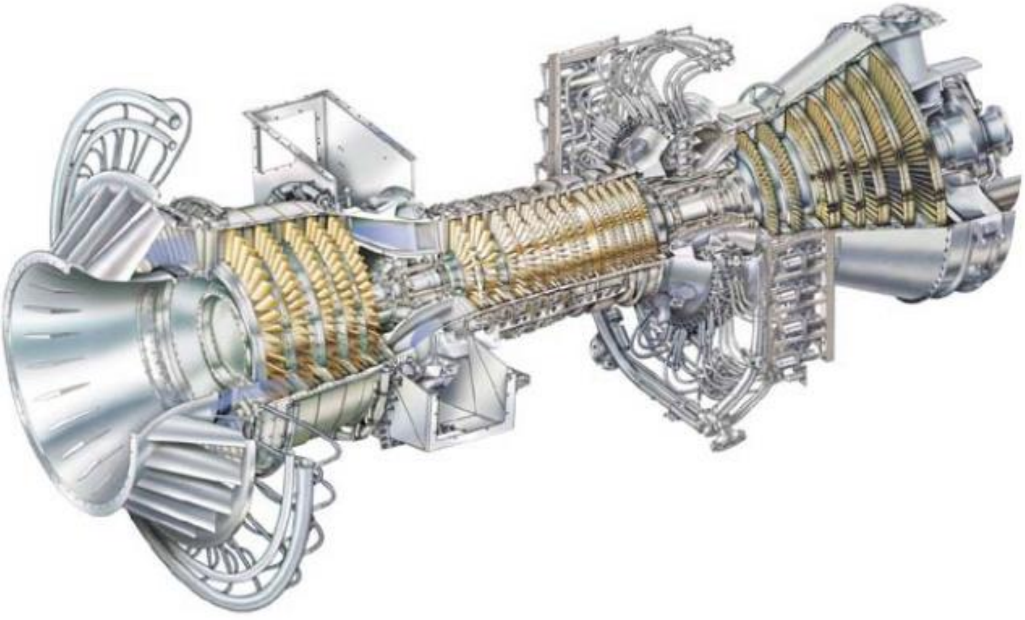
GAS TURBINE POWER PLANTS



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INTRODUCTION

I have written these notes for Italian speaking students in the last year of high school for the course of Applied Mechanics and Fluid Machinery. The main aim is to give them the chance to follow one module of their mechanics course all in English. The knowledge of basic thermodynamics studied during the fourth year of school is assumed, however, revision of the main concepts concerning gas turbo machinery is included in the first two chapters.

Some exercises, that I will solve with the students during the course, are included as well as all the mathematical demonstrations that they are able to understand with their present mathematical knowledge.

I hope that my work will be useful for improving the students' speaking, understanding and reading abilities in both technical and everyday English.

CONTENTS

1 FIRST LAW OF THERMODYNAMICS FOR AN OPEN SYSTEM

Thermodynamic systems can be open, if they exchange mass with external environment, or closed if they don't exchange mass. They usually exchange energy as mechanical energy (work) and/or thermal energy (heat), if the system doesn't exchange mass and energy it is called isolated. Gas turbine and turbo compressors are open systems so calculations are done with the first law of thermodynamics for an open system:

$$L_i + Q_e = \Delta i + \Delta E_k + \Delta E_g \quad (1)$$

- L_i internal work (it is the mechanical energy exchanged between the gas and the blades of the turbine)
- Q_e heat exchanged through control volume
- Δi change in enthalpy between the outlet and inlet of the machine
- ΔE_k change in kinetic energy
- ΔE_g change in potential energy

All the terms in equation (1) are measured in $\frac{J}{kg}$ (energy per mass unit). Figure 1 shows a picture of a gas turbine.

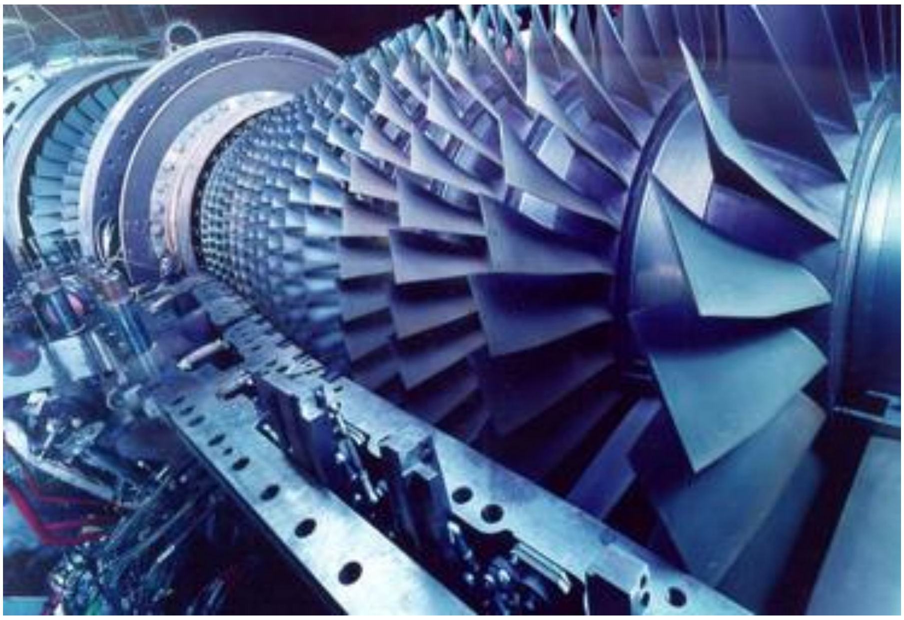


fig. 1 - Gas turbine

We consider, by convention, the work and the heat coming into the system as positive and the work and the heat coming out as negative.

Equation (1) can be simplified:

- Since turbo machines are so fast heat transfer is insignificant so $Q_e \cong 0$
- For gases the change in potential energy is small in comparison with the change in enthalpy so $\Delta E_g \cong 0$
- Normally the change in kinetic energy between the inlet and outlet gas is small and so $\Delta E_k \cong 0$.

We consider all gases to be perfect (or ideal) so the perfect gas equation (2) is always true.

$$P \cdot v = R \cdot T \quad (2)$$

- P gas pressure (P_a)
- v specific volume ($\frac{m^3}{kg}$)
- R individual gas constant depending on its molecular weight ($\frac{J}{kg \cdot K}$)
- T gas temperature (K)

For perfect gases the enthalpy is the product of pressure specific heat of the gas C_p ($\frac{J}{K \cdot kg}$) and the temperature T (K) so equation (1) becomes:

$$L_i = C_p \cdot \Delta T \quad (3)$$

For the sign convention mentioned above the work calculated with (3) is positive for a compressor and negative for a turbine. For turbines we defined the positive "internal work obtained " $(L_i)_{obt} = - L_i$. The mechanical power produced by a turbine and that absorbed by a turbo compressor are calculated by equations (4) and (5) respectively:

$$P_T = G.(L_i)_{obt} \cdot \eta_m^T \quad (4)$$

$$P_C = \frac{G \cdot L_i}{\eta_m^C} \quad (5)$$

Where G is the mass flow rate and $\eta_m^T$and η_m^C are the mechanical efficiencies of the two turbo machines. Mechanical efficiencies take into consideration the work lost by mechanical friction and they are quite high ($\eta_m \cong 0.96 \div 0.99$).

In table 1 the main constants for some gasses are included.

	C_p $(J/kg \cdot K)$	C_V $(J/kg \cdot K)$	$R = C_p - C_V$ $(J/kg \cdot K)$	$k = \frac{C_p}{C_V}$
Air	1003	716	287	1.4
O_2	908	649	260	1.4
N_2	1034	737	297	1.4
CO_2	842	653	183	1.29
H_2	14240	10100	4140	1.4
NH_3	2060	1560	500	1.32
CH_4	2253	1735	518	1.3

tab. 1 - Table of gas constants

2 THE ISENTROPIC EFFICIENCY FOR GAS TURBO MACHINERY

As we said before turbo machines are adiabatic, if the thermodynamic transformation of the gas in the machines was reversible it would be isentropic. Since the fluid-dynamic friction can never be neglected in fast machines, the real thermodynamic transformation is irreversible with increasing entropy. The isentropic efficiency compares a real machine with an ideal one. Il figure 2 are represented the real (irreversible) and ideal (reversible) transformations for turbines and turbo compressors in temperature-entropy diagram.

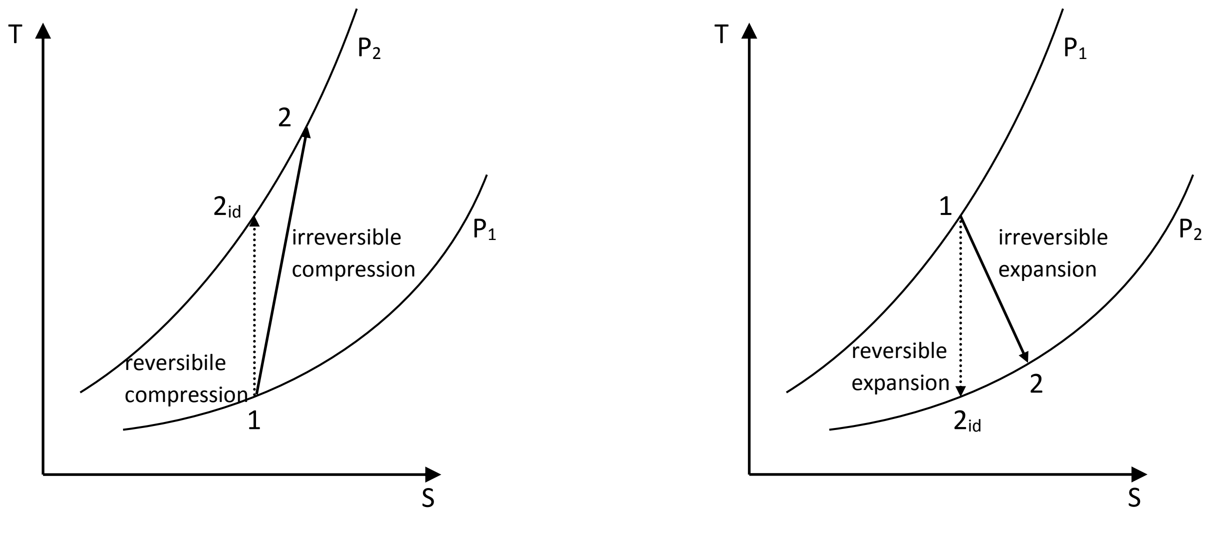


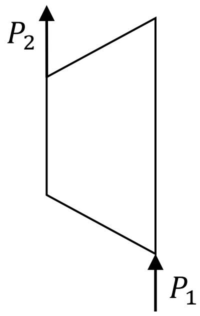
fig. 2 Thermodynamic transformations of the gas in turbo compressors and turbines

The isentropic efficiencies are defined as below:

<u>turbo compressor</u>

compression ratio

$\beta_c = \frac{P_2}{P_1}$

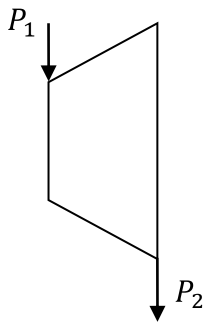


$$\eta_{is}^{C} = \frac{L_i^{id}}{L_i} \qquad (7)$$

<u>gas turbine</u>

expansion ratio

$\beta_e = \frac{P_1}{P_2}$



$$\eta_{is}^{T} = \frac{(L_i)_{obt}}{(L_i)_{obt}^{id}} \qquad (8)$$

The compression and expansion ratios are usually between 15 and 40 or sometimes more.

It is useful to remember the equation for isentropic transformations studied in general thermodynamics:

$$p \cdot v^k = constant \qquad (9)$$

with $k = \frac{c_P}{c_V}$

If we combine (9) with (2) we get:

$$\frac{T^k}{p^{k-1}} = constant \qquad (10)$$

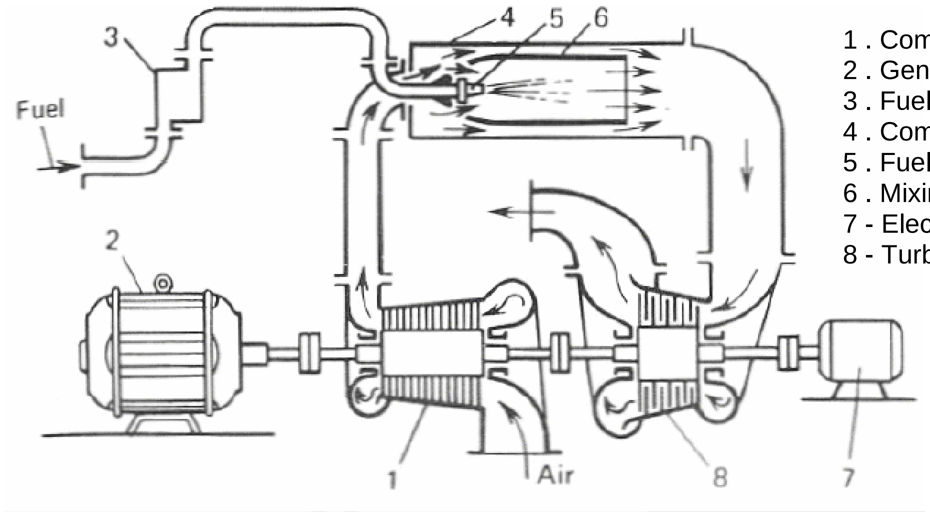
The real transformation in a turbo machine can be considered a polytropic transformation:

$$p \cdot v^m = constant \qquad (11)$$

Where the m is exponent of the polytropic.

3 GENERALITIES ABOUT GAS-TURBINE POWER PLANTS

The purpose of gas-turbine power plants is to produce mechanical power from the expansion of hot gas in a turbine. In these notes we will focus on stationary plants for electric power generation, however, gas turbines are also used as jet engines in aircraft propulsion. The simplest plant is the open turbine gas cycle used to produce electrical power as shown in figure 3.



1 . Compressor
2 . Generator
3 . Fuel flow control
4 . Combustion chamber
5 . Fuel nozzle
6 . Mixing tube
7 - Electrical starter
8 - Turbine

fig. 3 - Open turbine gas cycle

The net power available on the shaft is transformed into electrical power by the generator while the electrical starter is an electrical engine which is only used when the plant is turned on. The schematic picture of the previous plant, non including electrical starter and generator, is shown in figure 4.

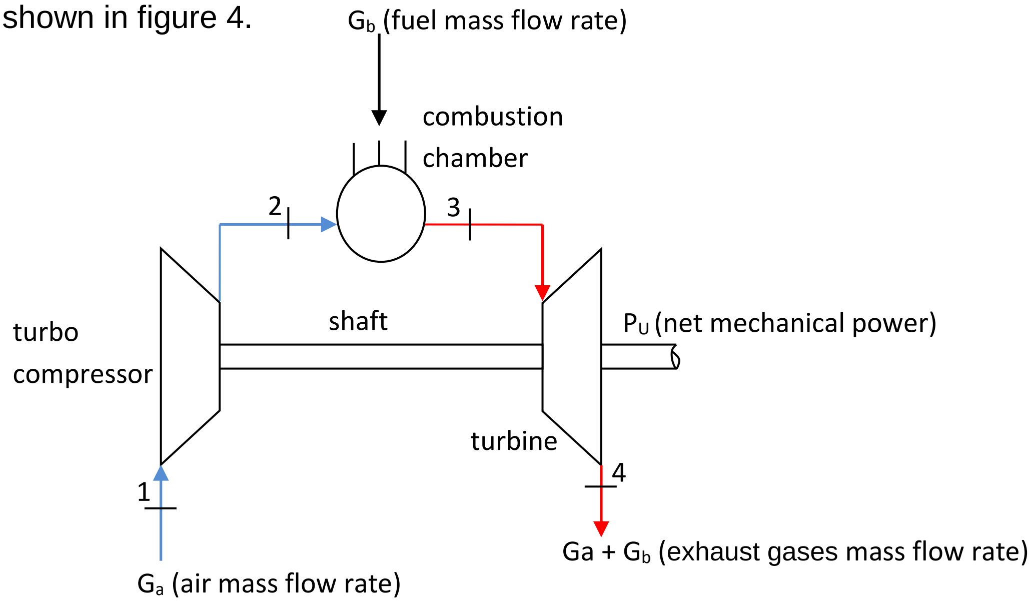


fig. 4 - Open turbine gas cycle

- Air at room pressure and temperature is compressed to a high pressure in the turbo compressor
- Fuel is added in the combustion chamber where combustion takes place resulting in high-temperature combusted gases
- The hot gases expand in the turbine back to the atmospheric pressure producing mechanical power.

The cycle is said to be open because fresh air enters the compressor continuously and exhaust is expelled, but thermodynamically it is as if the operating fluid returns to its initial state. Part of the mechanical power generated by the turbine is used to drive the compressor.

An alternative plant is the so called closed turbine gas cycle, schematically shown in figure 5.

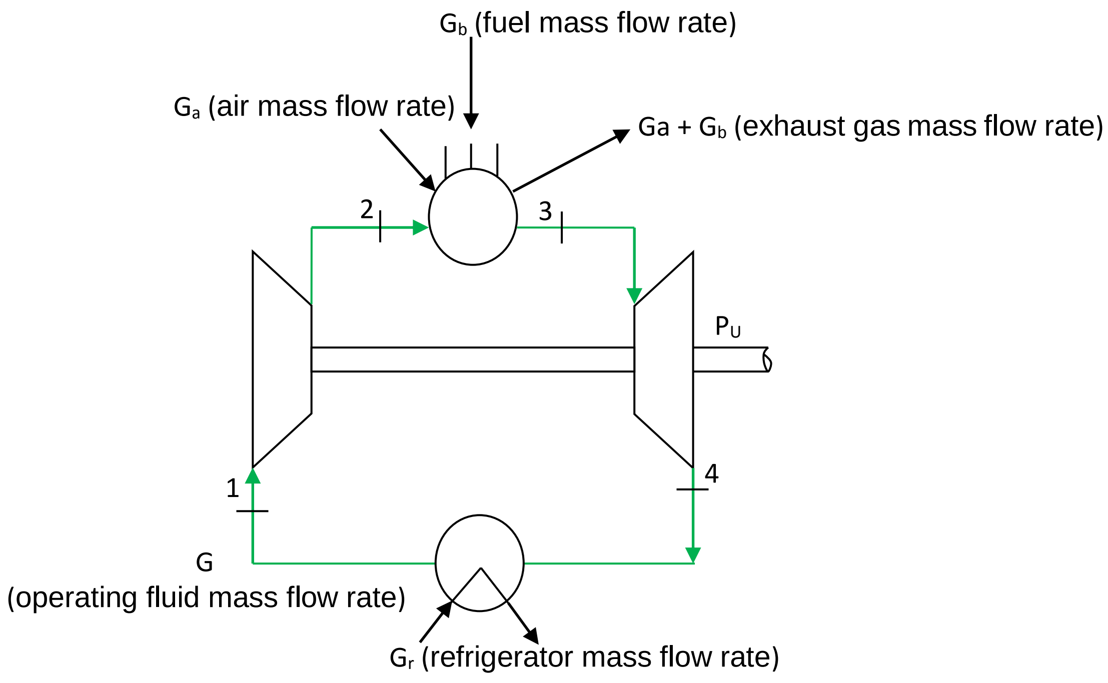


fig. 5 - Close turbine gas cycle

In this case the operating fluid does not undergo any chemical transformations (because it is not involved in combustion processes) but only thermodynamic transformations. Combustion takes place between air and fuel in the combustion chamber, which is also a heat exchanger. Both plants have advantages and disadvantages. In the first case it is not necessary to cool down the operating fluid (this requires a huge amount of refrigerator mass flow rates) and so this plant can be adopted on sites with low water availability. One of the advantages of the closed turbine gas cycle is that the turbine stays clean because the combustion products do not pass through it. Another advantage is that it is possible to use gases with $k > k_{air}$ ($k_{air} = 1.4$), which increases the efficiency of the cycle (as we will see in the next chapter).

The net power available on the shaft in both cases is:

$$P_U = P_T - P_C \qquad (12)$$

and it can be transformed into electric power (P_{el}) by an electrical machine called a turbo alternator:

$$P_{el} = P_U \cdot \eta_{el} \qquad (13)$$

where η_{el} is the electrical efficiency of the alternator.

The thermal power produced by the combustion of the fuel is:

$$\dot{Q}_1 = \eta_b \cdot G_b \cdot H_i \qquad (14)$$

where η_b and H_i are the combustion efficiency and the inferior calorific value of the fuel. The thermal efficiency (how heat is transformed in work) of the plant is:

$$\eta_u = \frac{P_U}{\dot{Q}_1} \qquad (15)$$

and the total efficiency of the electrical power transformation is:

$$\eta_{tot} = \frac{P_{el}}{\dot{Q}_1} = \eta_u \cdot \eta_{el} \qquad (16)$$

We will only consider the open-turbine-gas-cycle plant in these notes since they are the most common type.

4 THE JOULE CYCLE

The Joule (or Brayton-Joule) cycle, shown in figure 6, is the ideal cycle for gas-turbine engines. The ideal cycle is made up of the following four reversible processes:

- 1-2 Isentropic compression
- 2-3 Constant pressure combustion
- 3-4 Isentropic expansion
- 4-1 Constant pressure heat rejection

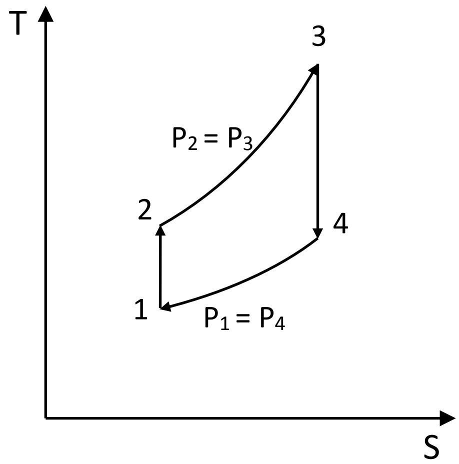


fig. 6 - The Brayton - Joule cycle in T-S chart

The efficiency of the ideal Joule cycle can easily be calculated assuming that in ideal cycles just one kilo of fluid, that undergoes only thermodynamic transformations, is considered. Therefore, the (15) can be written in terms of energy as below:

$$\eta_u = \frac{L_U}{Q_1} = \frac{(L_T)_{obt} - L_C}{Q_1} \qquad (17)$$

The terms of the fraction can be calculated with the first law of thermodynamics for open systems:

$$(L_T)_{obt} = C_P \cdot (T_3 - T_4) \qquad (18)$$

$$L_C = C_P \cdot (T_2 - T_1) \qquad (19)$$

$$Q_1 = C_P \cdot (T_3 - T_2) \qquad (20)$$

If we put (18), (19) and (20) in (17), and simplify the specific heat, we get:

$$\eta_u = \frac{(T_3 - T_4) - (T_2 - T_1)}{T_3 - T_2} \qquad (21)$$

(21) can be rearranged as follow:

$$\eta_u = \frac{(T_3 - T_2) - (T_4 - T_1)}{T_3 - T_2} = 1 - \frac{(T_4 - T_1)}{T_3 - T_2} = 1 - \frac{T_1 \cdot \left(\frac{T_4}{T_1} - 1\right)}{T_2 \cdot (\frac{T_3}{T_2} - 1)} \qquad (22)$$

but using (10) for the isentropic compression and expansion we see that $\frac{T_4}{T_1} = \frac{T_3}{T_2}$ so (22) becomes:

$$\eta_u == 1 - \frac{T_1}{T_2} \qquad (23)$$

using (10) again we get:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \beta_C{}^{\frac{k-1}{k}} \qquad (24)$$

so (22) becomes:

$$\eta_u == 1 - \frac{1}{\beta_C{}^{\frac{k-1}{k}}} \qquad (25)$$

Therefore, the efficiency of the ideal cycle depends on the type of gas (though the constant k) and the compression ratio and it increases if k and/or β_C increase. In figure 7 the plot of the efficiency versus the compression ratio (with k fixed) can be seen.

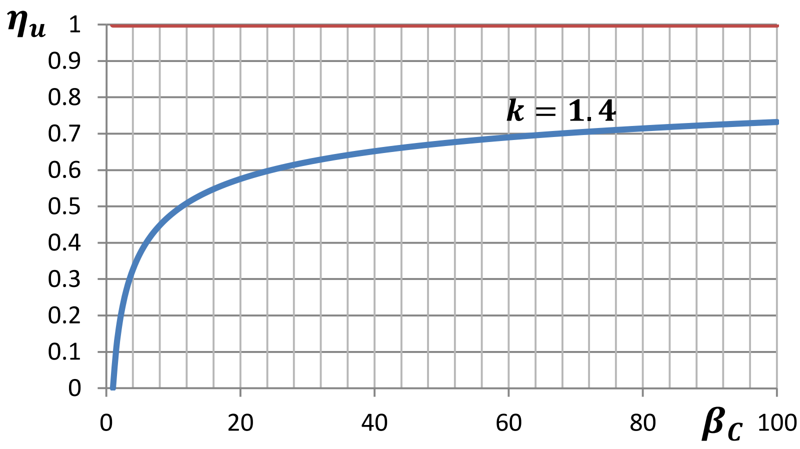


fig. 7 - Graph of the thermal efficiency versus the compression ratio for the ideal cycle

5 THE TRUE CYCLE

In gas turbine power plants the true cycle is quite different from the ideal Brayton-Joule, which was described in chapter 4, due to the following fluid dynamics and mechanical losses:

- compression and expansion are not isentropic
- pressure drop in the combustion chamber
- variation of average gas parameters (caused mainly by temperature variation)
- imperfect combustion
- bearing friction
- drive power of auxiliary equipment

We consider the non-isentropicity in the thermodynamic transformation in the turbo machines using the isentropic efficiencies (7) and (8).

To consider the pressure drop in the combustion chamber we define the pneumatic combustor efficiency as:

$$\eta_{\pi b} = \frac{P_3}{P_2} \qquad (26)$$

were P_2 is the turbo-compressor outlet pressure and P_3 is the combustor outlet pressure. The pneumatic combustor efficiency in smaller than one, as are all efficiencies, and so the true cycle expansion ratio is smaller than the compression ratio

$$\beta_c = \frac{P_2}{P_1} \qquad (27)$$

$$\beta_e = \frac{P_3}{P_4} = \frac{\eta_{\pi b} \cdot P_2}{P_1} = \eta_{\pi b} \cdot \beta_c \qquad (28)$$

In these notes we do not consider the variation of gas parameters so some average values are assumed. Imperfect and incomplete combustion are considered assuming a combustion efficiency $\eta_{\pi b}$, while mechanical friction (and the drive power of auxiliary equipment, if present) is considered assuming the mechanical efficiencies of the two turbo machines η_m^T and η_m^C.

The variations with respect to the ideal cycle are shown in figure 8.

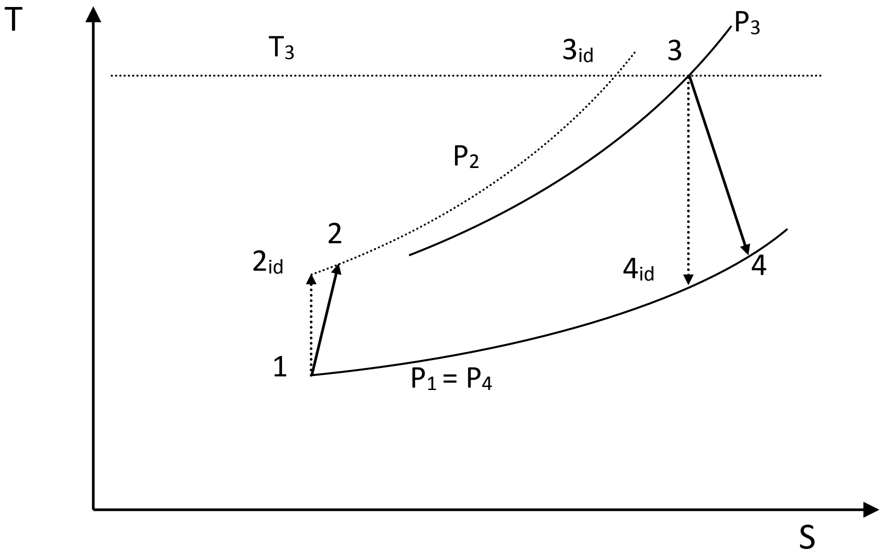


fig. 8 - The true Joule cycle in T-S chart

The mechanical power produced by the turbines in this case can be written as:

$$P_T = (G_a + G_b).(L_i)_{obt} \cdot \eta_m^T = (G_a + G_b).C_p \cdot (T_3 - T_4) \cdot \eta_m^T \quad (29)$$

While the power absorbed by the compressor is:

$$P_C = \frac{G_a \cdot L_i}{\eta_m^C} = \frac{G_a \cdot C_p \cdot (T_2 - T_1)}{\eta_m^C} \quad (30)$$

Approximately 55 to 65 percent of the power produced by the turbine is used to drive the compressor.

The plant thermal efficiency (15) can be written as follow:

$$\eta_u = \frac{P_U}{\dot{Q}_1} = \frac{(G_a + G_b) \cdot \ C_p \cdot (T_3 - T_4) \cdot \eta_m^T - \dfrac{G_a \cdot C_p \cdot (T_2 - T_1)}{\eta_m^C}}{\eta_b \cdot G_b \cdot H_i} \quad (31)$$

For the true cycle the efficiency strongly depends on the gas turbine inlet temperature T_3, higher temperatures lead to higher efficiency as shown in figure 9.

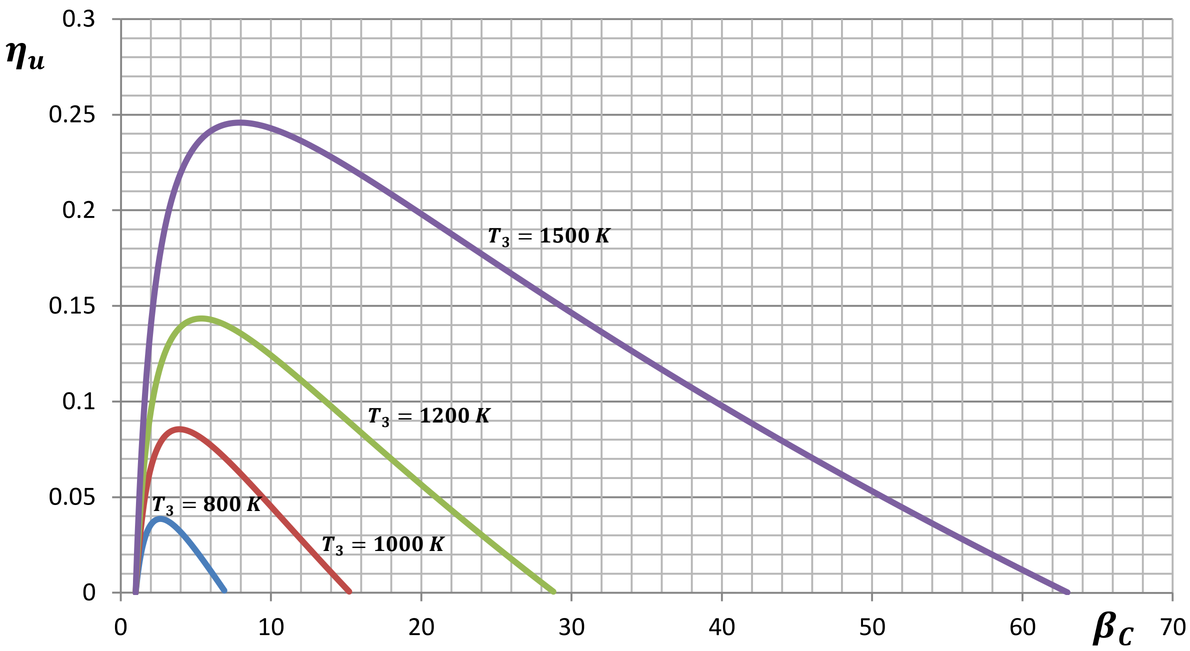


fig. 9 - Graph of thermal efficiency versus compression ratio and temperature T_3

Turbine inlet temperature is limited by the thermal conditions that can be tolerated by the metal alloy of the turbine blades. Gas temperatures at the turbine inlet can be between 1200ºC and 1400ºC, but some manufacturers have been able to boost inlet temperatures as high as 1600ºC by engineering blade coatings and cooling systems to protect metallurgical components from thermal damage.

6 THE COMBUSTION CHAMBER

As we already know the combustion chamber (or combustor or burner) is the component in a gas turbine power plant where combustion takes place. The picture of a typical combustor is shown in figure 10.

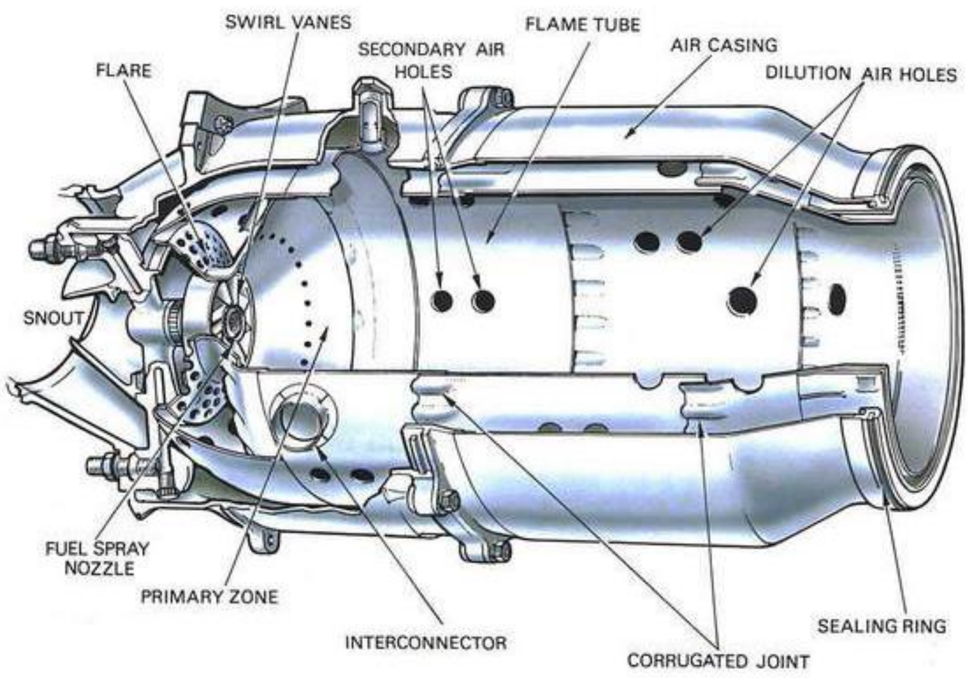
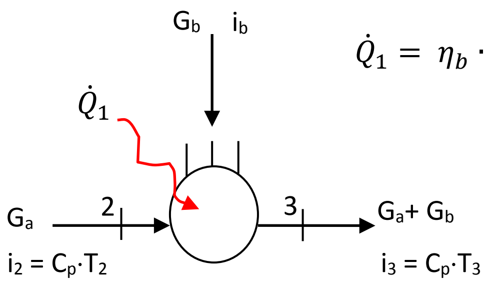


fig. 10 - Main parts of a combustion chamber

The energy balance in a combustion chamber is:



$$\dot{Q}_1 = \eta_b \cdot G_b \cdot H_i = (G_a + G_b) \cdot i_3 - (G_a \cdot i_2 + G_b \cdot i_b) \qquad (32)$$

were i_b is the fuel enthalpy. Since the fuel is usually preheated is possible to assume that $i_b \cong i_2$ and so (32) becomes:

$$\dot{Q}_1 = \eta_b \cdot G_b \cdot H_i = (G_a + G_b) \cdot (i_3 - i_2) \qquad (33)$$

The ratio of the mixture between air and the fuel is defined in the following way:

$$\alpha = \frac{G_a}{G_b} \qquad (34)$$

for gas turbines this ratio is usually bigger than 50, much higher than the stoichiometric ratio (about 15÷16), which means that combustion in gas-turbine power plants takes place with air excess. This is necessary to keep the T_3 temperature down in order not to damage the turbine blades.

7 GAS TURBINE PLANTS WITH REGENERATIVE INNER HEAT EXCHANGER

If outlet flue-gas temperature (T_4) is higher than the temperature at the end of compression (T_2) the exhausted gas can be used for pre-heating compressed air. In figure 11 the schematic picture of a plant with regenerative heat exchanger is shown.

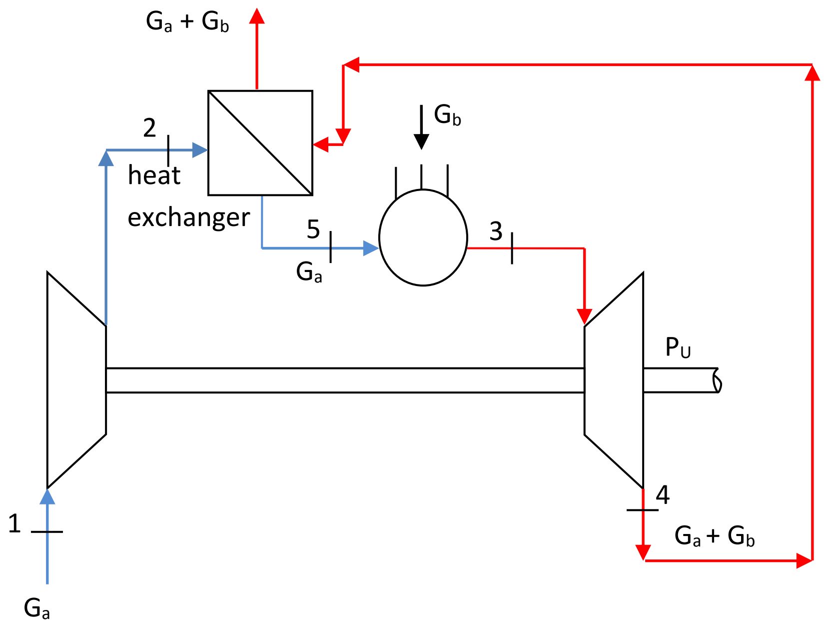


fig. 11 - Gas turbine plant with a regenerative inner exchanger

From figure 11 and 12 (regenerative cycle) it can be seen that heat is supplied starting from point 5 (heat exchanger outlet) instead of point 2 (turbo compressor outlet).

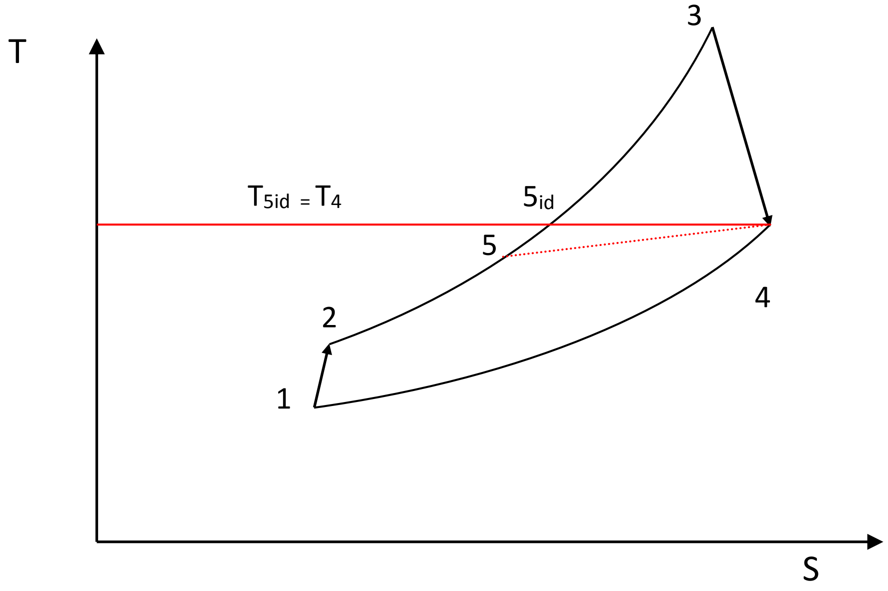


fig. 12 - Gas turbine plant regenerative cycle

So the actual thermal power supplied in the combustor is:

$$\dot{Q}_1 = \eta_b \cdot G_b \cdot H_i = (G_a + G_b) \cdot (i_3 - i_5) \qquad (35)$$

The use of a heat exchanger can increase the efficiency of the plant significantly because of the thermal power saved. If the heat exchanger was ideal, at the exchanger outlet the air would reach temperature $T_5 = T_4$, but for a real heat exchanger the outlet temperature $T_5 < T_4$ because of the finite exchange surface. The efficiency of the heat exchanger is defined as follows:

$$\mathcal{R} = \frac{T_5 - T_2}{T_4 - T_2} \qquad (36)$$

This is the ratio between the actual and the theoretical heat exchanged, it is approximately in the range of 0.6 - 0.8.

8 CYCLES WITH INTER COOLING AND REHEATING

To increase the cycle efficiency (15) it is possible to:

- Reduce the heat supplied
- Reduce the compressor power
- Increase the turbine power

The reduction of heat is achieved with regenerative cycles as seen in the last chapter.

It is possible to demostrate that the minimum work needed to compress a gas is spent with isothermal compression. It is very difficult to achieve such compression because, during compression, countinous heat subtraction would be necessary to keep the temperature constant. It is possible to approximate the isothermal compression with multiple stage compression with inter cooling in between.

The power of a gas turbine is increased with re-heating (re-combustion) during expansion. It is possible to have further combustion, after the first expansion, because of the large amount of excess air used.

In figure 13 a scheme of a plant with regeneration, inter cooling and re-heating is shown, while in figure 14 there is the corresponding thermodynamic cycle.

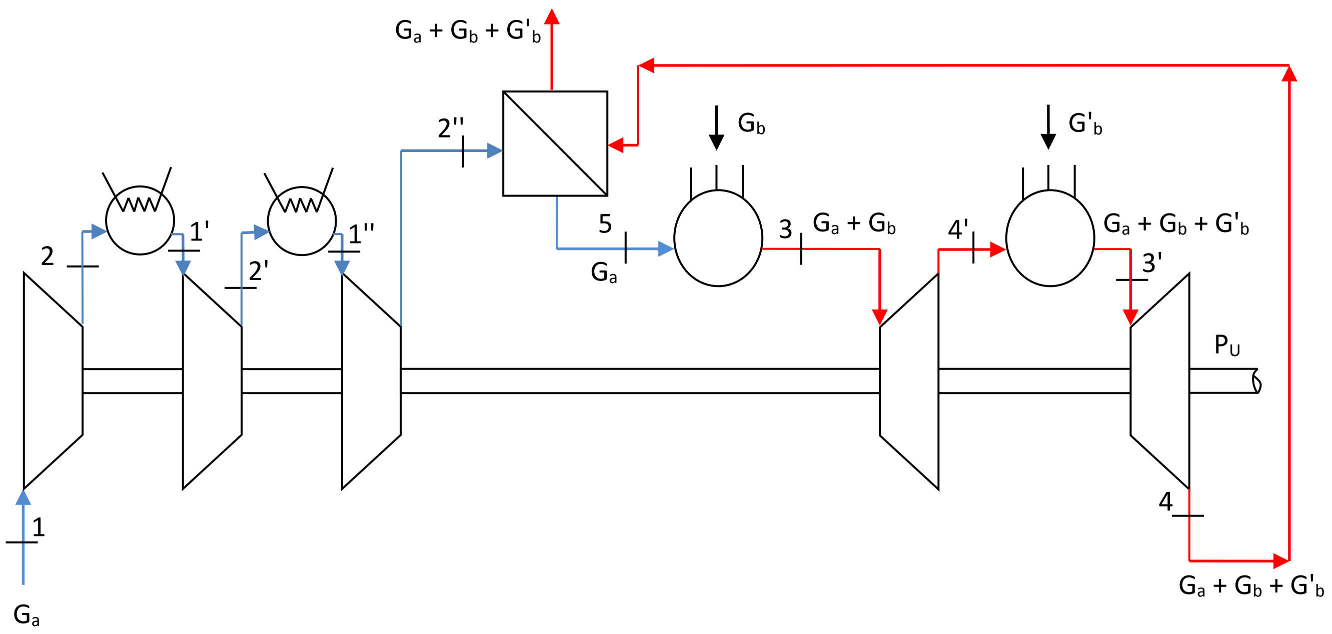


fig. 13 - Gas turbine plant with regenerative cycle, reheating and inter cold compression

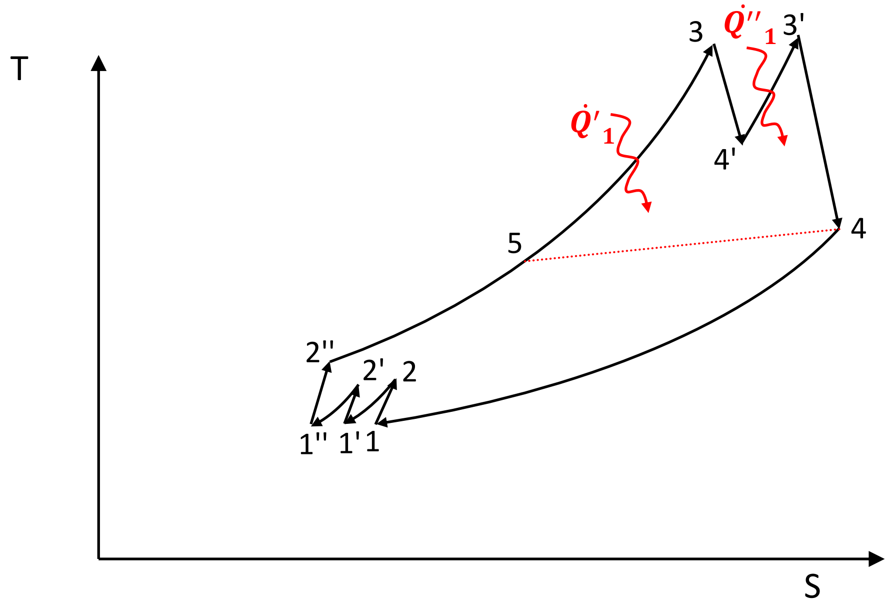


fig. 14 - Cycle of plant with regenerative cycle, reheating and inter cold compression

All the formulas necessary for the calculations are written below:

$$P_C = \frac{G_a \cdot C_p \cdot (T_2 - T_1)}{\eta_m^c} + \frac{G_a \cdot C_p \cdot (T_{2'} - T_{1'})}{\eta_m^c} + \frac{G_a \cdot C_p \cdot (T_{2''} - T_{1''})}{\eta_m^c} \quad (37)$$

$$P_T = (G_a + G_b) \cdot C_p \cdot (T_3 - T_{4'}) \cdot \eta_m^T + (G_a + G_b + G'_b) \cdot C_p \cdot (T_{3'} - T_4) \cdot \eta_m^T \quad (38)$$

$$\dot{Q}_1 = \dot{Q}'_1 + \dot{Q}''_1 \quad (39)$$

$$\dot{Q}'_1 = \eta_b \cdot G_b \cdot H_i = (G_a + G_b) \cdot C_p \cdot (T_3 - T_5) \quad (40)$$

$$\dot{Q}''_1 = \eta_b \cdot G'_b \cdot H_i = (G_a + G_b + G'_b) \cdot C_p \cdot (T_{3'} - T_{4'}) \quad (41)$$

$$\alpha' = \frac{G_a}{G_b} \quad (42)$$

$$\alpha'' = \frac{G_a + G'_a}{G'_b} \quad (43)$$

EXERCISES

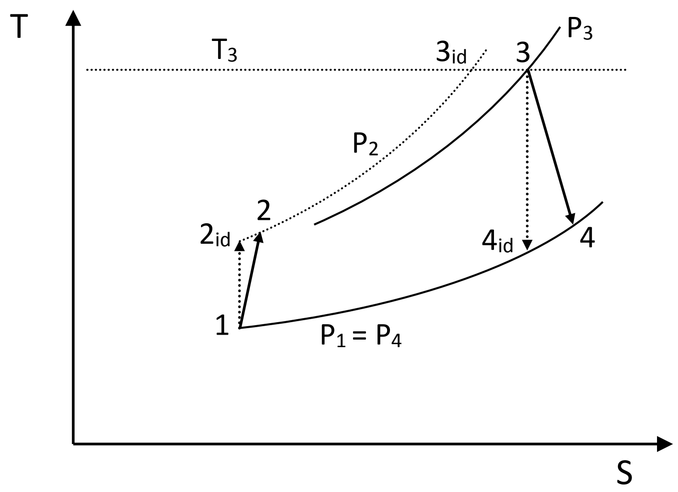
1) *For the gas-turbine power plant shown in fig. 3 the following data are given:*

$P_1 = 1\ bar$	$T_1 = 15°$	$T_3 = 1200°$	$Hi = 50\ MJ/kg$
$\eta_b = 0.95$	$\eta_{is}^T = \eta_{is}^C = 0.85$	$\beta_C = 20$	$\eta_m^T = \eta_m^C = 0.98$
$Gb = 1.6\ \frac{kg}{s}$	$\eta_{\pi b} = 0.95$	$Cp = 1090\ \frac{J}{kgK}$	$k = 1.4$

The plant is used to produce alternating current by way of an alternator with an electrical efficiency η_{el} *= 0.9, calculate:*

1. *The air flow rates*
2. *The net power available on the shaft*
3. *The electrical power*

When we have a cycle the first step is always to calculate all pressure and temperature values for the points of the real cycle.



point 1

$P_1 = 1\ bar = 100000\ Pa$

$T_1 = 15° = 288\ K$

point 2

From the compression ratio: $P_2 = P_1 \cdot \beta_C = 20\ bar = 2000000\ Pa$.

To calculate T_2 we can first calculate the final temperature of the isentropic compression with (10) $T_{2is} = T_1 \cdot \beta_C^{\frac{k-1}{k}} \cong 677.82\ K$. Then with the compressor isentropic efficiencies (7) it is possible to calculate the actual temperature T_2.

$$\eta_{is}^{C} = \frac{L_i^{id}}{L_i} = \frac{C_p \cdot (T_{2is} - T_1)}{C_p \cdot (T_2 - T_1)} \qquad T_2 \cong 746.61\ K$$

point 3

From the pneumatic combustor efficiency: $P_3 = P_2 \cdot \eta_{\pi b} = 19\ bar = 1900000\ Pa$. While T_3 is given: $T_3 = 1200° = 1473\ K$

point 4

$P_4 = P_1 = 1\ bar = 100000\ Pa$. The expantion ratio: $\beta_e = \eta_{\pi b} \cdot \beta_c = 19$.

T_4 can be calculated in the same way as T_2.

$$T_{4is} = T_3 \cdot \left(\frac{1}{\beta_e}\right)^{\frac{k-1}{k}} \cong 635.1\ K \qquad \eta_{is}^{t} = \frac{(L_i)_{obt}}{(L_i)_{obt}^{id}} = \frac{C_p \cdot (T_3 - T_4)}{C_p \cdot (T_3 - T_{4is})} \qquad T_4 \cong 760.78\ K$$

Now we can calculate the air flow rate with the energy balance of the combustion chamber (33) :

$$\eta_b \cdot G_b \cdot H_i = (G_a + G_b) \cdot C_p \cdot (T_3 - T_2) \quad \text{so} \quad G_a = \frac{\eta_b \cdot G_b \cdot H_i}{C_p \cdot (T_3 - T_2)} - G_b \cong 89.33\ \frac{kg}{s}$$

The work required by the compressor and that obtained by the turbine is:

$$L_c = C_p \cdot (T_2 - T_1) \cong 499889\ \frac{J}{kg}$$

$$(L_T)_{obt} = C_p \cdot (T_3 - T_4) \cong 776314.35\ \frac{J}{kg}$$

The net mechanical power on the shaft is:

$$P_U = (G_a + G_b) \cdot (L_T)_{obt} \cdot \eta_m^T - \frac{G_a \cdot L_c}{\eta_m^C} \cong 23612046\ W \cong 23.6\ MW$$

The thermal power is: $\dot{Q}_1 = \eta_b \cdot G_b \cdot H_i \cong 72000000\ W \cong 72\ MW$

The thermal efficiency is: $\eta_u = \frac{P_U}{\dot{Q}} \cong 0.327$

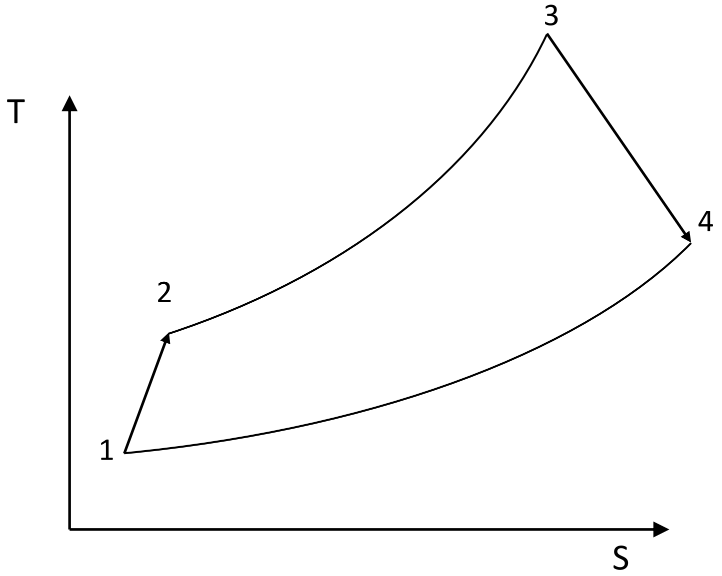
The electrical power is: $P_{el} = P_U \cdot \eta_{el} \cong 21.24\ MW$

The total efficiency is: $\eta_{tot} = \frac{P_{el}}{\dot{Q}_1} = \eta_u \cdot \eta_{el} \cong 0.2943$

2) *For an open cycle of a gas-turbine power plant the characteristic points are given:*

$P_1 \; = \; 1 \, bar$ $\quad$ $P_2 \; = \; 25 \, bar$ $\quad$ $P_3 \; = \; 24 \, bar$ $\quad$ $P_4 \; = \; P_1$

$T_1 \; = \; 293 \, k$ $\quad$ $T_2 \; = \; 750 \, k$ $\quad$ $T_3 \; = \; 1500 \, k$ $\quad$ $T_4 \; = \; 850 \, k$



Further data are known:

$Hi \; = \; 50 \, MJ/kg$ $\quad$ $\eta_b \; = \; 0.96$ $\quad$ $\eta_m^T \; = \; \eta_m^C = \; 0.98$ $\quad$ $Cp \; = \; 1090 \, \frac{J}{kgK}$

1. *Calculate the power plant efficiency*
2. *To improve the efficiency a regenerative exchanger with* $\mathcal{R} = 0.7$ *is used. Calculate the new plant efficiency.*

The efficiency is calculated with (31) but we don't know either the air flow rate or the fuel flow rate. By dividing the numerator and the denominator by G_b, (31) can be written as follows:

$$\eta_u = \frac{(\alpha + 1) \cdot \; C_p \cdot (T_3 - T_4) \cdot \eta_m^T - \dfrac{\alpha \cdot C_p \cdot (T_2 - T_1)}{\eta_m^C}}{\eta_b \cdot H_i}$$

where the only unknown value is $\alpha = \frac{G_a}{G_b}$ and it can be calculated by rearranging (33) as follows:

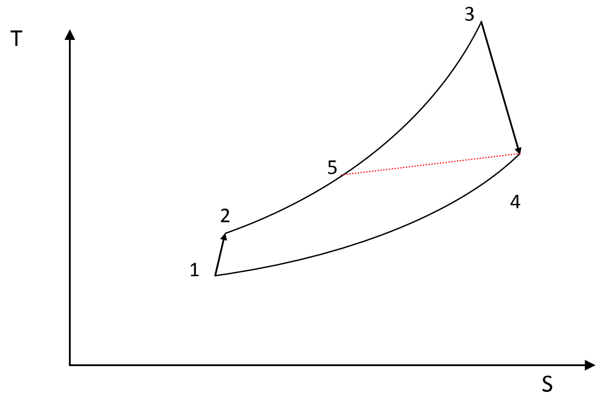
$$\eta_b \cdot H_i = (\alpha + 1) \cdot C_p \cdot (T_3 - T_2)$$

$$\alpha = \frac{\eta_b \cdot H_i}{C_p \cdot (T_3 - T_2)} - 1 = \frac{0.96 \cdot 50000000}{1090 \cdot (1500 - 750)} - 1 \cong 57.71$$

And so the efficiency is:

$$\eta_u = \frac{(57.71 + 1) \cdot\ 1090 \cdot (1500 - 850) \cdot 0.98 - \frac{57.71 \cdot 1090 \cdot (750 - 293)}{0.98}}{0.96 \cdot 50000000} \cong \quad 0.238$$

If the regenerative cycle is used the air and fuel flow rates ratio is different:



$$\alpha^* = \frac{\eta_b \cdot H_i}{C_p \cdot (T_3 - T_5)} - 1$$

where T_5 is calculated by (36) $\mathcal{R} = \frac{T_5 - T_2}{T_4 - T_2}$:

$$0.7 = \frac{T_5 - 750}{850 - 750} \qquad\qquad T_5 = 820\ K$$

$$\alpha^* = \frac{0.96 \cdot 50000000}{1090 \cdot (1500 - 820)} - 1 \ \cong 64.76$$

The new efficiency becomes:

$$\eta_u = \frac{(64.76 + 1) \cdot\ 1090 \cdot (1500 - 850) \cdot 0.98 - \frac{64.76 \cdot 1090 \cdot (750 - 293)}{0.98}}{0.96 \cdot 50000000} \cong \quad 0.265$$

the efficiency increases by about 11%.

N.B. In the calculation we assumed that the cycle points do not change in the regenerative cycle but this is only an approximation.

BIBLIOGRAPHY

[1] A. Capetti**,** *Motori Termici*, UTET, 1967

[2] A. Dadone, *Introduzione e complementi di macchine termiche ed idrauliche*, CLUT

[3] W. W. Bathie, *Fundamentals of Gas Turbines*, John Wiley & Sons, 1996

[4] R. Kurz, *Introduction to Gas Turbines and Applications*

[5] E. A. Baskharone, *Principles of Turbomachinery****,*** Cambridge University Press, 2006