INVENTORY MODELS

I. Basic EOQ Models

II. Quantity Discounts

III. Economic Lot Size (ELS)

Inventory

Inventory is any stored resource that is used to satisfy a current or future need.

Raw materials, work-in-process, and finished goods are examples of inventory. Inventory levels for finished goods, such as clothes dryers, are a direct function of market demand.

Inventory

By using this demand information, it is possible to determine how much raw materials (e.g., sheet metal, paint, and electric motors in the case of clothes dryers) and work-in-process are needed to produce the finished product.

Types of Inventory

Raw Materials Purchased but not processed Work-In-Process Undergone some change but not completed A function of cycle time for a product Maintenance/Repair/Operating (MRO) Necessary to keep machinery and processes productive Finished Goods

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Completed product awaiting shipment

The Material Flow Cycle



Importance of Inventory Control

Inventory control serves several important functions and adds a great deal of flexibility to the operation of a firm.

Five main uses of Inventory are as follows:

- 1. Decoupling Function
- 2. Storing Resources
- 3. Irregular Supply and Demand
- 4. Quantity Discounts

5. Avoiding Stockouts and Shortages

Inventory Control Decisions

Even though there are literally millions of different types of products manufactured in our society, there are only two fundamental decisions that you have to make when controlling inventory:

1. How Much to Order

2. When to Order

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Purpose of Inventory Models

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The purpose of all inventory models is to determine how much to order and when to order. As we know, inventory fulfills many important functions in an organization. But as the inventory levels go up to provide these functions, the cost of storing and holding inventory also increases. Thus, we must reach a fine balance in establishing inventory levels.

A major objective in controlling inventory is to minimize total inventory costs.

Components of Total Cost

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Some of the most significant inventory costs are as follows:

- 1. Cost of the items
- 2. Cost of ordering
- 3. Cost of carrying, or holding, inventory
- 4. Cost of stockouts

5. Cost of safety stock, the additional inventory that may be held to help avoid stockouts

Holding, Ordering, and Setup Costs

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Holding Cost

The cost of holding or "carrying" inventory over time.

Ordering Cost

The cost of placing an order and receiving goods.

Setup Cost

The cost to prepare a machine or process for manufacturing an order.

Inventory Cost Factors

ORDERING COST FACTORS

Developing and sending purchase orders Processing and inspecting incoming inventory Bill paying Inventory inquiries Utilities, phone bills, and so on for the purchasing department Salaries and wages for purchasing department employees Supplies such as forms and paper for the purchasing department

CARRYING COST FACTORS Cost of capital Taxes Insurance Spoilage Theft Obsolescence Salaries and wages for warehouse employees Utilities and building costs for the warehouse Supplies such as forms and papers for the warehouse

Independent vs Dependent Demand

Independent Demand

The demand for item is independent of the demand for any other item in inventory.

Dependent Demand

The demand for item is dependent upon the demand for some other item in the inventory.

INVENTORY MODELS

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I. Basic EOQ Models

Basic EOQ Model

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The Economic Order Quantity (EOQ) model is one of the oldest and most commonly known inventory control techniques.

Research on its use dates back to a 1915 publication by Ford W. Harris. This model is still used by a large number of organizations today.

This technique is relatively easy to use, but it makes a number of assumptions.

Basic EOQ Assumptions

Demand is known and constant.

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The lead time - that is, the time between the placement of the order and the receipt of the order - is known and constant.

The receipt of inventory is instantaneous. In other words, the inventory from an order arrives in one batch, at one point in time.

Basic EOQ Assumptions

Quantity discounts are not possible.

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The only variable costs are the cost of placing an order, ordering cost, and the cost of holding or storing inventory over time, carrying, or holding, cost.

If orders are placed at the right time, stockouts and shortages can be avoided completely.

Basic EOQ Assumptions Summary

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Important Assumptions:

- 1. Demand is known, constant, and independent
- 2. Lead time is known and constant
- 3. Receipt of inventory is instantaneous and complete
- 4. Quantity discounts are not possible
- 5. Only variable costs are setup and holding
- 6. Stockouts can be completely avoided

Inventory Usage Over Time



Inventory Usage Over Time

[Refer to Graph in Slide 19]

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With these assumptions, inventory usage has a sawtooth shape. In the graph, Q represents the amount that is ordered. If this amount is 500 units, all 500 units arrive at one time when an order is received. Thus, the inventory level jumps from 0 to 500 units. In general, the inventory level increases from 0 to Q units when an order arrives.

Inventory Usage Over Time

[Refer to Graph in Slide 19]

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Because demand is constant over time, inventory drops at a uniform rate over time. Another order is placed such that when the inventory level reaches 0, the new order is received and the inventory level again jumps to Q units, represented by the vertical lines. This process continues indefinitely over time.

Objective of Basic EOQ Model

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The objective of most inventory models is to minimize the total cost. With the assumptions just given, the significant costs are the ordering cost and the inventory carrying cost. All other costs, such as the cost of the inventory itself, are constant. Thus, if we minimize the sum of the ordering and carrying costs, we also minimize the total cost.

Total Cost as a Function of Order Quantity



Objective of Basic EOQ Model

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[Refer to Graph in Slide 23]

To help visualize this, Slide 23 graphs total cost as a function of the order quantity, Q. As the value of Q increases, the total number of orders placed per year decreases. Hence, the total ordering cost decreases. However, as the value of Q increases, the carrying cost increases because the firm has to maintain larger average inventories.

Objective of Basic EOQ Model

[Refer to Graph in Slide 23]

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The optimal order size, Q*, is the quantity that minimizes the total cost. Note in Slide 23 that Q* occurs at the point where the ordering cost curve and the carrying cost curve intersect. This is not by chance. With this particular type of cost function, the optimal quantity always occurs at a point where the ordering cost is equal to the carrying cost.

Average Inventory Level

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Now that we have a better understanding of inventory costs, let us see how we can determine the value of Q^* that minimizes the total cost. In determining the *annual* carrying cost, it is convenient to use the average inventory.

Referring to Slide 19, we see that the on-hand inventory ranges from a high of Q units to a low of zero units, with a uniform rate of decrease between these levels. Thus, the average inventory can be calculated as the average of the minimum and maximum inventory levels.

Average Inventory Level

That is:

Average Inventory Level = (0 + Q2)/2 = Q/2

We multiply this average inventory by a factor called the **Annual Inventory Carrying Cost Per Unit** to determine the annual inventory cost.

The EOQ Model



P = Purchase Cost per Unit of the Inventory Item

EOQ - Annual Setup Cost

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- Q = Number of Units in each Order
- Q* = Optimal Number of Pieces per Order (EOQ)
 - D = Annual Demand in Units for the Inventory Item
 - S = Setup or Ordering Cost per Order
 - H = Holding or Carrying Cost per Unit per Year

Annual Setup Cost = (Number of Orders Placed per Year) x (Setup or Order Cost per Order)



EOQ - Annual Holding Cost

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- Q = Number of Units in each Order
- Q* = Optimal Number of Pieces per Order (EOQ)
 - D = Annual Demand in Units for the Inventory Item
 - S = Setup or Ordering Cost per Order
 - H = Holding or Carrying Cost per Unit per Year

Annual Holding Cost =(Average Inventory Level)

x (Holding Cost per Unit per Year)

$$= \left(\frac{\text{Order Quantity}}{2} \right) \text{(Holding Cost per Unit per Year)}$$
$$= \left(\frac{\mathbf{Q}}{2} \right) \text{(H)}$$

The EOQ Model Annual Setup Cost =

Annual Holding Cost =

- Q = Number of Units in each Order
- Q* = Optimal Number of Pieces per Order (EOQ)
 - D = Annual Demand in Units for the Inventory Item
 - S = Setup or Ordering Cost per Order
 - H = Holding or Carrying Cost per Unit per Year

Optimal Order Quantity is found when Annual Setup Cost Equals Annual Holding Cost

$$\frac{D}{Q}S = \frac{Q}{2}H$$

Solving for Q* (EOQ)

 $Q^2 = 2DS/H$

 $2DS = Q^2H$

$$Q^* = \sqrt{2DS/H}$$

EOQ Example

Let us now apply these formulas to the case of SBC, a company that buys alarm clocks from a manufacturer and distributes to retailers. SBC would like to reduce its inventory cost by determining the optimal number of alarm clocks to obtain per order. The annual demand is 1,000 units, the ordering cost is \$10 per order, and the carrying cost is \$0.50 per unit per year. Each alarm clock has a purchase cost of \$5. How many clocks should SBC order each time?

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EOQ Example

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Determine the Optimal Number of Alarm Clocks to Order

- D = 1,000 Units
- S = \$10 per Order
- H = \$0.50 per Unit per Year

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$Q^* = \sqrt{\frac{2(1,000)(10)}{0.50}} = \sqrt{40,000} = 200 \text{ units}$$

EOQ - Expected Number of Orders

D = 1,000 units S = \$10 per order H = \$0.50 per unit per year

Expected = N = $\frac{Demand}{EOQ}$ = $\frac{D}{Q^*}$ Orders N = $\frac{1,000}{200}$ = 5 Orders per Year

EOQ - Expected Time Between Orders

D = 1,000 units $Q^* = 200$ units S = \$10 per orderN = 5 Orders per Year H =\$0.50 per unit per year **250 Working Days per Year** Number of Working **Expected Time** = T = **Between Orders** Days per Year N $T = \frac{250}{5} = 50$ Days Between Orders

 $TBO_{EOQ} = \frac{EOQ}{D}$ (Working Days per Year)

EOQ - Total Cost



= (D/Q* x S) + (Q*/2 x H) + (P x D)

EOQ - Total Cost

Total Cost =

Total Setup/Ordering Cost + Total Holding/Carrying Cost + Total Purchase Cost (D/Q* x S) (Q*/2 x H) (P x D)

Observe that the total purchase cost ($P \times D$) does not depend on the value of Q. This is so because regardless of how many orders we place each year, or how many units we order each time, we will still incur the same annual total purchase cost.

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EOQ - Total Cost

38						
D=1	,000 Units		Q* =	200 un	nits	
S = \$10 per Order $N = 5 orders per year$						
H = S	60.50 per Unit	per	Year T =	50 day	/S	
P = \$5						
Total Annual Cost						
=	Setup Cost	+	Holding Cost	+	Purchase Cost	
TC =	D Q*S	+	<u>_Q*</u> 2 Н	+	(P x D)	
TC =	<u>1,000</u> (\$10) 200	+	<u>200</u> (\$.50) 2	+	\$5 x 1000	
TC =	\$50	+	\$50	+	\$5,000	

TC = \$5,100

Purchase Cost of Inventory Items

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It is often useful to know the value of the average inventory level in monetary terms. We know that the average inventory level is Q/2, where Q is the order quantity. If we order Q* (the EOQ) units each time, the value of the average inventory can be computed by multiplying the average inventory by the unit purchase cost, P. That is:

Average Monetary Value of Inventory = $P \times (Q^*/2)$

Purchase Cost of Inventory Items

- D = 1,000 Units $Q^* = 200$ units
- S = \$10 per Order N = 5 orders per year
- H =\$0.50 per Unit per Year T =50 days
- P = \$5

Average Monetary Value of Inventory

- $= P \times (Q^*/2)$
- = \$5 x (200/2)

= \$500

Calculating the Ordering Costs (S) and Carrying Cost (H) for a Given Value of EOQ

EOQ Formula: $Q^* = \sqrt{2DS/H}$

In using these formulas, we assumed that the values of the Ordering Cost (S) and Carrying Cost (H) are known constants.

$$S = Q^{* 2} \times H/(2D)$$

H = 2DS/Q* ²

Calculating the Ordering Costs (S) and Carrying Cost (H) for a Given Value of EOQ

- 42
- D = 1,000 Units
- S = \$10 per Order
- H = \$0.50 per Unit per Year P = \$5
- $Q^* = 200$ units
 - N = 5 orders per year
 - T = 50 days

- $S = Q^{*2} \times H/(2D)$
 - $= 200^2 \times (\$0.50/2 \times 1,000)$
 - = 40,000 x 0.00025
 - = \$10 per Order

- $H = 2DS/Q^{* 2}$
 - $= 2 \times (1,000 \times 10) / 200^{2}$
 - = 20,000 / 40,000
 - = \$0.50 per Unit per Year

Reorder Point (ROP)

Now that we have decided how much to order, we look at the second inventory question: when to order. In most simple inventory models, it is assumed that we have instantaneous inventory receipt. That is, we assume that a firm waits until its inventory level for a particular item reaches zero, places an order, and receives the items in stock immediately.

Reorder Point (ROP)

In many cases, however, the time between the placing and receipt of an order, called the Lead Time, or Delivery Time, is often a few days or even a few weeks. Thus, the when to order decision is usually expressed in terms of a reorder point (ROP), the inventory level at which an order should be placed.

Reorder Point (ROP)

□ EOQ answers the "How Much" question.

The Reorder Point (ROP) tells "When" to order.

$$ROP = \begin{pmatrix} Demand \\ per Day \end{pmatrix} \begin{pmatrix} Lead Time for a New \\ Order in Days \end{pmatrix}$$
$$= d \times L$$

Where d = D Number of Working Days in a Year

Reorder Point Curve



Reorder Point Example

Demand = 1,000 Alarm Clocks per Year 250 Working Days in the Year Lead Time for Orders is 3 Working Days

> d = Number of Working Days in a Year = 1,000/250= 4 Units per Day $ROP = d \times L$ = 4 Units per Day x 3 Days = 12 Units

INVENTORY MODELS

II. Quantity Discounts

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To increase sales, many companies offer quantity discounts to their customers. A quantity discount is simply a decreased unit cost for an item when it is purchased in larger quantities. It is not uncommon to have a discount schedule with several discounts for large orders. See example below:

Discount Number	Discount Quantity	Discount	Discount Cost
1	0 to 999	0%	\$5.00
2	1,000 to 1,999	4%	\$4.80
3	2,000 and over	5%	\$4.75

As can be seen in Slide 49 Table, the normal cost for the item in this example is \$5. When 1,000 to 1,999 units are ordered at one time, the cost per unit drops to \$4.80, and when the quantity ordered at one time is 2,000 units or more, the cost is \$4.75 per unit. As always, management must decide when and how much to order. But with quantity discounts, how does a manager make these decisions?

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As with previous inventory models discussed so far, the overall objective is to minimize the total cost. Because the unit cost for the third discount in Slide 49 Table is lowest, we might be tempted to order 2,000 units or more to take advantage of this discount. Placing an order for that many units, however, might not minimize the total inventory cost. As the discount quantity goes up, the item cost goes down, but the carrying cost increases because the order sizes are large. Thus, the major trade-off when considering quantity discounts is between the reduced item cost and the increased carrying cost.

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Recall that we computed the total cost (including the total purchase cost) for the EOQ model as follows:

Total Annual Cost

= Setup Cost + Holding Cost + Purchase Cost = $\frac{D}{Q^*}S$ + $\frac{Q^*}{2}H$ + (P x D)

Next, we illustrate the four-step process to determine the quantity that minimizes the total cost.

4 Steps to Analyze Quantity Discount Models

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- For each discount price, calculate a Q* value, using the EOQ formula. In quantity discount EOQ models, the unit carrying cost, H, is typically expressed as a percentage (I) of the unit purchase cost (P). That is, H = I x P. As a result, the value of Q* will be different for each discounted price.
- 2. For any discount level, if the Q* computed in step 1 is too low to qualify for the discount, adjust Q* upward to the lowest quantity that qualifies for the discount. For example, if Q* for discount 2 in Slide 49 Table turns out to be 500 units, adjust this value up to 1,000 units.

Total Cost Curve for the Quantity Discount Model



Total Cost Curve for the Quantity Discount Model

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As seen in Slide 54, the total cost curve for the discounts shown in Slide 49 is broken into three different curves. There are separate cost curves for the first ($0 \le Q \le 999$), second (1,000 $\le Q \le 1,999$), and third ($Q \ge 2,000$) discounts. Look at the total cost curve for discount 2. The Q* for discount 2 is less than the allowable discount range of 1,000 to 1,999 units. However, the total cost at 1,000 units (which is the minimum quantity needed to get this discount) is still less than the lowest total cost for discount 1.

Thus, step 2 is needed to ensure that we do not discard any discount level that may indeed produce the minimum total cost. Note that an order quantity compute in step 1 that is greater than the range that would qualify it for a discount may be discarded.

4 Steps to Analyze Quantity Discount Models

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- 3. Using the Total Cost Equation, compute a total cost for every Q* determined in steps 1 and 2. If a Q* had to be adjusted upward because it was below the allowable quantity range, be sure to use the adjusted Q* value.
- Select the Q* that has the lowest total cost, as computed in step 3. It will be the order quantity that minimizes the total cost.

Analyzing a Quantity Discount Summary

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1. For each discount, calculate Q*

2. If Q* for a discount doesn't qualify, choose the smallest possible order size to get the discount

3. Compute the total cost for each Q* or adjusted value from Step 2

4. Select the Q* that gives the lowest total cost

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GSB Department Store stocks toy cars. Recently, the store was given a quantity discount schedule for the cars, as shown in Slide 49. Thus, the normal cost for the cars is \$5.00. For orders between 1,000 and 1,999 units, the unit cost is \$4.80, and for orders of 2,000 or more units, the unit cost is \$4.75. Furthermore, the ordering cost is \$49 per order, the annual demand is 5,000 race cars, and the inventory carrying charge as a percentage of cost, I, is 20%, or 0.2. What order quantity will minimize the total cost?

Discount Number	Discount Quantity	Discount	Discount Cost
1	0 to 999	0%	\$5.00
2	1,000 to 1,999	4%	\$4.80
3	2,000 and over	5%	\$4.75

D = 5,000 Units S = \$49 per Order I = 20% of Cost

$$Q^* = \sqrt{\frac{2DS}{H}}$$

 $H = I \times P$ (Cost)

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Calculate Q* for every Discount

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_{1}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}} = 700 \text{ cars/order}$$
$$Q_{2}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}} = 714 \text{ cars/order}$$
$$Q_{3}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}} = 718 \text{ cars/order}$$

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In the GSB Department Store example, observe that the Q* values for discounts 2 and 3 are too low to be eligible for the discounted prices (Slide 49 Table). They are, therefore, adjusted upward to 1,000 and 2,000, respectively.

With these adjusted Q* values, we find that the lowest total cost of \$24,725 results when we use an order quantity of 1,000 units [See Slide 61 and Slide 62].

Calculate Q* for every Discount

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_{1}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}} = 700 \text{ cars/order}$$
$$Q_{2}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}} = 714 \text{ cars/order}$$
$$1,000 - \text{ Adjusted}$$
$$Q_{3}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}} = 718 \text{ cars/order}$$
$$2,000 - \text{ Adjusted}$$

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Discount Number	Unit Price	Order Quantity	Annual Product Cost	Annual Ordering Cost	Annual Holding Cost	Total Cost
1	\$5.00	700	\$25,000	\$350.00	\$350	\$25,700.00
2	\$4.80	1,000	\$24,000	\$245.00	\$480	\$24,725.00
3	\$4.75	2,000	\$23,750	\$122.50	\$950	\$24,822.50

Choose the Price and Quantity that gives the Lowest Total Cost Buy 1,000 Units at \$4.80 per Unit

INVENTORY MODELS

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III. Economic Lot Size (ELS)

Economic Lot Size (ELS)

ELS is the quantity of material or units of a manufactured good that can be produced or purchased within the lowest unit cost range. It is determined by reconciling the decreasing unit cost of larger quantities with the associated increasing unit cost of handling, storage, insurance, interest, etc.

(www.businessdictionary.com)

Economic Lot Size (ELS)

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Economic Lot Size (ELS)

A manufacturer must determine the production lot size that will result in minimum production and storage cost.

Economic Order Quantity (EOQ)

A purchaser must decide what quantity of an item to order that will result in minimum reordering and storage cost.

ELS Model

- 67
- D = Annual Demand in Units for the Inventory Item
- S = Setup or Ordering Cost per Order
- H = Holding or Carrying Cost per Unit per Year
- p = production rate
- d = daily demand

$$ELS = \sqrt{\frac{2DS}{H}} \times \sqrt{\frac{p}{p-d}}$$

ELS - Total Annual Cost

- D = Annual Demand in Units for the Inventory Item
- S = Setup or Ordering Cost per Order
- H = Holding or Carrying Cost per Unit per Year
- p = production rate
- d = daily demand

$$C = \frac{ELS}{2} \left(\frac{p-d}{p} \right) (H) + \frac{D}{ELS} (S)$$

ELS - Time Between Orders (TBO)

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- D = Annual Demand in Units for the Inventory Item
- S = Setup or Ordering Cost per Order
- H = Holding or Carrying Cost per Unit per Year
- p = production rate
- d = daily demand

$$TBO_{ELS} = \frac{ELS}{D} (Work Days/Year)$$

ELS – Production Time per Lot

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D = Annual Demand in Units for the Inventory Item

ELS

p

- S = Setup or Ordering Cost per Order
- H = Holding or Carrying Cost per Unit per Year
- p = production rate
- d = daily demand

ELS - Example

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A plant manager of a chemical plant must determine the lot size for a particular chemical that has a steady demand of 30 barrels per day. The production rate is 190 barrels per day, annual demand is 10,500 barrels, setup cost is \$200, annual holding cost is \$0.21 per barrel, and the plant operates 350 days per year.

a. Determine the Economic Production Lot Size (ELS)

- b. Determine the Total Annual Setup and Inventory Holding Cost for this item (Total Annual Cost)
- c. Determine the time between orders (TBO), or cycle length, for the ELS
- d. Determine the Production Time per Lot

ELS - Example

a. Solving first for the ELS, we get

ELS =
$$\sqrt{\frac{2DS}{H}} \times \sqrt{\frac{p}{p-d}} = \sqrt{\frac{2(10,500)(\$200)}{\$0.21}} \sqrt{\frac{190}{190-30}} = 4,873.4$$
 barrels

b. The total annual cost with the ELS is

$$C = \frac{ELS}{2} \left(\frac{p-d}{p} \right) (H) + \frac{D}{ELS} (S)$$
$$= \frac{4,873.4}{2} \left(\frac{190-30}{190} \right) (\$0.21) + \frac{10,500}{4,873.4} (\$200)$$

= \$430.91 + \$430.91 = \$861.82

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ELS - Example

c. Applying the TBO formula to the ELS, we get

$$TBO_{ELS} = \frac{ELS}{D} (Work Days/Year) = \frac{4,873.4}{10,500} (350)$$

= 162.4 or 162 days

d. The production time during each cycle is the lot size divided by the production rate:

$$\frac{\text{ELS}}{p} = \frac{4,873.4}{190} = 25.6 \text{ or } 26 \text{ days}$$

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