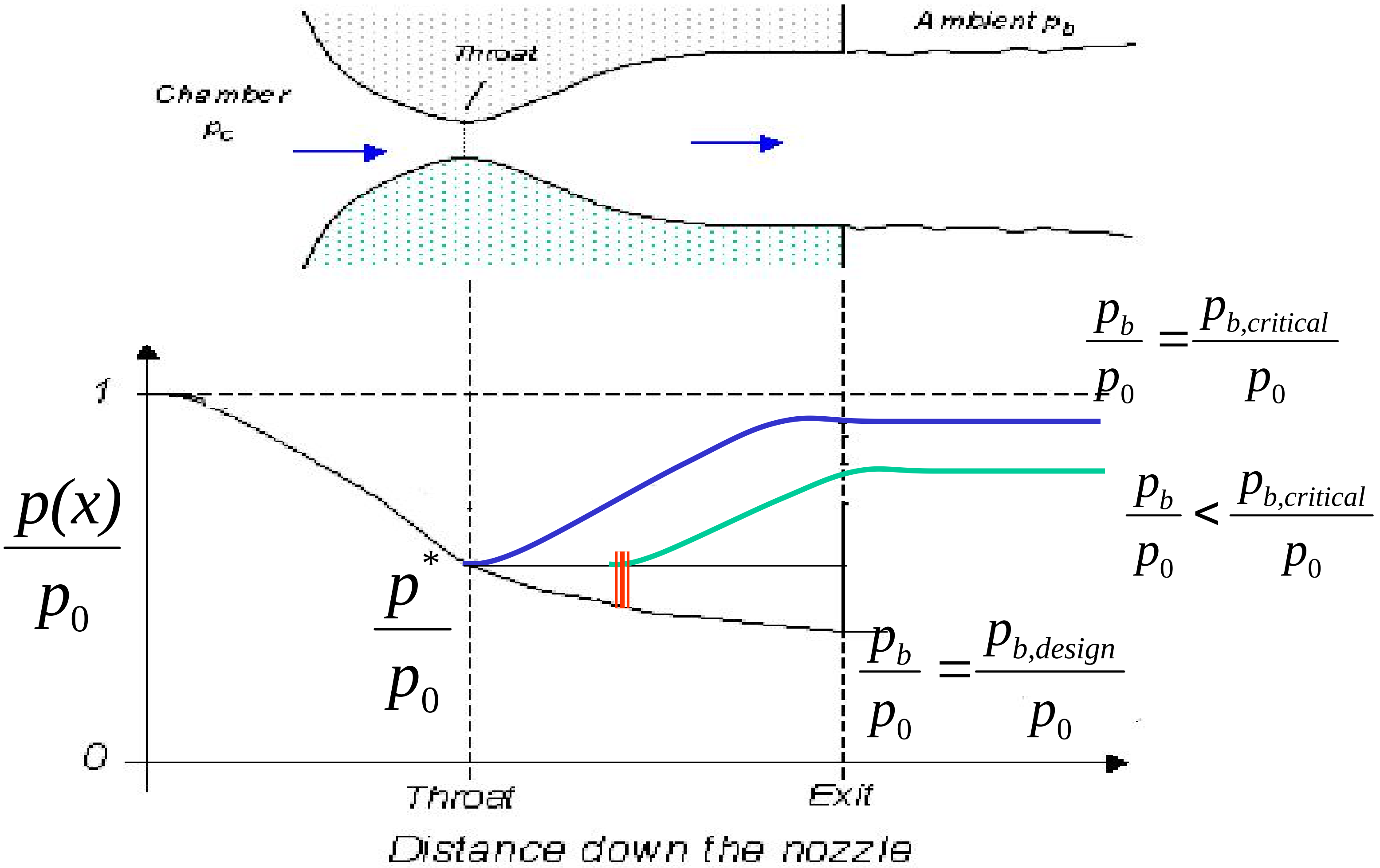
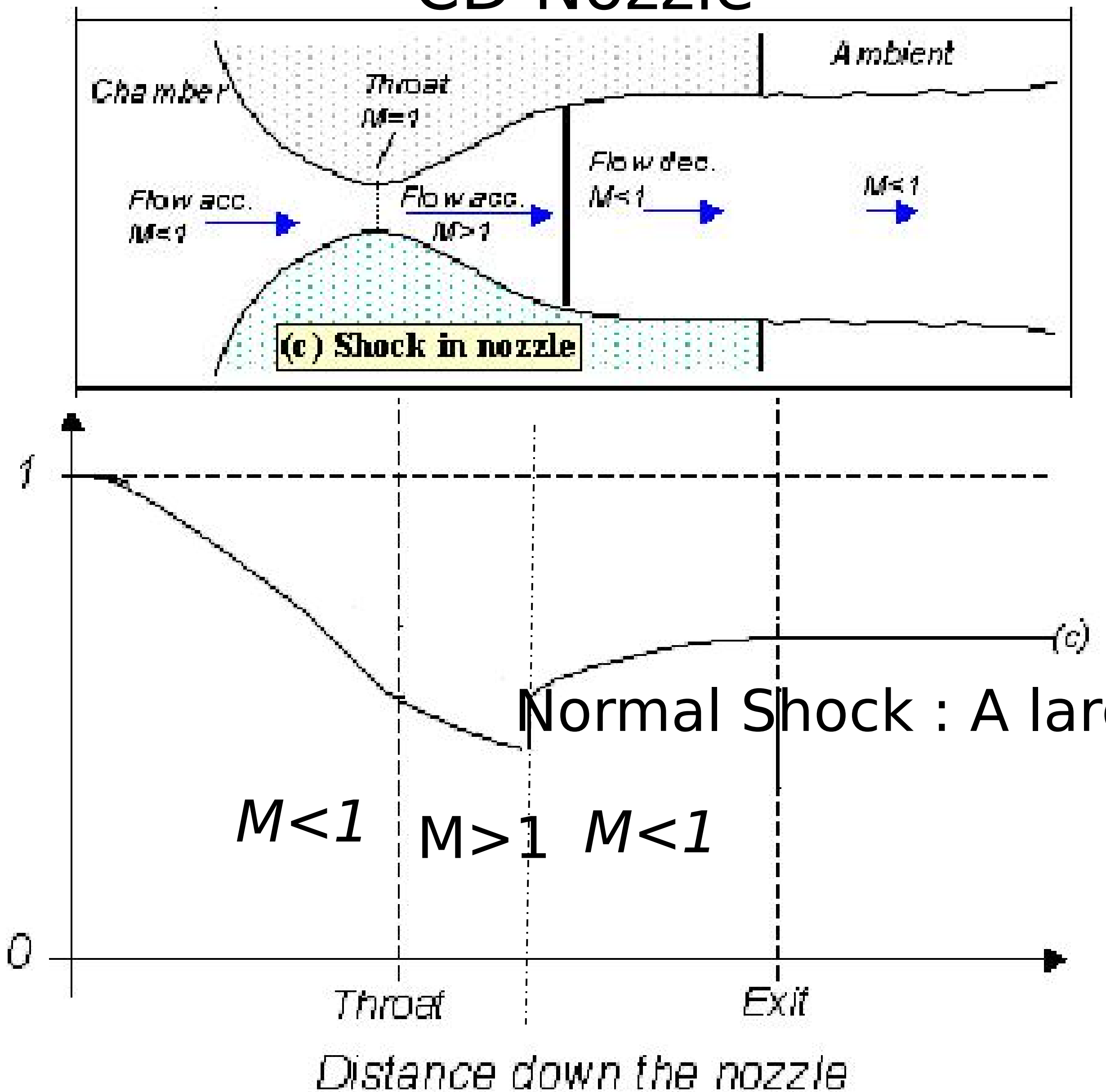
Gas Dynamics for Study of Nozzles@ Critically Off-Design Conditions

Convergent-Divergent Nozzle : $p_{b,design} < p_{bt} < p_{b,critical}$

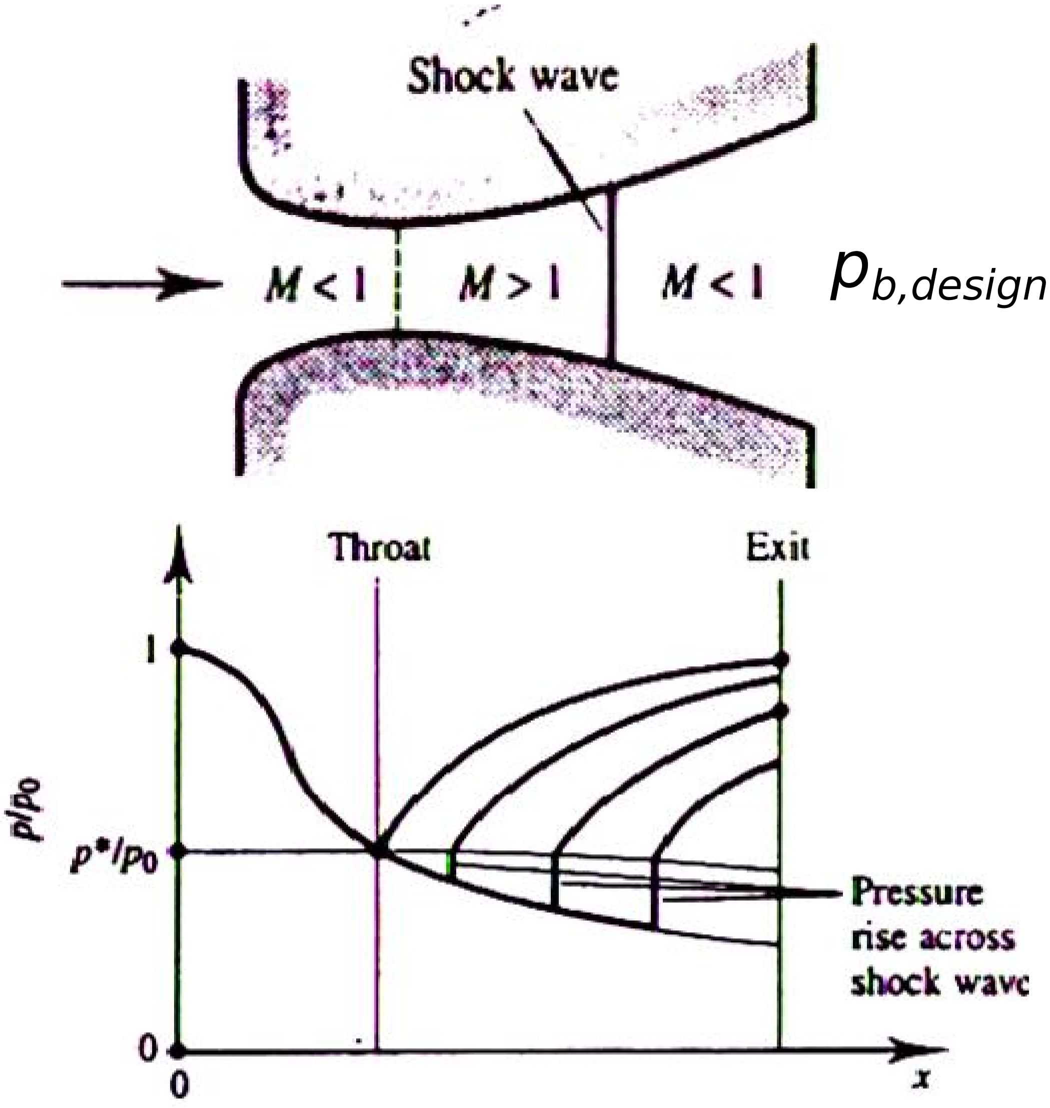


Standing Normal Shock in Diverging Section of CD Nozzle



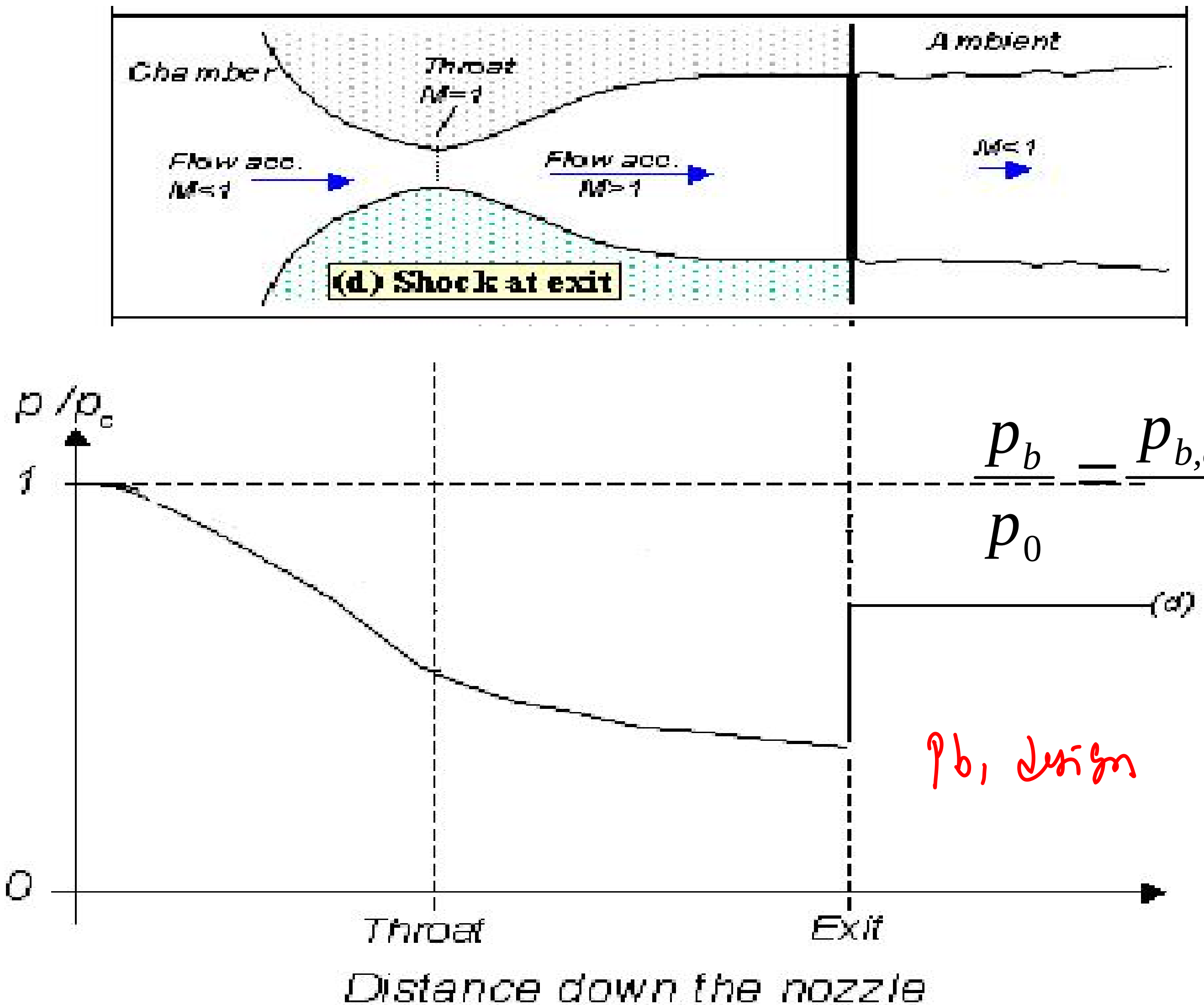
Normal Shock : A large disconti

Movement of Normal Shock Towards Exit



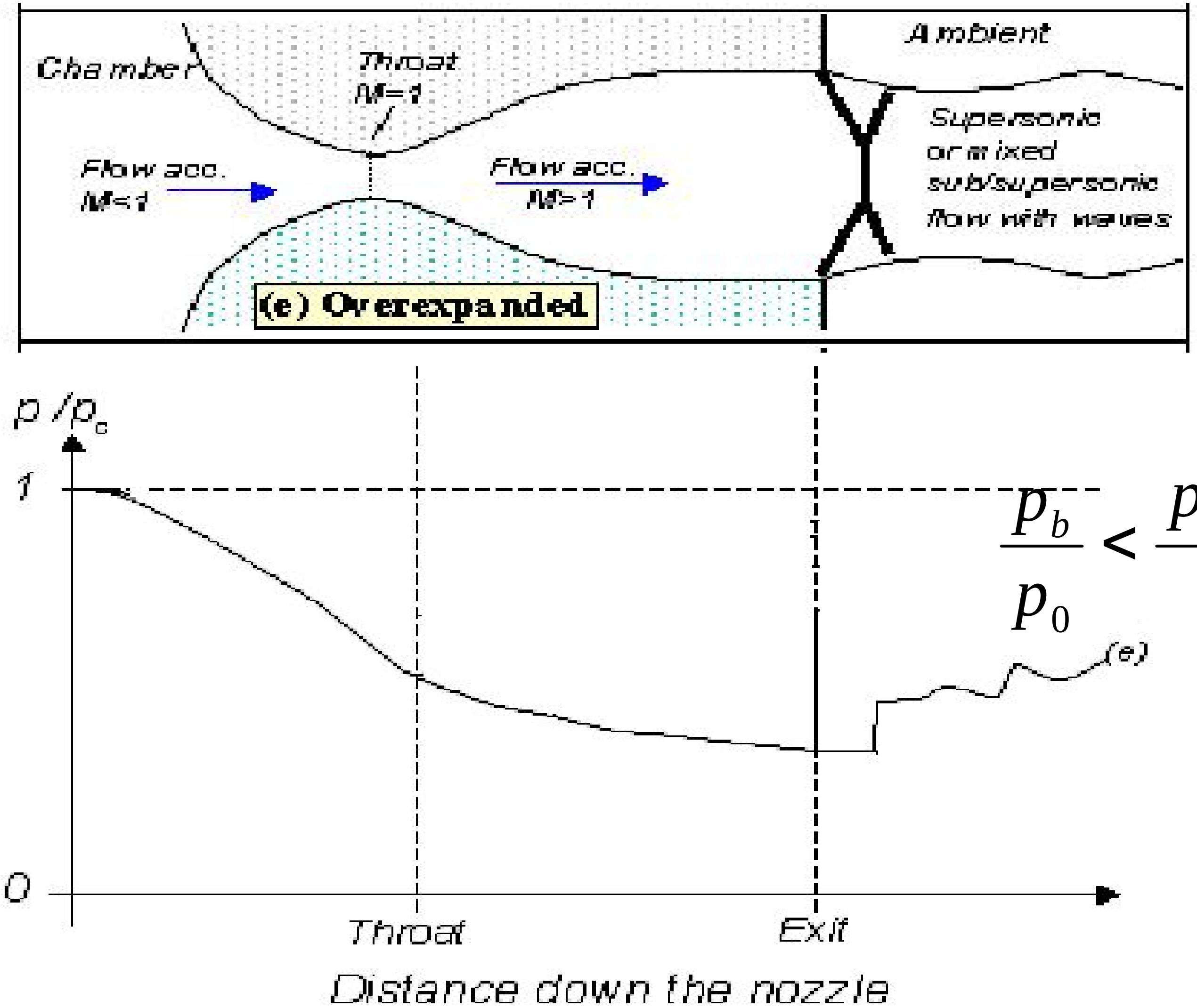
$p_{b,design} < p_{bt} < p_{b,critical}$

Back Pressure Responsible for NS at Nozzle Exit



$$\frac{p_b}{p_0} = \frac{p_{b,exitshock}}{p_0}$$

Occurrence of Attached Oblique Shock

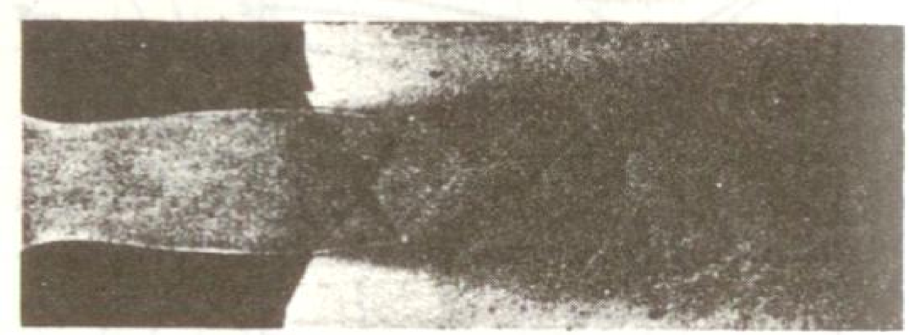


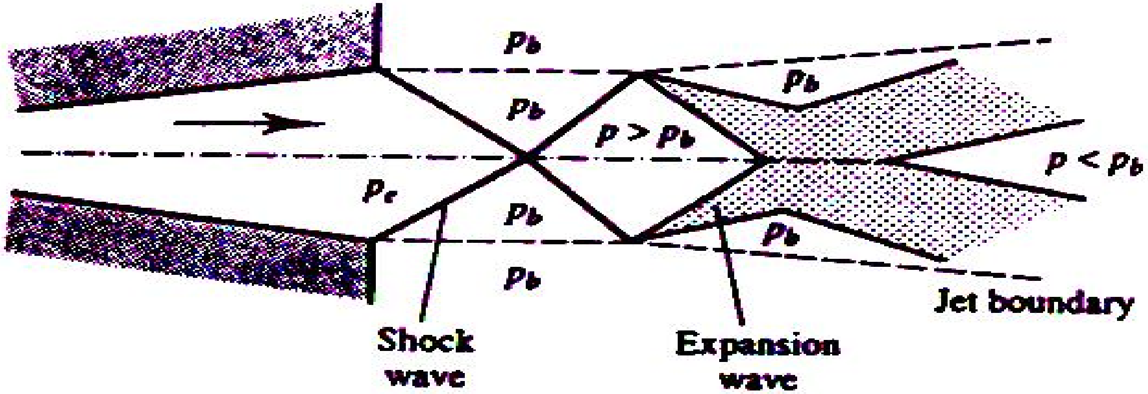
$$\frac{p_b}{p_0} < \frac{p_{b,exitshock}}{p_0}$$

Flow Visualization Studies

$$\frac{p_b}{p_{b,design}} = 0.66$$

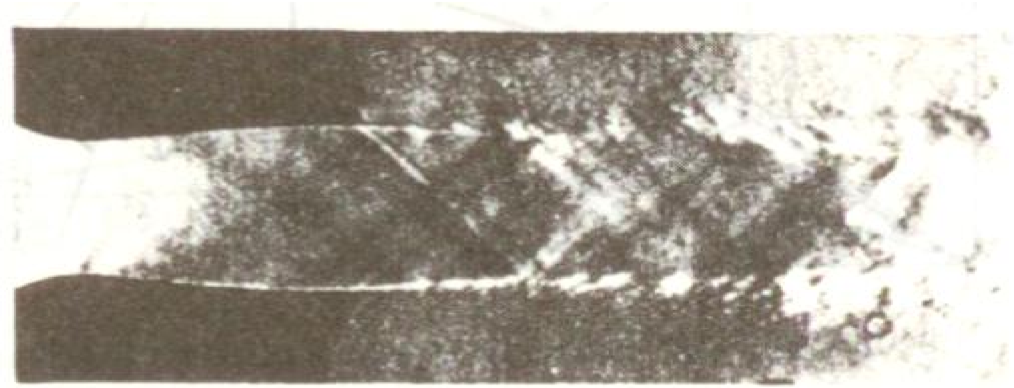
$$\frac{p_b}{p_{b,design}} = 0.85$$

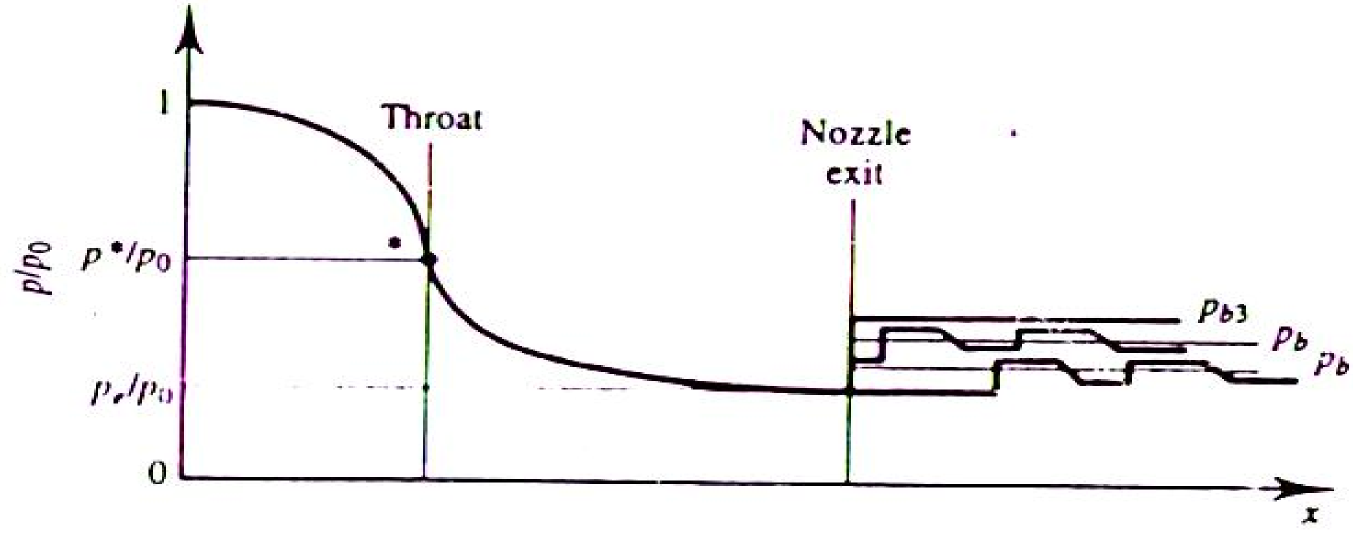




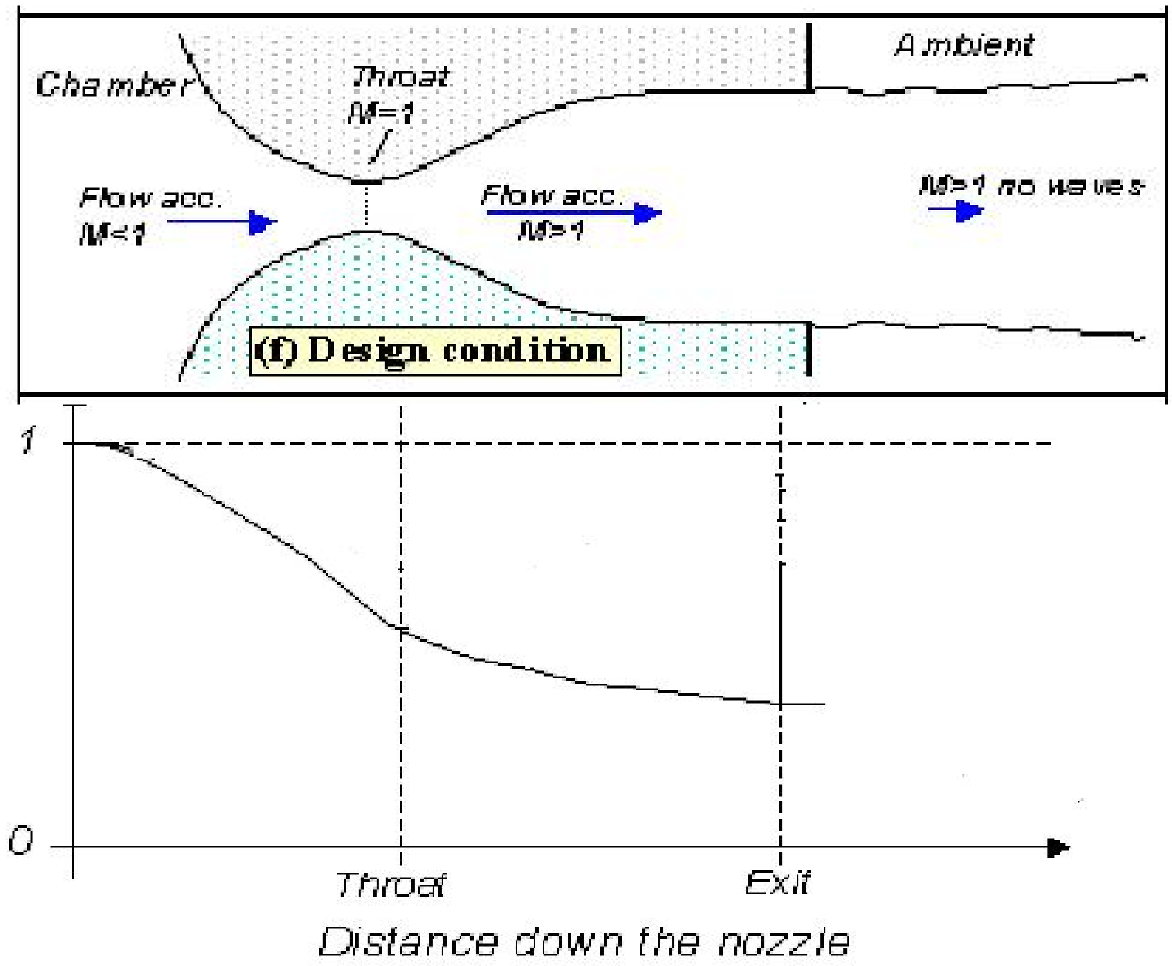
Approaching to Design Back Pressure

$$\frac{p_b}{p_{b,design}} = 1.0$$

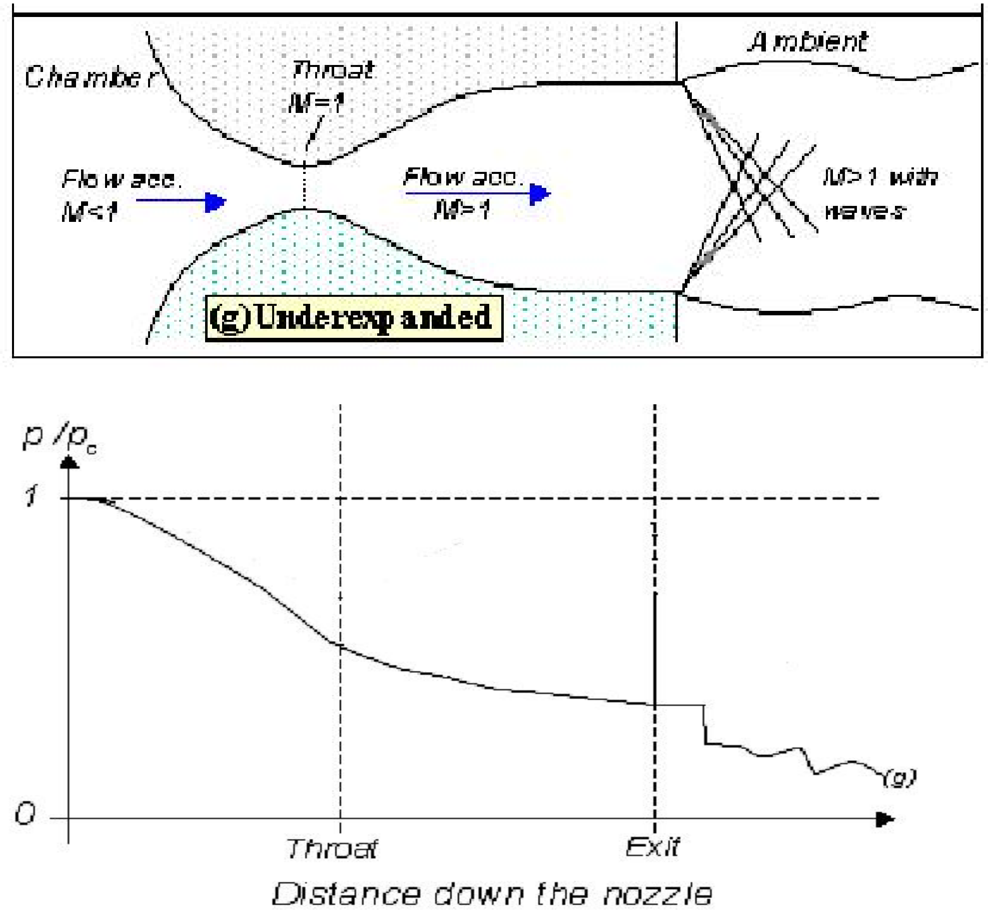




Steady Cruising Design Conditions

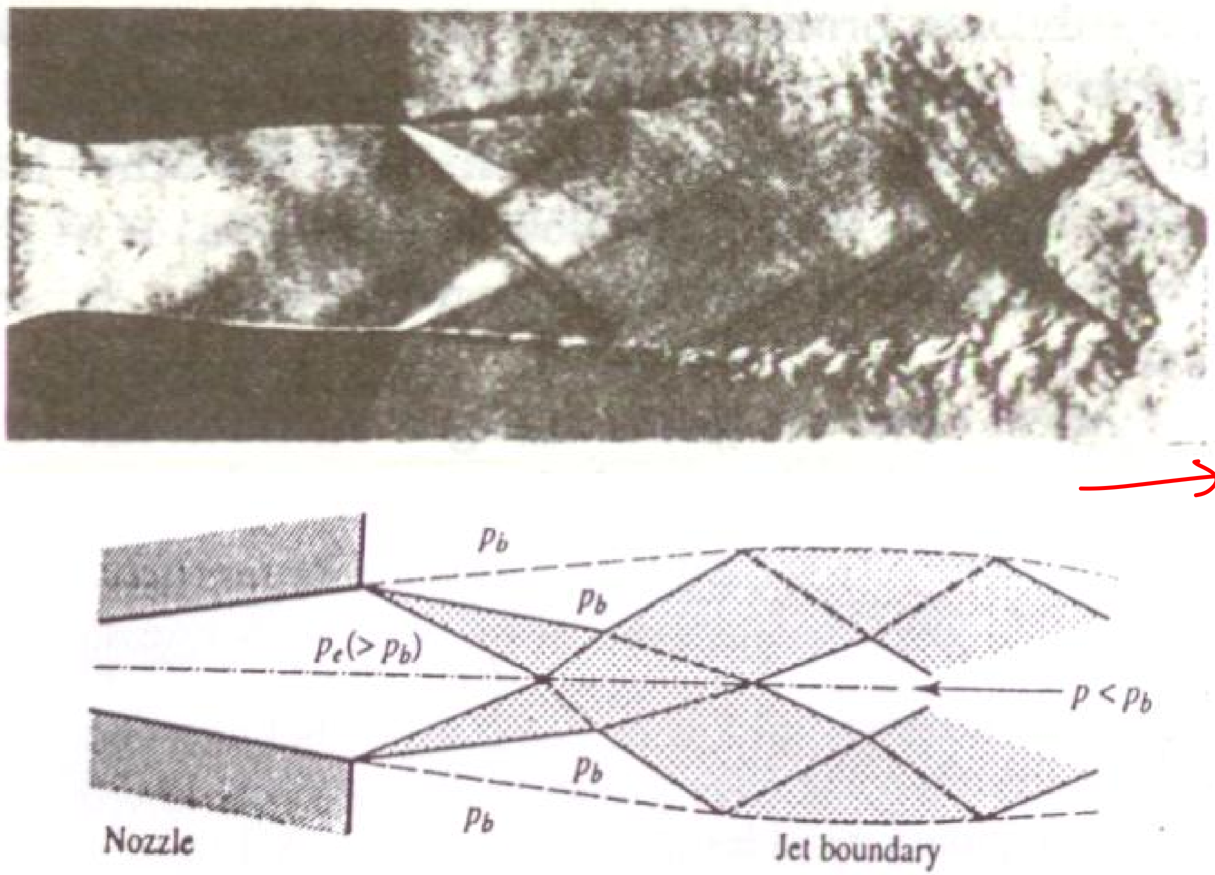


Local Turbulence leading to Back Pressure Lower than the design conditions

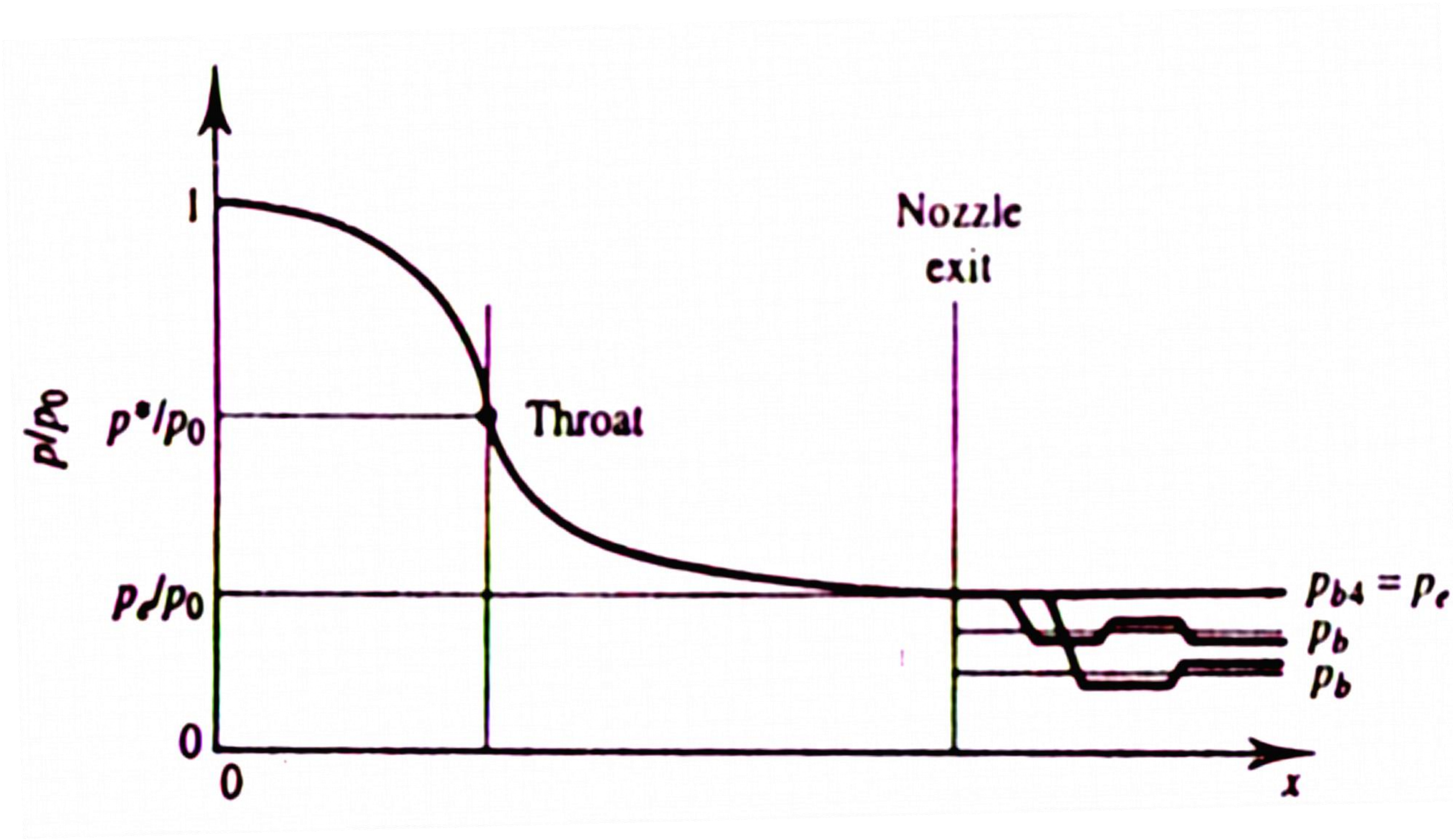


$$\frac{p_b}{p_0} < \frac{p_{b,design}}{p_0}$$

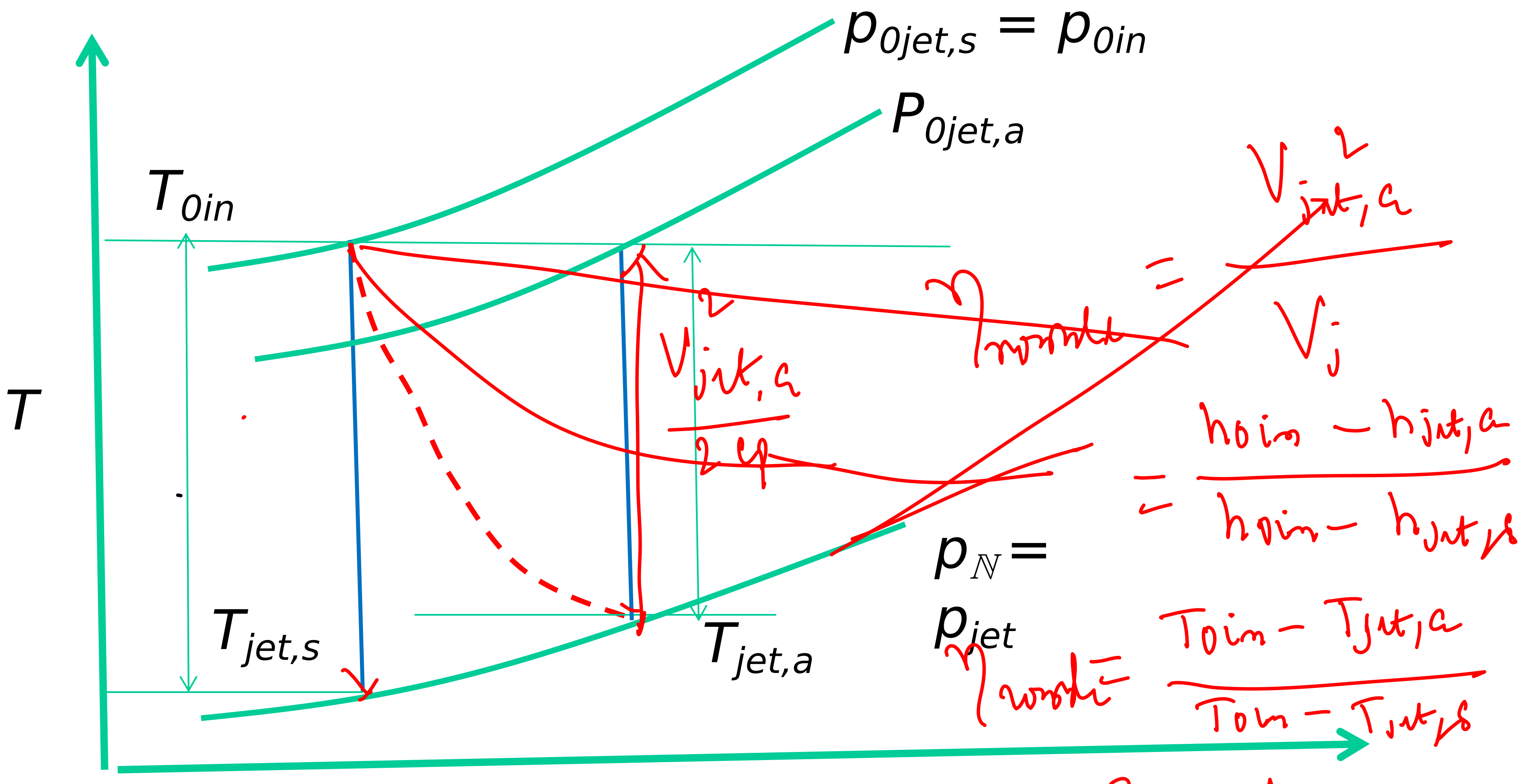
Back Pressure Lower than the design conditions



Back Pressure Lower than the design conditions



Temperature Entropy Diagram for A Real Nozzle



Main Irreversibility : Friction

Bad design
flow separation
shocks

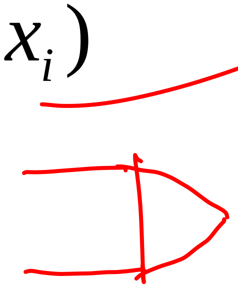
Comprehensive Design Methods for Efficient Nozzles

Design of Nozzle : First Step

- The nozzle deals with expansion of turbine exit gas from a low subsonic flow to supersonic outlet condition and hence this is a convergent-divergent (CD) nozzle.
- The first step in the design of a nozzle is to determine the dimensions of inlet area:

$$\dot{m}_{core} \times (1+f) = \rho(x_i) A(x_i) V(x_i)$$

$$A(x_i) = \frac{\dot{m}_{core} \times (1+f)}{\rho(x_i) V(x_i)} = \frac{\dot{m}_{core} \times (1+f) R T(x_i)}{p(x_i) V(x_i)}$$



One dimensional Gas Dynamics Equations

Non reaction, no body forces, viscous work negligible

Conservation of mass for steady flow:

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

Conservation of momentum for frictional steady flow:

$$\frac{\gamma}{2} M^2 \frac{f dx}{D_h} + \frac{dp}{p} + \frac{\gamma}{2} M^2 \frac{dV^2}{V^2} = 0$$

Conservation of energy for ideal steady flow:

$$\frac{\delta q'''}{C_p T} - \frac{dT}{T} - \frac{(\gamma - 1) M^2}{2} \frac{dV^2}{V^2} = 0$$

Other Equations

Ideal Gas law : $$\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0$$

Mach number equation : $$\frac{dT}{T} + \frac{dM^2}{M^2} - \frac{dV^2}{V^2} = 0$$

Theory of Nozzle Design

Momentum equations for Adiabatic steady one dimensional frictional flow :

$$\frac{dp}{p}+\frac{g}{2}M^{2}\frac{4fdx}{D}+gM^{2}\frac{dV}{V}=0$$

- The conventional design practices are based on dictating the variation of cross-sectional area (geometric theory)of the ejector.
- It is well known fact that the nozzle establishes continuous momentum and energy changes in the flow of gas from inlet to exit.
- Final flow outlet conditions are achieved for a given inlet conditions of flow.
- A physics based design theory is essential.

$\frac{dA}{dx}=C_1$

$\frac{dD}{dx}=C_2$

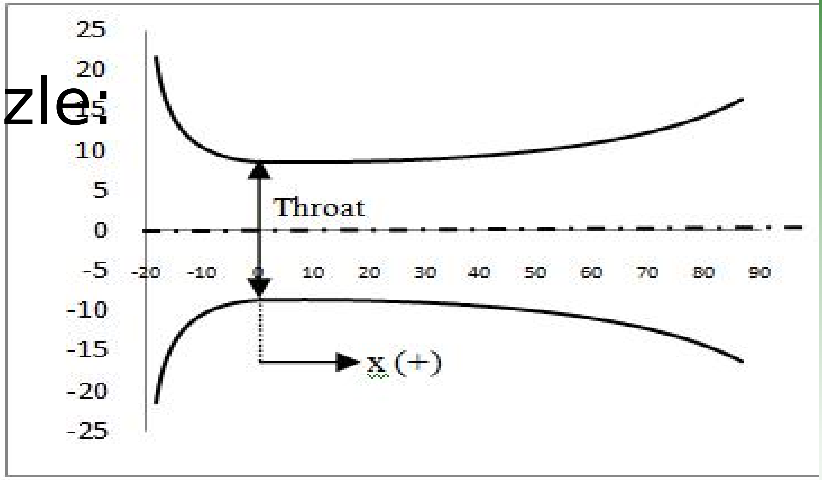
$\frac{dM}{dx}=C_3$

Physics based Design Theories

- Constant Rate of Momentum Change (CRMC)
- Constant Rate of Kinetic Energy Change (CRKEC)
- CRMC theory:
- Define *M* as rate Momentum of gas flow.
- The change in rate of momentum of gas flow is defined as: $\frac{d\dot{M}}{dx} = \dot{m}\frac{dV}{dx}$
- The basis for the CRMC theory is to keep the value of *dM/dx* constant.
- Select an appropriate value for this constant. $\frac{d\dot{M}}{dx} = \dot{m}_p \frac{dV}{dx} = l$

$$‘ \quad \frac{dV}{dx} = \frac{l}{\dot{m}}$$

Design Equations for CRMC Nozzle



Base equation for CRMC nozzle:

$$\frac{dV}{V} = \frac{l \quad dx}{\dot{m} V}$$

Momentum equation:

$$\frac{dp}{p} + \frac{g}{2} M^2 \frac{4 f dx}{D} + g M^2 \frac{dV}{V} = 0$$

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

$$\frac{dp}{p} + \frac{g}{2} M^2 \frac{4 f dx}{D} + g M^2 \frac{l \quad dx}{\dot{m} V} = 0$$

$$\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0$$

$$\frac{dA}{A} = \frac{g}{2} M^2 \frac{4 f dx}{D} + \left(M^2 - 1\right) \frac{l \quad dx}{\dot{m} \; V}$$

$$\frac{dT}{2T} + \frac{dM}{M} - \frac{dV}{V} = 0$$

Contd

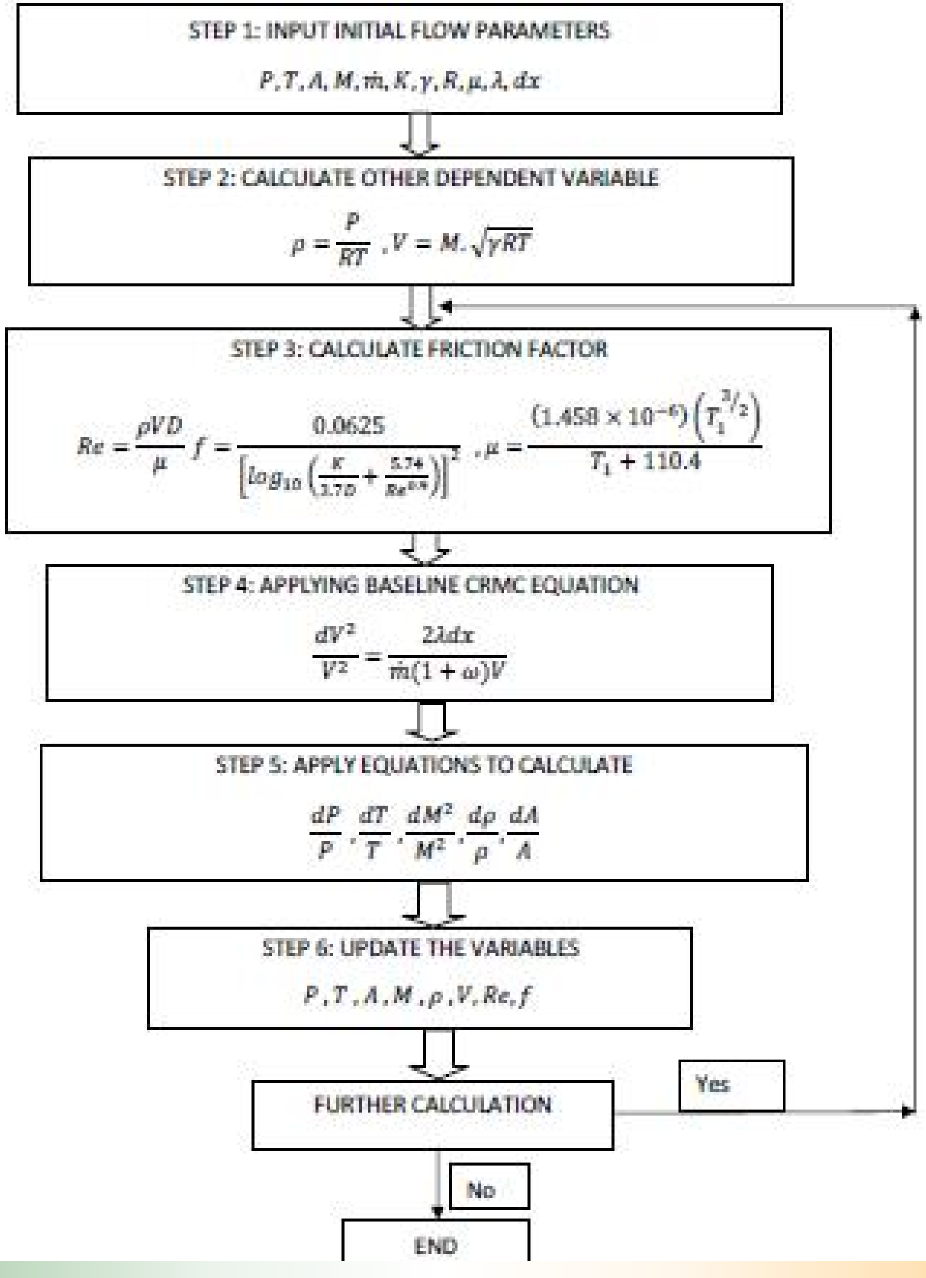
Estimation of Friction Term

$$f = \frac{0.0625}{\left[\log\left(\frac{k}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

$$Re = \frac{\rho V D}{\mu}$$

$$\mu = 3.3 \times 10^{-5} \left(\frac{T}{293}\right)^{1.5} \frac{384.1}{T + 111} \frac{N.s}{m^2}$$

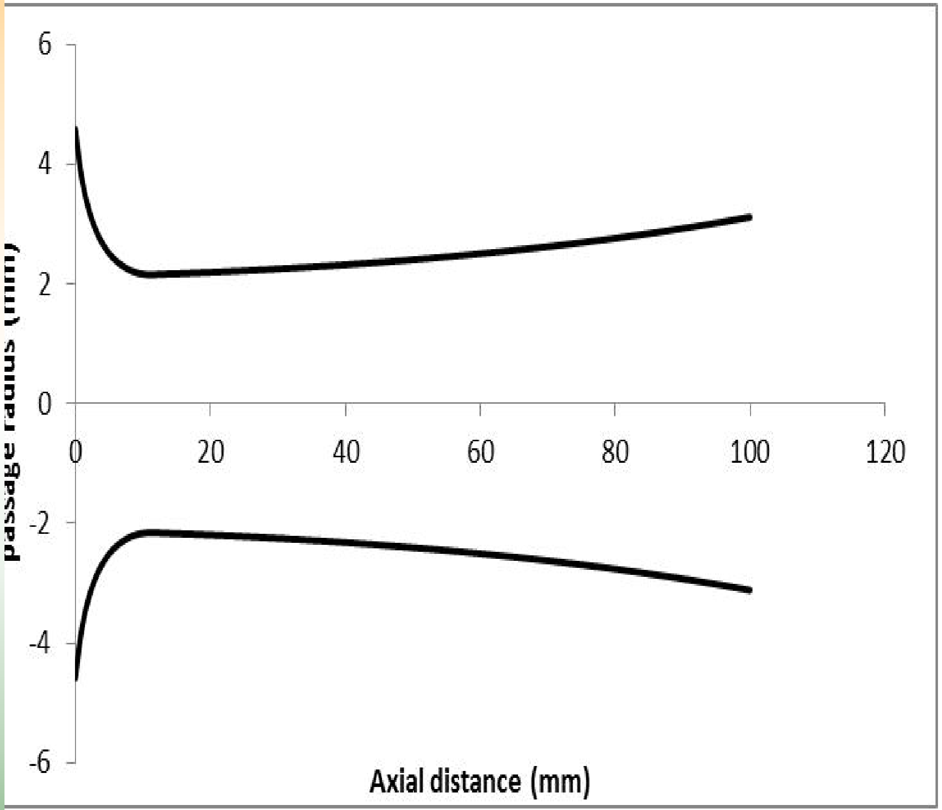
Flow Chart for Geometry generation

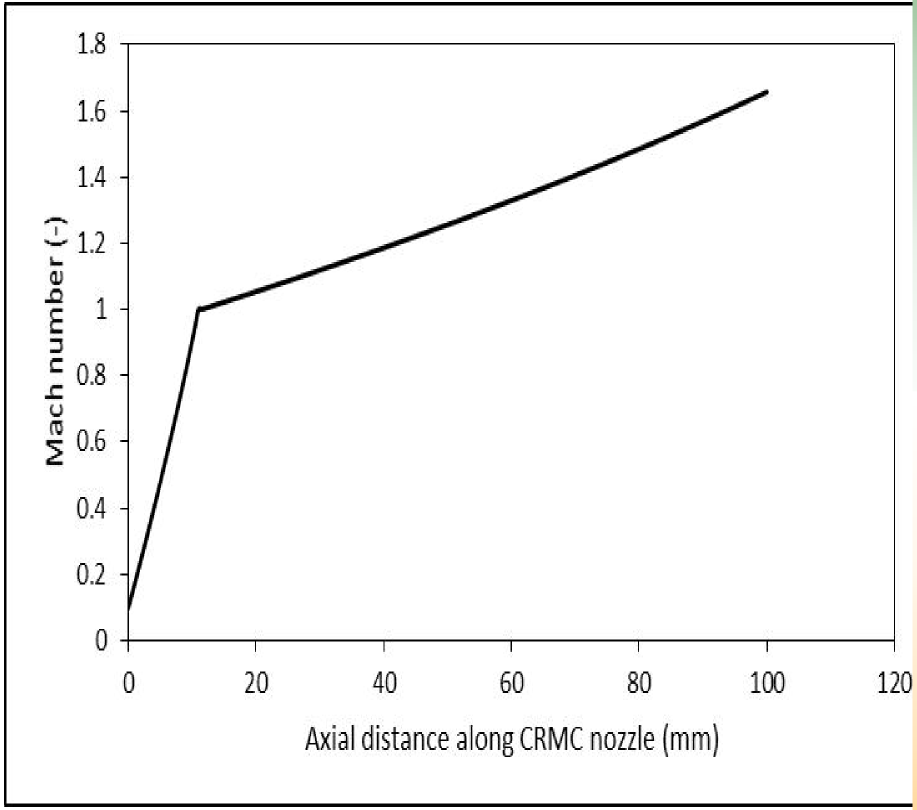


Model Nozzle

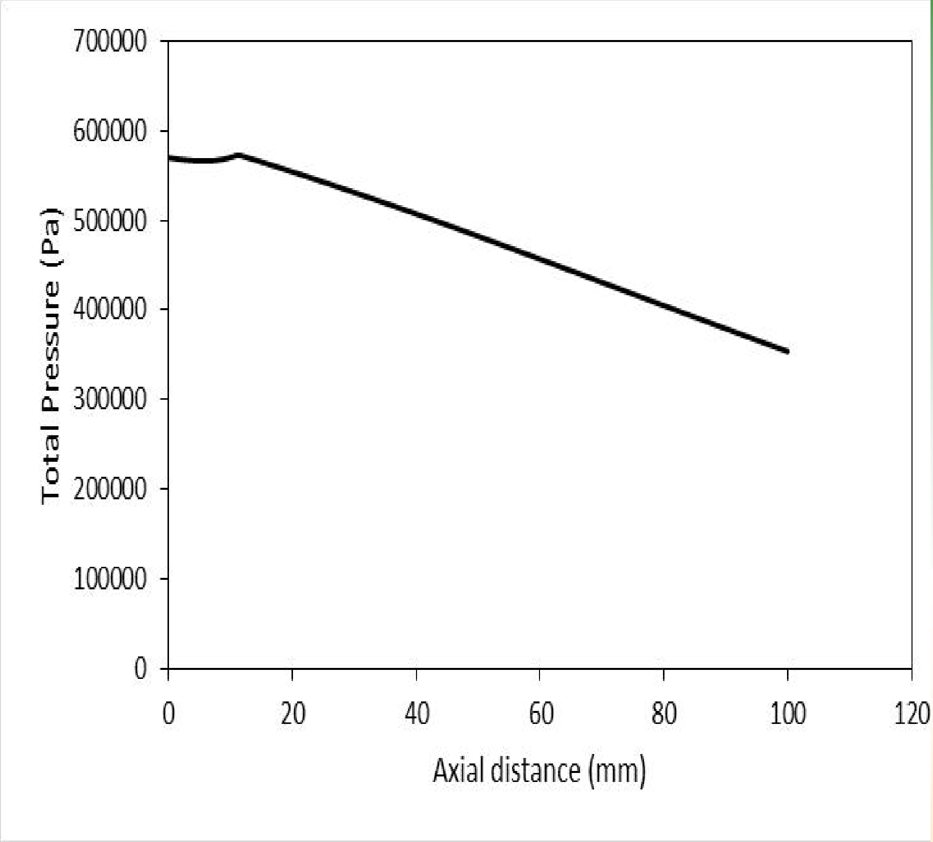
- λ = 45658
- Inlet temperature = 298K
- Inlet Pressure= 400kPa
- Inlet Area = 0.000176625 m^2
- Mass flow rate = 0.03767 kg/s
- Thickness of the pipe = 0.0125 m
- Coeffecient of condcution of the material = 54 W/m2K (Mild steel)
- Tamb = 298 K

Shape of CRMC Nozzle

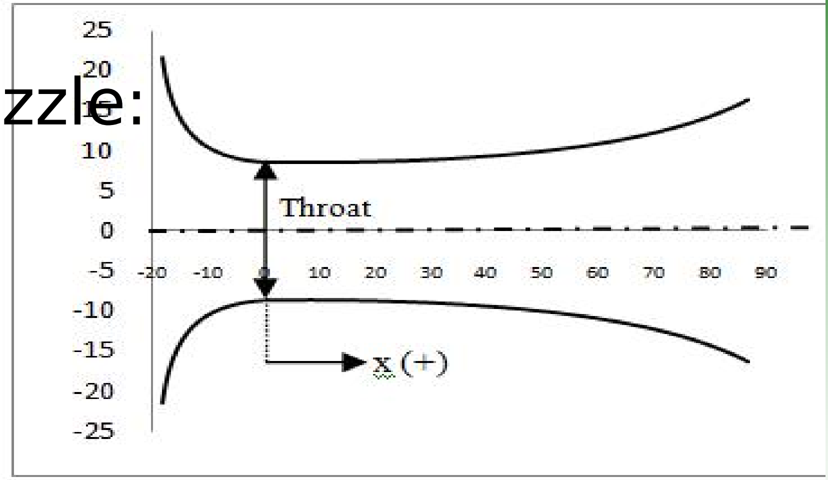




Pressure variation in CRMC Nozzle



Design Equations for CRKEC Nozzle



Base equation for CRKEC nozzle:

$$\frac{d(\overset{\forall}{KE})}{dx} = \dot{m}V\frac{dV}{dx} = f$$

$$\frac{dV}{V} = \frac{f \; dx}{\dot{m}V^2}$$

Incremental change in local cross sectional area :

$$\frac{dA}{A} = \frac{g}{2}M^2\frac{4f \; dx}{D} + \left(M^2 - 1\right)\frac{f \; dx}{\dot{m}V^2}$$

Road Map to Test Novel Ideas

Gas Dynamic Description & Formulation

→

Introducing Friction Model

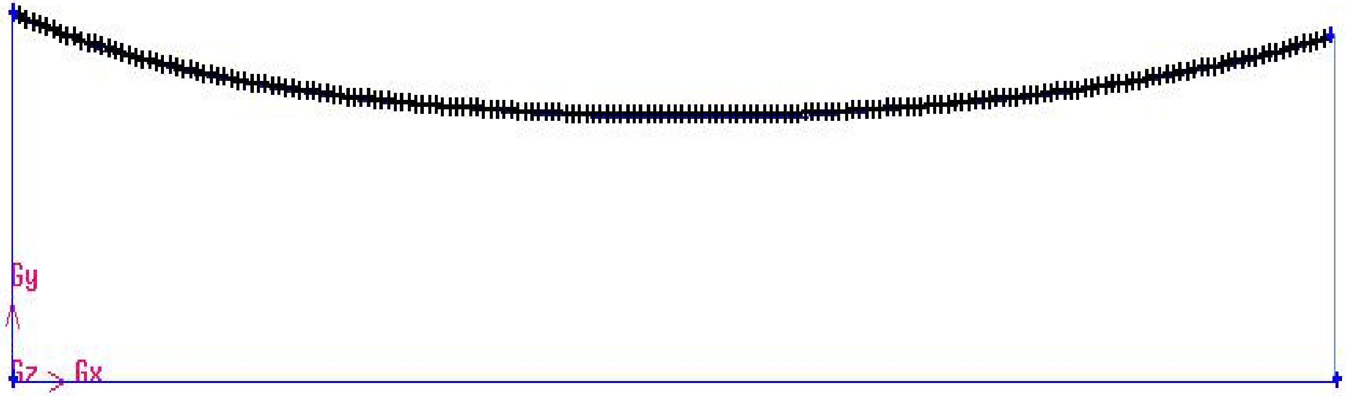
→

Develop a MATLAB code to generate nozzle profile using Gas Dynamic formulaiton

↓

nozzle

Computerized Gas Dynamic Analysis of Nozzle



COMPUTATIONAL STUDIES - Governing Equations

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Continuity equation

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{zr})+\frac{\partial}{\partial z}(\tau_{zz})\right)$$

R- momentum equation

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_z}{\partial r}\right)=-\frac{\partial p}{\partial r}+\rho g_r-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr})-\frac{\tau_{\theta\theta}}{r}+\frac{\partial}{\partial z}(\tau_{rz})\right)$$

Z- momentum equation

$$\tau_{rr}=-\mu\left[2\frac{\partial v_r}{\partial r}-\frac{2}{3}(\nabla\cdot v)\right]\qquad \tau_{zz}=-\mu\left[2\frac{\partial v_z}{\partial z}-\frac{2}{3}(\nabla\cdot v)\right]\qquad \tau_{\theta\theta}=\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r}{r}\right)-\frac{2}{3}(\nabla\cdot v)\right]$$

$$\tau_{zr}=\tau_{rz}=-\mu\left[\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right]\qquad \tau_{r\theta}=\tau_{r\theta}=-\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]\qquad \tau_{\theta z}=\tau_{z\theta}=-\mu\left[\frac{\partial v_\theta}{\partial z}+\frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$$

$$\rho\left(v_r\frac{\partial T}{\partial r}+v_z\frac{\partial T}{\partial z}\right)=\left(v_r\frac{\partial p}{\partial r}+v_z\frac{\partial p}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(k_r\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)+\varphi$$

Energy equation

$$\varepsilon=\frac{P}{RT}$$

Equation of state

$$\frac{\partial\rho k}{\partial t}+\frac{\partial}{\partial x_i}(\rho k v_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+G_k+G_b-\rho\varepsilon-Y_M+S_K$$

$$\frac{\partial\rho\varepsilon}{\partial t}+\frac{\partial}{\partial x_i}(\rho\varepsilon v_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]+C_{1\varepsilon}\frac{\varepsilon}{k}+(G_k+C_{3\varepsilon}G_b)-C_{2\varepsilon}\frac{\varepsilon^2}{k}+S_K$$

Turbulence equations

Boundary Conditions

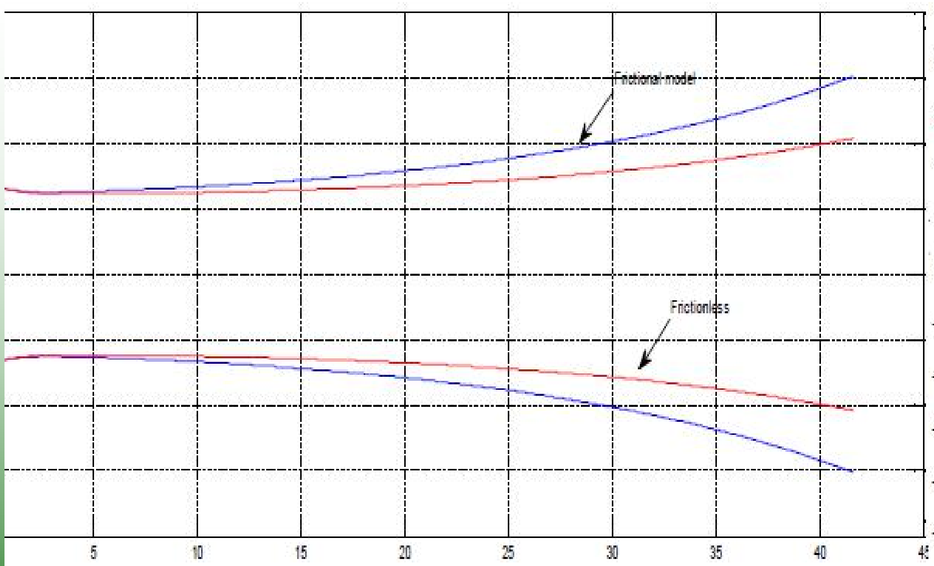
S.N.	Boundary	Boundary type	Stagnation pressure	Static pressure	Temperature (K)	Wall roughness
1.	Inlet	Pressure inlet	25 torr	18 torr	300	NA
2	Outlet	Pressure outlet	NA	0	NA	NA
3	Wall	Wall	NA	NA	Adiabatic	1.5e-6
4	Symmetry	Symmetry	NA	NA	NA	NA

Grid Size independence check

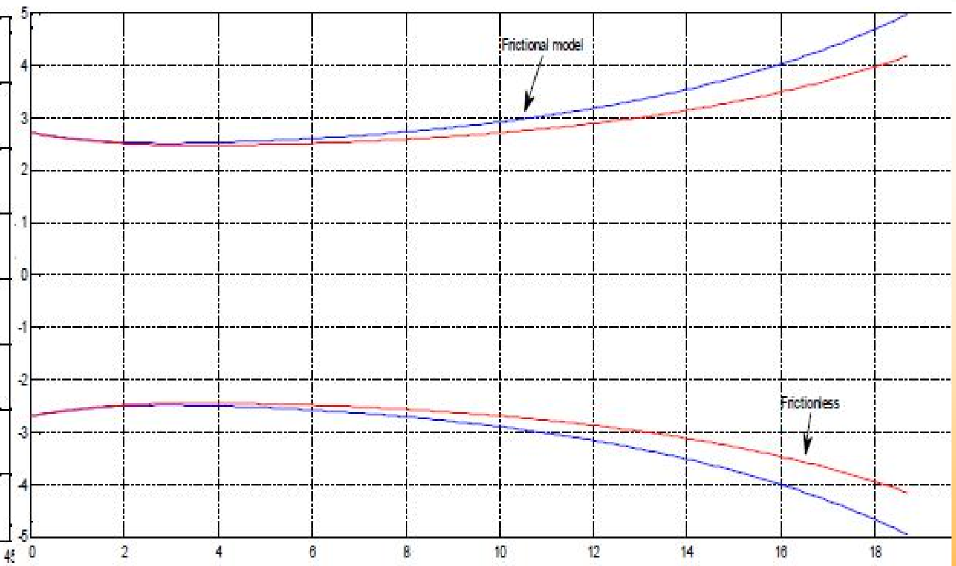
S.N.	Grid Size(mm)	Nozzle Length(mm)	Exit Size(mm)	Static Pressure(torr)	Mach No.	Static Temp(K)
1	1	25	10.2238	2.2241	2.0288	164.7045
2	0.5	25.5	10.5321	2.323	2.0261	164.8076
3	0.3	25.5	10.513	2.4165	2.012	165.8192
4	0.1	25.2	10.5128	2.4974	2.0021	166.5263
5	0.08	25.2	10.5103	2.507	2.0006	166.6294

We have selected grid size of 0.1mm

Comparison of Friction vs. frictionless model

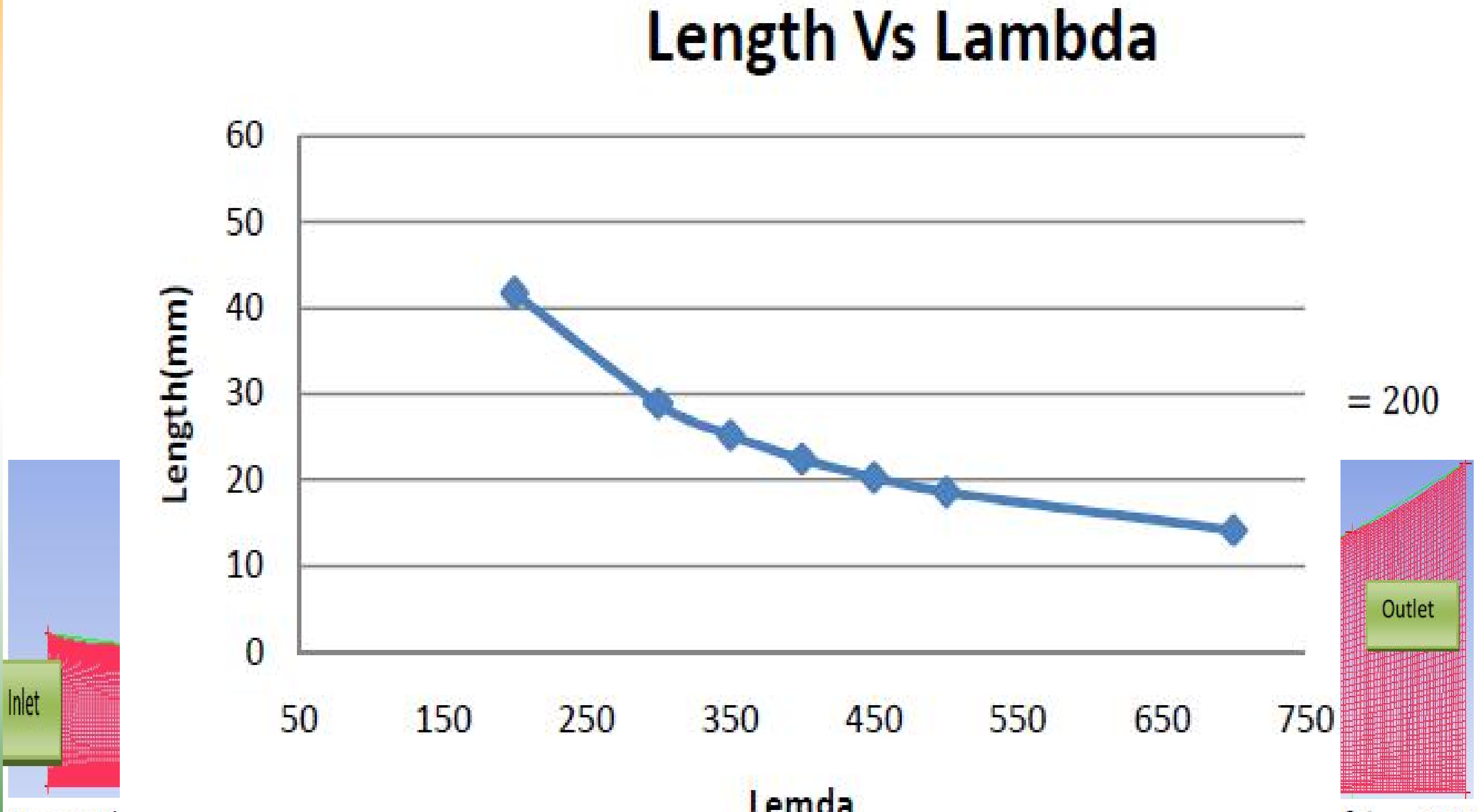


Lambda 200



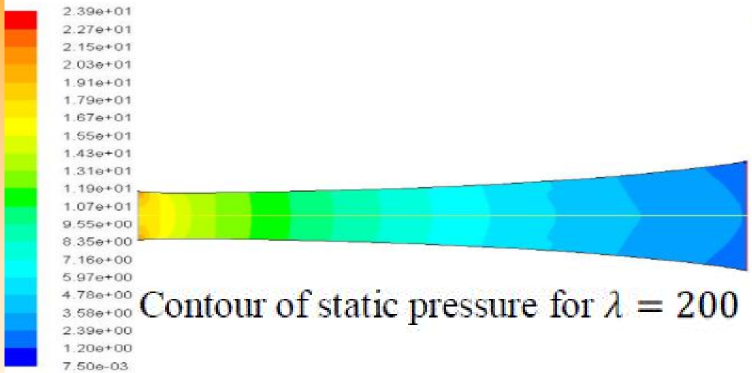
Lambda 500

Optimization of CRMC nozzle length

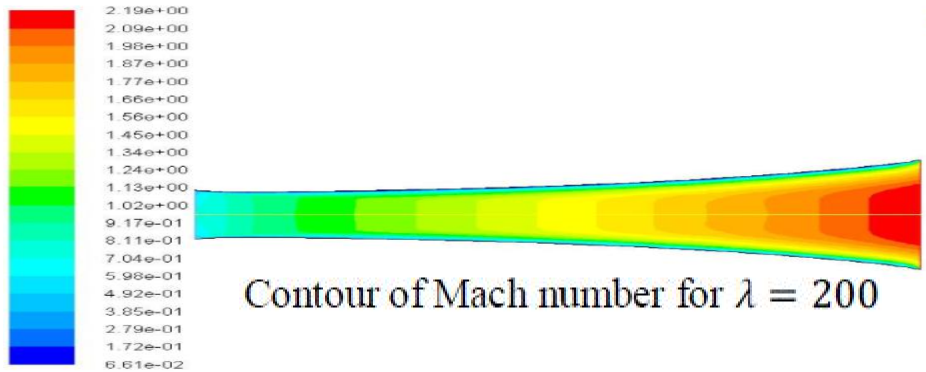


Geometrical model with mesh of 2D nozzle of $\lambda = 350$ Geometrical model with mesh of 2D nozzle of $\lambda = 700$

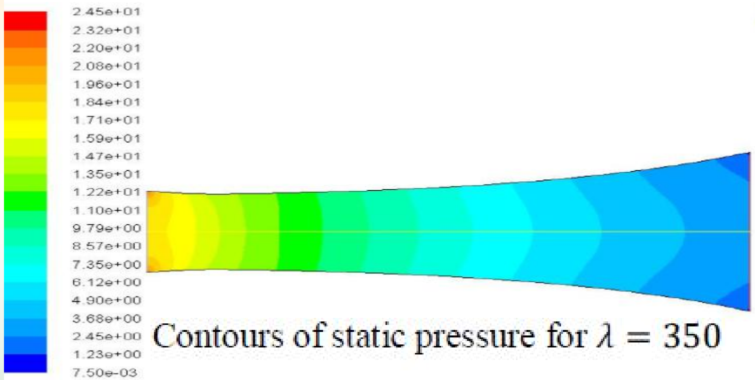
Simulation Results



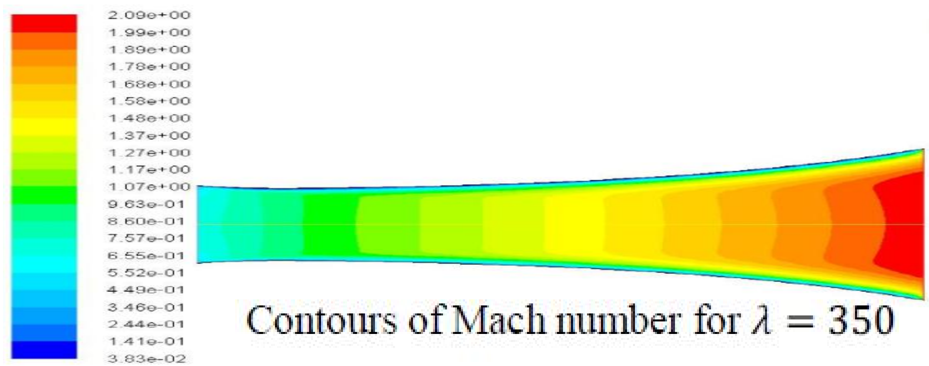
Contour of static pressure for $\lambda = 200$



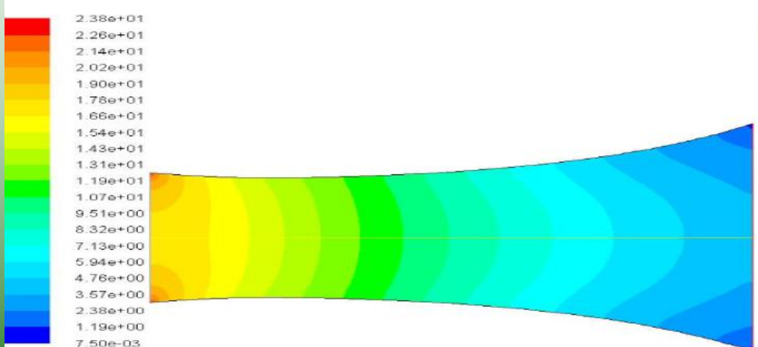
Contour of Mach number for $\lambda = 200$



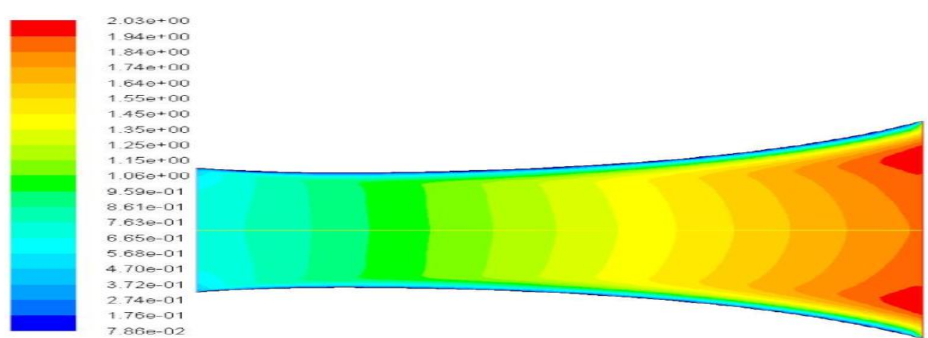
Contours of static pressure for $\lambda = 350$



Contours of Mach number for $\lambda = 350$



Contours of Static Pressure for $\lambda = 700$

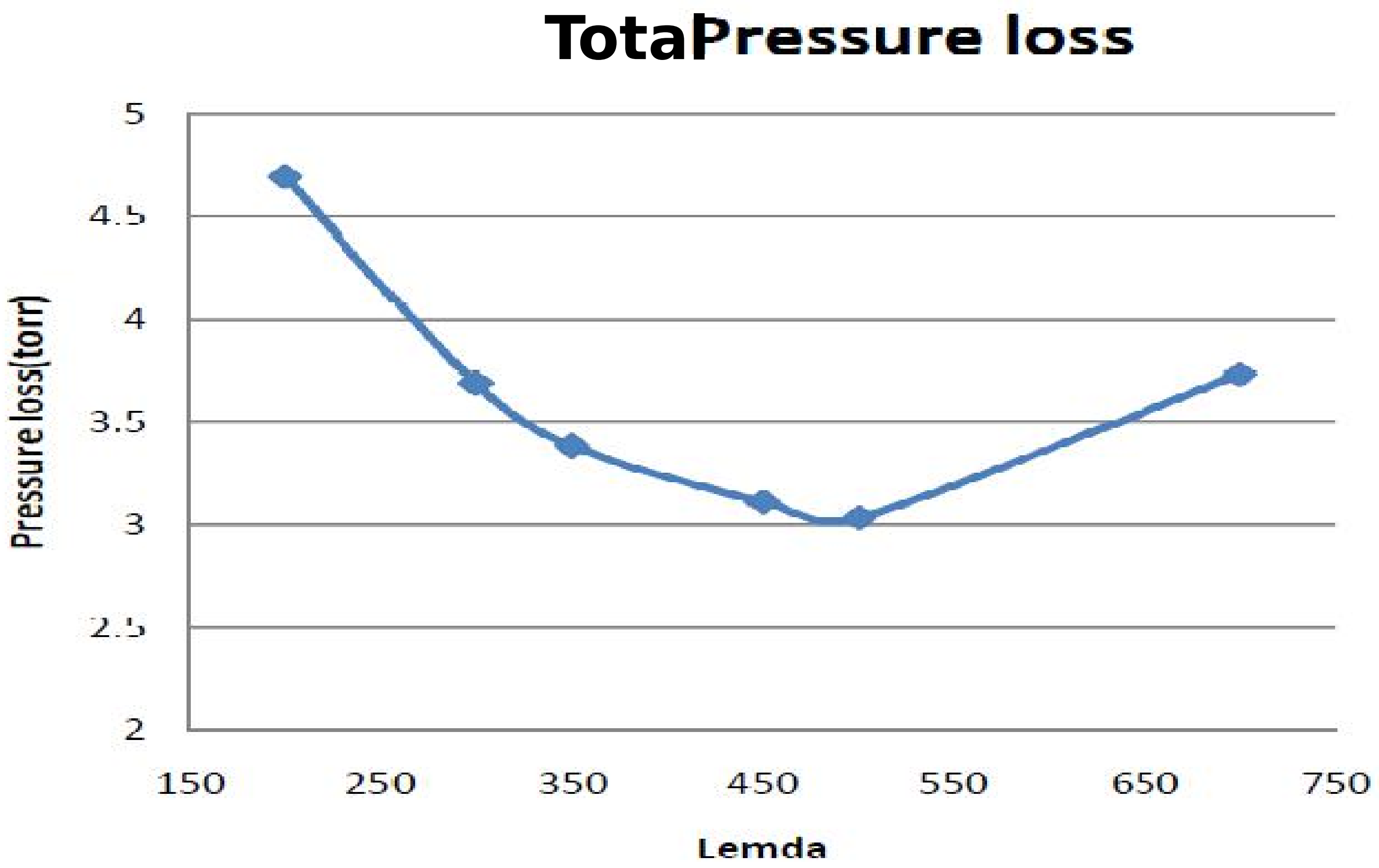


Contours of Mach number for $\lambda = 700$

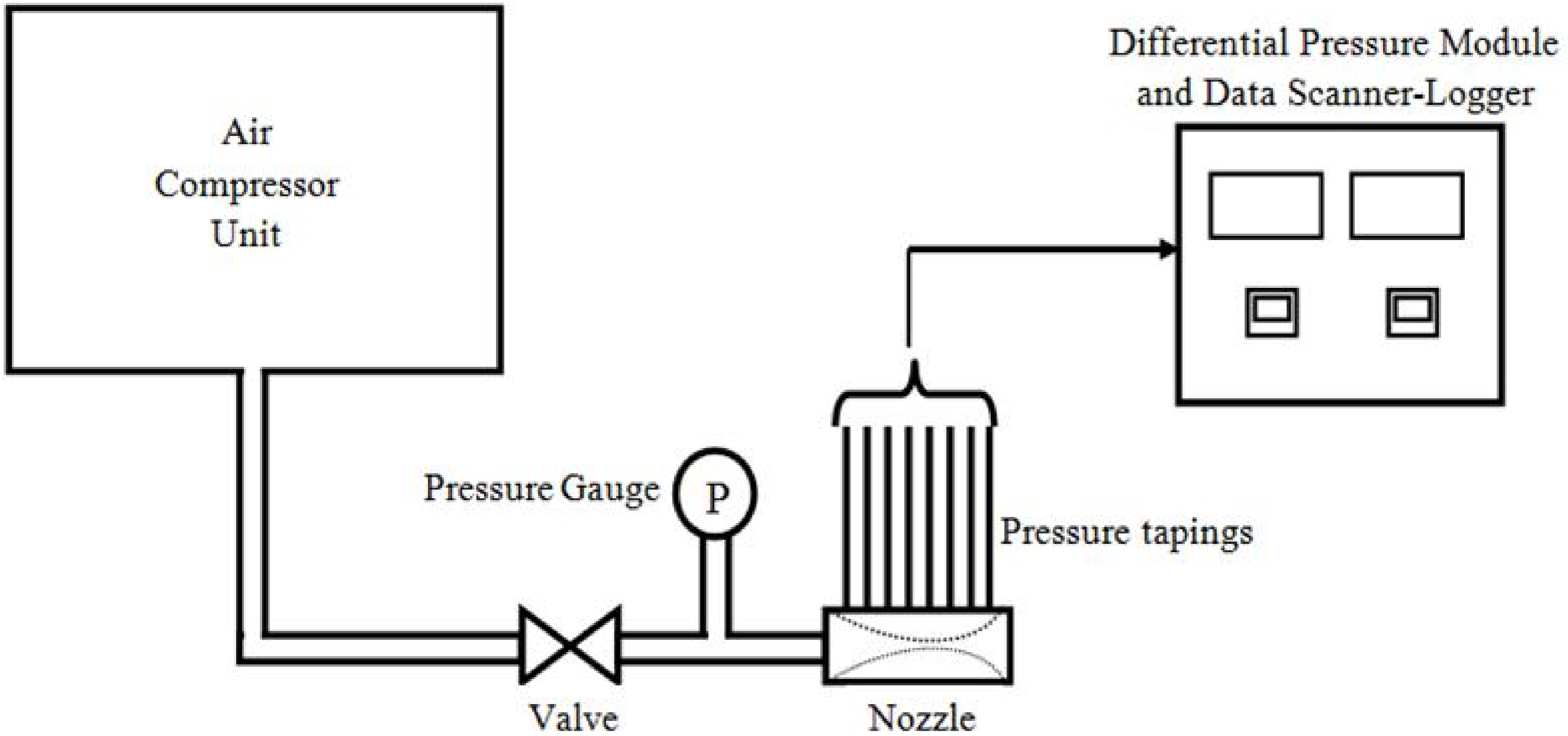
Analysis of Simulation data

lambda	length	M	Ps	Pt	Ploss	To
200	41.33	2.0629	2.179	20.31	4.69	300.017
300	28.8	2.0159	2.514	21.31	3.69	300
350	25.2	1.9986	2.6335	21.619	3.381	300.02
400	22.5	1.97086	2.7656	21.65	3.35	299.99
450	20.4	1.9648	2.8464	21.89	3.11	300
500	18.7	1.948	2.944	21.97	3.03	300.01
700	14.3	1.85	3.3837	21.27	3.73	300

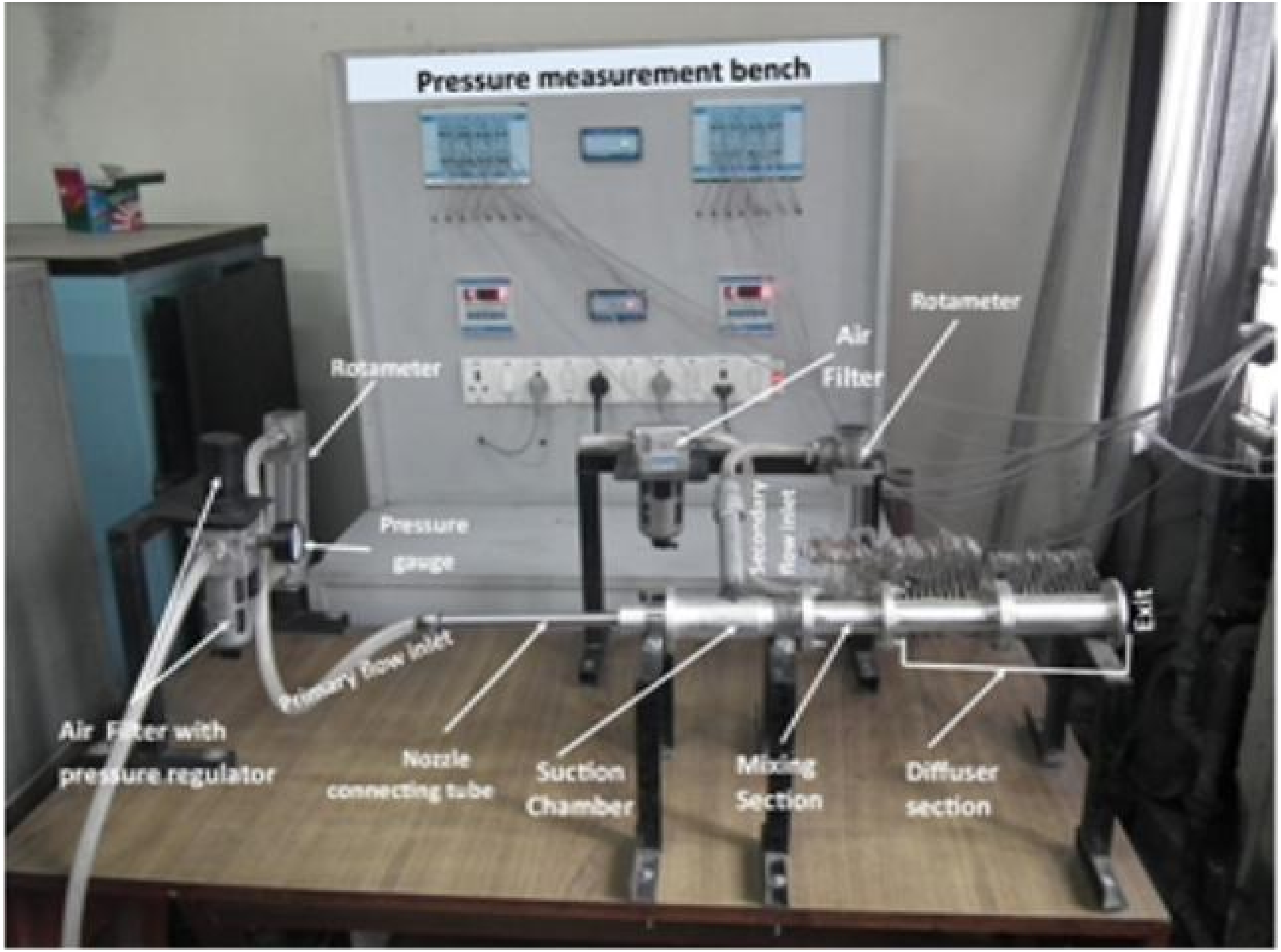
Optimal design of CRMC Design



Experimental Set-up



Schematic of the Experimental Setup



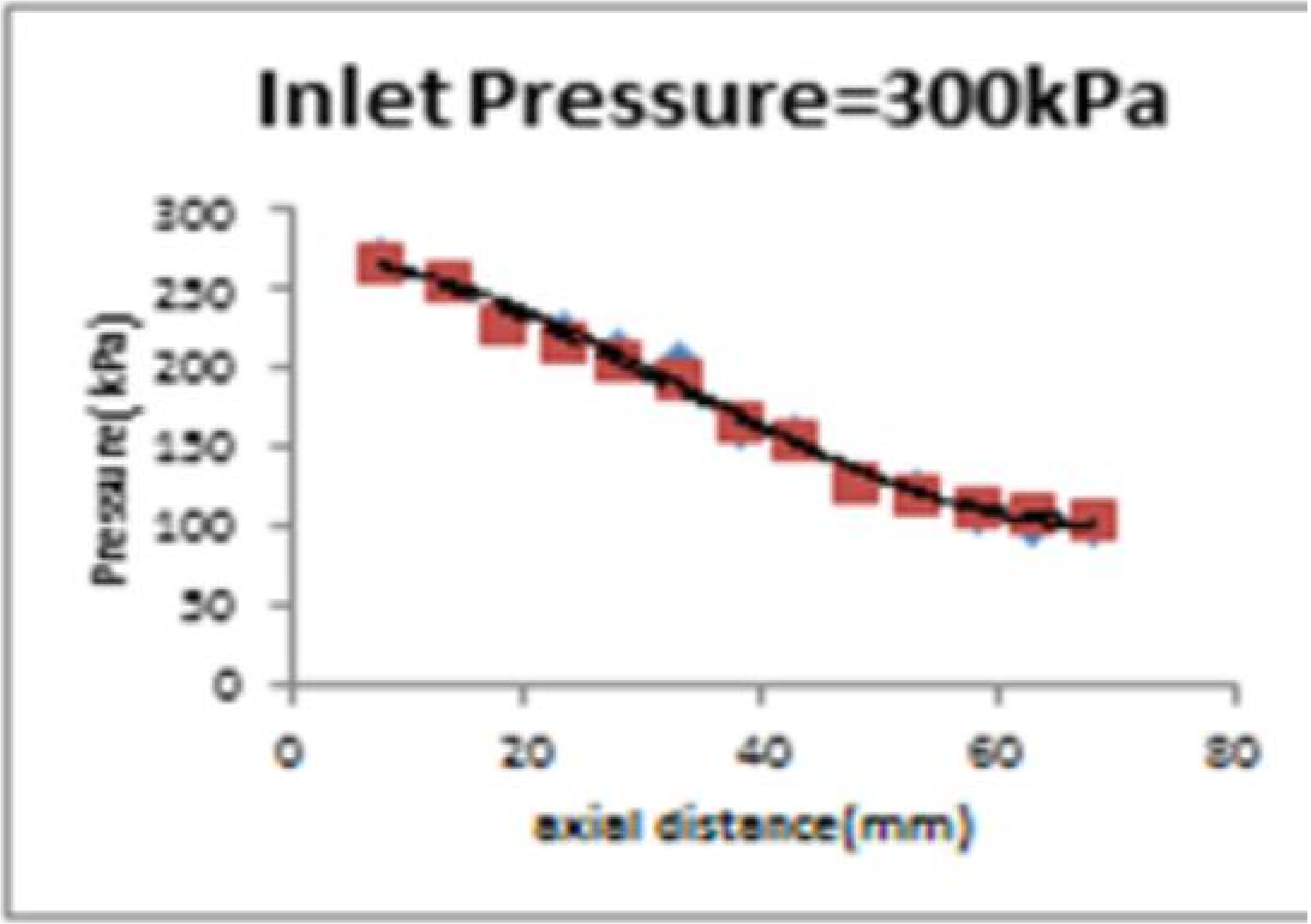
Pressure measurement bench
Rotameter
Air
Filter
Rotameter
Pressure
gauge
Secondary
flow inlet
Primary flow inlet
Exit
Nozzle
connecting tube
Suction
Chamber
Mixing
Section
Diffuser
section

Comparison of Experimental and Fluent data

Inlet Pressure=300kPa

Axial Position (in mm)	Mean Pressure Experimental (in kPa)	Pressure from Fluent® (in kPa)	%Error
8	270.82	267	-1.43097
13	254.46	255	0.210745
18	234.12	230	-1.7937
23	224.84	218	-3.13816
28	212.46	205	-3.64004
33	205.38	195	-5.32734
38	163.83	167	1.898255
43	160.29	155	-3.41486
48	129.78	130	0.163544
53	123.59	121	-2.14698
58	109	115	5.210268
63	101.93	109	6.482182
68	102.81	106	3.001281

Comparison of Experimental and Fluent data



F-14 Engine

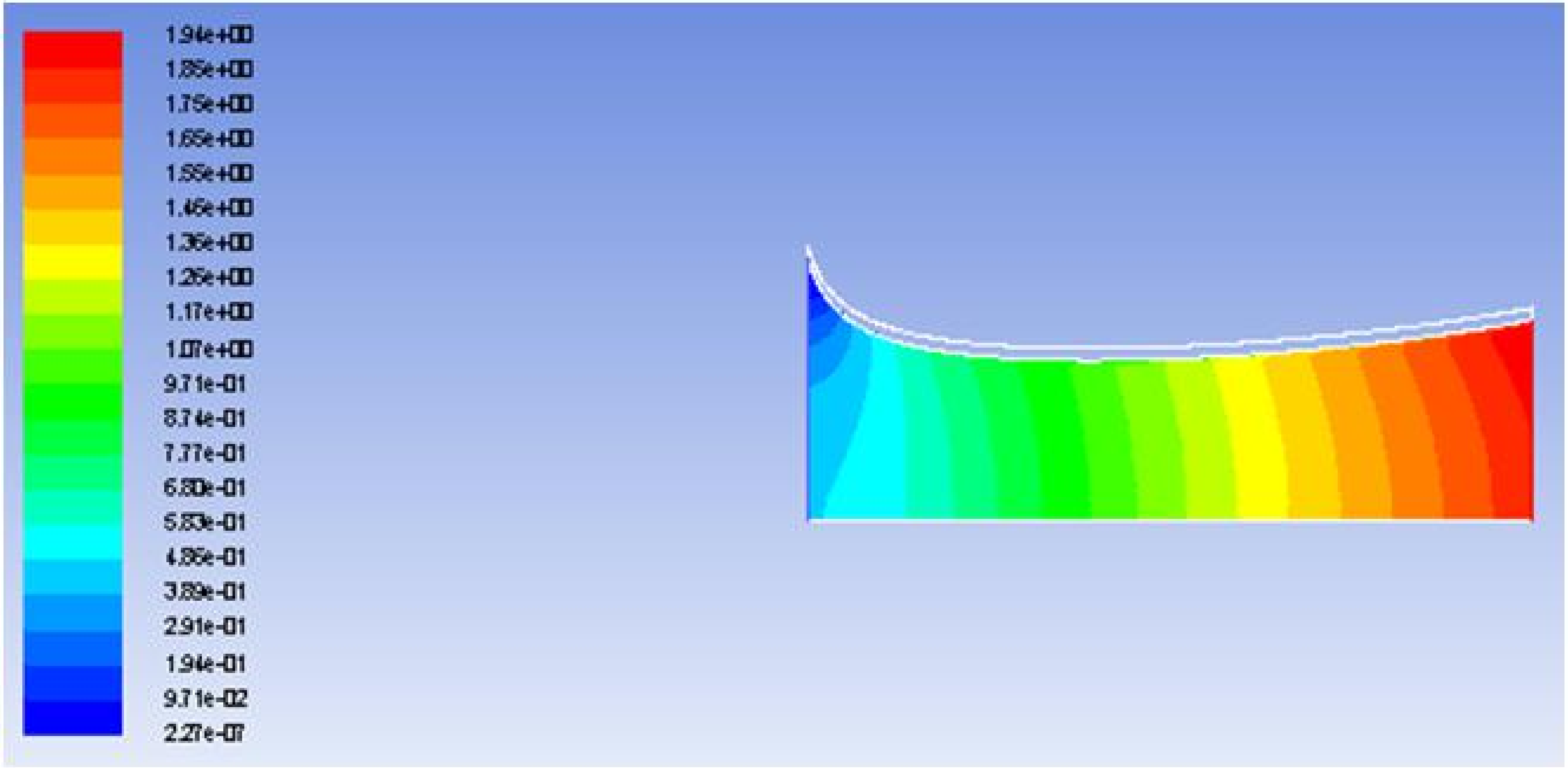
TF-30-P-414 engine manufactured by Pratt and Whitney

F-14 Engine Performance Rating

Performance ratings at standard altitude conditions

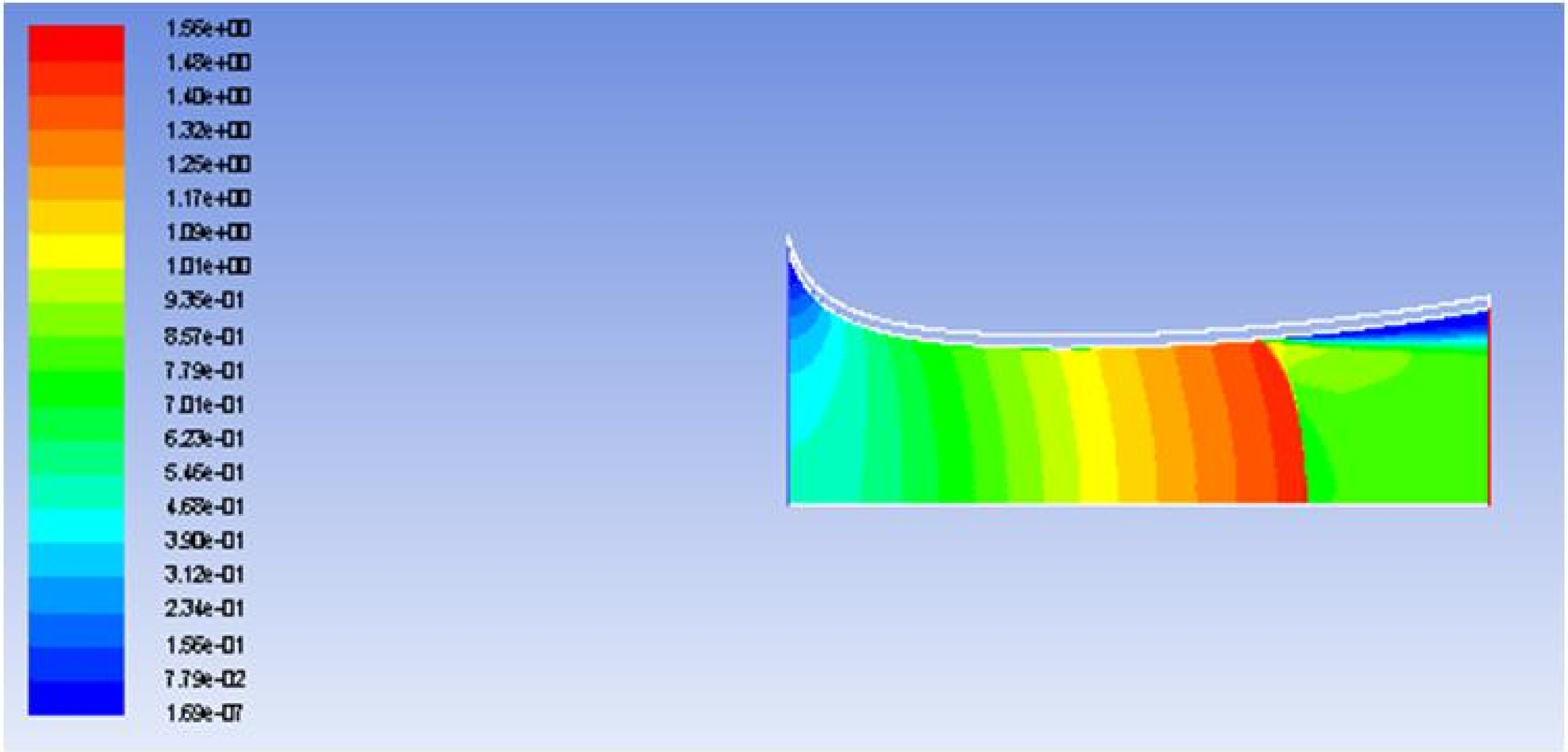
Rating	Altitude (m)	Mach	Thrust (N)	Turbine Inlet temp (^{0}C)	Airflow (kg/s)
Maximum	10668	1.8	95147	1154.4	110.72
Intermediate	6096	0.6	23517.7	1070	65.68
Max Continuous	11582	0.75	12010	1007.2	35.01

Simulation at Design Condition (CRMC)



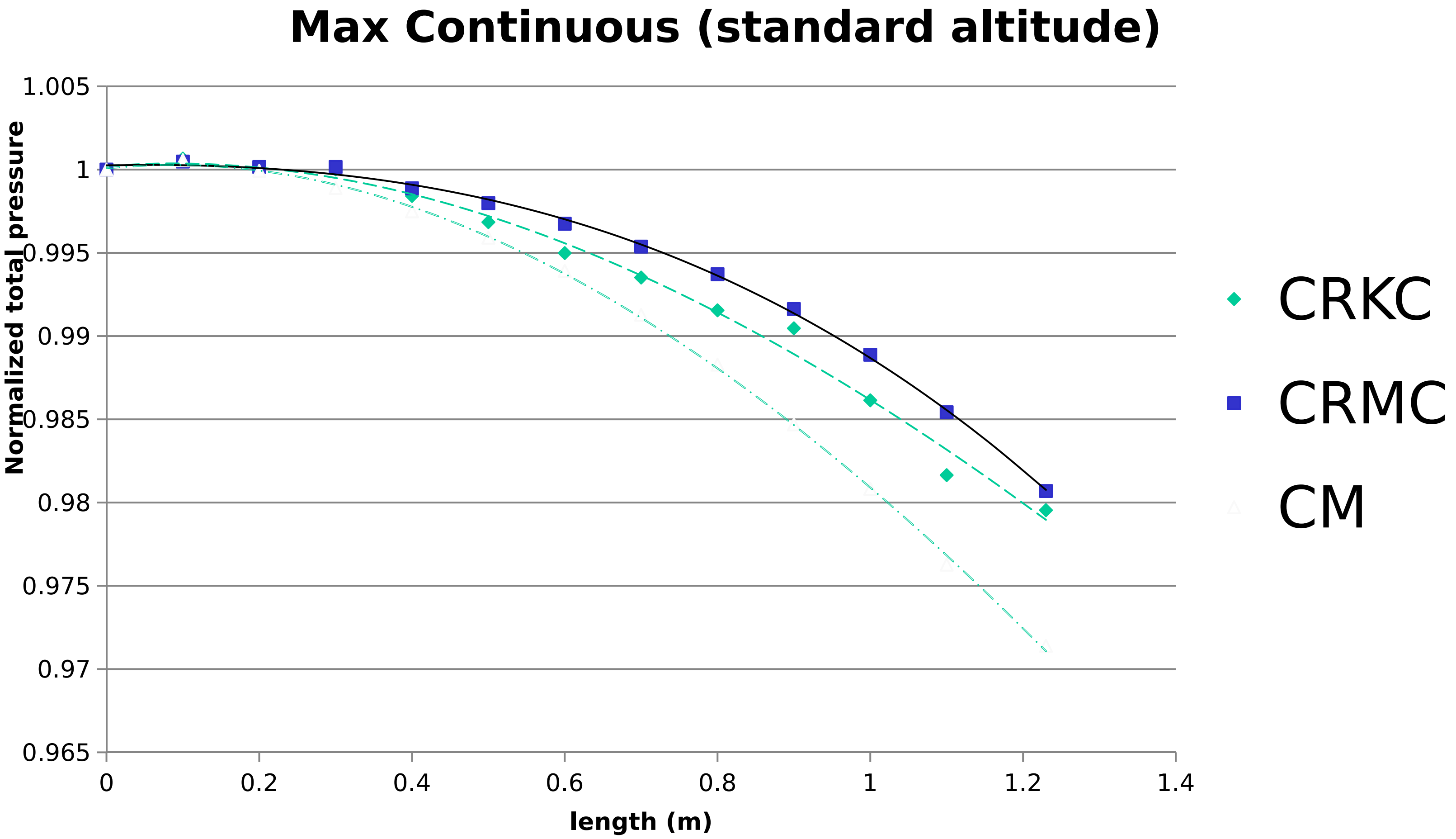
Static Pressure variation along nozzle length
Static Temperature variation along nozzle length
Mach number variation along nozzle length

Simulation at Intermediate at Sea Level Conditions (CRKC)



Static Pressure variation along nozzle length
Static Temperature variation along nozzle length
Mach number variation along nozzle length

Comparison of Normalized total Pressure



Comparison of Normalized total Pressure