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INDUS UNIVERSITY

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M.Tech (Structural Engineering)

Design Stress formulas and Design of Section for Axial Tension

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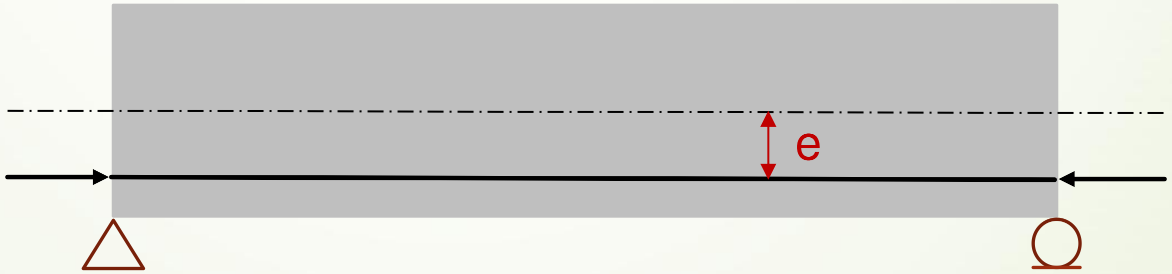
Design Stress Formulas

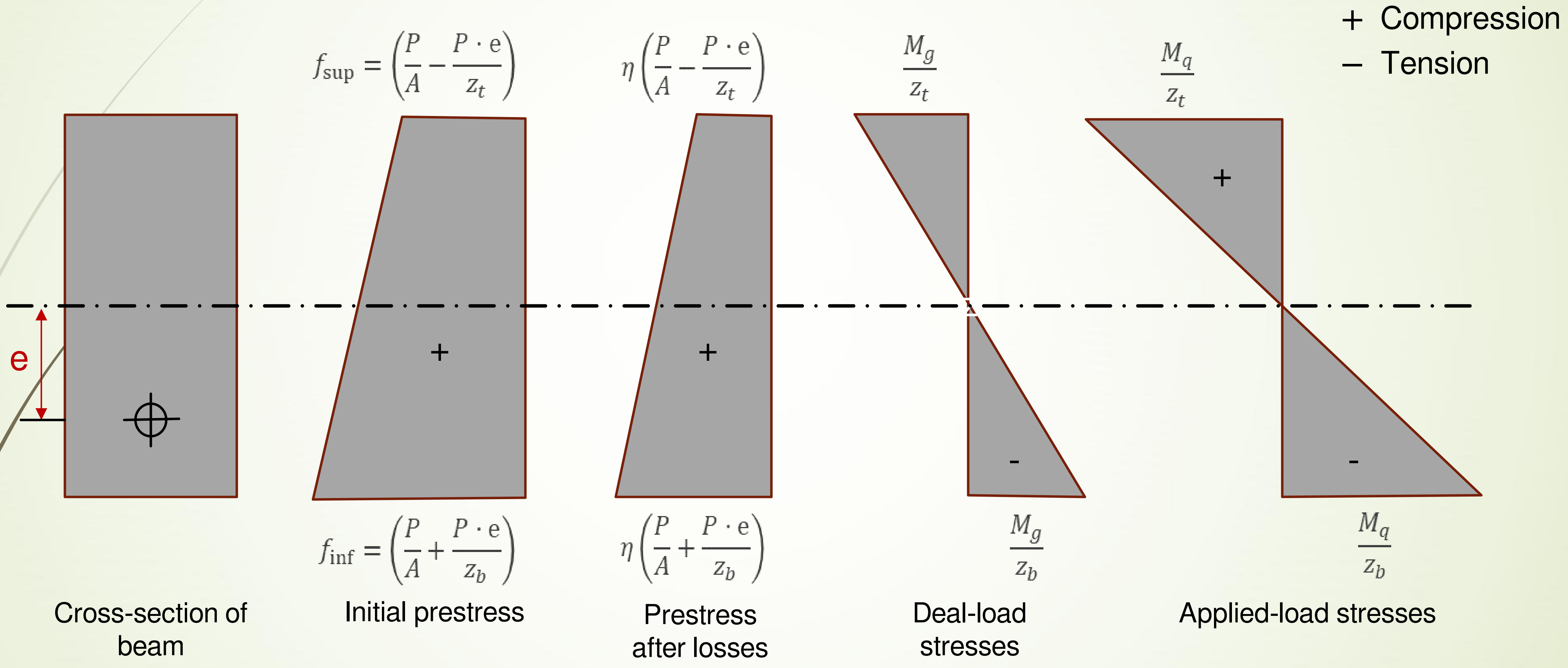
Minimum section modulus of prestressed section under the action of flexure should satisfy the limits specified for permissible stress at the transfer stage of prestress and service load.

The general critical combinations considered are as follows:

1. The maximum prestressing force at transfer together with the minimum moments sustained by section.
2. The minimum prestressing force after all losses in combination with the maximum design moment for the serviceability limit state.



Stress due to prestress and applied load



The total longitudinal stress at top (f_{sup}) and bottom (f_{inf}) of the section are:

$$f_{sup} = \left(\frac{P}{A} - \frac{P \cdot e}{z_t}\right) \quad \text{..... (a)}$$

$$f_{inf} = \left(\frac{P}{A} + \frac{P \cdot e}{z_b}\right) \quad \text{..... (b)}$$

Where,

P = Prestressing force

e = Eccentricity

A = Cross-sectional area of beam

z_t, z_b = Section modulus for the top and bottom fibres

If P_1 is initial prestressing force and β are the immediate and time dependent loss factors, then the prestressing force and bending moment due to applied loading at the transfer loading stage are βP_1 and M_g respectively.

Similarly ηP_1 and M_q donates the prestressing force and bending moment at the service condition.

Due to the upward deflection of the beam at transfer of prestress the stresses at the top and bottom fibres of the beam are tensile and compressive in nature.

But at the service condition, the bottom fibre is in tension and the top fibre in compression.

At the transfer:

Top fibre:

$$\left(\frac{\beta P_1}{A} - \frac{\beta P_1 \cdot e}{z_t} + \frac{M_g}{z_t}\right) \geq f_{tt}$$

$$\left(f_{\sup} + \frac{M_g}{z_t}\right) \geq f_{tt} \qquad \text{..... (1.1)}$$

Where,

f_{tt} = Permissible/allowable tensile stress in concrete at transfer of prestress

β = Reduction factor for loss of prestress or loss ratio

Bottom fibre:

$$\left(\frac{\beta P_1}{A} + \frac{\beta P_1 \cdot e}{z_b} - \frac{M_g}{z_b}\right) \leq f_{ct}$$

$$\left(f_{\inf} - \frac{M_g}{z_b}\right) \leq f_{ct} \qquad \text{.....} \quad (1.2)$$

Where,

f_{ct} = Permissible/allowable compressive stress in concrete at transfer of prestress

β = Reduction factor for loss of prestress or loss ratio

- At service condition:

Top fibre:

$$\left(\frac{\eta P_1}{A} - \frac{\eta P_1 \cdot e}{z_t} + \frac{M_g}{z_t} + \frac{M_q}{z_t}\right) \leq f_{cw}$$

$$\left(\eta f_{sup} + \frac{M_g}{z_t} + \frac{M_q}{z_t}\right) \leq f_{cw} \qquad \text{..... (1.3)}$$

Where,

f_{cw} = Permissible/allowable compressive stress in concrete under service condition

η = Reduction factor for loss of prestress or loss ratio

Bottom fibre:

$$\left(\frac{\eta P_1}{A} + \frac{\eta P_1 \cdot e}{z_b} - \frac{M_g}{z_b} - \frac{M_q}{z_b}\right) \geq f_{tw}$$

$$\left(\eta f_{\inf} - \frac{M_g}{z_b} - \frac{M_q}{z_b}\right) \geq f_{tw} \qquad \text{.....} \quad (1.4)$$

Where,

f_{tw} = Permissible/allowable tensile stress in concrete under service condition

η = Reduction factor for loss of prestress or loss ratio

In the above equation, it should be noted that the permissible/allowable tensile stress is a negative quantity.

Hence the calculated stress should be greater than the allowable/permissible stress.

Similarly, the calculated stress must be less than allowable/permissible compressive stress.

Minimum section modulus for top fibres

From equation (1.1) and (1.3) we have the top fibre stress,

- For equation (1.1)

$$\left(\frac{\beta P_1}{A} - \frac{\beta P_1 \cdot e}{z_t} + \frac{M_g}{z_t}\right) \geq f_{tt}$$

- Multiplying both sides of the equation by η,

$$\left(\frac{\eta\beta P_1}{A} - \frac{\eta\beta P_1 \cdot e}{z_t} + \frac{\eta M_g}{z_t}\right) \geq \eta f_{tt}$$

- Multiplying the above equation by -1 gives

$$-\beta\eta\left[\frac{P_1}{A} - \frac{P_1 \cdot e}{z_t}\right] - \frac{\eta M_g}{z_t} \leq -\eta f_{tt} \qquad \text{..... (2.1)}$$

- Similarly for equation (1.3)

$$\left(\frac{\eta P_1}{A} - \frac{\eta P_1 \cdot e}{z_t} + \frac{M_d}{z_t}\right) \leq f_{cw}$$

Where,

$$M_d = M_g + M_q$$

- Multiplying the equation by loss factor β gives,

$$\beta\eta\left[\frac{P_1}{A} - \frac{P_1 \cdot e}{z_t}\right] + \frac{\beta M_d}{z_t} \leq \beta f_{cw} \qquad ..... \quad (2.2)$$

By adding equation (2.1) and (2.2) the required section modulus z_t can be found by

$$\frac{\beta M_d - \eta M_g}{z_t} \leq (\beta f_{c\omega} - \eta f_{tt}) \leq f_{tr}$$

$$z_t \geq \frac{(\beta M_d - \eta M_g)}{(\beta f_{c\omega} - \eta f_{tt})} \quad \text{..... (3.0(a))}$$

This equation can also be written as

$$z_t \geq \left[\frac{M_q + (1-\eta) M_g}{f_{tr}}\right] \quad \text{..... (3.0(b))}$$

Minimum section modulus for Bottom fibres

The bottom fibre stress meet the criteria given by equation (1.2) and (1.4)

- From equation (1.2)

$$\left(\frac{\beta P_1}{A} + \frac{\beta P_1 \cdot e}{z_b} - \frac{M_g}{z_b}\right) \leq f_{ct}$$

$$\beta\eta\left[\frac{P_1}{A} + \frac{P_1 \cdot e}{z_b}\right] - \frac{\eta M_g}{z_b} \leq \eta f_{ct} \qquad \text{..... (2.3)}$$

- From equation 1.4

$$\left(\frac{\eta P_1}{A} + \frac{\eta P_1 \cdot e}{z_b} - \frac{M_d}{z_b}\right) \geq f_{tw}$$

- Multiply with loss factor β gives,

$$\beta\eta\left[\frac{P_1}{A} + \frac{P_1 \cdot e}{z_b}\right] - \frac{\beta M_d}{z_b} \geq \beta f_{tw}$$

- Multiply above equation with -1 gives,

$$-\beta\eta\left[\frac{P_1}{A} + \frac{P_1 \cdot e}{z_b}\right] + \frac{\beta M_d}{z_b} \leq -\beta f_{tw} \quad \text{.....} \quad (2.4)$$

By adding 2.2 and 2.4

$$\frac{\beta M_d - \eta M_g}{z_b} \leq (\eta f_{ct} - \beta f_{tw}) \leq f_{br}$$

$$z_b \geq \frac{(\beta M_d - \eta M_g)}{(\eta f_{ct} - \beta f_{tw})}$$

This equation can also be written as

$$z_b \geq \left[\frac{M_q + (1-\eta) M_g}{f_{br}}\right]$$

What is Axial Prestressing?

Axial prestressing is defined as a member, in which the entire cross-section of concrete has a uniform compressive prestress.

In this type of prestressing, the centroid of the tendons coincides with that of the concrete section.

No Eccentricity of CGS with respect to CGC is considered.

Prestressed members under axial loads only are uncommon.

Members like piles may be subjected to axial compression or tension

Design of Section for Axial Tension

Due to the presence of precompression, prestressed concrete is ideally suited for the design of members subjected to axial tension.

Various members in which axial tension is primary force are tie members of trusses, walls of cylindrical tanks, silos and pipes subjected to internal pressure.

The design essentially consists of determining the cross-sectional area of the member and the required prestressing force to safely support the axial tensile load conforming to the limit state of serviceability and collapse.

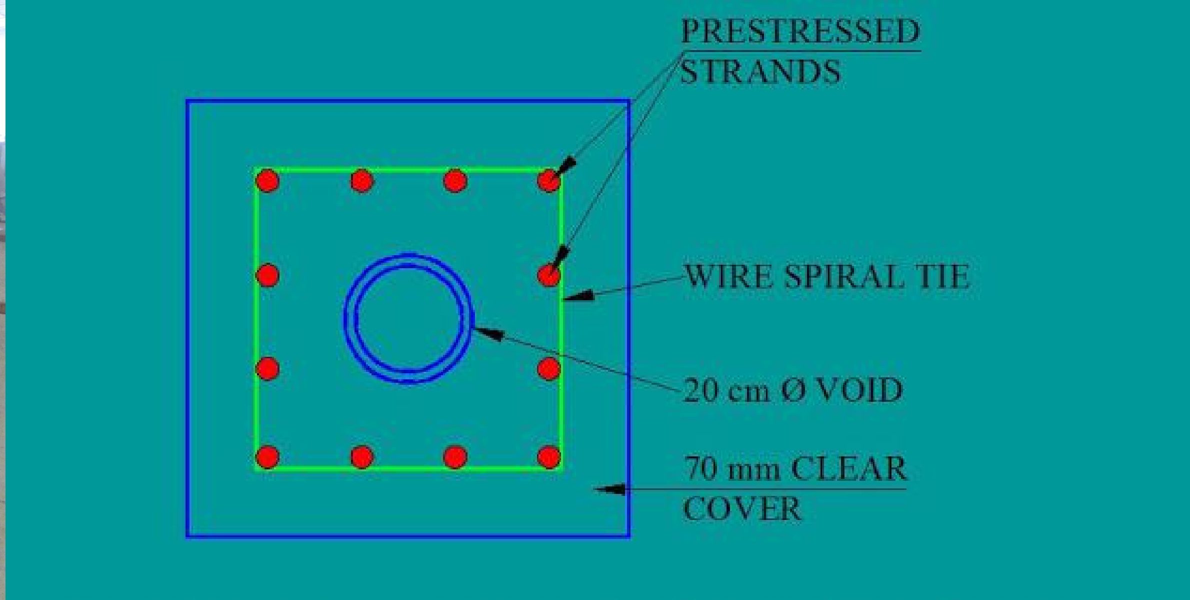
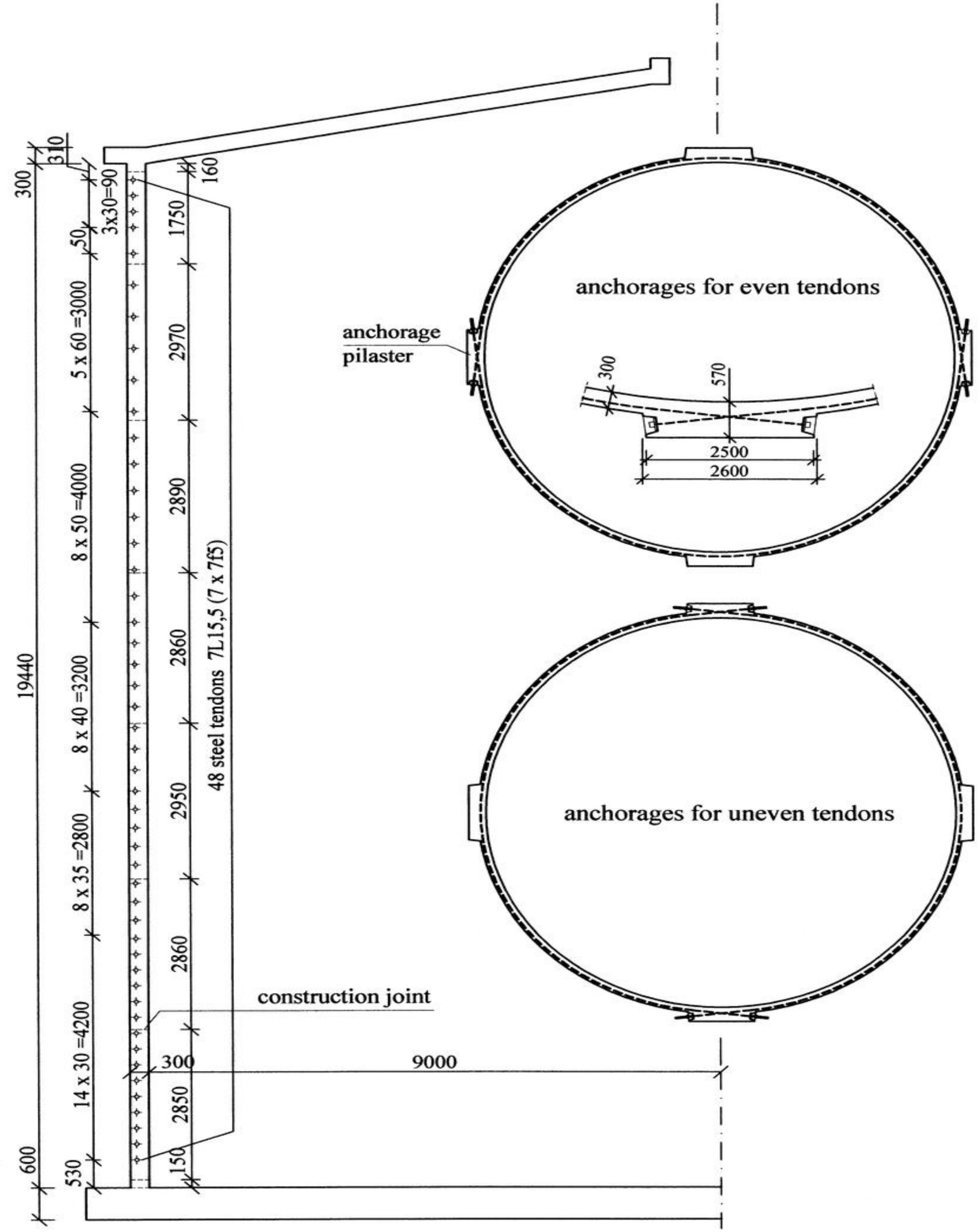
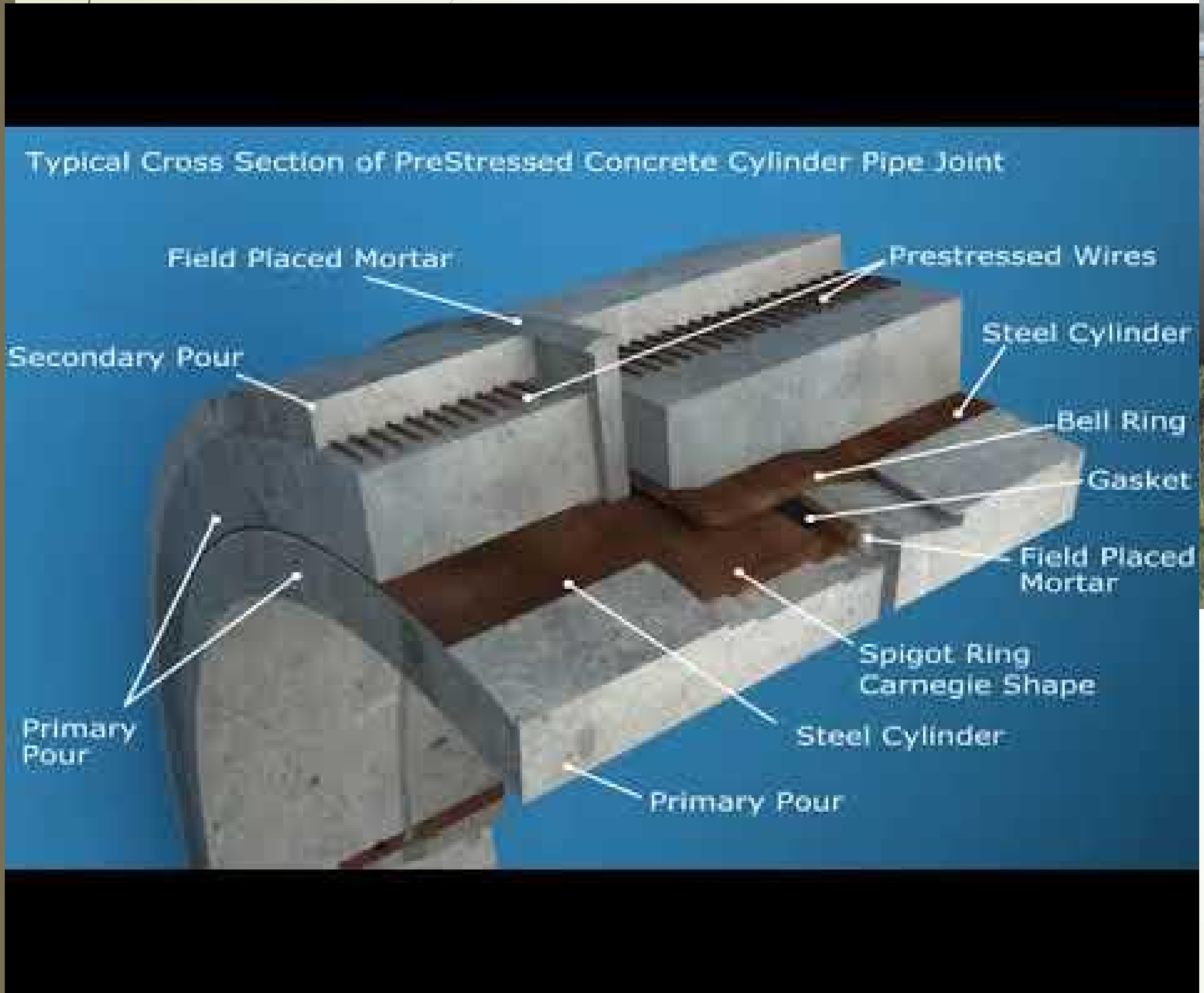


FIG- CROSS-SECTION OF A TYPICAL PRESTRESSED CONCRETE PILE



anchorages for even tendons
anchorage pilaster
300
570
2500
2600
anchorages for uneven tendons
construction joint
300
9000
48 steel tendons 7L15,5
19440
300
310
3x30=90
50
5 x 60 =3000
8 x 50 =4000
8 x 40 =3200
8 x 35 =2800
14 x 30 =4200
600
530
160
1750
2970
2890
2860
2950
2860
2850
150


Typical Cross Section of PreStressed Concrete Cylinder Pipe Joint
Field Placed Mortar
Prestressed Wires
Steel Cylinder
Secondary Pour
Bell Ring
Gasket
Field Placed Mortar
Spigot Ring Carnegie Shape
Primary Pour
Steel Cylinder
Primary Pour

As per Clause 24.2 of IS: 1343 – 2012

No tensile stress are permitted in Type-1 members.

Tensile stress up to 3 N/mm^2 are permitted at serviceability limit states in Type-2 members.

Type-3 members are not generally recommended for resisting direct tension.

The cross-sectional area of concrete is determined from the stress conditions at transfer and working loads as follows

At transfer stage,

$$\left[f_c - \frac{N_{min}}{A}\right] \leq f_{ct} \qquad \text{..... (1)}$$

At working load stage,

$$\left[\eta f_c - \frac{N_d}{A}\right] \geq f_{tw} \qquad \text{..... (2)}$$

From the two stress conditions, the area of the section is given by the relation,

$$A = \left(\frac{N_d - \eta N_{min}}{\eta f_{ct} - f_{tw}}\right) \qquad \text{..... (3)}$$

If the minimum load and permissible tensile stresses are zero, the cross-sectional area is obtained from the expression,

$$A = \left(\frac{N_d}{\eta f_{ct}}\right) \qquad \text{..... (4)}$$

Where,

N_d = design tensile load

N_{min} = minimum tensile load (generally zero)

f_c = compressive prestress in concrete

f_{ct} = permissible compressive stress in concrete at transfer of prestress

f_{tw} = permissible tensile stress in concrete under working load

η = prestress loss ratio

A = equivalent area of concrete

The cross-sectional area determined by the expressions are sometimes not practicable, especially in the case of circular water tank.

In such cases, the thickness required is fixed, based on practical considerations of housing the vertical cables and proper compaction of the concrete in the walls of the tank.

The minimum thickness for cast in situ walls is 100 mm and for precast walls, 125 mm.

However in the case of large tanks with walls having vertical cables, a minimum thickness of 1500 mm is generally required,

Ultimate Tensile Strength

Ultimate tensile strength of a section can be calculated as per Clause 23.3 of IS: 1343 – 2012.

Ultimate Tensile Strength (P_{uR}) is calculated as below,

When reinforcements are not present, Stress is given by

$$P_{\mathrm{uR}} = 0.87 * f_{pk} * A_p$$

Here, f_{pk} = Characteristic Strength of Tendons

A_p = Area of Prestressed Tendons

When reinforcements are present, Stress is given by

$$P_{uR} = 0.87 * f_y * A_s + 0.87 * f_{pk} * A_p$$

Here, f_{pk} = Characteristic Strength of Tendons

A_p = Area of Prestressed Tendons

f_y = Characteristic Strength of Reinforcement

A_s = Area of Non-Prestressed Reinforcements

Load Factor – Limit State of Cracking and Collapse

Members subjected to direct tension, such as the walls of liquid-retaining structures, are generally designed to have suitable global load factors against cracking and collapse.

According to Clause 3.2.2 of IS: 3370 (Part 3) – 1967, the load factors against cracking and collapse should be not less than 1.2 and 2, respectively.

The cracking load generally corresponds to the stage where the tensile stress developed in the member reaches a value equal to the tensile strength of concrete.

However, the cracking load of a prestressed member assembled from precast blocks is computes on the assumptions that concrete is incapable of resisting any tensile stress.

$$N_{cr} = A(f_c + f_t)$$

Where,

N_{cr} = cracking load

f_t = tensile strength of concrete

$$=0.24\sqrt{f_{ck}}$$

So, Load Factor against cracking = $\left(\frac{N_{cr}}{N_d}\right)$

For ordinary tension members, such as ties of trusses, ring beams, and rail-road ties, a minimum load factor of 1.5 to 2, depending upon the importance of the tension member in the structure.

At the limit state of collapse, the concrete member is completely cracked at the critical section and the entire axial load is resisted by the tendons.

If the area of steel provided in the cross-section based on the serviceability limit state is insufficient to provide the desired load factor against collapse, additional reinforcements are designed and distributed uniformly in the section.

Steps for solving example:

1. Find area of required concrete section - A
2. Assume suitable section - $b * d$
3. Find permissible stress of that section - f_{ct}
4. Ultimate strength of section - P_{uR}
5. Load factor against collapse
6. Load factor against cracking

Design Examples

Q. Design a suitable section for the tie member of a prestressed concrete truss to carry a design tensile force of $600\ kN$.Assume the permissible compressive stress in concrete at transfer as $15\ N/mm^2$ and tension is not allowed under service loads. Loss of prestress is 20%. High tensile wires of $8\ mm$ diameter with an ultimate tensile strength of $1400\ N/mm^2$ with an initial stress of $800\ N/mm^2$ are available for use. The direct tensile strength of concrete is $3\ N/mm^2$. A load factor of 2 against collapse and 1.25 against cracking is to be ensured in the design.

A. We are given,

$N_d =$ 600 kN $f_t =$ 3 N/mm^2

$f_{ct} =$ 15 N/mm^2 f_{pk} = 1400 N/mm^2

f_{tw} = 0

η = 0.8

Area of concrete section, A

$$= \left(\frac{N_d}{\eta f_{ct}}\right)$$

$$= \left[\frac{600*10^3}{0.8*15}\right]$$

$$= 50000\ mm^2$$

Assume section size as (200×250) $(A = 50000\ mm^2)$

Compressive Prestress,

$$f_{ct} = \left[\frac{600*10^3}{0.8*50000}\right]$$

$$= 15\ N/mm^2$$

Prestressing Force,

$$P = 15 * 50000$$

$$= 750\ kN$$

No of 8 mm wires

$$= \left[\frac{750*10^3}{50*800}\right]$$

$$= 18.75 \text{ (take no. of wire as 20)}$$

Ultimate tensile strength of tie (Tendons)

$$P_{uR} = 0.87 * f_{pk} * A_{st}$$

$$= 0.87 * 1400 * 20 * 50$$

$$= 1218\ kN$$

Load factor against collapse,

$$= \left(\frac{P_{ur}}{N_d}\right) = \left(\frac{1218}{600}\right)$$

$$= 2.05 \quad \text{O.K.}$$

Cracking load,

$$= [50000((0.8 * 15) + 3)]$$

$$= 750\ kN$$

Load factor against cracking,

$$= \left(\frac{750}{600}\right)$$

$$= 1.25 \quad \text{O.K.}$$

Q. Design the thickness and circumferential wire winding for a cylindrical water tank wall subjected to a design tensile force of $600\ kN/m$. $f_{ct} = 16\ N/mm^2$, $f_{tw} = 0\ N/mm^2$, direct tensile strength of concrete is 3 N/mm2 and N=0.85. High tensile wires of $7\ mm$ diameter($f_p = 1600\ N/mm^2$) initially stressed to $1000\ N/mm^2$ are available for use. A load factor of 2 against collapse and 1.25 against cracking is required.

A. We are given,

$N_d = 600\ kN$

$f_{ct} = 16\ N/mm^2$

$f_{tw} = 0$

$\eta = 0.85$

$f_t = 3\ N/mm^2$

$f_{pk} = 1600\ N/mm^2$

Area of concrete section, A

$$= \left[\frac{N_d}{\eta f_{ct}}\right]$$

$$= \left[\frac{600*10^3}{(0.85*16)}\right]$$

$$= 44118\ mm^2/m$$

Thickness of wall

$$= \frac{44118}{1000}$$

$$= 44.118\ mm$$

Based on practical considerations adopt a minimum thickness of $100\ mm$.

Are of concrete section, A

$$= b * d$$

$$= 1000 * 100$$

$$= 10^5\ mm^2$$

Prestress required

$$= \left[\frac{N_d}{\eta A}\right]$$

$$= \left[\frac{600*10^3}{0.85 * 10^5}\right]$$

$$= 7.05\ N/mm^2$$

Prestressing force

$$= 7.05 * 10^5$$

$$= 705\ kN$$

No. of 7 mm diameter H.T. wires

$$= \left[\frac{705*10^3}{38.5*10^3}\right]$$

$$= 19$$

Pitch of wires

$$= \left(\frac{1000}{19}\right)$$

$$= 52\ mm$$

Ultimate tensile force (tendons)

$$= 0.87 * F_{pk} * A_p$$

$$= 19 * 38.5 * 0.87 * 1600$$

$$= 1018\ kN$$

Required ultimate force

$P_{uR} = (2 \times 600)$

$= (\text{load factor} * N_d)$

$= 1200\ kN$

Additional tensile strength required (reinforcements)

$= (1200 - 1018)$

$= 182\ kN$

Area of HYSD bars

$= \frac{P}{0.87 * f_y}$

$= \left[\frac{182 * 10^3}{0.87 * 415}\right]$

$= 504\ mm^2$

No. of 6 mm diameter bars on each face

$$= \frac{504}{28*2}$$

$$= 9$$

Spacing of 6 mm diameter bars on each face

$$= \left[\frac{1000}{9}\right]$$

$$= 111\ mm$$

Cracking Load

$$= (10^5)((0.85 * 7.05) + 3)$$

$$= 899\ kN$$

Load factor against cracking

$$= \left(\frac{899}{600}\right)$$

$$= 1.49$$ O.K.

Thank You