$= 2084.39$ VAr.

Apparent Power $= VI = 230 \times 23.28 = 5354.4$ VA.

PARALLEL AC CIRCUITS



Calculate the following:

i) Z_1 and Z_2

ii) Voltage drop across Z_1 and Z_2

iii) Currents I_1 and I_2

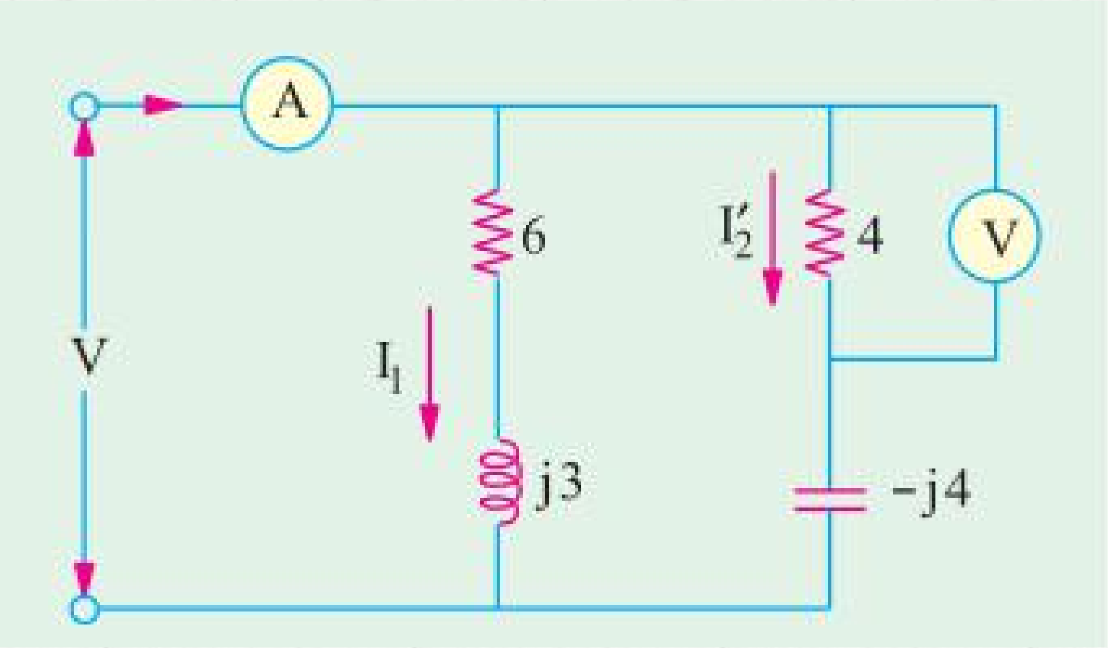
iv) Real Power, Reactive Power and Apparent Power of the circuit

v) Draw the complete phasor

PARALLEL AC CIRCUITS

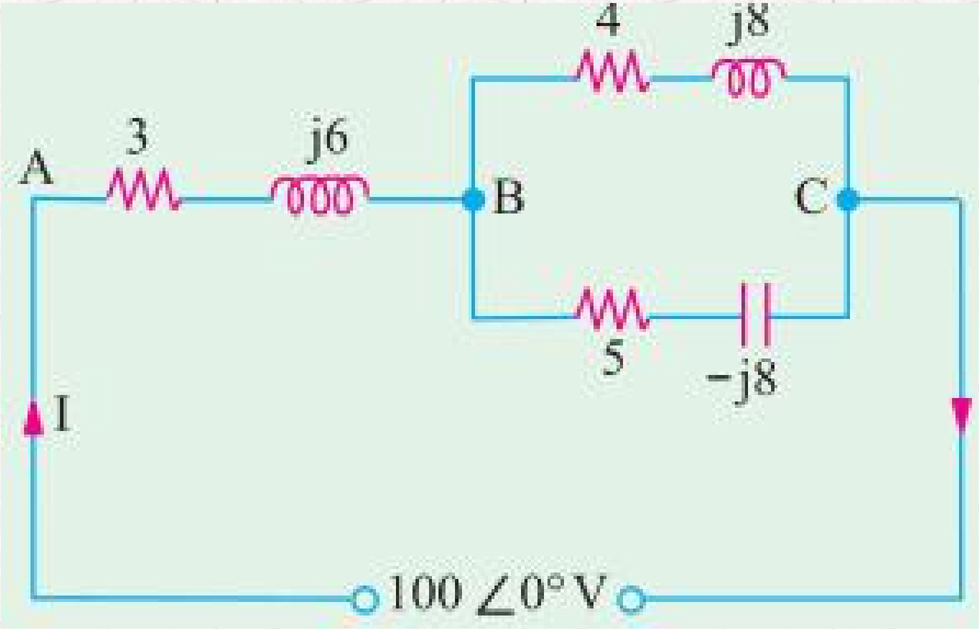
Example 14.1. *Two circuits, the impedance of which are given by $Z_1 = 10 + j\,15$ and $Z_2 = 6 - j8$ ohm are connected in parallel. If the total current supplied is 15 A, what is the power taken by each branch ? Find also the p.f. of individual circuits and of combination. Draw vector diagram.*

PARALLEL AC CIRCUITS



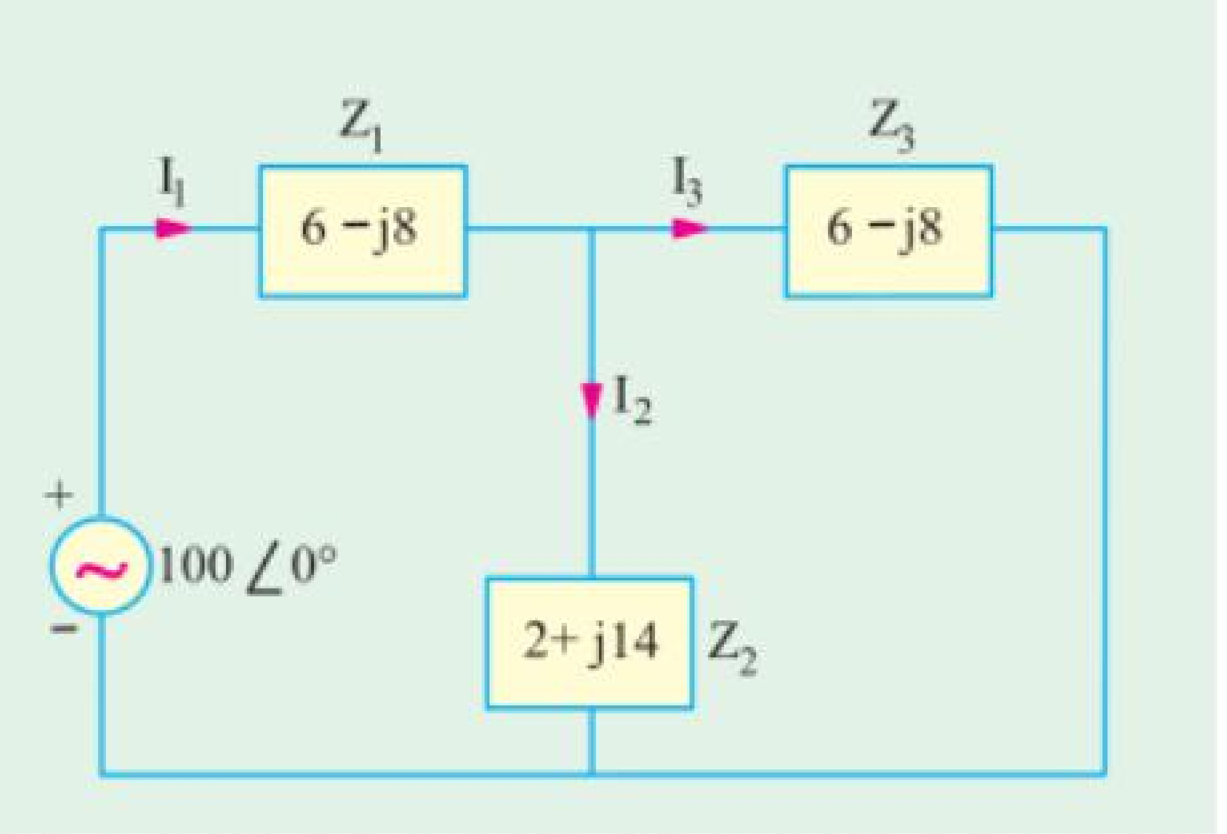
If the voltmeter in Fig. 14.14 reads 60 V, find the reading of the ammeter.

PARALLEL AC CIRCUITS



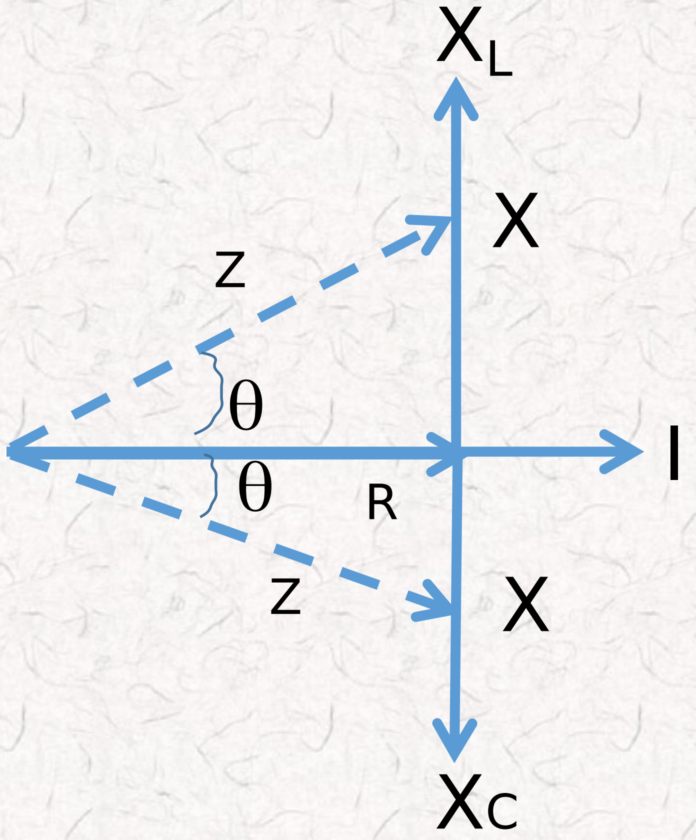
Example 14.31. *For the circuit shown in Fig. 14.37 (a), find* **(i)** *total impedance* **(ii)** *total current* **(iii)** *total power absorbed and power-factor. Draw a vector diagram.*

PARALLEL AC CIRCUITS



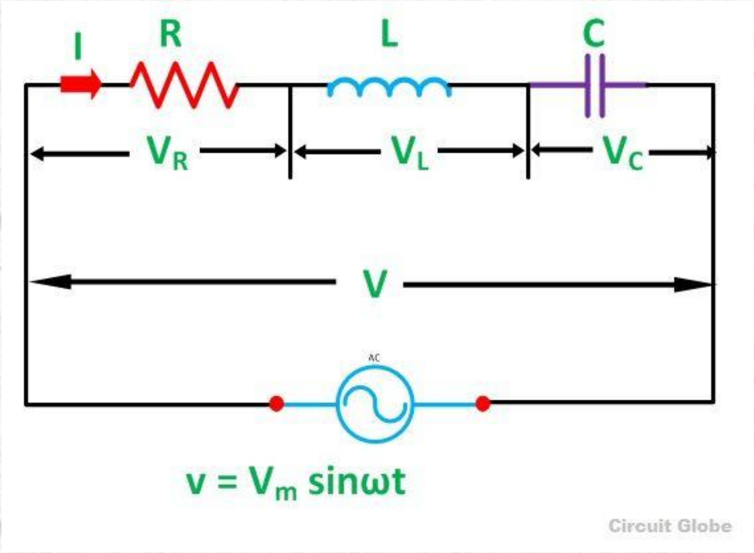
Calculate the branch currents and branch voltage

SERIES RESONANCE



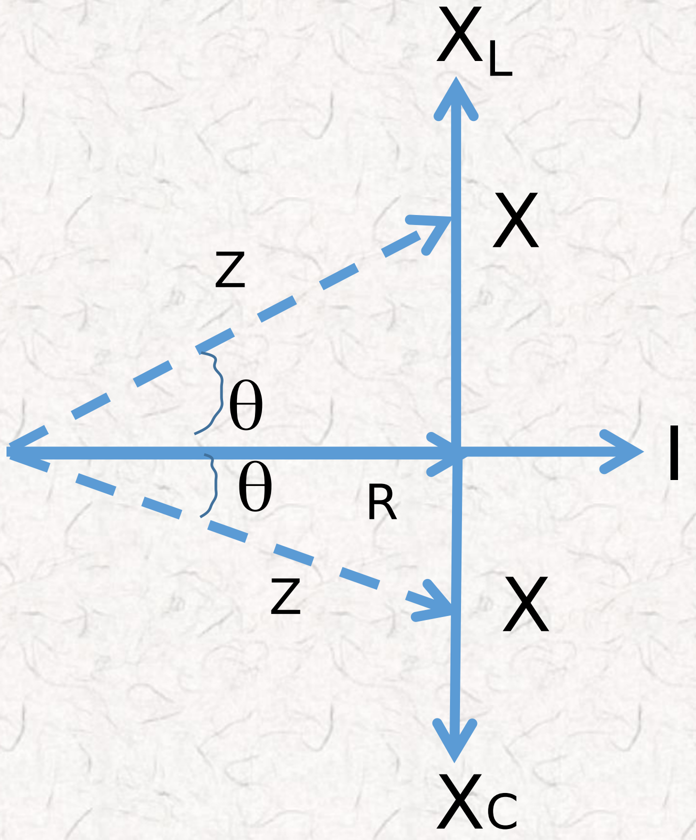
$$X = X_L \sim X_C$$

$$Z = R \pm jX$$



- Resonance is a condition in series RLC circuit in which the capacitative reactance becomes equal to inductive reactance. There by resulting in a pure resistive impedance

SERIES RESONANCE



$$X = X_L \sim X_C$$

$$Z = R \pm jX$$

- Thus at resonance

$X_L = X_C$ and so Z=R

Since at resonance

$$X_L = X_C \text{ we can write } X_L - X_C = 0$$

$$or \quad 2\pi f_0 L - \frac{1}{2\pi f_0 C} = 0$$

$$or \quad 4\pi^2 f_0^2 LC - 1 = 0$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad Hz$$

SERIES RESONANCE

Properties of the circuit at Resonance

At resonance

1. The circuit is a purely resistive circuit.
2. the value of the current is maximum since the total impedance is minumum
3. Voltage and currents are in Phase
4. Maxumum power occurs at resonance since the power factor is unity

SERIES RESONANCE

At resonance

Z=R

Power factor = 1

$Current\ at\ resonance,\ I_0 = \frac{V}{Z} = \frac{V}{R}$

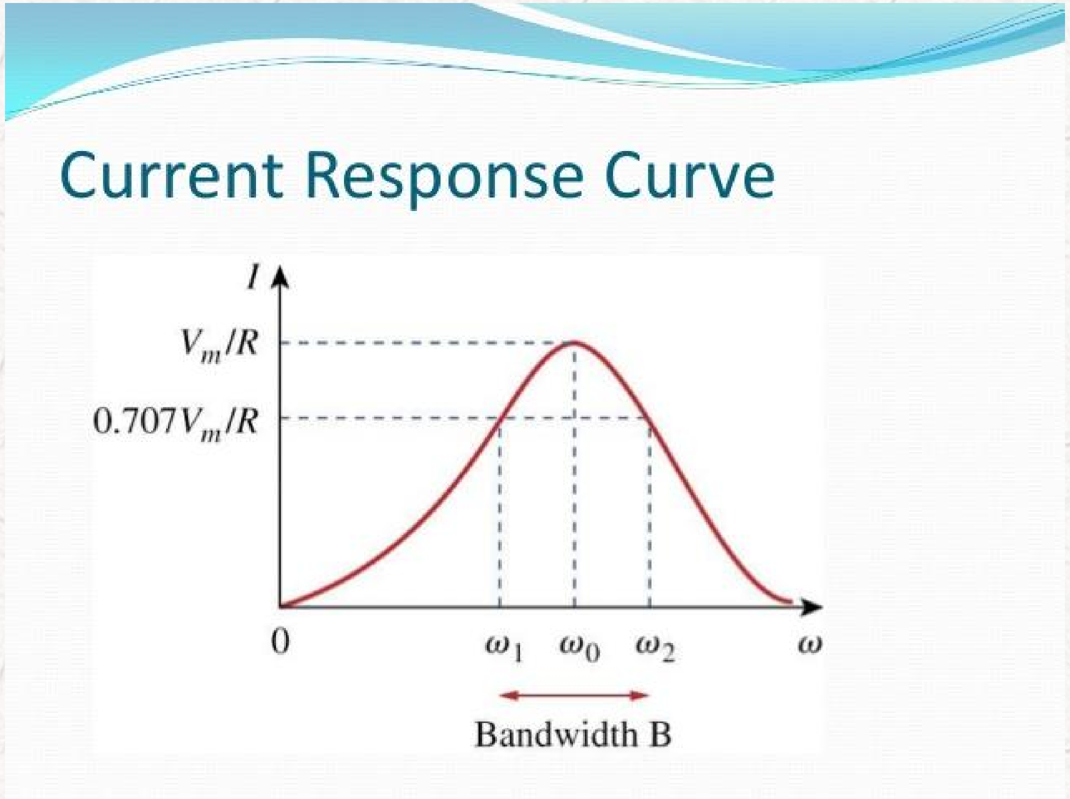
Resonance frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad Hz$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad radian/sec$$

SERIES RESONANCE

Current response at resonance



- ω_0 *is the resonance frequency*
- *At this frequency the current is maximum*

$$I_m = \frac{V}{R}$$

- ω_1 *and* ω_2 *are called half power frequencies*
- *At these frequencies the current through the circuit is* $0.707\, I_m = \frac{1}{\sqrt{2}} I_m$

SERIES RESONANCE

Current response at resonance

At resonance frequency ω_0 the current is I_m so the power will be

$$P_m = I_m^2 R$$

At frequencies ω_1 *and* ω_2 the current through the circuit is $\frac{1}{\sqrt{2}} I_m$ *so the power will be*

$$\left(\frac{1}{\sqrt{2}} I_m\right)^2 R = \frac{I_m^2 R}{2} = \frac{P_m}{2}$$

This shown that at frequencies ω_1 *and* ω_2 the power is halfed, so they are called half power frequencies

SERIES RESONANCE

Expression for half power frequencies

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \text{rad/s} \qquad \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \text{rad/s}$$

SERIES RESONANCE

Band width

Bandwidth, β, is defined as the difference between the two half power frequencies

$$\beta = \omega_2 - \omega_1 = \frac{R}{L} \quad rad/sec$$

Quality Factor

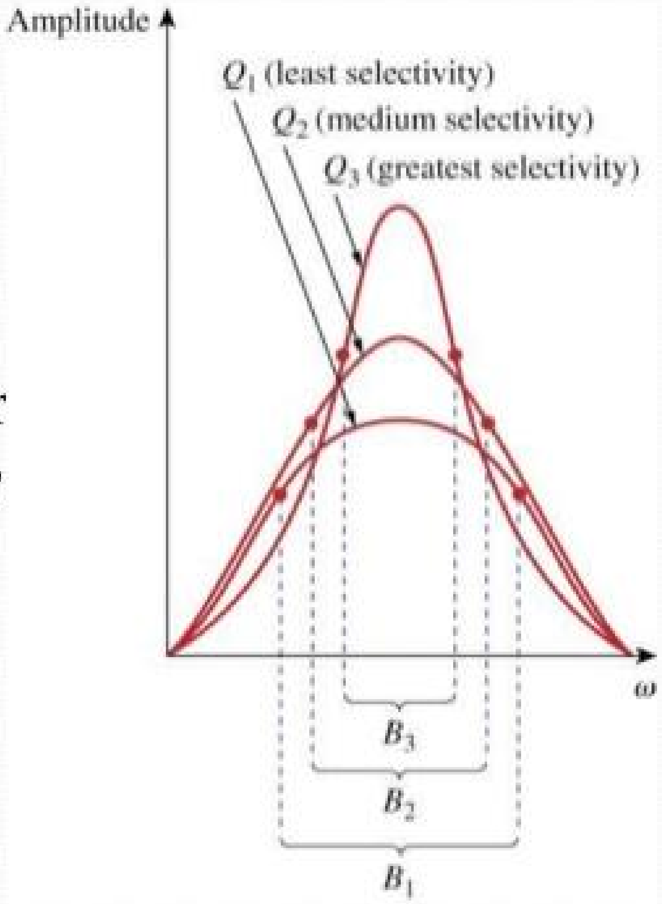
It is defined as the ratio of resonance frequency to bandwidth.

$$Q = \frac{\omega_0}{\beta} = \frac{\omega_0 L}{R}$$

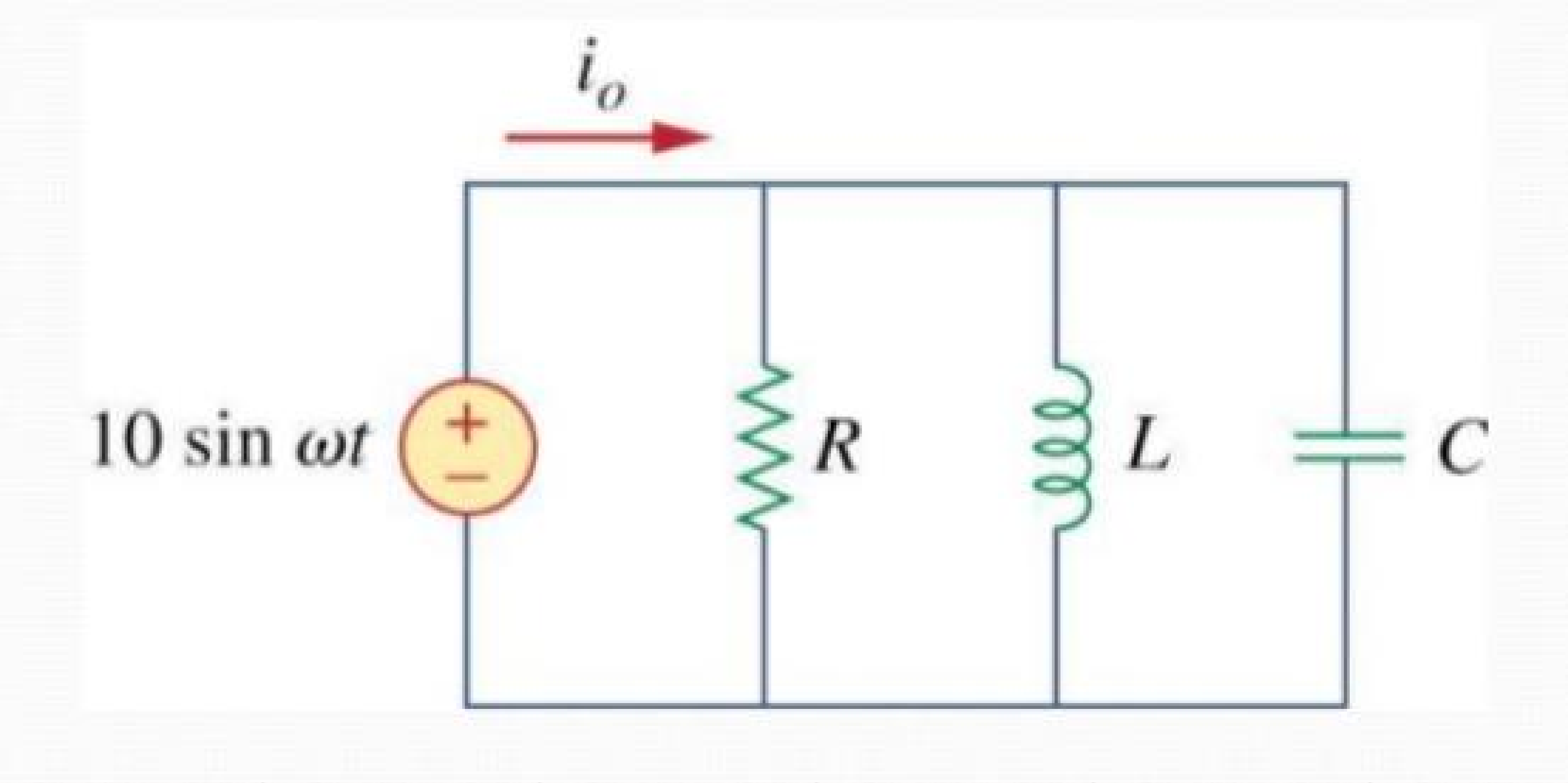
SERIES RESONANCE

Q-Factor Vs Bandwidth

- Higher value of Q, smaller the bandwidth. (Higher the selectivity)
- Lower value of Q larger the bandwidth. (Lower the selectivity)



PARALLEL RESONANCE



PARALLEL RESONANCE

The total admittance

$$Y_{Total} = Y_1 + Y_2 + Y_3$$

$$Y_{Total} = \frac{1}{R} + \frac{1}{(j\omega L)} + \frac{1}{(-j/\omega C)}$$

$$Y_{Total} = \frac{1}{R} + \frac{-j}{\omega L} + j\omega C$$

$$Y_{Total} = \frac{1}{R} + j(\omega C - 1/\omega L)$$

PARALLEL RESONANCE

- In case of parallel resonance the imaginary part of equivalent impedance or admittance is equated to zero
- So equating the imaginary part to zero we get the expression for resonance frequency.

COMPARISON BETWEEN SERIES AND PARALLEL RESONANCE

S.No.	Property	Series circuit	Parallel circuit
1.	Impedance at resonance	$Z_0 = R$, the minimum	$Z_0 = L/CR$, the maximum
2.	Current at resonance	$I_0 = V/R$, the maximum	$I_0 = V/(L/CR)$, the minimum
3.	Resonance frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$,	$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - R^2 C/L}$,
4.	Magnification of	Voltage (not affected by R)	Current (affected by R)
5.	Nature of the circuit:		
	(*i*) Below f_0	(*i*) Capacitive	(*i*) Inductive
	(*ii*) Above f_0	(*ii*) Inductive	(*ii*) Capacitive

RESONANCE

- **Problem-1:** What is the resonance frequency of a series RLC circuit with R$= 10$ ohm, L $= 25$ mH and C $= 100$μF. Also calculate the Q factor.

RESONANCE

- **Problem-2:** A series RLC circuit has inductance of 10 mH and resistance of 2 ohm. What is the value of C that will provide resonance $?$. Find the current at resonance if V$=$ 230 V and 10000 Hz.

RESONANCE

- **Problem-3:** A coil has a resistance of 20 ohm and inductance of 80 mH ans is connected in series with 100 pF capacitance. Determine the resonance frequency, the circuit impedance at resonance, current at resonance, voltage drop across L and C at resonance.

RESONANCE

- **Problem-4:** A coil is at resonance at 10kHz with a capacitance. If the resistance and inductance of the coil is 200 ohm and 5 H respectively find the Q factor.

RESONANCE

- **Problem-5:** For a series RLC circuit, R $= 10$ ohm, L $= 100$ mH and C $= 10$ μF. calculate the resonance frequency, half power frequency and band width.