Unit 3

Digital Modulation

Syllabus

- Amplitude Shift Keying (ASK)
- Frequency Shift Keying (FSK), FSK Detection Using PLL
- Binary Phase Shift Keying (PSK)- Transmitters, Coherent and non coherent detection, Bit and Baud Rate, Bandwidth and Frequency Spectrum.
- Quadrature Phase Shift Keying (QPSK), QPSK Demodulator,, BPSK, 8 PSK & 16 PSK
- Quadrature Amplitude Modulation (QAM); 8 QAM transmitter and receiver
- Carrier Recovery; Squaring Loop & Costas Loop,
- Differential PSK, DBPSK transmitter and receiver

Binary Modulation

- ASK
- FSK
- PSK

M-ary Modulation schemes

- QPSK
- QAM

Bit and

- Bit rate is the reciprocal of the time of one data element (bit).
- Bit rate is expressed as $f_b = 1/t_b$.
- Mathematically, baud is the reciprocal of the time of one output signaling element, and a signaling element may represent several information bits.
- Baud is expressed as baud $= 1/t_s$.
- In addition, since baud is the encoded rate of change, it also equals the bit rate f_b divided by the number of bits encoded into one signaling element.
- Hence if N data bits are encoded in each signal element then

$$\text{Baud} = \left(\frac{f_b}{N}\right)$$

- In M-ary communication, N= $\log 2\, M$

AMPLITUDE-SHIFT KEYING

- Mathematically, amplitude-shift keying is

$$v_{(ask)}(t) = [1 + v_m(t)]\left[\frac{A}{2}\cos(\omega_c t)\right]$$

- Modulating signal [$v_m(t)$] *is* normalized where + 1 V = logic 1 and -1 V = logic 0.

- Therefore for a logic 1 input, $v_m(t) = +1$ V, Equation *reduces to*

$$v_{(ask)}(t) = [1 + 1]\left[\frac{A}{2}\cos(\omega_c t)\right]$$

$$= A\cos(\omega_c t)$$

- And for a logic 0 input, $v_m(t)$ = -1 V, Equation *reduces to*

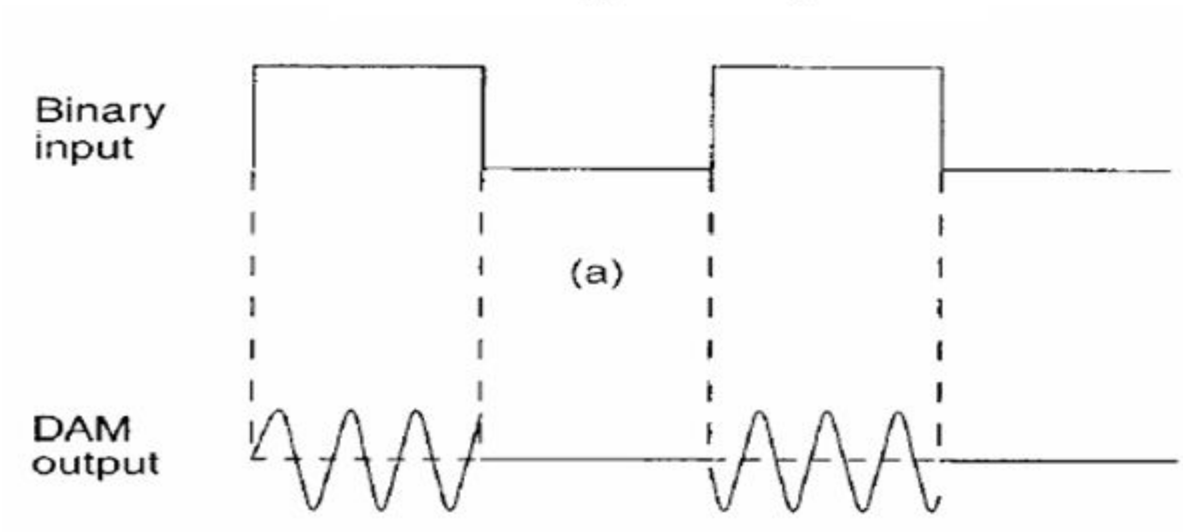
$$v_{(ask)}(t) = [1 - 1]\left[\frac{A}{2}\cos(\omega_c t)\right]$$

- Thus, the modulated wave *is either A cos($w_c t$) or 0*.

- Hence, the carrier is either *"on" or "off" which is why* amplitude-shift keying is sometimes referred to as *on- off keying(OOK)*.

ASK (also called Digital amplitude modulation)

$$\text{baud} = f_b / 1 = f_b$$



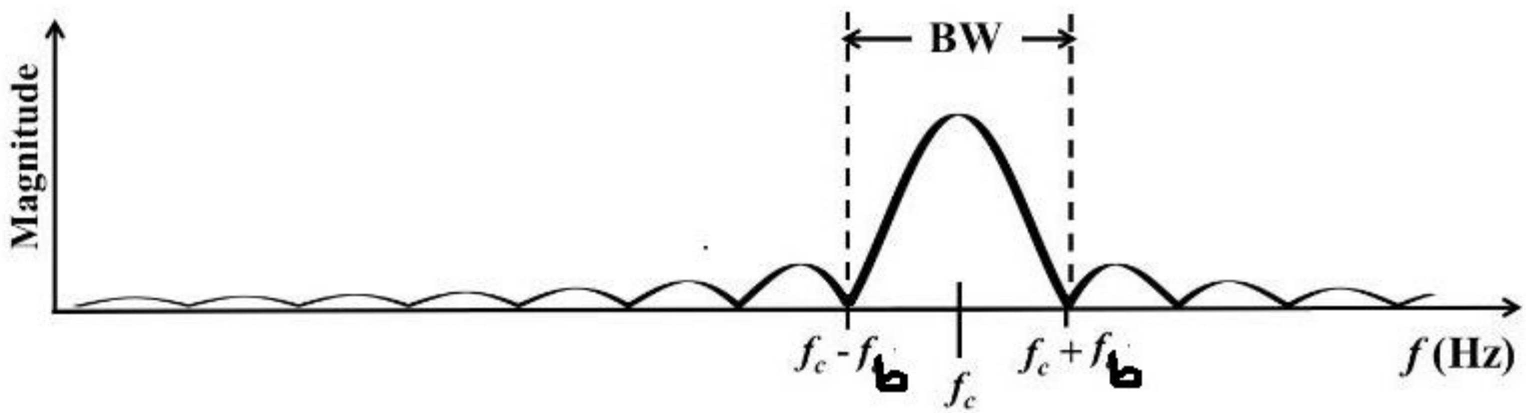
- fb is the input bit rate.

Performance of BASK/OKK

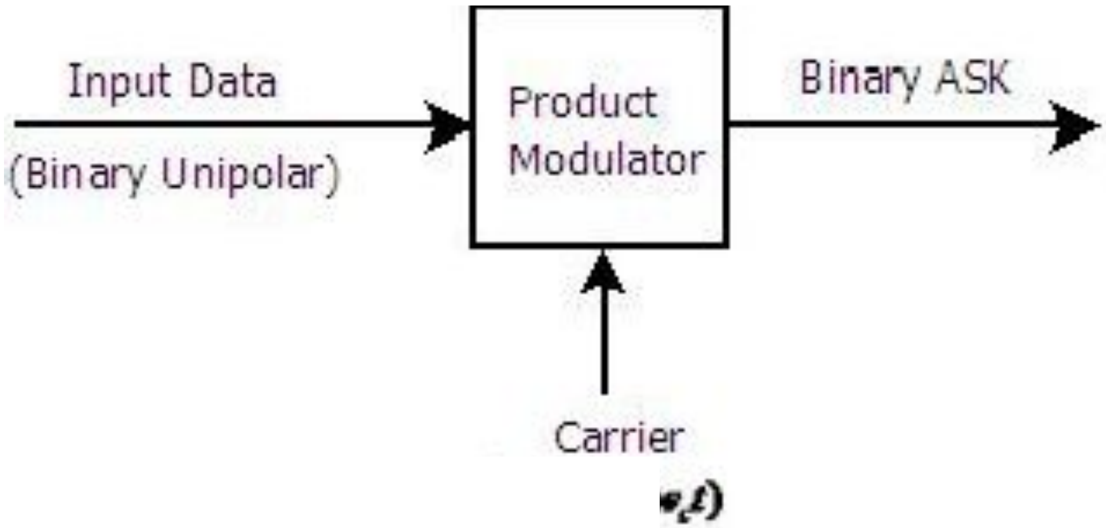
- As noise affects only the amplitude of the signal, the noise performance of ASK is poor.

Bandwidth of ASK

- Bandwidth of ASK signal is given by B=2fb since the message signal spectrum is shifted by fc Hz and there is a mirror image of the frequency content on the left side of fc in ASK modulation.



ASK Modulator



FREQUENCY-SHIFT KEYING

- FSK is a form of constant-amplitude angle modulation similar to standard frequency modulation (FM) except the modulating signal is a binary signal.
- FSK is sometimes called *binary FSK (BFSK)*.
- The general expression for FSK is

$$v_{fsk}(t) = V_c \cos\{2\pi[f_c + v_m(t)\,\Delta f]t\}$$

- Themodulating signal is a normalized binary waveform where a logic 1 = + 1 V and a logic 0 = -1 V.

- Thus, for a logic l input, we can write

$$v_{fsk}(t) = V_c \cos[2\pi(f_c + \Delta f)t]$$

- For a logic 0 input, vm(t) = -1,

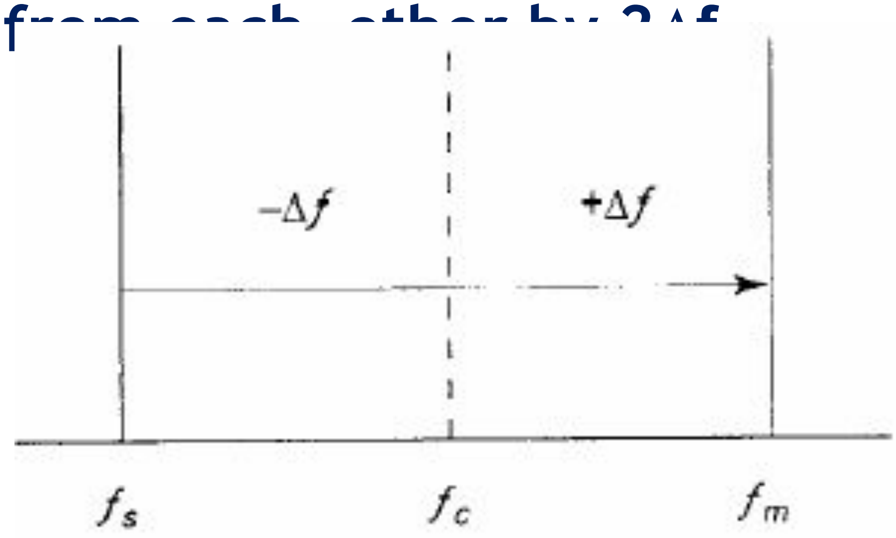
$$v_{fsk}(t) = V_c \cos[2\pi(f_c - \Delta f)t]$$

- Thus With binary FSK, the carrier center frequency (f_c) is shifted (deviated) up and down in the frequency domain by the binary input signal.

•As the binary input signal changes from a logic 0 to a logic 1 and vice versa, the output frequency shifts between two frequencies

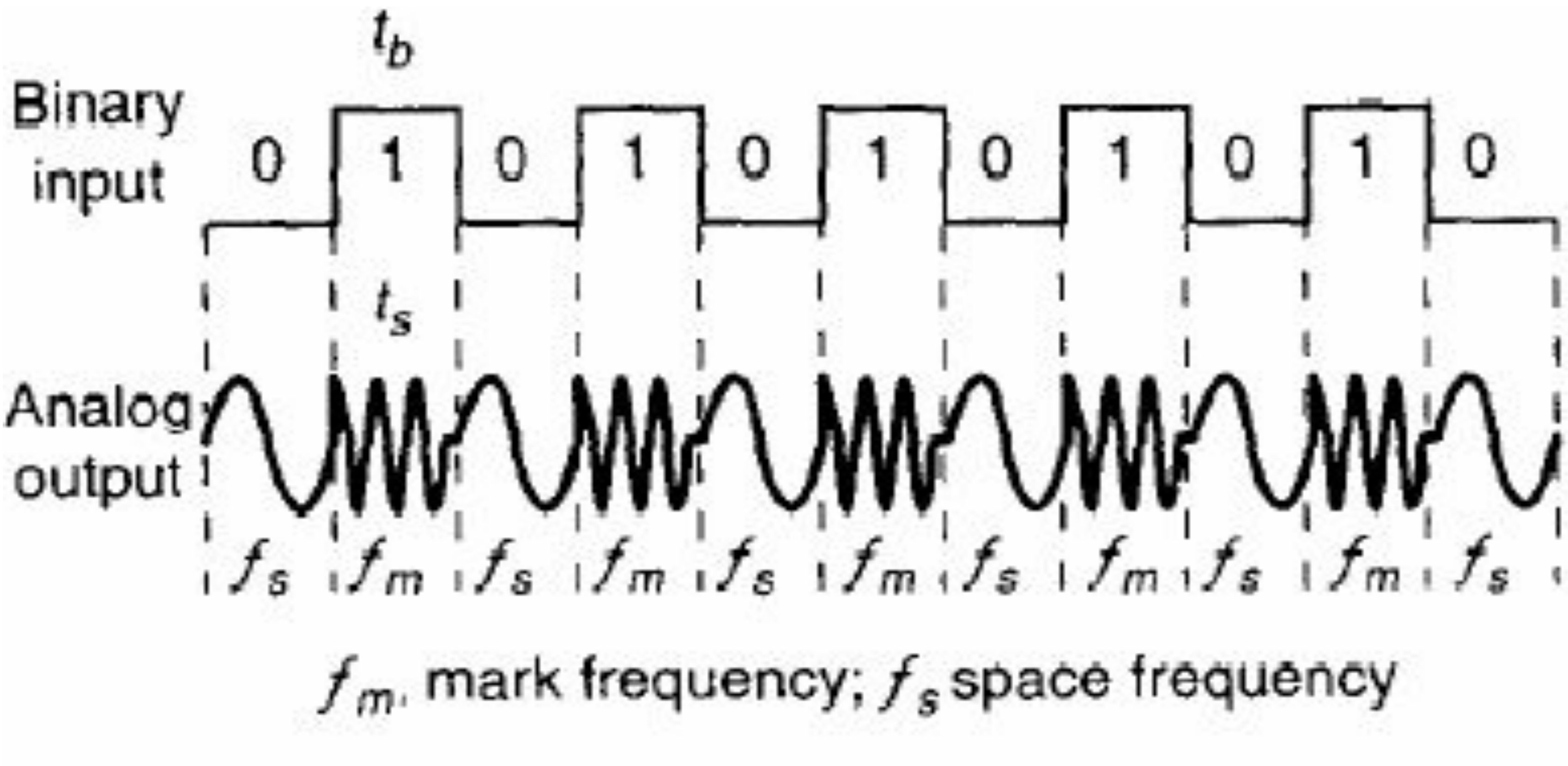
(a) mark, or logic 1 frequency (fm),
(b) space, or logic 0 frequency (fs).

*The mark and space frequencies are separated from the carrier frequency by the peak frequency Δf deviation and from each other by 2Δf



- Frequency deviation is expressed mathematically as

$$\Delta f = |f_m - f_s| / 2$$



FSK Bit Rate, Baud, and Bandwidth

- The bit time equals the time of an FSK signaling element, and the bit rate equals the baud.
- The baud for binary FSK can also be determined by substituting $N = 1$ in

$$\text{Baud} = \left(\frac{f_b}{N}\right)$$

$$\text{baud} = f_b / 1 = f_b$$

- The minimum bandwidth for FSK is given as

$$B = |(f_s - f_b) - (f_m - f_b)|$$

$$= |(f_s - f_m)| + 2f_b$$

$$B = 2(\Delta f + f_b)$$

Example 2-2

Determine (a) the peak frequency deviation, (b) minimum bandwidth, and (c) baud for a binary FSK signal with a mark frequency of 49 kHz, a space frequency of 51 kHz, and an input bit rate of 2 kbps

Solution

a. The peak frequency deviation is determined from Equation 2.14:

$$\Delta f = |149kHz - 51\ kHz| \ /\ 2 = 1\ kHz$$

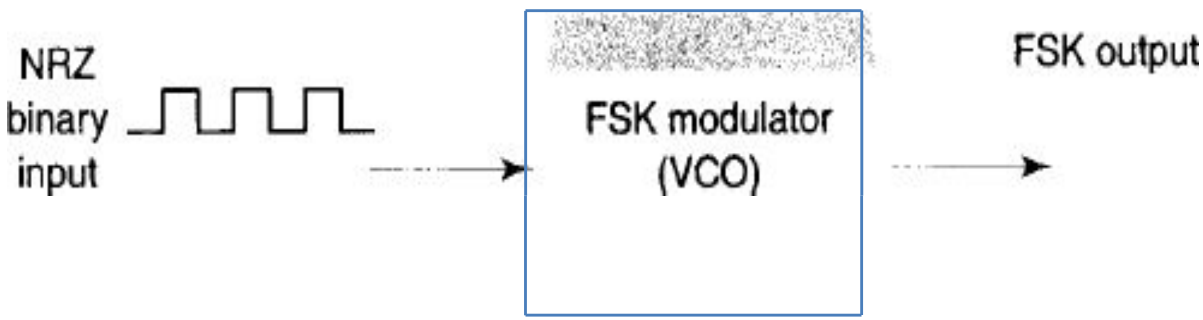
b. The minimum bandwidth is determined from Equation 2.15:

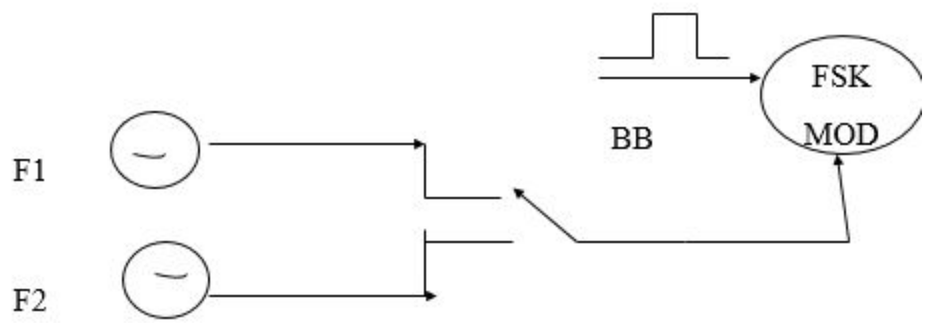
$$B = 2(1000 + 2000)$$
$$= 6\ kHz$$

c. For FSK, $N = 1$, and the baud is determined from Equation 2.11 as

$$baud = 2000 \ /\ 1 = 2000$$

FSK Transmitter





Performan

- Binary FSK has a poorer error performance than PSK or QAM and, consequently, is seldom used for high-performance digital radio systems.

- Its use is restricted to low-performance, low-cost asynchronous data modems that are used for data communications over analog, voice-band telephone lines.

PHASE-SHIFT KEYING

- The simplest form of PSK is binary phase-shift keying (BPSK), where N = 1 and M = 2.
- Therefore, with BPSK, two phases (2^1 = 2) are possible for the carrier.
- One phase represents a logic 1, and the other phase represents a logic 0.
- As the input digital signal changes state (i.e., from a 1 to a 0 or from a 0 to a 1), the phase of the output carrier shifts between two angles that are separated by 180°.
- Other names for BPSK are phase reversal keying (PRK) and bi-phase modulation.

- BPSK is a form of square-wave modulation of a continuous wave (CW) signal.

Equation of BPSK and comparison with ASK & FSK

A Binary Phase-Shift Keying (BPSK) signal is represented by

$$s(t) = A\, m(t) \cos(2 * pi * f_c * t), \quad 0 \le t \le T$$

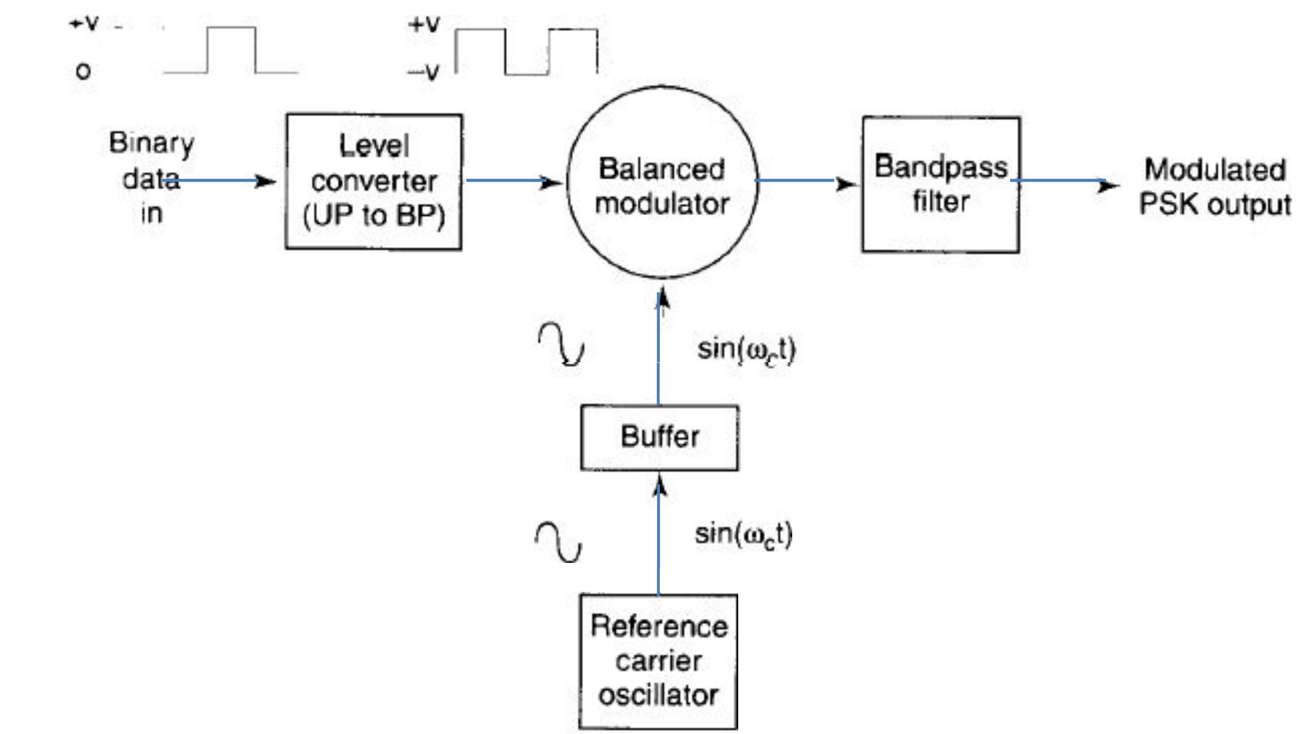
where A is constant, f_c is the carrier frequency, m(t) = +1 or -1 and T is the bit duration.

PSK is superior to FSK as it does not need 2 carrier signals.

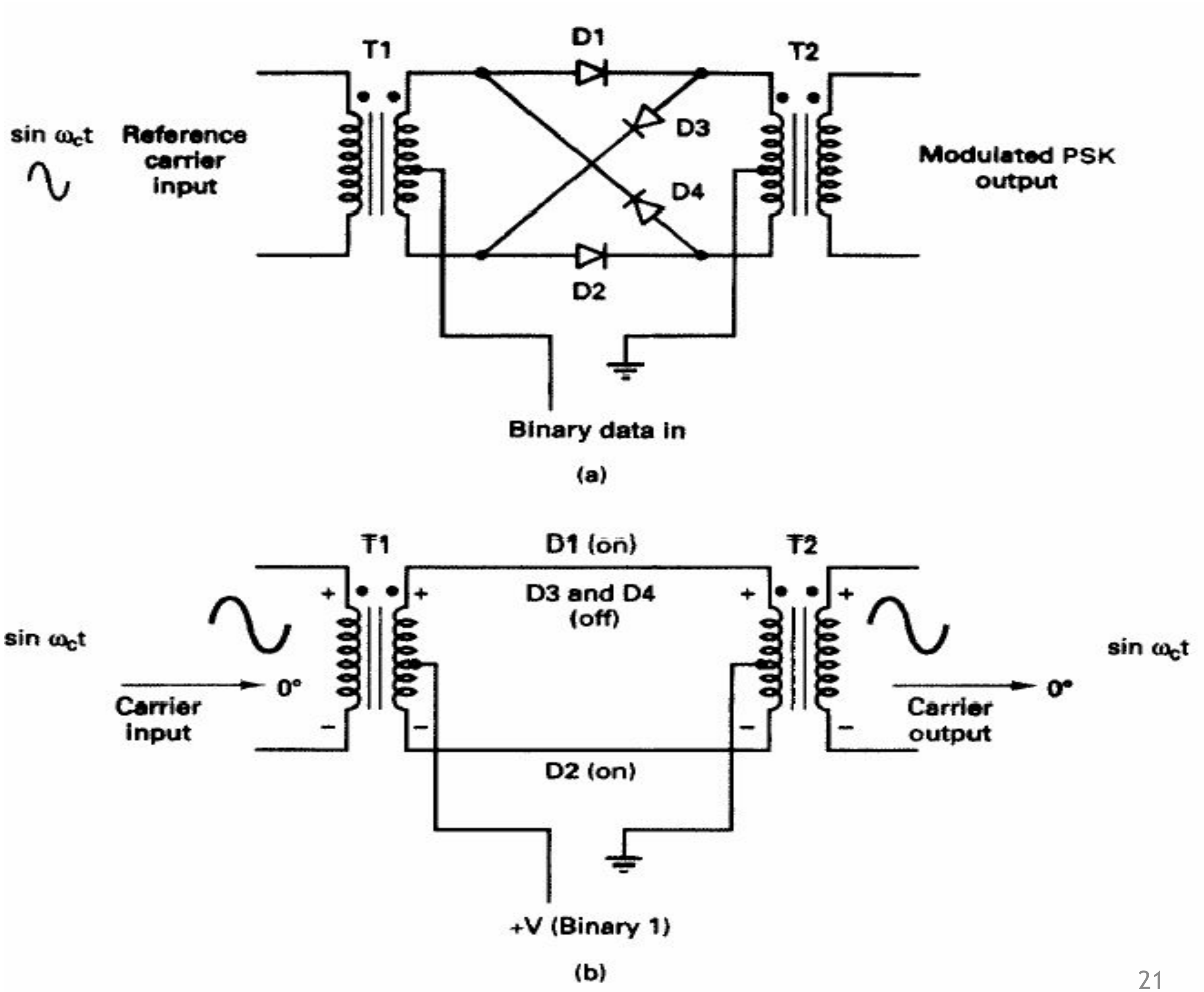
It is as simple as ASK & less susceptible to noise. (In ASK criterion for detection is the amplitude of the signal; in PSK it is the phase & noise affects the amplitude easier)

However PSK need more sophisticated hardware to distinguish between phases.

BPSK Modulator



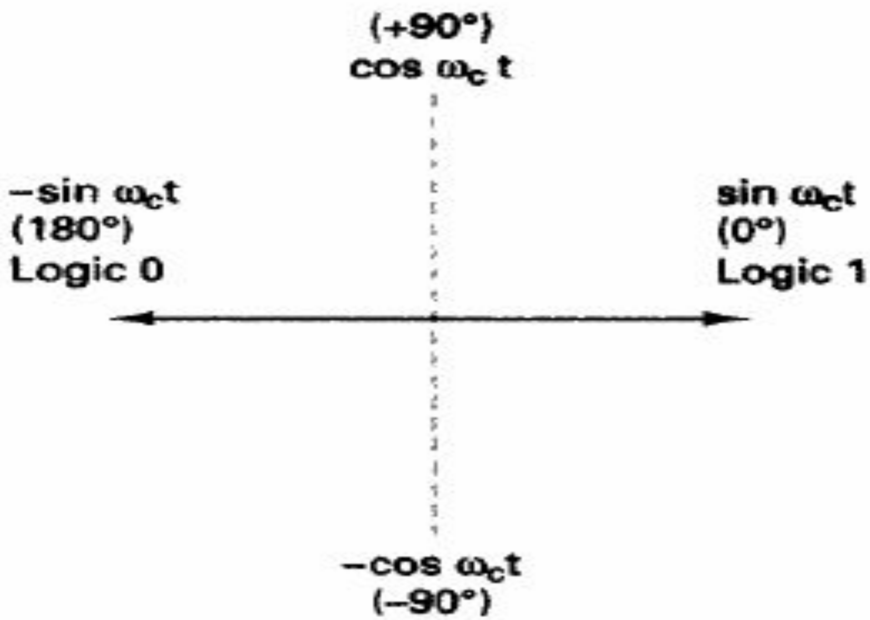
Balanced Modulator



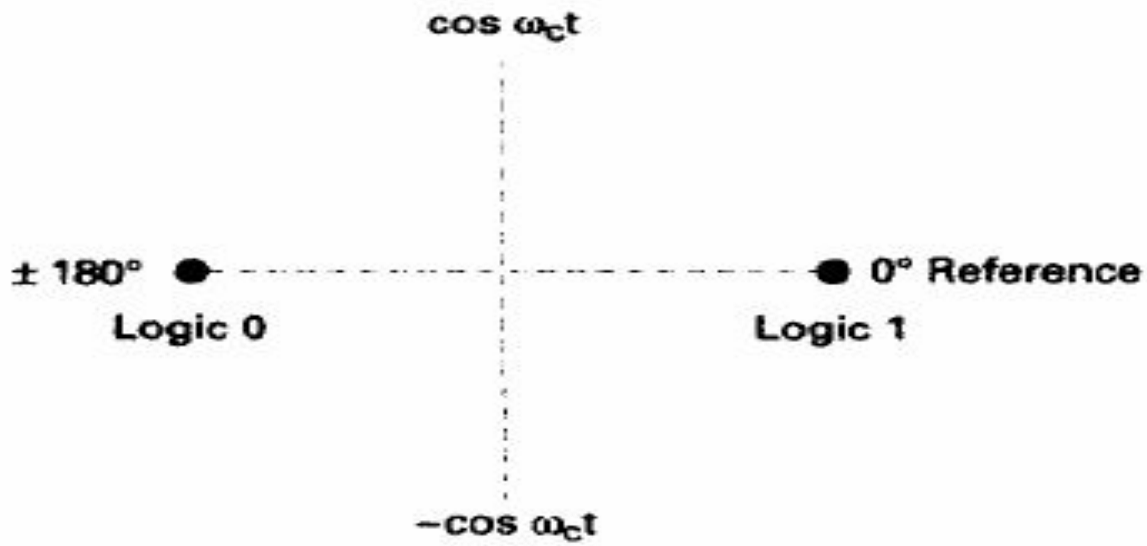
BPSK modulator: truth table phasor and constellation d

Binary input	Output phase
Logic 0 Logic 1	180° 0°

(a)



(b)



<u>Constellation Diagram</u>

- **It displays the signal as a two-dimensional xy-plane scatter diagram in the complex plane to designate the amplitude & phase of the signal element.**

- ***The angle of a point*, measured counterclockwise from the horizontal axis, represents the phase shift of the carrier wave from a reference phase.**

- ***The distance of a point from the origin* represents a measure of the amplitude or power of the signal.**

Bandwidth considerations of BPSK

$output = \cos w_m t . \cos w_c t$ (assuming unit amplitude) (*neglecting higher frequencies*)

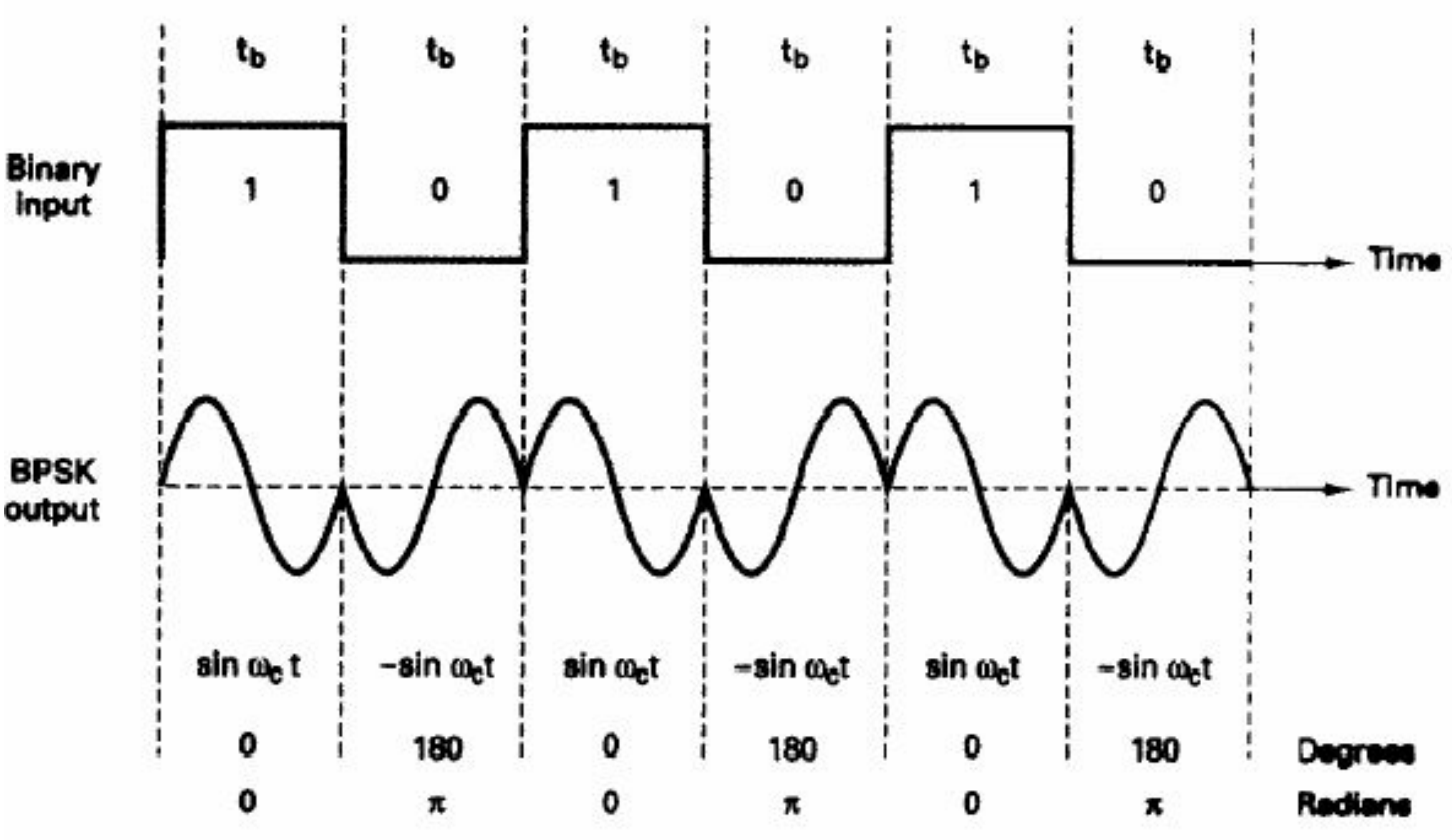
$$= \frac{1}{2}[\cos(w_c + w_m)t + \cos(w_c - w_m)t]$$

But here $w_m = 2\pi f_b$ putting this in above eq. weget

$$= \frac{1}{2}[\cos(f_c + f_b)t + \cos(f_c - f_b)t]$$

$$BW = (f_c + f_b) - (f_c - f_b) = 2f_b$$

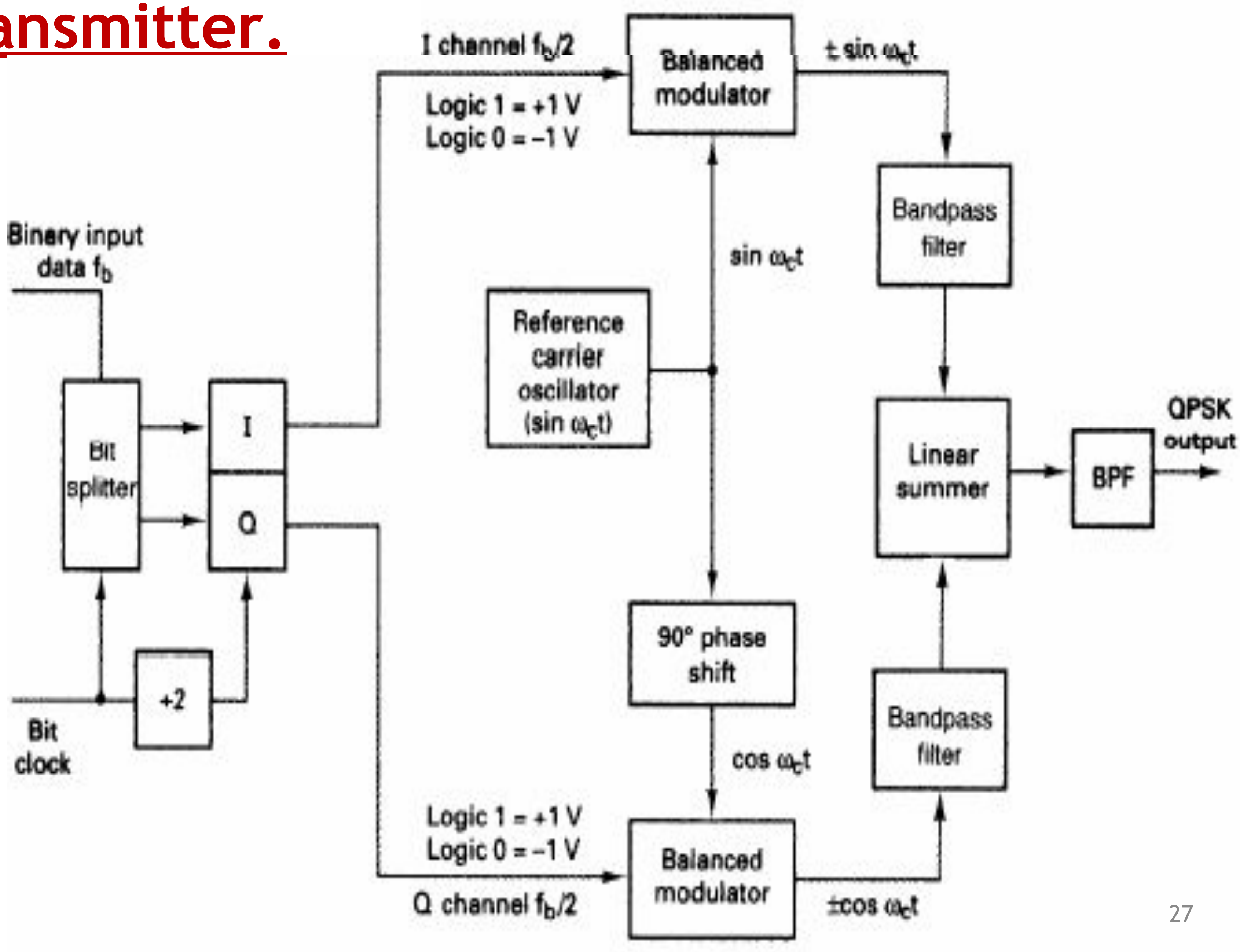
Output phase-versus-time relationship for a BPSK modulator



Quaternary Phase-Shift

- QPSK is an *M*-ary encoding scheme where N = *2 and M= 4*.
- *which means there are 4 symbols (00,01,10 and 11) and 2 bits per symbol*.
- Therefore, with QPSK, the binary input data are combined into groups of two bits, called *dibits*.
- each dibit code generates one of the four possible output phases (+45°, +135°, -45°, and -135°).

QPSK transmitter.



QPSK transmitter.

- Two bits (di-bit) are clocked into the bit splitter.
- After both bits have been serially inputted, they are simultaneously parallel outputted.
- The I bit modulates a carrier that is in phase with the reference oscillator (hence the name "I" for "in phase" channel)
- The Q bit modulate, a carrier that is 90° out of phase i.e. cosine wave .

- **For a logic 1 = + 1 and a logic 0= - 1 , two phases are possible at the output of the –I modulator (+sinw_ct and -sinw_ct)**

- **Similarly two phases are possible at the output of the Q balanced modulator (+cosw_ct), and (-cosw_ct).**

- **For input of Q =I= 1, the two inputs to the I balanced modulator are**
  +1 and sinw_ct, and The two inputs to the Q balanced modulator are
  +1 and cosw_ct.

- **Outputs are**

Output **of the linear summer for 1,1 input dibits is**

$$= sin\, w_c t + \cos w_c t$$

$$= \left\{ sin\, w_c t + sin(90^0 + w_c t) \right\}$$

$$= \left\{ 2\sin\left(\frac{2w_c t + 90^0}{2}\right)\cos\left(\frac{90^0}{2}\right) \right\}$$

$$= \sqrt{2}\sin\left(\frac{2w_c t + 90^0}{2}\right)$$

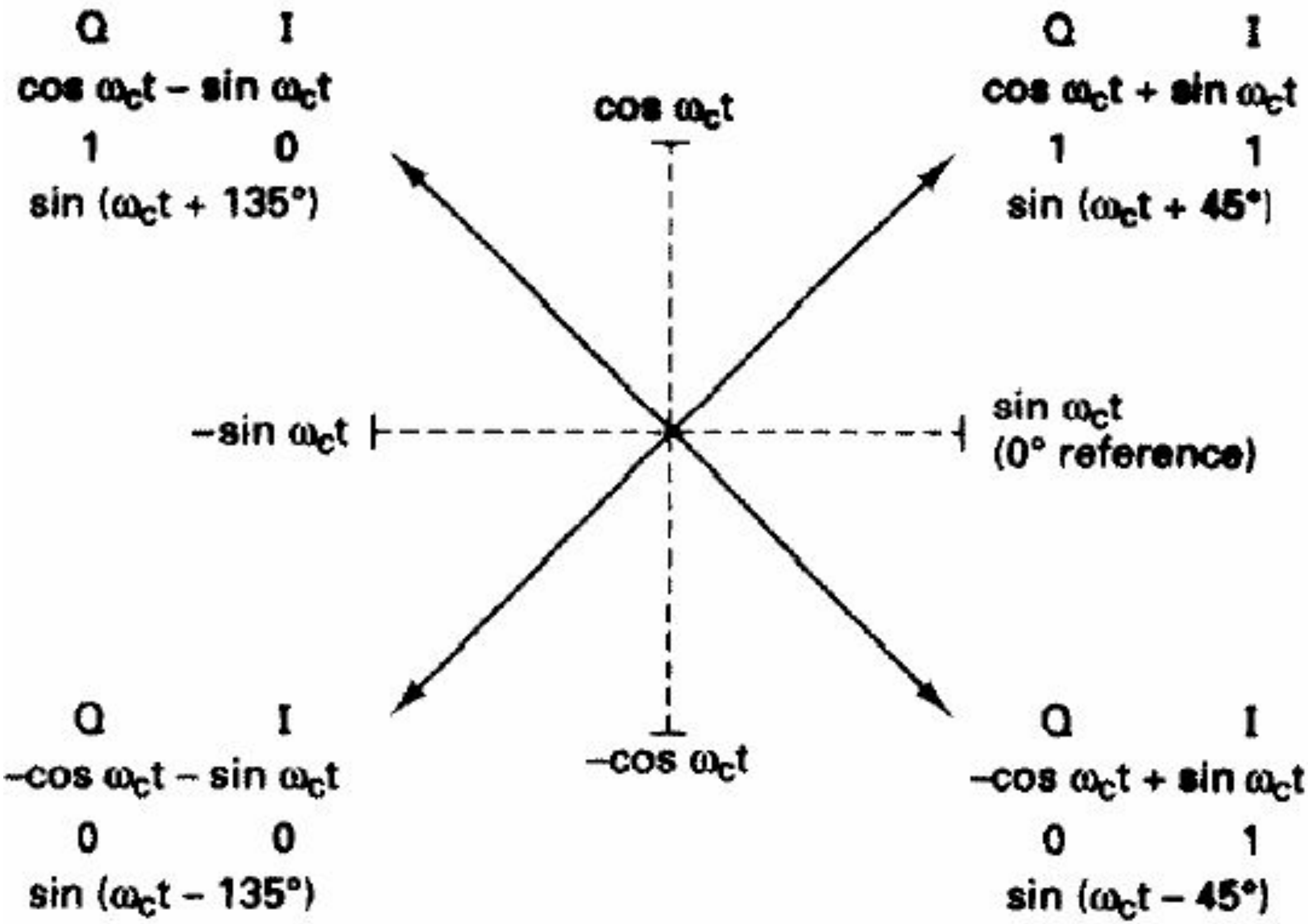
$$= 1.414\sin(w_c t + 45^o\}$$

(Similarly it can be calculated for all other input combinations)

Truth table

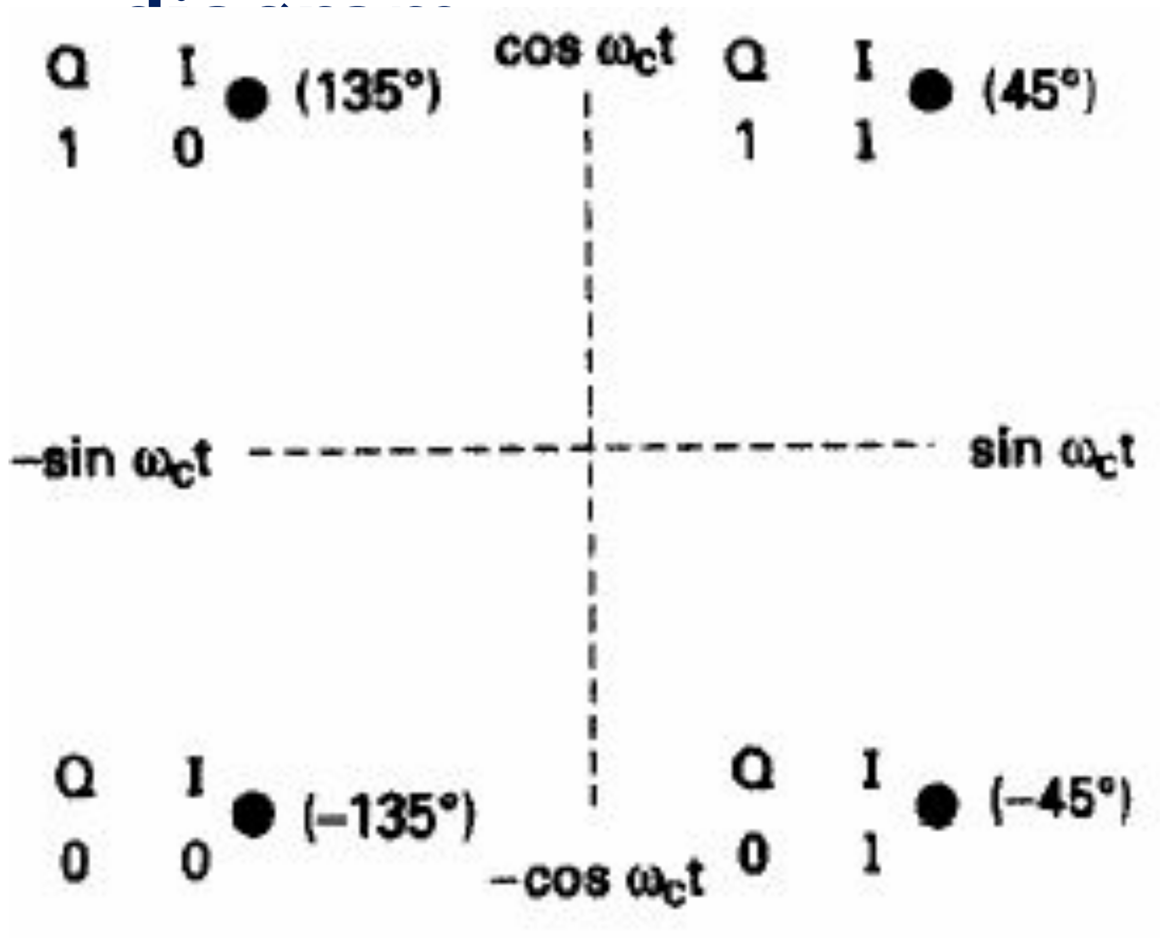
Binary input		QPSK output phase
Q	I	
0	0	−135°
0	1	−45°
1	0	+135°
1	1	+45°

Phasor

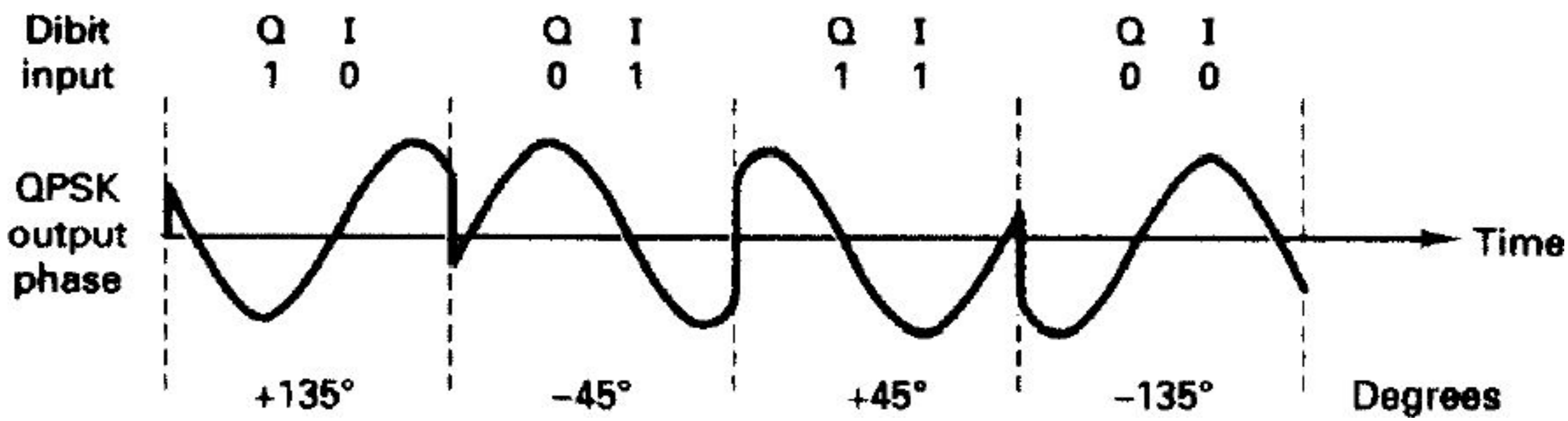


Q I
cos ωct − sin ωct
1 0
sin (ωct + 135°)
cos ωct
Q I
cos ωct + sin ωct
1 1
sin (ωct + 45°)
−sin ωct
sin ωct
(0° reference)
Q I
−cos ωct − sin ωct
0 0
sin (ωct − 135°)
−cos ωct
Q I
−cos ωct + sin ωct
0 1
sin (ωct − 45°)

Constellation diagram



Output phase-versus-time relationship for a QPSK modulator.



Bandwidth of QPSK

$out\ put = cos\ w_m t . \cos w_c t$ (assuming unit amplitude) (*neglecting higher frequencies*)

$$= \frac{1}{2}[\cos(w_c + w_m)t + \cos(w_c - w_m)t]$$

But here $w_m = \frac{2\pi f_b}{2}$ putting this in above eq. we get

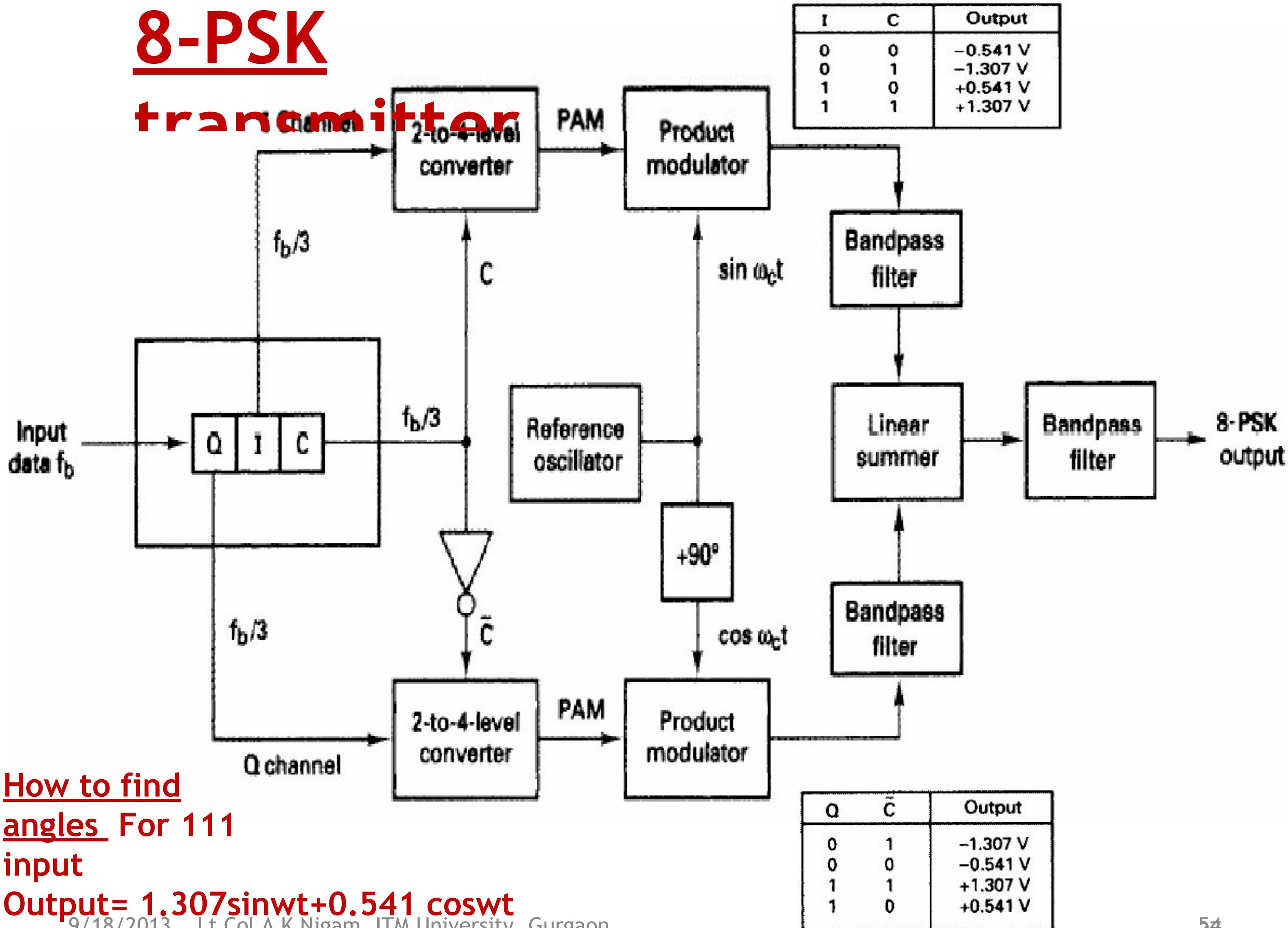
$$= \frac{1}{2}[\cos(f_c + \frac{f_b}{2})t + \cos(f_c - \frac{f_b}{2})]$$

$$BW = (f_c + \frac{f_b}{2}) - (f_c - \frac{f_b}{2}) = f_b$$

8-PSK Modulation

- With *8-PSK, three bits are encoded, forming tribits and* producing eight different output phases.
- To encode eight different phases, the incoming bits are encoded in groups of three, called tribits (2^3 = 8).
- With 8-PSK, the angular separation between adjacent output phases is only 45° (360 / 8).

8-PSK transmitter



I	C	Output
0	0	−0.541 V
0	1	−1.307 V
1	0	+0.541 V
1	1	+1.307 V

Q	$\bar{C}$	Output
0	1	−1.307 V
0	0	−0.541 V
1	1	+1.307 V
1	0	+0.541 V

How to find angles For 111 input

Output= 1.307sinwt+0.541 coswt

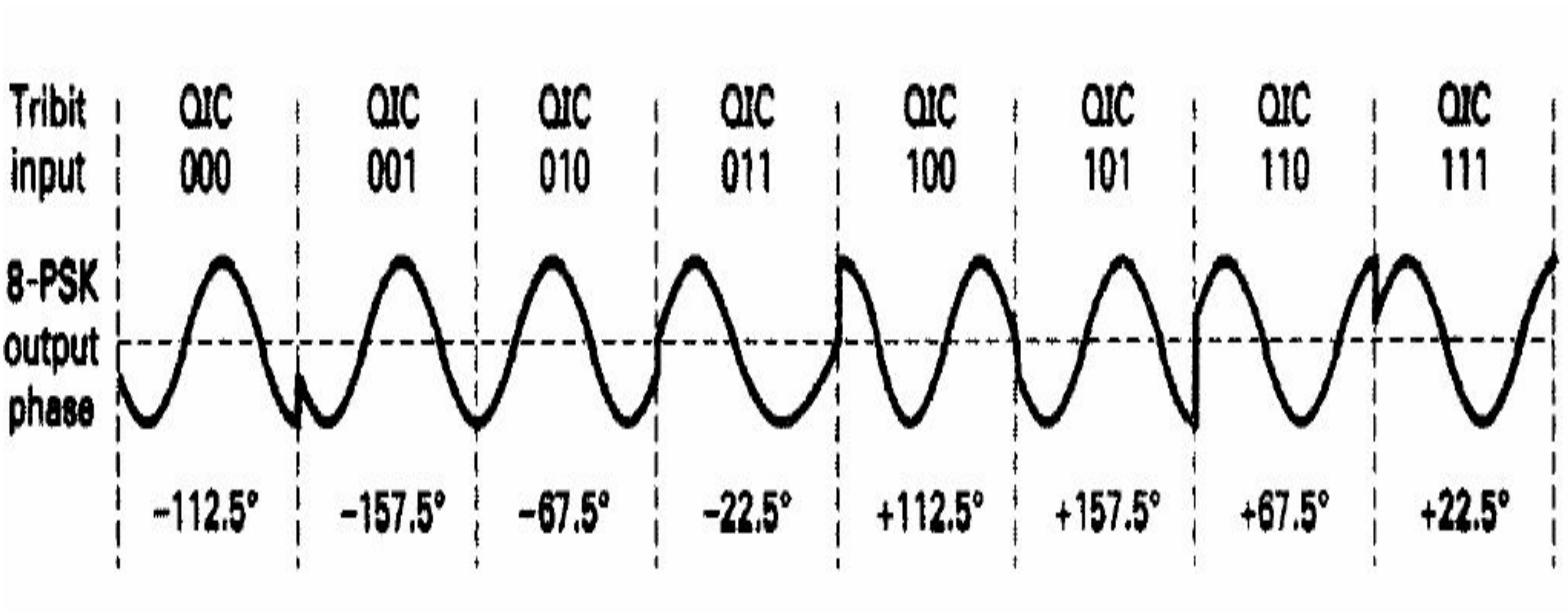
Angle= tan^-1(.541/1.307) in 1st quadrangle

Output Phases

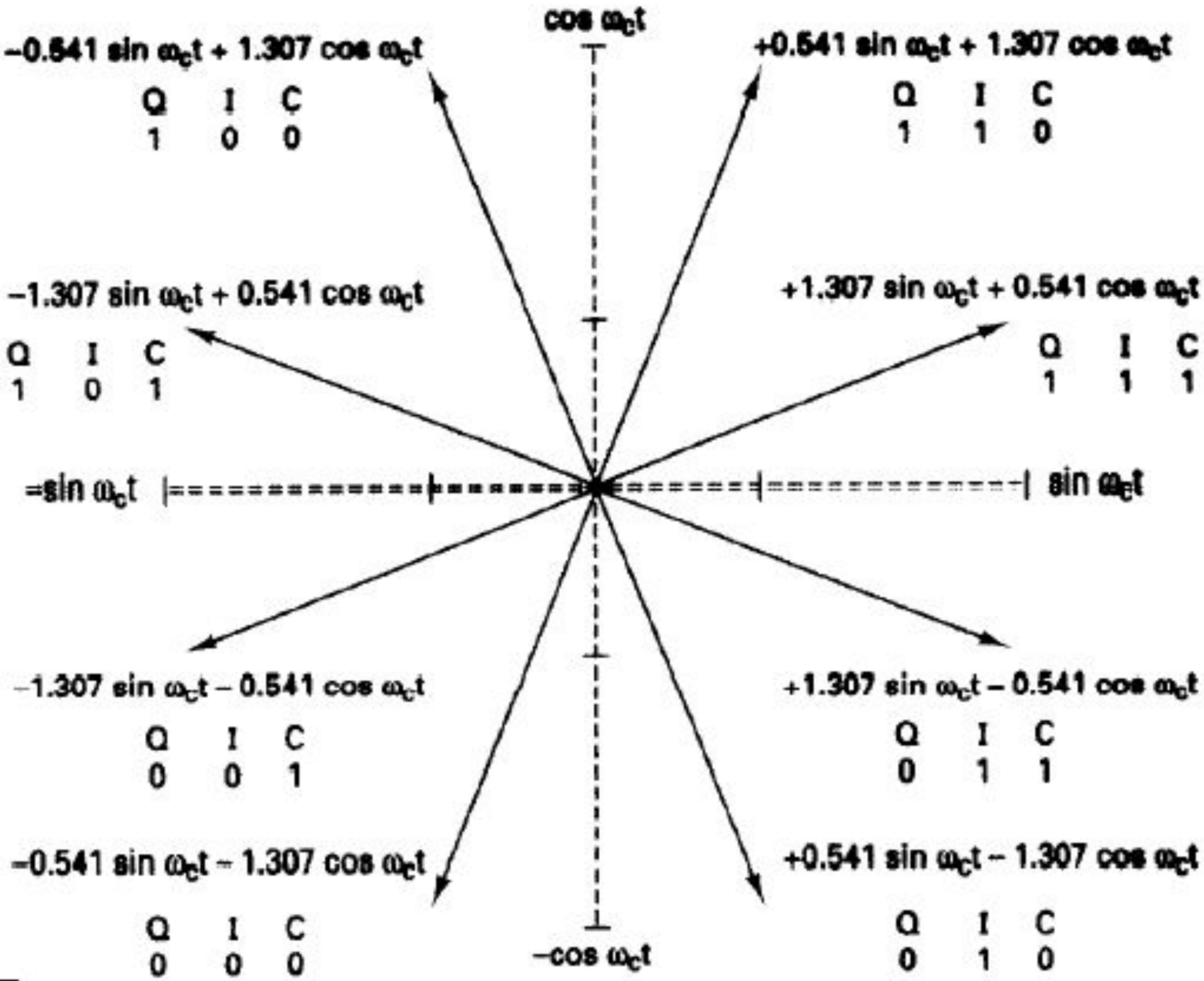
Note
Phases are +/-(22.5° +45°)

Binary input			8-PSK output phase
Q	I	C	
0	0	0	−112.5°
0	0	1	−157.5°
0	1	0	−67.5°
0	1	1	−22.5°
1	0	0	+112.5°
1	0	1	+157.5°
1	1	0	+67.5°
1	1	1	+22.5°

Output phase-versus-time relationship for an 8- PSK modulator

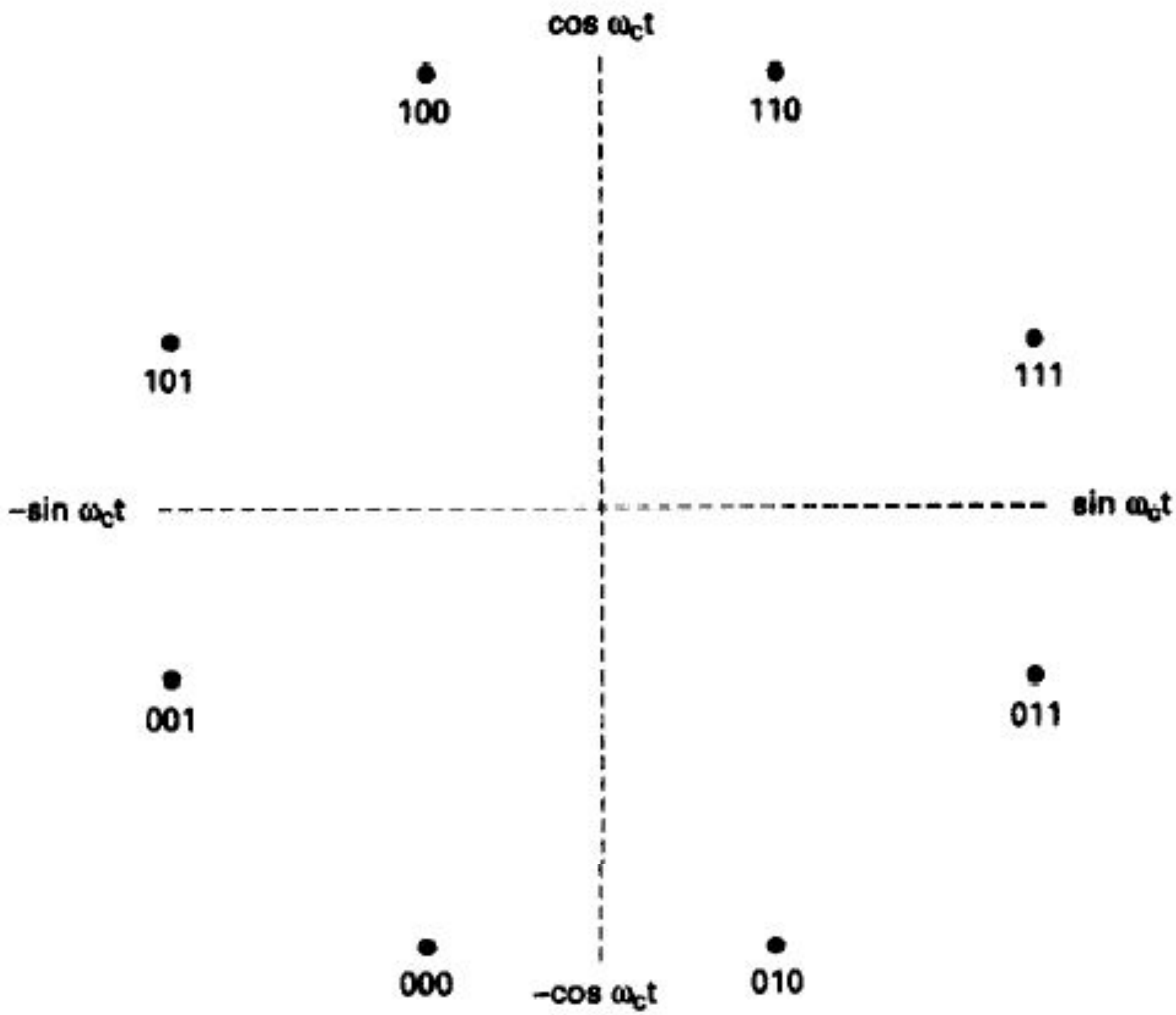


Phasor



cos ωct
−0.541 sin ωct + 1.307 cos ωct
Q I C
1 0 0
+0.541 sin ωct + 1.307 cos ωct
Q I C
1 1 0
−1.307 sin ωct + 0.541 cos ωct
Q I C
1 0 1
+1.307 sin ωct + 0.541 cos ωct
Q I C
1 1 1
−sin ωct
sin ωct
−1.307 sin ωct − 0.541 cos ωct
Q I C
0 0 1
+1.307 sin ωct − 0.541 cos ωct
Q I C
0 1 1
−0.541 sin ωct − 1.307 cos ωct
Q I C
0 0 0
+0.541 sin ωct − 1.307 cos ωct
Q I C
0 1 0
−cos ωct

Constellation Diagram: 8 PSK



Bandwidth considerations of 8-PSK

- With 8-PSK, the data are divided into three channels,
- the bit rate in the I, Q, or C channel is equal to one-third of the binary input data rate *(fb /3)*.

Analysis

$$out\ put = \cos w_c t . \cos w_m t \quad (amplitude\ assumed\ to\ be\ unity)$$

$$= \frac{1}{2}[\cos(w_c + w_m)t + \cos(w_c - w_m)] \quad (neglecting\ higher\ frequencies)$$

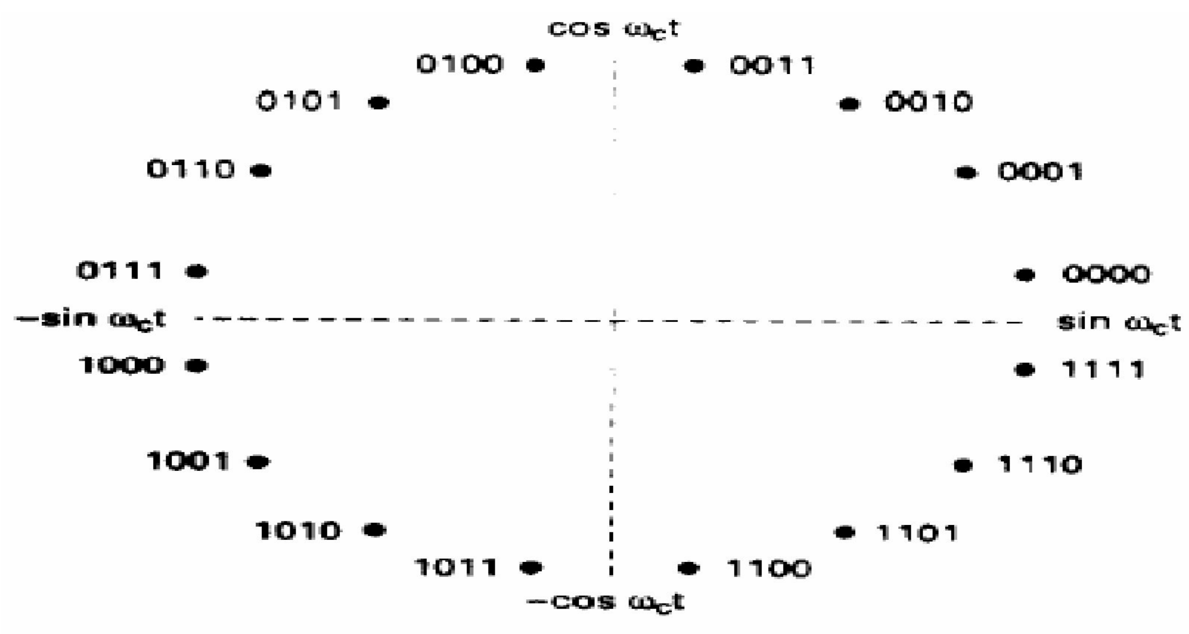
$$But\ here\ w_m = \frac{2\pi f_b}{3} \text{ putting this in above eq. we get}$$

$$= \frac{1}{2}[\cos(f_c + \frac{f_b}{3})t + \cos(f_c - \frac{f_b}{3})]$$

$$BW = (f_c + \frac{f_b}{3})t - (f_c - \frac{f_b}{3}) = \frac{2 * f_b}{3}$$

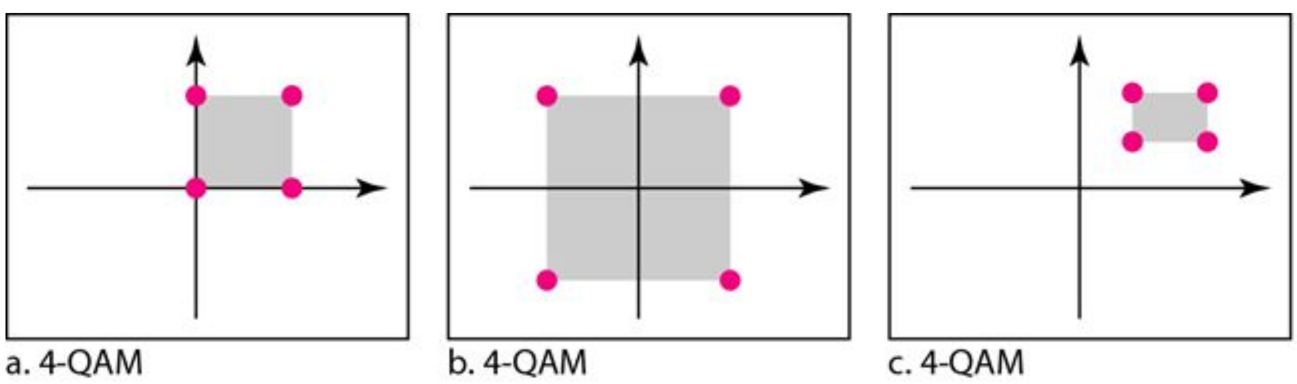
16 PSK

- With 16-PSK, the angular separation between adjacent output phases is only 22.5° (360 / 16).



Quadrature Amplitude Modulation

- Combination of ASK and PSK
- 4-QAM is an M-ary encoding technique where M = 4.
- Two carriers, one with in-phase and the other quadrature, with different amplitude levels for each carrier.

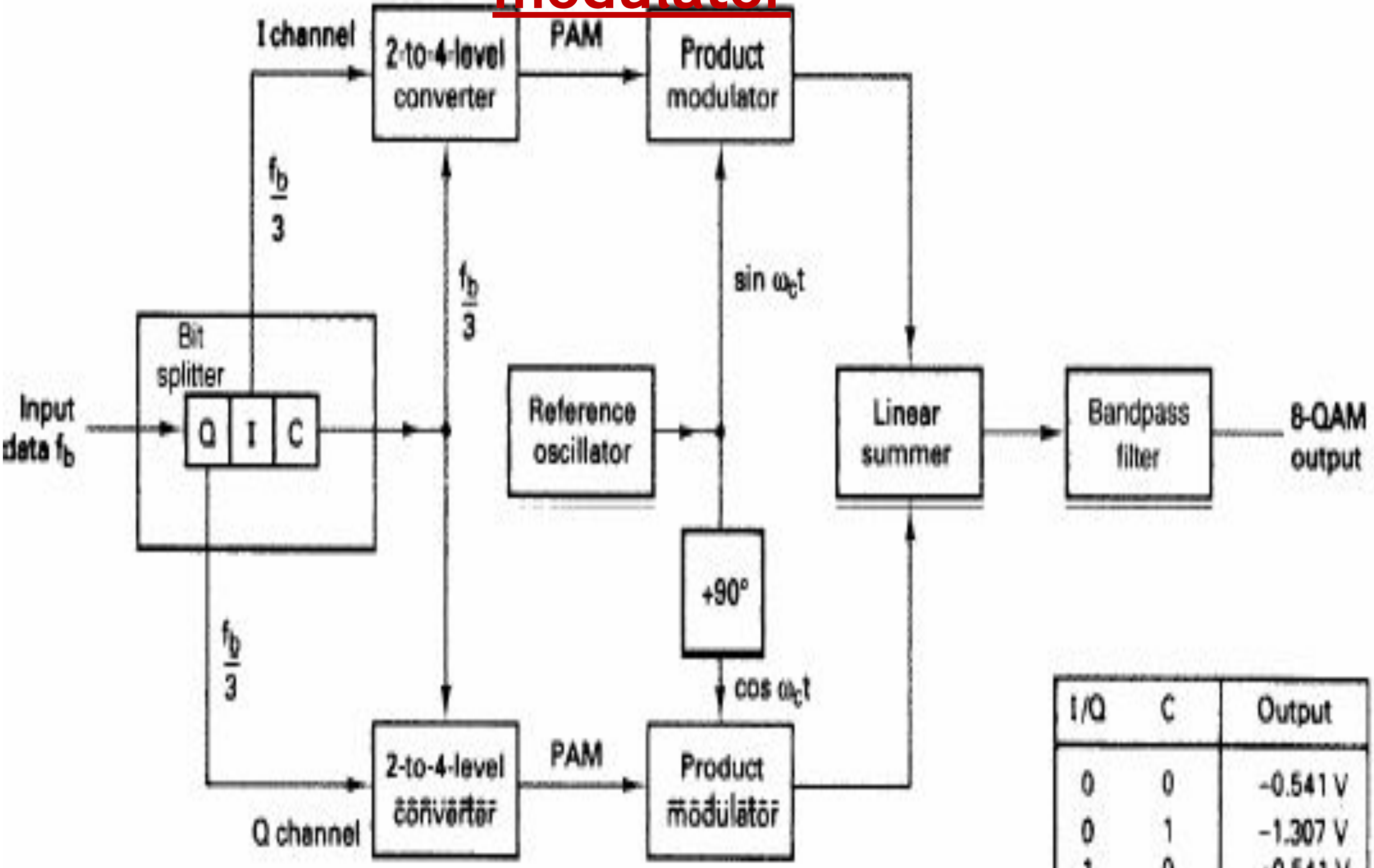


QUADRATURE – AMPLITUDE MODULATION

8-QAM

- 8-QAM is an M-ary encoding technique where M = 8.
- Unlike 8-PSK, the output signal from an 8- QAM modulator is not a constant-amplitude signal.

8-QAM modulator



I/Q	C	Output
0	0	-0.541 V
0	1	-1.307 V
1	0	+0.541 V
1	1	+1.307 V

8-QAM modulator

- The incoming data are divided into groups of three bits (tribits): the I, Q, and C bit streams.

- Each stream has a bit rate equal to one-third of the incoming data rate.

- The I and Q bits determine the polarity of the PAM signal at the output of the 2-to-4-level converters.

- The C channel determines the magnitude.

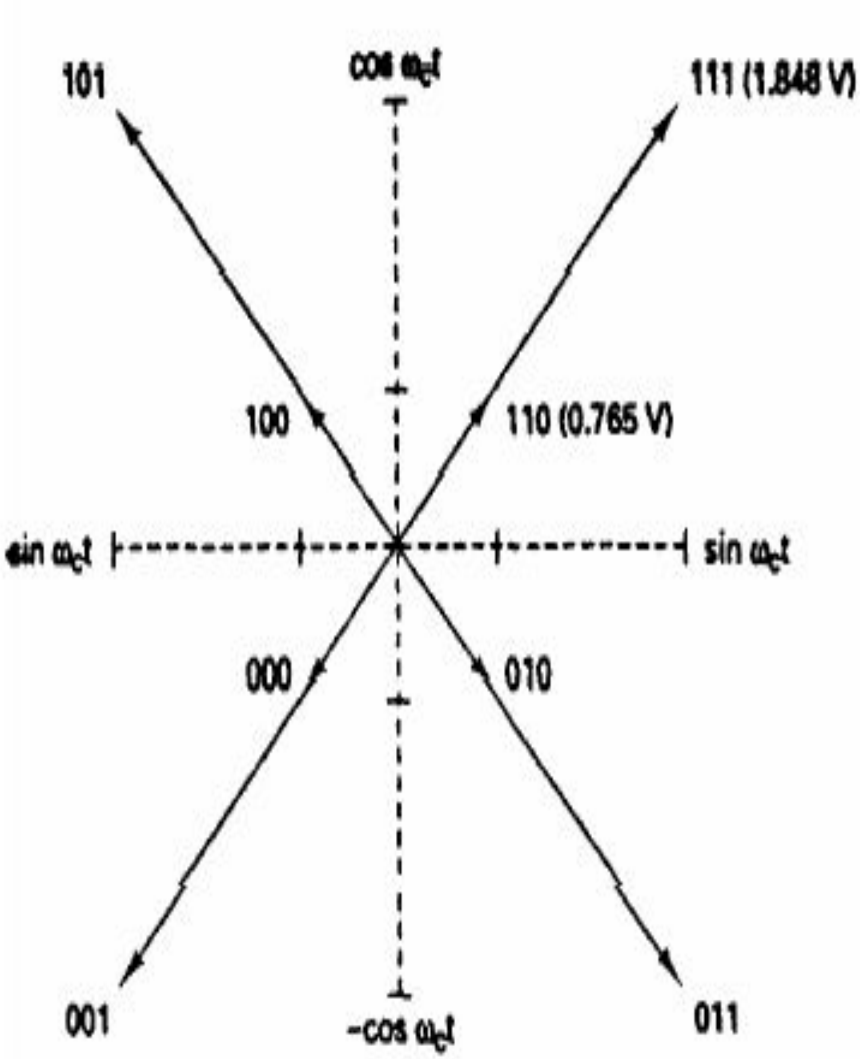
- Because the c bit is fed un-inverted to both the i and the q channel 2-to-4-level converters, the magnitudes of the I and Q PAM signals are always equal.

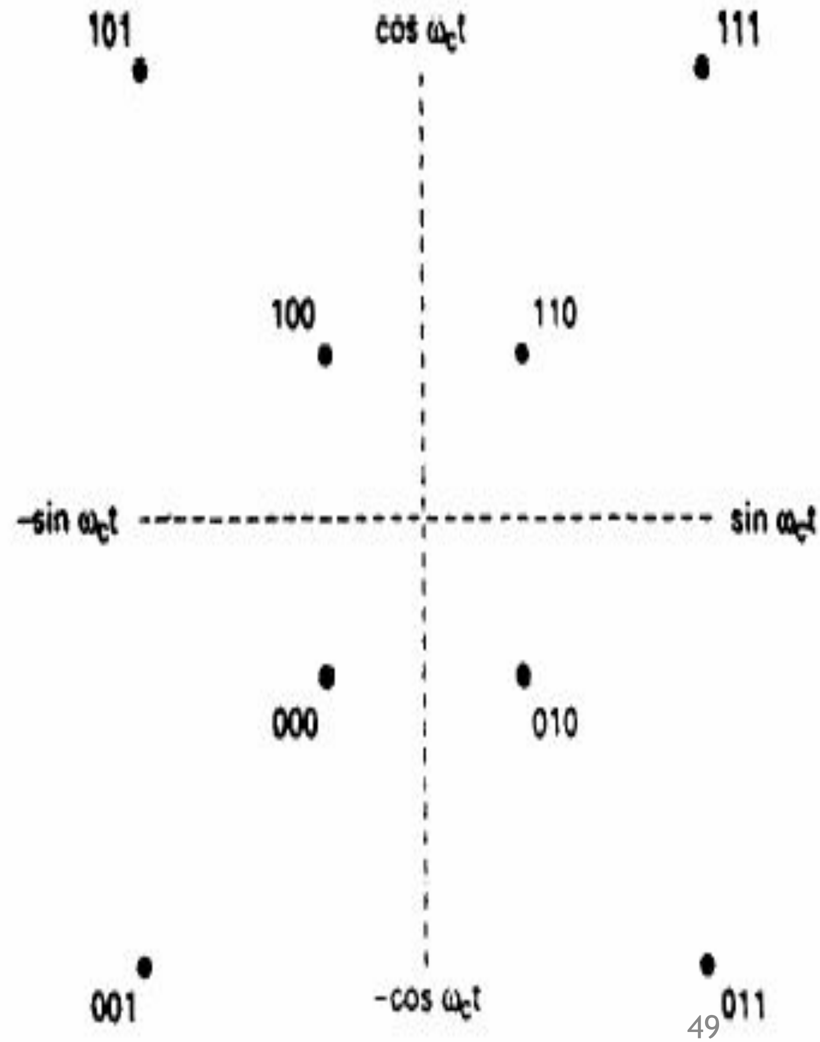
- Their polarities depend on the logic condition of the i and q bits and therefor may be different.

Truth

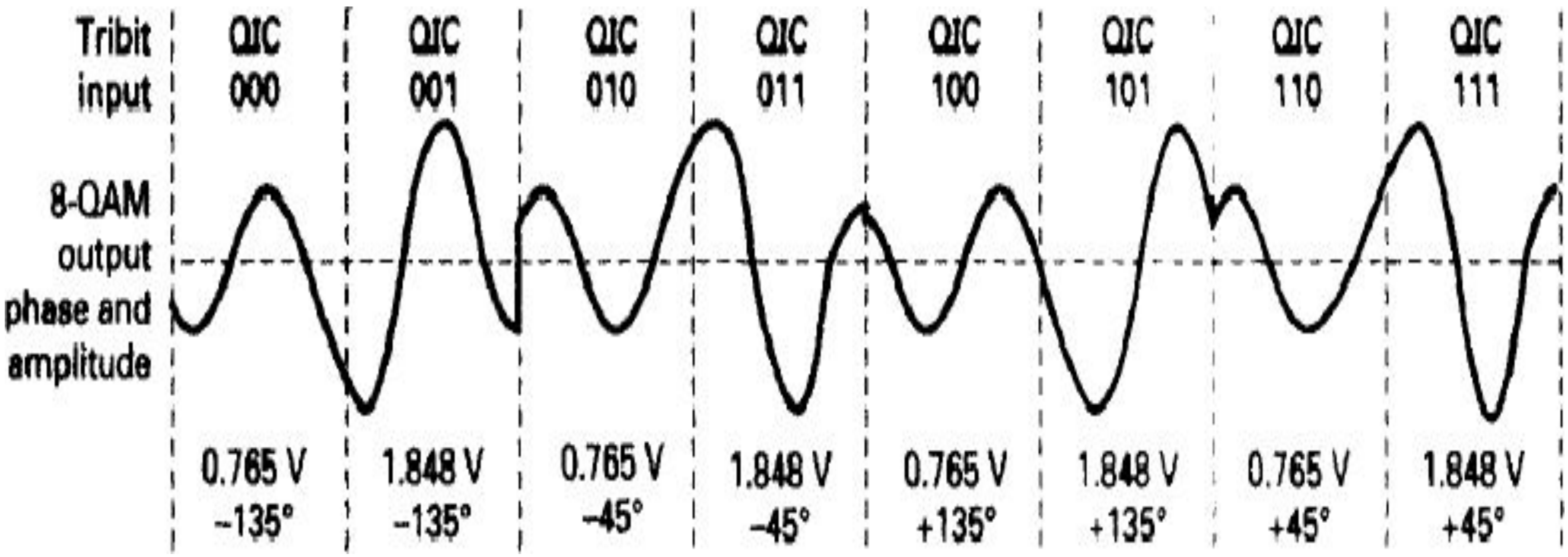
Binary input			8-QAM output	
Q	I	C	Amplitude	Phase
0	0	0	0.765 V	−135°
0	0	1	1.848 V	−135°
0	1	0	0.765 V	−45°
0	1	1	1.848 V	−45°
1	0	0	0.765 V	+135°
1	0	1	1.848 V	+135°
1	1	0	0.765 V	+45°
1	1	1	1.848 V	+45°

Phasor and constellation





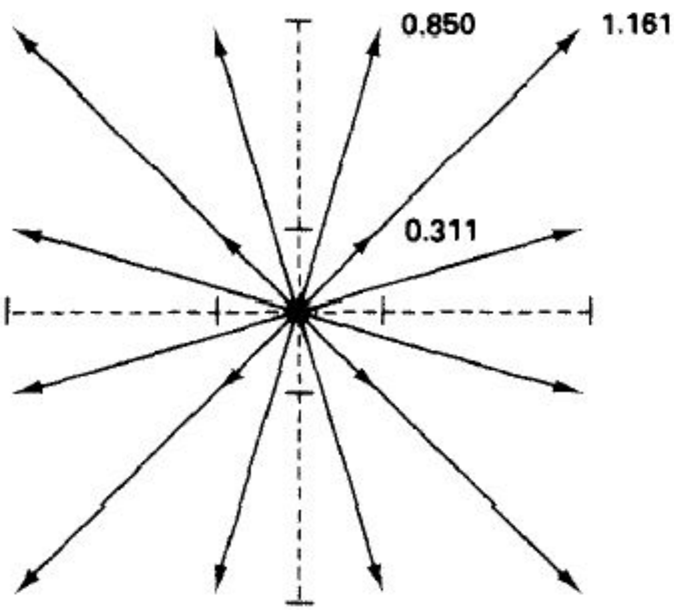
Output phase and amplitude-versus-time relationship for 8-QAM

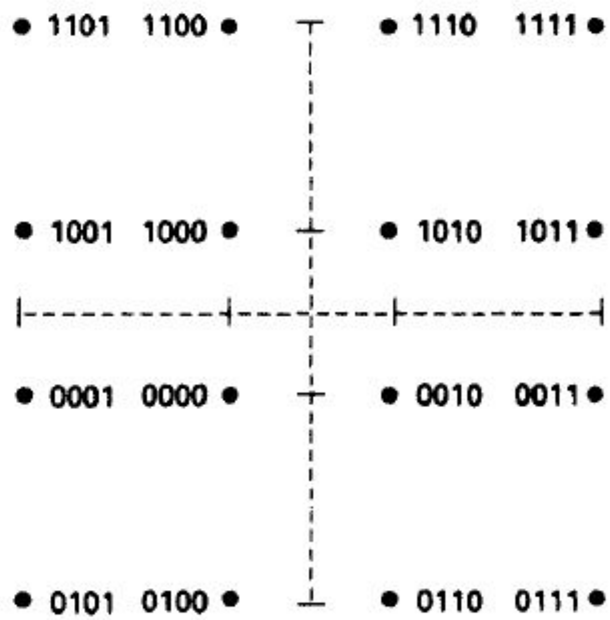


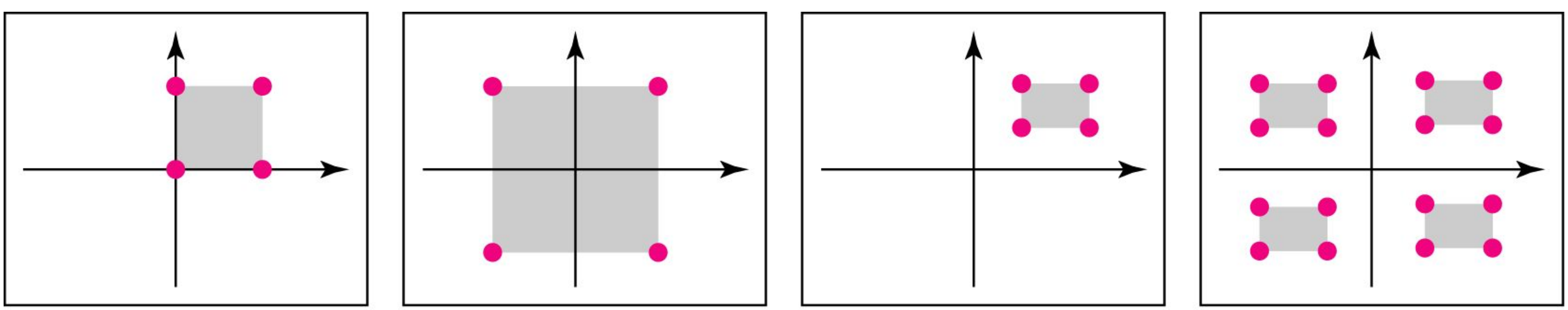
Bandwidth considerations of 8-QAM.

- N=3
- Thus the minimum bandwidth required for 8- QAM is *fb / 3*, *the* same as in 8-PSK.

16-QAM Phasor and constellation diagram







a. 4-QAM b. 4-QAM c. 4-QAM d. 16-QAM

Exampl
e

Que 1. For 8-PSK operating with information bit rate of 64 kbps, determine baud and the minimum bandwidth.

Que 2. For 16-PSK and a transmission system with a 10 kHz bandwidth, determine the maximum bit rate.

ASK, FSK, PSK and QAM summary

Modulation	# Symbols (M)	# Bits/symbol ($N=\log_2 M$)	Bandwidth (BW)
FSK	2	1	$f_{mark}-f_{space}+2R_b$
ASK	2	1	$2R_{sym}= 2R_b/N=2R_b$
OOK	2	1	$2R_{sym}= 2R_b/N=2R_b$
BPSK	2	1	$2R_{sym}= 2R_b/N=2R_b$
QPSK	4	2	$2R_{sym}= 2R_b/N= R_b$
8PSK	8	3	$2R_{sym}= 2R_b/N = 2R_b/3$
8QAM	8	3	$2R_{sym}= 2R_b/N = 2R_b/3$
16PSK	16	4	$2R_{sym}= 2R_b/N = R_b/2$
16QAM	16	4	$2R_{sym}= 2R_b/N = R_b/2$
⋮	⋮	⋮	⋮

Where R_b is the input bit rate in bps. (in earlier slides the notation f_b is used for R_b)

Modem modulation methods:

FSK,PSK, QAM

Modem is abbreviation for Modulator – Demodulator. Modems are used for data transfer from one computer network to another computer network through telephone lines.

Modulator converts information from digital mode to analog mode at the transmitting end and demodulator converts the same from analog to digital at receiving end.