8
STEAM NOZZLES

8.1 Introduction

In the impulse steam turbine, the overall transformation of heat into mechanical work is accomplished in two distinct steps. The available energy of steam is first changed into kinetic energy, and this kinetic energy is then transformed into mechanical work. The first of these steps, viz., the transformation of available energy into kinetic energy is dealt with in this chapter.

A nozzle is a passage of varying cross-sectional area in which the potential energy of the steam is converted into kinetic energy. The increase of velocity of the steam jet at the exit of the nozzle is obtained due to decrease in enthalpy (total heat content) of the steam. The nozzle is so shaped that it will perform this conversion of energy with minimum loss.

8.2 General Forms of Nozzle Passages

A nozzle is an element whose primary function is to convert enthalpy (total heat) energy into kinetic energy. When the steam flows through a suitably shaped nozzle from zone of high pressure to one at low pressure, its velocity and specific volume both will increase.

The equation of the continuity of mass may be written thus :

$$m = \frac{AV}{v} = A\left(\frac{V}{v}\right) \qquad \ldots(8.1)$$

where m = mass flow in kg/sec.,

V = velocity of steam in m/sec.,

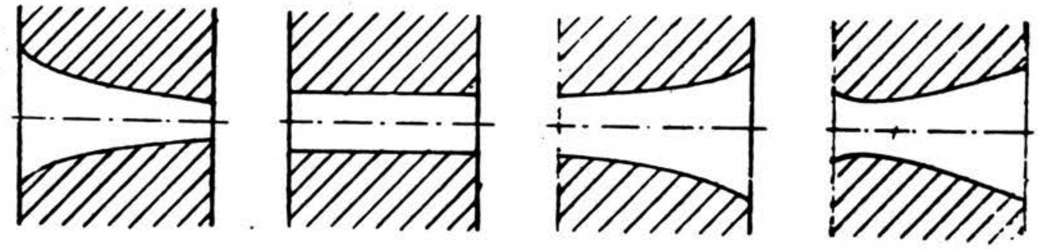
A = area of cross-section in m^2, and

v = specific volume of steam in m^3/kg.

In order to allow the expansion to take place properly, the area at any section of the nozzle must be such that it will accomodate the steam whatever volume and velocity may prevail at that point.

As the mass flow (m) is same at all sections of the nozzle, area of cross-section (A) varies as $\frac{V}{v}$. The manner in which both V and v vary depends upon the properties of the substance flowing. Hence, the contour of the passage of nozzle depends upon the nature of the substance flowing.

For example, consider a *liquid*– a substance whose specific volume v remains almost constant with change of pressure. The value of $\frac{V}{v}$ will go on increasing with change of pressure. Thus, from eqn. (8.1), the area of cross-section should decrease with the decrease of pressure. Fig. 8-1(a) illustrates the proper contour of longitudinal section of a nozzle suitable for liquid. This also can represent convergent nozzle for a fluid whose peculiarity is that while both velocity and specific volume increase, the rate of specific volume increase is less than that of the velocity, thus resulting in increasing value of $\frac{V}{v}$.



(a) (b) (c) (d)

Fig. 8-1. General forms of Nozzles.

Fig. 8-1(b) represents the correct contour for some hypothetical substance for which both velocity and specific volume increase at the same rate, so that their ratio $\frac{V}{v}$ is a constant at all points. The area of cross-section should therefore, be constant at all points, and the nozzle becomes a plain tube.

Fig. 8-1(c) represents a divergent nozzle for a fluid whose peculiarity is that $\frac{V}{v}$ decreases with the drop of pressure, i.e., specific volume increases at a faster rate than velocity with the drop of pressure. The area of cross-section should increase as the pressure decreases.

Table 8-1

Properties of steam at various pressures when expanding dry saturated steam from 14 bar to 0.15 bar through a nozzle, assuming frictionless adiabatic flow.

Pressure p bar	Dryness fraction x	Enthalpy drop $H_1 - H_2$ kJ	Velocity V m/sec.	Specific Volume v_s m^3/kg	Discharge per unit area kg/m^2	Area A m^2	Diameter D metre
14	1·000	-	-	-	-	-	-
12	0·988	38·6	278	0·1633	1,723	0·00058	0·0272
10	0.974	84·1	410	0·1944	2,165	0·00046	0·0242
7	0·950	164·7	574	0·2729	*2,214	0·00045	x0·0239
3·5	0·908	309	786	0·5243	1,651	0·00061	0·0279
1·5	0·872	441·2	939	1·1593	929	0·0011	0·0374
0·70	0·840	555·6	1,054	2·365	531	0·00188	0·049
0·15	0·790	736·7	1,214	10·022	153	0·0065	0·091

* Maximum discharge per unit area x Smallest diameter

Fig. 8-1(d) shows the general shape of convergent-divergent nozzle suitable for gases and vapours. It can be shown that in practice, while velocity and specific volume both increase from the start, velocity first increases faster than the specific volume, but after a certain critical point, specific volume increases more rapidly than velocity. Hence the value of $\frac{V}{v}$ first increases to maximum and then decreases, necessitating a nozzle of *convergent-divergent* form. The above statement may be verified by referring to table 8-1, which shows the properties of steam at various pressures when expanding dry saturated steam from 14 bar to 0·15 bar through a nozzle, assuming frictionless adiabatic flow.

8.3 Steam Nozzles

The mass flow per second for wet steam, at a given pressure during expansion is given by

$$m = \frac{AV}{v} = \frac{AV}{xv_s} \text{ kg/sec.} \qquad \text{...(8.2)}$$

where A = Area of cross-section in m^2,

V = Velocity of steam in m/sec,

v_s = Specific volume of dry saturated steam, m^3/kg,

x = Dryness fraction of steam, and

$v = x\, v_s$ = Specific volume of wet steam, m^3/kg.

As the mass of steam per second (m) passing through any section of the nozzle must be constant, the area of cross-section (A) of nozzle will vary according to the variation of $\frac{V}{xv_s}$ i.e., product of A and $\frac{V}{xv_s}$ is constant. If the factor $\frac{V}{xv_s}$ increases with the drop in pressure, the cross-sectional area should decrease and hence a *convergent* shaped nozzle. The decrease of the factor $\frac{V}{xv_s}$ with pressure drop will require increasing cross-sectional area to maintain mass flow constant and hence the *divergent* shaped nozzle.

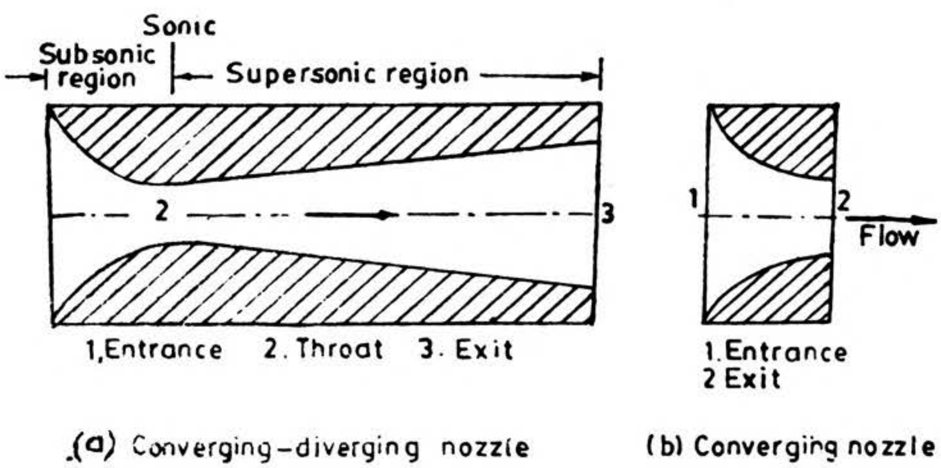


Fig. 8-2. Longitudinal sectional view of steam nozzles.

In practice at first the nozzle cross-section tapers to a smaller section in order to allow for increasing value of $\frac{V}{xv_s}$; after this smallest diameter is reached, it will diverge to a larger cross-section. The smallest section of the nozzle is known as the *throat.*

A nozzle which first converges to throat and then diverges, as in fig. 8-2(a), is termed as *converging-diverging nozzle.* It is used for higher pressure ratio $\left(\frac{p_2}{p_1}\right)$.

Some form of nozzles finish at the throat and no diverging portion is fitted; this type shown in fig. 8-2(b), is known as *converging nozzle.* In this the greatest area is at the entrance and minimum area is at the exit which is also the throat of the nozzle. This nozzle is used when the pressure ratio, $\frac{P_2}{p_1}$ is less than 0.58 (critical).

8.4 Flow Through Steam Nozzles

From the point of view of thermodynamics, the steam flow through nozzles may be spoken as adiabatic expansion. During the flow of steam through the nozzle, heat is neither supplied nor rejected. Moreover, as the steam expands from high pressure to low pressure, the heat energy is converted into kinetic energy, i.e., work is done in expanding to increase the kinetic energy. Thus the expansion of steam through a nozzle is an adiabatic, and the flow of steam through nozzle is regarded as an adiabatic flow.

It should be noted that the expansion of steam through a nozzle is not a free expansion, and the steam is not throttled, because it has a large velocity at the end of the expansion. Work is done by the expanding steam in producing this kinetic energy.

In practice, some kinetic energy is lost in overcoming the friction between the steam and the side of the nozzle and also internal friction, which will tend to regenerate heat. The heat thus formed tends to dry the steam. About 10% to 15% of the enthalpy drop from inlet to exit is lost in friction. The effect of this friction, in resisting the flow and in drying the steam, must be taken into account in the design of steam nozzles, as it makes an appreciable difference in the results.

Another complication in the design of steam through a nozzle is due to a phenomenon known as *supersaturation;* this is due to a time lag in the condensation of the steam during the expansion. The expansion takes place very rapidly and if the steam is initially dry or superheated, it should become wet as the pressure falls, because the expansion is adiabatic. During expansion the steam does not have time to condense, but remains in an unnatural dry or superheated state, then at a certain instant, it suddenly condenses to its natural state. See illustrative problem no. 14.

Thus, the flow of steam through a nozzle may be regarded as either an ideal adiabatic (isentropic) flow, or adiabatic flow modified by friction and supersaturation.

If friction is negligible, three steps are essential in the process of expansion from pressure p_1 to p_2 :

(i) Driving of steam upto the nozzle inlet from the boiler. The 'flow-work' done on the steam is p_1v_1 and results in similar volume of steam being forced through the exit to make room for fresh charge (steam).

(ii) Expansion of steam through the nozzle while pressure changes from p_1 to p_2, the work done being $\frac{1}{n-1}(p_1v_1 - p_2v_2)$

where n is the index of the isentropic expansion,

v_1 = volume occupied by 1 kg of steam at entrance to nozzle, and

v_2 = volume occupied by 1 kg of steam as it leaves the nozzle.

Alternatively, this work done is equal to the change of internal energy, $\mu_1 - \mu_2$ as during isentropic expansion work is done at the cost of internal energy.

(iii) Displacement of the steam from the low pressure zone by an equal volume discharged from the nozzle. This work amounts to p_2v_2 which is equal to the final flow work spent in forcing the steam out to make room for fresh charge (steam).

Thus, the new work done in increasing kinetic energy of the steam,

$$W = p_1v_1 + \left[\frac{1}{n-1}(p_1v_1 - p_2v_2)\right] - p_2v_2$$

$$W = \frac{n}{n-1}(p_1v_1 - p_2v_2) \qquad \text{... (8.3)}$$

This is same as the work done during Rankine cycle.

Alternatively, $W = p_1v_1 + (\mu_1 - \mu_2) - p_2v_2$

$$= (p_1v_1 + \mu_1) - (\mu_2 + p_2v_2) = H_1 - H_2 \qquad \text{...(8.4)}$$

where, H_1 and H_2 are the values of initial and final enthalpies allowing for the states of superheating or wetness as the case may be. This is exactly equivalent to the enthalpy drop equivalent to the work done during the Rankine cycle. The value of $H_1 - H_2$ may be found very rapidly from the Mollier chart ($H - \Phi$ chart) or more slowly but with greater accuracy from the steam tables.

In the design of steam nozzles the calculations to be made are :

(i) the actual velocity attained by the steam at the exit,

(ii) the minimum cross-sectional area (throat area) required for a given mass flow per second,

(iii) the exit area, if the nozzle is converging-diverging, and

(iv) the general shape of the nozzle – axial length.

8.4.1 Velocity of steam leaving nozzle : The gain of kinetic energy is equal to the enthalpy drop of the steam. The initial velocity of the steam entering the nozzle (or velocity of approach) may be neglected as being relatively very small compared with exit velocity.

For isentropic (frictionless adiabatic) flow and considering one kilogram of steam

$$\frac{V^2}{2 \times 1{,}000} = H_1 - H_2 = H$$

where H is enthalpy drop in kJ/kg and V = velocity of steam leaving the nozzle in m/sec.

$$\therefore V = \sqrt{2 \times 1{,}000H} = 44{\cdot}72\sqrt{H} \text{ m/sec.} \qquad \text{... (8.5)}$$

Let the available enthalpy drop after deducting frictional loss be kH,

i.e. $(1 - k)H$ is the friction loss,

Then, $V = 44{\cdot}72\sqrt{kH}$ m/sec. ... (8.6)

If the frictional loss in the nozzle is 15 per cent of the enthalpy drop, then $k = 0.85$.

8.4.2 Mass of steam discharged : The mass flow of steam in kg per second through a cross-sectional area A and at a pressure p_2 can be written as

$$m = \frac{AV_2}{v_2}$$ where v_2 = specific volume of steam at pressure p_2.

But $v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = v_1 \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$ ...(8.7)

where, v_1 = specific volume of steam at pressure p_1.

Using the value of velocity V from eqns. (8.3.) and (8.5),

$$m = \frac{A}{v_2}\sqrt{\left[2{,}000\frac{n}{n-1}(p_1v_1 - p_2v_2)\right]} = \frac{A}{v_2}\sqrt{2{,}000\frac{n}{n-1}p_1v_1\left[1 - \frac{p_2v_2}{p_1v_1}\right]}$$

Putting the value of v_2 from eqn. (8.7), we get,

$$m = \frac{A}{v_1\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}}\sqrt{2{,}000\frac{n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

$$m = A\sqrt{2{,}000\frac{n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]} \quad \text{... (8.8)}$$

8.4.3 Critical pressure ratio : Using eqn. (8.8), the rate of mass flow per unit area is given by

$$\frac{m}{A} = \sqrt{2{,}000\frac{n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]}$$

The mass flow per unit area has the maximum value at the throat which has minimum area, the value of pressure ratio $\left(\frac{p_2}{p_1}\right)$ at the throat can be evaluated from the above expression corresponding to the maximum value of $\frac{m}{A}$.

All the items of this equation are constant with the exception of the ratio $\left(\frac{p_2}{p_1}\right)$.

Hence, $\frac{m}{A}$ is maximum when $\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]$ is the maximum.

Differentiating the above expression with respect to $\left(\frac{p_2}{p_1}\right)$ and equating to zero for a maximum discharge per unit area

$$\frac{d}{d\left(\frac{p_2}{p_1}\right)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right] = 0$$

$$\therefore \frac{2}{n}\left[\frac{p_2}{p_1}\right]^{\frac{2}{n}-1} - \frac{n+1}{n}\left[\frac{p_2}{p_1}\right]^{\frac{n+1}{n}-1} = 0$$

Hence, $\left[\frac{p_2}{p_1}\right]^{\frac{2-n}{n}} = \frac{n+1}{n}\left[\frac{p_2}{p_1}\right]^{\frac{1}{n}}$ or $\left[\frac{p_2}{p_1}\right]^{2-n} = \left[\frac{n+1}{2}\right]^{n}\left(\frac{p_2}{p_1}\right)$

from which $\left[\frac{p_2}{p_1}\right]^{1-n} = \left[\frac{n+1}{2}\right]^{n}$ or $\frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$ ... (8.9)

$\frac{p_2}{p_1}$ is known as *critical pressure ratio* and depends upon the value of index *n*.

The following approximate values of index *n* and corresponding values of critical pressure ratios may be noted :

Initial condition of steam	Value of index *n* for isentropic expansion	Nozzle critical pressure ratio $\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$
Superheated or supersaturated	1·300	0·546
Dry saturated	1·135	0·578
Wet	1·113	0·582

Dr. Zeuner has suggested a well known equation for value of *n* in the adiabatic expansion of steam viz. $n = 1{\cdot}035 + 0{\cdot}1x_1$, where x_1 is the initial dryness fraction of steam.

The eqn. (8.9) gives the ratio between the throat pressure (p_2) and the inlet pressure (p_1) for a maximum discharge per unit area through the nozzle. The mass flow being constant for all sections of nozzle, maximum discharge per unit area occurs at the section

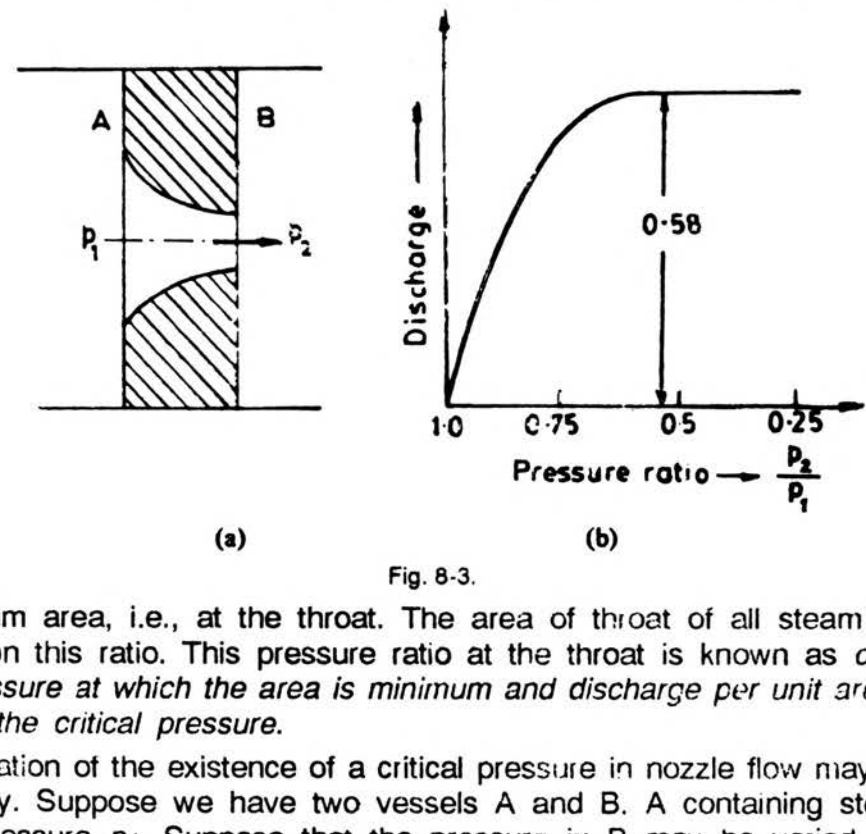


Fig. 8-3.

having minimum area, i.e., at the throat. The area of throat of all steam nozzle should be designed on this ratio. This pressure ratio at the throat is known as *critical pressure ratio. The pressure at which the area is minimum and discharge per unit area is maximum is termed as the critical pressure.*

The implication of the existence of a critical pressure in nozzle flow may be expressed in another way. Suppose we have two vessels A and B. A containing steam at a high and steady pressure p_1. Suppose that the pressure in B may be varied at will. A and B are connected by a diaphragm containing a convergent nozzle, as shown in fig. 8-3(a).

Assume at first that p_2 is equal to p_1, then there is no flow of steam through the nozzle. Now let p_2 be gradually reduced. The discharge *m* through the nozzle will increase as shown by the curve of fig. 8-3(b). As the pressure p_2 approaches the critical value, the discharge rate gradually approaches its maximum value, and when p_2 is reduced below the critical value, the discharge rate does not increase but remains at the same value as that at the critical pressure. The extraordinary result that p_2 can be reduced well below the critical pressure without influencing the mass flow was first discovered by R.D. Napier.

Another explanation can be visualised as follows : the critical pressure will give velocity of steam at the throat equal to the velocity of the sound *(sonic velocity)*. The flow of steam in the convergent portion of the nozzle is *sub-sonic*. Thus, to increase the velocity of steam above sonic velocity *(super sonic)* by expanding steam below critical pressure, divergent portion is necessary [fig· 8-2(a)]·

8.4.4 Areas of throat and exit for maximum discharge : The first step is to estimate the critical pressure or throat pressure for the given initial condition of steam.

(1) If the nozzle is *convergent*, the nozzle terminates at the throat, hence the throat is the exit end or mouth of the nozzle.

Next, using the Mollier ($H - \Phi$) chart, the enthalpy drop can be calculated by drawing a vertical line to represent the isentropic expansion from p_1 to p_2 (p_2 is throat pressure). Read off from the $H - \Phi$ chart the value of enthalpies H_1 and H_2 or enthalpy drop $H_1 - H_2$ and dryness fraction x_2 as shown in fig. 8-4.

Then, for throat, enthalpy drop from entry to throat, $H_t = H_1 - H_2$ kJ/kg, and velocity at throat, $V_2 = 44{\cdot}72\sqrt{H_t}$ m/sec.

Then, mass flow, $m = \dfrac{A_2 V_2}{x_2 v_{s2}}$ kg/sec. (if steam is wet at throat) ... (8.10)

where A_2 = throat area.

The value of v_{s2} (specific volume of dry saturated steam) at pressure p_2 can be obtained from the steam tables.

If the steam is superheated at throat, $m = \dfrac{A_2 V_2}{v_{sup}}$ kg/sec. ... (8.11)

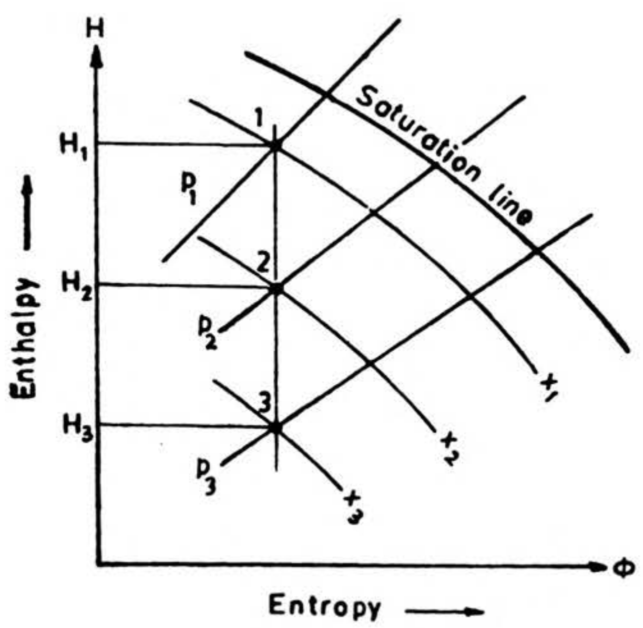


Fig. 8-4. H - φ diagram.

where, $v_{sup} = v_{s2}\left(\dfrac{T_{sup\,2}}{T_{s_2}}\right)$

As the mass of discharge m is known, the area A_2 (throat area) can be calculated.

(2) If the nozzle is *convergent-divergent*, calculation of throat area is the same as for the convergent nozzle in which case the value of p_2 is critical pressure. As the back pressure in this nozzle is lower than critical pressure, the vertical line on the $H - \Phi$ chart is extended up to the given back pressure p_3 at the exit as shown in fig. 8-4.

The value of enthalpy H_3 and the dryness fraction x_3 at exit are read off directly from the $H - \Phi$ chart.

For the exit or mouth of the nozzle, enthalpy drop from entry to exit,

$H_e = H_1 - H_3$ kJ/kg and velocity at exit,

$V_3 = 44{\cdot}72\sqrt{H_e}$ m/sec.

Then, mass flow, $m = \dfrac{A_3 V_3}{x_3 v_{s_3}}$ kg/sec. ... (8.12)

The value of v_{s3} at pressure p_3 can be obtained directly from the steam tables. As the mass of discharge m is known, the exit area A_3 can be calculated by using eqn. (8.12).

Similarly, for any pressure p along the nozzle axis, steam velocity and then the cross-sectional area can be evaluated.

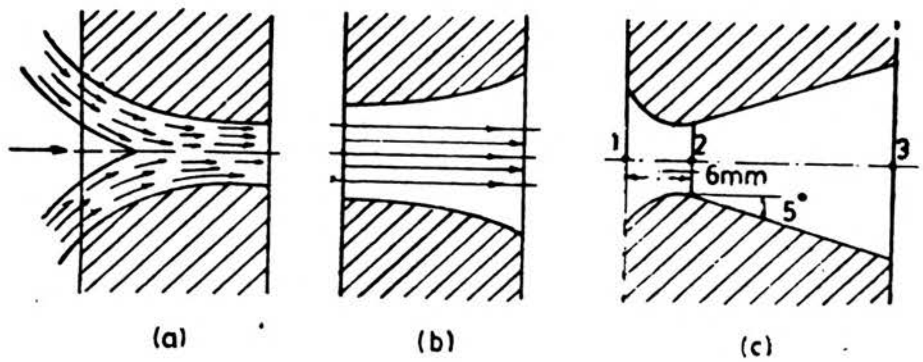


Fig. 8-5.

8.4.5 Length of nozzle : The length of the convergent portion should be short in order to reduce the surface friction, and normally a length of about 6 mm will be found adequate. This rapid change in the area is possible because the convergence of the walls of a passage tends to stabilize the flow as shown in fig. 8-5 (a).

In the divergent portion, high velocity steam has tendency, on account of inertia, to flow along the axis in a form of a circular jet of sectional area equal to throat area. If the divergence is rapid, steam will not occupy the increased area provided. Thus, steam may pass out through the divergent point without drop of pressure as shown in fig. 8.5(b). To avoid this, divergent portion should have sufficient length so that steam has enough time to occupy the full cross-sectional area provided, thus resulting in desired drop of pressure and increase in kinetic energy. This necessitates gradual increase in area. It is found satisfactory in practice to make the length of the nozzle from throat to exit such that the included cone angle is about 10° as shown in fig. 8-5(c).

Problem – 1 : *A convergent-divergent nozzle for a steam turbine has to deliver steam under a supply condition of 11 bar with 100°C superheat and a back pressure of 0.15 bar. If the outlet area of the nozzle is 9.7 cm², determine using steam tables, the mass of steam discharged per hour. If the turbine converts 60% of the total enthalpy drop into useful work, determine the power delivered by the turbine. Neglect the effect of friction in the nozzle. Take K_p of superheated steam as 2.3 kJ/kg K.*

From Steam Tables

p	t_s	v_s	h	L	H	Φ_w	Φ_s
bar	°C	m³/kg	kJ/kg	kJ/kg	kJ/kg	kJ/kg K	kJ/kg K
11	184·09	—	781·34	2,000·4	2,781·7	2·1792	6·5536
0·15	53·97	10·022	225·94	2,373·1	2,599·1	0·7549	8·0085

Let suffixes 1 and 3 represent conditions at entry and exit of the nozzle.

Entropy before expansion = Entropy after expansion

$$\Phi_1 = \Phi_3$$

i.e. $\Phi_{s1} + k_p \log_e \frac{T_{sup}}{T_s} = \Phi_{w3} + x_3 (\Phi_{s3} - \Phi_{w3})$

i.e., $6{\cdot}5536 + 2{\cdot}3 \log_e \frac{184{\cdot}09 + 100 + 273}{184{\cdot}09 + 273} = 0{\cdot}7549 + x_3 (8{\cdot}0085 - 0{\cdot}7549)$

$\therefore\ 6{\cdot}5536 + 2{\cdot}3 \times 0{\cdot}1976 = 0{\cdot}7549 + 7{\cdot}2536 \times x_3$

$\therefore\ x_3 = \frac{6{\cdot}5536 + 0{\cdot}4545 - 0{\cdot}7549}{7{\cdot}2536} = \frac{6{\cdot}2432}{7{\cdot}2536}$

i.e., x_3 = **0·862** (dryness fraction at exit)

Enthalpy, $H_1 = H_s + k_p (T_{sup} - T_s) = 2{,}781{\cdot}7 + 2{\cdot}3\ (100) = 3{,}011{\cdot}7$ kJ/kg

Enthalpy, $H_3 = h_3 + x_3 L_3 = 225{\cdot}94 + 0{\cdot}862 \times 2{,}373{\cdot}1 = 2{,}271{\cdot}6$ kJ/kg

Enthalpy drop from inlet to exit,

$H_e = H_1 - H_3 = 3{,}011{\cdot}7 - 2{,}271{\cdot}6 = 740{\cdot}1$ kJ/kg.

Using eqn. (8.5), Velocity at exit, $V_3 = 44{\cdot}72 \sqrt{H_e} = 44{\cdot}72 \sqrt{740{\cdot}1} = 1{,}216{\cdot}6$ m/sec.

For mass continuity, $m = \frac{A_3 V_3}{v_3}$ [Eqn. (8.2)]

$$= \frac{9{\cdot}7 \times 1{,}216{\cdot}6}{10^4 \times (10{\cdot}022 \times 0{\cdot}862)} = 0{\cdot}1366 \text{ kg/sec.}$$

Mass of steam discharged per hour = 0·1366 × 3,600 = **491·76 kg/hour.**

Useful work done per kg of steam = 0·6 × 740·1 = 444·06 kJ/kg.

Power delivered = 444·06 × 0·1366 = 60·66 kJ/sec. or **60·66 kW**

Note : *The enthalpy drop from inlet to exit (H_1 - H_3) and the final dryness fraction (x_3) can be found directly from H – Φ chart by the method as shown in fig. 8-6.*

Problem – 2 : *A convergent-divergent nozzle is required to discharge 350 kg of steam per hour. The nozzle is supplied with steam at 8.5 bar and 90% dry and discharges against a back pressure of 0.4 bar. Neglecting the effect of friction, find the throat and exit diameters.*

Let suffixes, 1,2 and 3 represent conditions at entry, throat and exit of the nozzle respectively as shown in fig. 8-6.

As the steam is initially wet, critical or throat pressure,

$p_2 = 0{\cdot}582 \times p_1 = 0{\cdot}582 \times 8{\cdot}5 = 4{\cdot}95$ bar.

As shown in fig. 8-6, vertical line 1-2-3 is drawn. The values read off from the H – Φ chart (Mollier chart) are :

Enthalpy drop from entry to throat, $H_t = H_1 - H_2 = 102$ kJ/kg,

Enthalpy drop from entry to exit, $H_e = H_1 - H_3 = 456$ kJ/kg,

Dryness fraction of steam at throat, $x_2 = 0{\cdot}87$ and

Dryness fraction of steam at exit, $x_3 = 0{\cdot}777$

Velocity at throat, $V_2 = 44.72 \sqrt{H_t} = 44{\cdot}72 \sqrt{102} = 452$ m/sec. [eqn. (8.5)]

Specific volume of dry saturated steam at 4.95 bar (by arithmetical interpolation from steam tables), $v_{s2} = 0.3785$ m^3/kg.

$\therefore$ Actual volume of wet steam at throat,

$v_2 = x_2 \times v_{s2} = 0{\cdot}87 \times 0{\cdot}3785 = 0{\cdot}33$ m^3/kg.

For mass continuity, $m = \dfrac{A_2 V_2}{v_2}$ kg/sec. [eqn. (8.2)]

i.e., $\dfrac{350}{3{,}600} = \dfrac{A_2 \times 452}{0{\cdot}33 \times 10^4}$

$\therefore\ A_2 = \dfrac{350 \times 0{\cdot}33 \times 10^4}{3{,}600 \times 452} = 0{\cdot}71\ \text{cm}^2$

$\therefore$ Throat diameter, $D_2 = \sqrt{\dfrac{0{\cdot}71 \times 4}{\pi}} = 0{\cdot}951$ cm i.e., **9·51 mm**

Similarly velocity at exit,

$V_3 = 44{\cdot}72\sqrt{456} = 955$ m/sec.

Specific volume of dry saturated steam at 0.4 bar (from steam tables),

$v_{s3} = 3{\cdot}993\ \text{m}^3/\text{kg}$.

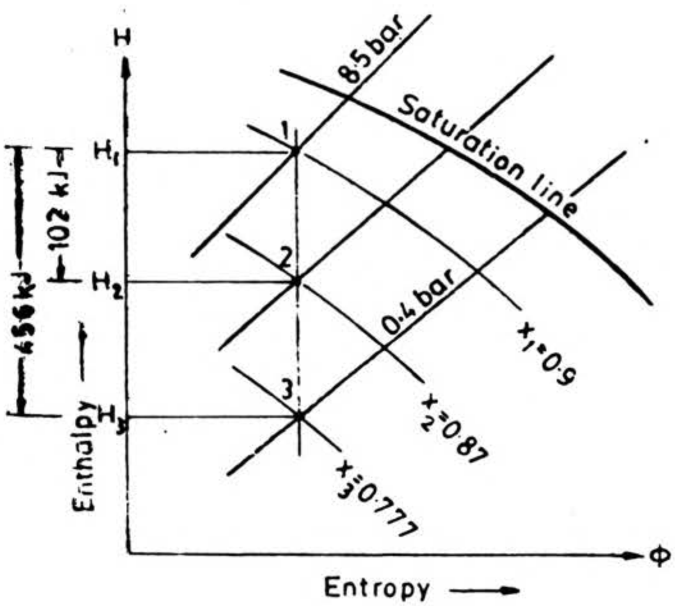


Fig. 8-6. H - φ diagram.

$\therefore$ Actual volume of wet steam at exit, $v_3 = x_3 \times v_{s3}$

$= 0{\cdot}777 \times 3{\cdot}993 = 3{\cdot}11\ \text{m}^3/\text{kg}$.

Again for mass continuity,

$m = \dfrac{A_3 V_3}{v_3}$ [eqn. (8.2)]

i.e., $\dfrac{350}{3{,}600} = \dfrac{A_3 \times 955}{3{\cdot}11 \times 10^4}$

$\therefore\ A_3 = \dfrac{350 \times 3{\cdot}11 \times 10^4}{955 \times 3{,}600}$

$= 3{\cdot}16\ \text{cm}^2$

$\therefore$ Exit diameter, $D_3 =$

$\sqrt{\dfrac{3{\cdot}16 \times 4}{\pi}} = 2{\cdot}01$ cm i.e., **20·1mm**

Problem – 3 : *An impulse turbine which is to develop 175 kW with probable steam consumption of 11 kg per kW-hour is supplied with dry saturated steam at 10 bar. Find the number of nozzles each of about 6 mm diameter at the throat that will be required for the purpose and estimate the exact diameters at the throat and exit of the nozzles. The condenser pressure is 0.15 bar. Neglect the effect of friction in nozzles. Assume index of expansion as 1.135.*

Let suffixes, 1, 2 and 3 represent conditions at entry, throat and exit of the nozzle.

From eqn. (8.9), $\dfrac{p_2}{p_1} = \left(\dfrac{2}{n+1}\right)^{\frac{n}{n-1}}$

Putting $n = 1{\cdot}135$, $\dfrac{p_2}{p_1} = \left(\dfrac{2}{2{\cdot}135}\right)^{\frac{1{\cdot}135}{0{\cdot}135}} = (0{\cdot}936)^{8{\cdot}4} = 0{\cdot}578$

Critical or throat pressure, $p_2 = 0{\cdot}578 \times p_1 = 0{\cdot}578 \times 10 = 5{\cdot}78$ bar.

Enthalpy drop from entry to throat, $H_t = H_1 - H_2 = 122$ kJ/kg and

Dryness fraction of steam at throat, $x_2 = 0{\cdot}957$ (from $H - \Phi$ chart).

Velocity at throat, $V_2 = 44{\cdot}72\sqrt{H_t} = 44{\cdot}72\sqrt{122} = 494$ m/sec.,

From steam tables at 5·78 bar, $v_{s2} = 0{\cdot}327$ m³/kg by arithmetical interpolation.

Specific volume at throat, $v_2 = x_2 \times v_{s2} = 0{\cdot}957 \times 0{\cdot}327$ m³/kg.

$$\text{For mass continuity, } m = \frac{A_2 V_2}{v_2} = \frac{\frac{\pi}{4}\left(\frac{6}{10}\right)^2 \times 494}{10^4 \times 0{\cdot}957 \times 0{\cdot}327} = 0{\cdot}0446 \text{ kg/sec.}$$

$$\text{Steam consumption per sec.} = \frac{11 \times 175}{3{,}600} = 0{\cdot}5347 \text{ kg/sec.}$$

$$\therefore \text{ Number of nozzles required} = \frac{0{\cdot}5347}{0{\cdot}0446} = 11{\cdot}99 \text{ say } 12$$

$$\therefore \text{ Exact diameter at throat, } D_2 = 6\sqrt{\frac{11{\cdot}99}{12}} = \mathbf{5{\cdot}997\ mm.}$$

For exit :

Enthalpy drop from entry to exit, $H_e = H_1 - H_3 = 655$ kJ/kg and

Dryness fraction of steam at exit, $x_3 = 0{\cdot}85$ (from $H - \Phi$ chart).

$\therefore$ Velocity at exit, $V_3 = 44{\cdot}72\sqrt{H_e} = 44{\cdot}72\sqrt{655} = 1{,}145$ m/sec.

From steam tables at 0.15 bar, $v_{s_3} = 10{\cdot}022$ m³/kg.

$\therefore$ Taking the number of nozzles as 12,

$$\text{Mass of steam per nozzle} = \frac{0{\cdot}5347}{12} = 0{\cdot}0446 \text{ kg/sec.}$$

$$\text{Again for mass continuity, } m = \frac{A_3 V_3}{v_3} \text{ or } A_3 = \frac{m v_3}{V_3} = \frac{m \times V_{s_3} \times x_3}{V_3}$$

$$\therefore A_3 = \frac{0{\cdot}0446 \times (10{\cdot}022 \times 0{\cdot}85) \times 10^4}{1{,}145} = 3{\cdot}646 \text{ cm}^2.$$

$$\therefore \text{ Exact diameter at exit, } D_3 = \sqrt{\frac{3{\cdot}646 \times 4}{\pi}} = 2{\cdot}155 \text{ cm i.e., } \mathbf{21{\cdot}55\ mm.}$$

Problem – 4 : *Steam expands from 17 bar and 80°C superheat to 0·7 bar in a convergent-divergent nozzle. Assuming that the expansion is frictionless adiabatic, and the steam discharged is 0·25 kg/sec., calculate the diameters of the sections of nozzle (i) at a point where the pressure is 9·5 bar, and (ii) at exit.*

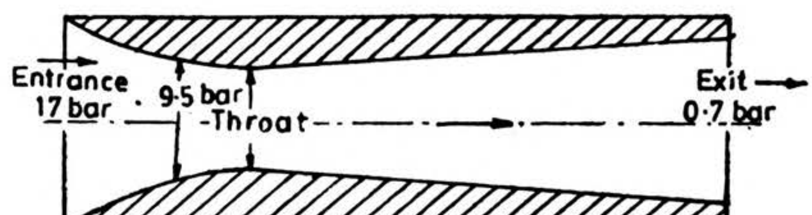


Fig. 8-7

Take K_p *of superheated steam as 2·3 kJ/kg K.*

Referring to fig. 8-7,

Let suffixes, 1,2 and 3 represent conditions at entrance, section of the nozzle where pressure is 9·5 bar and exit respectively.

As the steam is initially superheated,

critical or throat pressure = $0{\cdot}546 \times p_1 = 0{\cdot}546 \times 17 = 9{\cdot}28$ bar.

It means that the nozzle is still converging where the pressure is 9·5 bar.

(1) For section of the nozzle where the pressure is 9·5 bar :

Enthalpy drop from entry to section of nozzle, where the pressure is 9·5 bar,

$H_1 - H_2 = 140$ kJ/kg (from $H - \Phi$ chart);

Temperature of steam, $t_2 = 213°C$ (from $H - \Phi$ chart).

At 9·5 bar, saturation temperature, $t_s = 177{\cdot}69°C$ (from steam tables).

∴ Steam at section where pressure is 9·5 bar is superheated, i.e., steam is still superheated after expansion.

At 9.5 bar, $v_{s2} = 0.2042$ m^3/kg (from steam tables).

Specific volume at 9·5 bar and 213°C,

$$v_2 = V_{s_2} \times \frac{T_{sup2}}{T_{sat2}} = 0{\cdot}2042 \times \frac{(213 + 273)}{(177{\cdot}69 + 273)} = 0{\cdot}22 \text{ m}^3/\text{kg}$$

Velocity at section, where pressure is 9·5 bar, $V_2 = 44{\cdot}72\sqrt{140} = 529$ m/sec.

For mass continuity, $m = \dfrac{A_2 V_2}{v_2}$

$$\text{i.e., } 0{\cdot}25 = \frac{\frac{\pi}{4}(D_2)^2 \times 529}{0{\cdot}22 \times 10^4}$$

$$\therefore (D_2)^2 = \frac{0{\cdot}25 \times 0{\cdot}22 \times 10^4 \times 4}{\pi \times 529} = 1{\cdot}324$$

∴ Diameter, $D_2 = \sqrt{1{\cdot}324} = 1{\cdot}15$ cm i.e., **11·5 mm**

Diameter of the section of the nozzle at a point where the pressure is 9·5 bar = 11·5 mm.

(ii) For exit :

From $H - \Phi$ chart, Enthalpy drop from inlet to exit, $H_e = H_1 - H_3 = 600$ kJ/kg and dryness fraction, $x_3 = 0.89$.

Velocity at exit, $V_3 = 44{\cdot}72\sqrt{H_e} = 44{\cdot}72\sqrt{600} = 1{,}095$ m/sec.

From steam tables, at 0·7 bar, $v_{s3} = 2{\cdot}365$ m^3/kg.

Specific volume at exit, $v_3 = x_3 \times v_{s3} = 0{\cdot}89 \times 2{\cdot}365$ m^3/kg.

For mass continuity, $m = \dfrac{A_3 V_3}{v_3}$

$$\therefore A_3 = \frac{\pi}{4}(D_3)^2 \times \frac{1}{10^4} = \frac{m \times v_3}{V_3} = \frac{0{\cdot}25 \times (0{\cdot}89 \times 2{\cdot}365)}{1{,}095}$$

$$\therefore (D_3)^2 = \frac{0{\cdot}25 \times 0{\cdot}89 \times 2{\cdot}365 \times 10^4 \times 4}{\pi \times 1{,}095} = 6{\cdot}12$$

∴ Exit diameter, $D_3 = \sqrt{6{\cdot}12} = 2{\cdot}47$ cm, i.e., **24·7 mm.**

Problem – 5 : *A convergent-divergent nozzle is supplied with dry saturated steam at 11 bar. If the divergent portion of the nozzle is 6 cm long and the throat diameter is*

8 mm, determine the semi-cone angle of the divergent part of the nozzle so that the steam may leave the nozzle at 0·4 bar. Neglect the effect of friction in the nozzle.

For throat :

Let suffixes, 1, 2 and 3 represent conditions at entry, throat and exit of the nozzle.

As the steam supplied is initially dry saturated,

Critical or throat pressure, $p_2 = 0{\cdot}578 \times p_1 = 0{\cdot}578 \times 11 = 6{\cdot}36$ bar.

From $H - \Phi$ chart, Enthalpy drop from entry to throat, $H_t = H_1 - H_2 = 113$ kJ/kg and dryness fraction of steam at throat, $x_2 = 0{\cdot}96$.

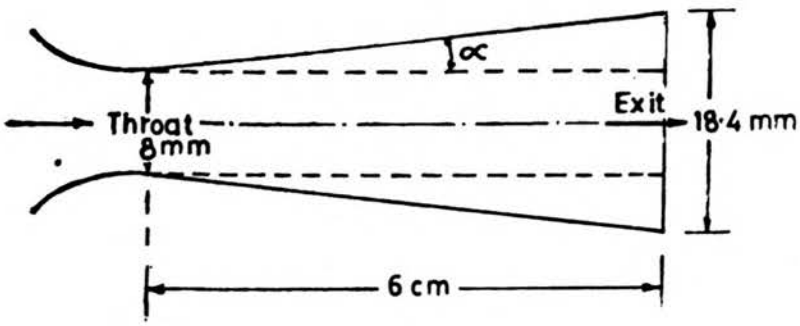


Fig. 8-8.

Velocity at throat,

$$V_2 = 44{\cdot}72\sqrt{H_t} = 44{\cdot}72 \times 113 = 475 \text{ m/sec.}$$

From steam tables at 6.36 bar, $v_{s2} = 0{\cdot}2995$ m^3/kg.

$\therefore$ Specific volume at throat, $v_2 = x_2 \times v_{s2} = 0{\cdot}96 \times 0{\cdot}2995$ m^3/kg.

For mass continuity,

$$m = \frac{A_2 V_2}{v_2} = \frac{\frac{\pi}{4}\left(\frac{8}{10}\right)^2 \times 475}{0{\cdot}96 \times 0{\cdot}2995 \times 10^4} = 0{\cdot}083 \text{ kg/sec.}$$

For exit :

From $H - \Phi$ chart, Enthalpy drop from entry to exit, $H_e = H_1 - H_3 = 540$ kJ/kg and dryness fraction of steam at exit, $x_3 = 0{\cdot}834$

Velocity at exit, $V_3 = 44{\cdot}72\sqrt{H_e} = 44{\cdot}72\sqrt{540} = 1{,}039$ m/sec.

From steam tables at 0·4 bar, $v_{s3} = 3{\cdot}993$ m^3/kg

Specific volume at exit, $v_3 = x_3 \times v_{s3} = 0{\cdot}834 \times 3{\cdot}993$ m^3/kg.

For mass continuity, $m = \frac{A_3 V_3}{v_3}$

$$\therefore\ A_3 = \frac{\pi}{4}(D_3)^2 \times \frac{1}{10^4} = \frac{m \times v_3}{V_3} = \frac{0{\cdot}083 \times (0{\cdot}834 \times 3{\cdot}993)}{1{,}039}$$

$$\therefore\ (D_3)^2 = \frac{0{\cdot}083 \times 0{\cdot}834 \times 3{\cdot}993 \times 10^4 \times 4}{\pi \times 1{,}039} = 3{\cdot}39$$

$\therefore$ Exit diameter, $D_3 = \sqrt{3{\cdot}39} = 1{\cdot}84$ cm, i.e., 18·4 mm.

$$\text{Referring to fig. 8-8, } \tan\alpha = \frac{\frac{D_3 - D_2}{2}}{l} = \frac{\frac{18{\cdot}4 - 8}{2}}{6 \times 10} = 0{\cdot}0866$$

$\therefore$ Semi-cone angle, $\alpha = $ **4° – 57'**

Problem – 6 : *Dry saturated steam at a pressure of 8.5 bar enters a convergent-divergent nozzle, and leaves at a pressure of 1·5 bar.*

If the flow is frictionless adiabatic and the corresponding expansion index is 1·135, find using steam tables, the ratio of the cross-sectional area at exit to that at the throat.

Let suffixes 1, 2 and 3 represent conditions at entry, throat and exit of the nozzle respectively.

From eqn. (8.9), $\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

Putting $n = 1{\cdot}135$, $\frac{p_2}{p_1} = \left(\frac{2}{2{\cdot}135}\right)^{\frac{1{\cdot}135}{0{\cdot}135}} = (0{\cdot}936)^{8{\cdot}4} = 0{\cdot}578$

$\therefore\ p_2 = 0{\cdot}578 \times p_1 = 0{\cdot}578 \times 8{\cdot}5 = 4{\cdot}91$ bar

At 8·5 bar, from steam tables, $\Phi_s = 6{\cdot}6421$ kJ/kg K, $H = 2{,}771{\cdot}6$ kJ/kg,

At 4.91 bar, from steam tables by arithmetical interpolation, h = 636 kJ/kg

$L = 2{,}110$ kJ/kg, $\Phi_w = 1{\cdot}8550$ kJ/kg K, $\Phi_s = 6{\cdot}860$ kJ/kg K, $v_s = 0{\cdot}38$ m^3/kg;

At 1.5 bar, from steam tables, $h = 467{\cdot}11$ kJ/kg, $L = 2{,}226{\cdot}5$ kJ/kg,

$\Phi_w = 1{\cdot}4336$ kJ/kg K, $\Phi_s = 7{\cdot}2233$ kJ/kg K, $v_s = 1{\cdot}1593$ m^3/kg.

For throat :

Entropy before expansion = Entropy after expansion

$$\Phi_1 = \Phi_{w2} + x_2(\Phi_{s2} - \Phi_{w2})$$

$$\text{i.e., } 6{\cdot}6421 = 1{\cdot}8550 + x_2(6{\cdot}860 - 1{\cdot}8550)$$

$\therefore\ x_2 = \frac{4{\cdot}7871}{5{\cdot}005} = 0{\cdot}956$

$H_2 = h_2 + x_2L_2 = 636 + 0{\cdot}956 \times 2{,}110 = 2{,}653{\cdot}16$ kJ/kg

$\therefore$ Enthalpy drop from entry to throat,

$$H_t = H_1 - H_2 = 2{,}771{\cdot}6 - 2{,}653{\cdot}16 = 118{\cdot}44 \text{ kJ/kg.}$$

Velocity at throat, $V_2 = 44{\cdot}72\sqrt{118{\cdot}44} = 487$ m/sec.

For mass continuity, $m = \frac{A_2V_2}{v_2} = \frac{A_2V_2}{(x_2 \times v_{s_2})}$

$\therefore\ A_2 = \frac{m \times (x_2 \times v_{s2})}{V_2} = \frac{m \times (0{\cdot}956 \times 0{\cdot}38)}{487} = 0{\cdot}000746\ m$

For exit :

Now, $\Phi_1 = \Phi_{w3} + x_3(\Phi_{s3} - \Phi_{w3})$

i.e., $6{\cdot}6421 = 1{\cdot}4336 + x_3(7{\cdot}2233 - 1{\cdot}4336)$

$\therefore\ x_3 = \frac{5{\cdot}2085}{5{\cdot}7897} = 0{\cdot}9$

$H_3 = h_3 + x_3L_3 = 467{\cdot}11 + 0{\cdot}9 \times 2{,}226{\cdot}5 = 2{,}470{\cdot}96$ kJ/kg.

$\therefore$ Enthalpy drop from entry to exit,

$H_e = H_1 - H_3 = 2{,}771{\cdot}6 - 2{,}470{\cdot}96 = 300{\cdot}64$ kJ/kg

Velocity at exit, $V_3 = 44{\cdot}72\sqrt{300{\cdot}64} = 775$ m/sec.

Now, $m = \frac{A_3V_3}{v_3} = \frac{A_3V_3}{(x_3 \times v_{s3})}$

$\therefore\ A_3 = \frac{m \times (x_3 \times v_{s3})}{V_3} = \frac{m \times (0{\cdot}9 \times 1{\cdot}1593)}{775} = 0{\cdot}001346\ m$

$\therefore \frac{\text{Area at exit}}{\text{Area at throat}} = \frac{0{\cdot}001346m}{0{\cdot}000746m} = \mathbf{1{\cdot}8}$ (ratio)

Problem – 7 : *Assuming frictionless adiabatic flow through a converging-diverging nozzle, show that the maximum discharge per unit area at the throat is given by :*

$$\sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}} \text{ kg per sec. per m}^2$$

where, n is the index of expansion,

p_1 is the initial pressure of steam in kPa, and

v_1 is the specific volume of steam in m^3/kg at the initial pressure.

The flow or discharge per unit area through the nozzle throat is given by eqn. (8.8),

$$\frac{m}{A} = \sqrt{2{,}000 \frac{n}{n-1} \cdot \frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]}$$

This will be maximum when $\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

Substituting this value of $\left(\frac{p_2}{p_1}\right)$ in the above equation, we get,

$$\frac{m}{A} = \sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n-1}\right)\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n} \times \frac{n}{n-1}} - \left(\frac{2}{n+1}\right)^{\frac{n+1}{n} \times \frac{n}{n-1}}\right]}$$

$$= \sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n-1}\right)\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}} - \left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\right]}$$

$$= \sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n-1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left\{\frac{\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}}{\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}} - 1\right\}}$$

$$= \sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n-1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1} - \frac{n+1}{n-1}} - 1\right]}$$

$$= \sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n-1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left[\left(\frac{2}{n+1}\right)^{\frac{2-n-1}{n-1}} - 1\right]}$$

$$= \sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n-1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left[\left(\frac{2}{n+1}\right)^{-\left(\frac{n-1}{n-1}\right)} - 1\right]}$$

$$\frac{m}{A} = \sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n-1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left(\frac{n+1}{2} - 1\right)}$$

$$= \sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n-1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left(\frac{n-1}{2}\right)}$$

$$= \sqrt{1{,}000\, n \frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}} \text{ kg per sec. per m}^2$$

Problem – 8 : *Assuming frictionless adiabatic flow through a converging-diverging nozzle, show that for the maximum discharge, the velocity of steam at throat is given by :*

$$V_2 = \sqrt{2{,}000 \frac{n}{n+1} p_1 v_1} \text{ m/sec.}$$

where n is the index of expansion,

p_1 is the initial pressure of steam in kPa, and

v_1 is the specific volume of steam in m^3/kg at the initial pressure.

For isentropic (frictionless adiabatic) flow, neglecting initial velocity of steam and considering one kilogram of steam, the velocity of steam at throat, V_2 is given by

$$\frac{V_2^2}{2{,}000} = H_1 - H_2 \text{ or } V_2 = \sqrt{2{,}000\,(H_1 - H_2)}$$

Considering pv^n = constant, a law of isentropic expansion of steam in the nozzle, the above expression of V_2 can be written as

$$V_2 = \sqrt{2{,}000 \frac{n}{n-1}(p_1 v_1 - p_2 v_2)}$$

$$= \sqrt{2{,}000 \frac{n}{n-1} p_1 v_1 \left(1 - \frac{p_2 v_2}{p_1 v_1}\right)}$$

But $\frac{v_2}{v_1} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$ from $p_1 v_1^n = p_2 v_2^n$

$$\therefore V_2 = \sqrt{2{,}000 \frac{n}{n-1} p_1 v_1 \left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}}$$

For maximum discharge, $\frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$

$$\therefore V_2 = \sqrt{2{,}000 \frac{n}{n-1} p_1 v_1 \left\{1 - \left(\frac{2}{n+1}\right)^{\frac{n}{n-1} \times \frac{n-1}{n}}\right\}}$$

$$= \sqrt{2{,}000 \frac{n}{n-1} p_1 v_1 \left\{1 - \frac{2}{n+1}\right\}}$$

$$= \sqrt{2{,}000 \frac{n}{n-1} p_1 v_1 \times \frac{n-1}{n+1}}$$

$$V_2 = \sqrt{2{,}000 \frac{n}{n+1} p_1 v_1} \quad \text{m/sec.}$$

Problem – 9 : *Dry saturated steam enters a nozzle at pressure of 10 bar and with an initial velocity of 90 m/sec. The outlet pressure is 6 bar and outlet velocity is 435 m/sec. The heat loss from the nozzle is 6·3 kJ per kg of steam flow. Calculate the dryness fraction and the area at the exit, if the area at the inlet is 12·56 cm².*

Steady flow energy equation per kg of steam flow through the nozzle at inlet and outlet can be written as

$$H_1 + \frac{V_1^{\,2}}{2{,}000} = H_2 + \frac{V_2^{\,2}}{2{,}000} + \text{ losses}$$

where, H_1 and H_2 are enthalpies at inlet and outlet, kJ/kg, and

V_1 and V_2 are velocities at inlet and outlet, m/sec.

H_1 = enthalpy of dry saturated steam at 10 bar = 2778·1 kJ/kg (from steam tables).

$$\therefore\ 2{,}778{\cdot}1 + \frac{90^2}{2{,}000} = H_2 + \frac{435^2}{2{,}000} + 6{\cdot}3$$

$$\therefore\ H_2 = 2{,}778{\cdot}1 + 4{\cdot}05 - 94{\cdot}61 - 6{\cdot}3 = 2{,}681{\cdot}24 \text{ kJ/kg.}$$

As steam is dry saturated at inlet it will be wet at outlet. Let its dryness fraction be x_2. $H_2 = h_2 + x_2 L_2$ [at 6 bar, h_2 = 670·56 kJ/kg, L_2 = 2,086·3 kJ/kg (from steam tables)].

i.e., $2{,}681{\cdot}24 = 670{\cdot}56 + x_2 \times 2086{\cdot}3$

$$\therefore\ x_2 = \frac{2{,}681{\cdot}24 - 670{\cdot}56}{2{,}086{\cdot}3} = \mathbf{0{\cdot}964}$$

$v_1 = v_{s1} = 0{\cdot}1944$ m³/kg at 10 bar (from steam tables).

$v_{s2} = 0{\cdot}3157$ m³/kg at 6 bar (from steam tables).

$v_2 = x_2 \times v_{s2} = 0{\cdot}964 \times 0{\cdot}3157 = 0{\cdot}3043$ m³/kg at 6 bar.

Now, for mass flow continuity,

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \quad \text{kg/sec.}$$

$$\therefore\ \frac{A_2}{A_1} = \frac{v_2}{v_1} \times \frac{V_1}{V_2}$$

$$\therefore\ \frac{A_2}{A_1} = \frac{0{\cdot}3043}{0{\cdot}1944} \times \frac{90}{435} = 0{\cdot}3239$$

But $A_1 = 12{\cdot}56$ cm² (area at inlet)

$\therefore\ A_2 = 0{\cdot}3239 \times 12{\cdot}56 =$ **4·068 cm²** (area at exit)

8.5 Effect of Friction in a Nozzle

As stated earlier, the length of the converging part of the converging-diverging nozzle is very small compared with that of divergent part. Thus, most of friction in the converging-diverging nozzle occurs in the divergent part, i.e., between the throat and the exit. The effect of friction is to reduce the available enthalpy drop for conversion into kinetic energy by about 10 to 15 per cent. The equation for the velocity is then written as

$V = 44{\cdot}72\sqrt{kH}$ m per sec.

where k is the coefficient which allows for friction loss,

i.e., $k = \dfrac{\text{Actual or useful enthalpy drop}}{\text{Isentropic enthalpy drop}}$

It is sometimes termed as *nozzle efficiency.*

Or, if the initial velocity of the steam entering the nozzle can be neglected compared with the final velocity, the *nozzle efficiency* can also be expressed as

$$\frac{V^2}{2{,}000 \times (\text{Isentropic enthalpy drop, } H)}$$

The kinetic energy lost in friction is transformed into heat which tends to dry or superheat the steam. Thus, it (friction) will affect the final condition of the steam issuing from the nozzle. Its effect can be represented on the $H - \Phi$ diagram.

Referring to fig. 8-9, let point A represent the initial condition of steam and expansion takes place from pressure p_1 to p_2. Vertical line AB represents isentropic expansion. The total enthalpy drop AB is reduced by friction to AC such that $\dfrac{AC}{AB} = k$.

From the known value of k, point C on the diagram can be obtained. But the expansion must end on the same pressure line p_2. Hence CD is drawn horizontally to meet p_2 pressure line at D. Then the point D represents the final condition of steam after expansion. It may be noted that dryness fraction at D is greater than that at B. Thus, the effect of friction has been partly to dry the steam.

Actually, drying effect of the friction will occur throughout the whole expansion, so that the actual expansion would be represented by the dotted line AD (fig. 8–9).

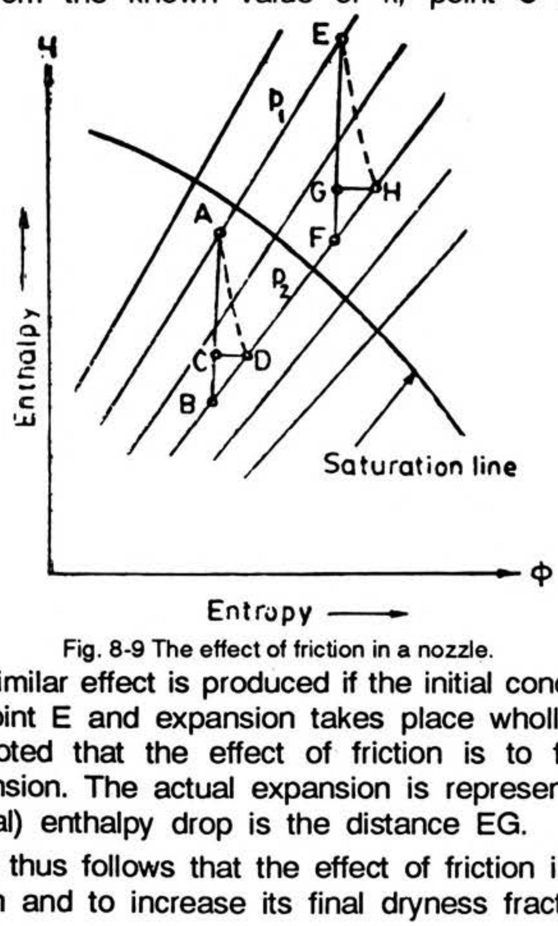


Fig. 8-9 The effect of friction in a nozzle.

Similar effect is produced if the initial condition of steam is superheated as represented by point E and expansion takes place wholly or partly in the superheated region. It will be noted that the effect of friction is to further superheat the steam at the end of expansion. The actual expansion is represented by the dotted line EH and the resulting (actual) enthalpy drop is the distance EG.

It thus follows that the effect of friction in the nozzle is to reduce the velocity of the steam and to increase its final dryness fraction or degree of superheat.

Problem – 10 : *Steam is supplied at a dryness fraction of 0.97 and 11 bar pressure to a convergent-divergent nozzle and expands down to a back pressure of 0.3 bar. The throat area of the nozzle is 5 cm² and 12 per cent of the total enthalpy drop is lost in the divergent part. Determine : (a) steam flow in kg per sec. and (b) the nozzle outlet area.*

Let suffixes, 1, 2 and 3 represent conditions at entry, throat and exit of the nozzle. As the steam is wet initially, critical or throat pressure,

$p_2 = 0{\cdot}582 \times p_1 = 0{\cdot}582 \times 11 = 6{\cdot}4$ bar.

From steam tables :

p bar	t_s °C	v_s m^3/kg	h kJ/kg	L kJ/kg	H kJ/kg	Φ_w kJ/kg K	Φ_s kJ/kg K
11	–	0·17753	781·34	2,000·4	2,781·7	2·1792	6·5536
6·4	–	0·297	681·6	2,078	2,759·6	1.9566	6.7383
0·3	–	5·229	289·23	2,336·1	2,625·3	0·9439	7·7686

(a) **For throat :**

$$\Phi_1 = \Phi_2$$

$\Phi_{w1} + x_1(\Phi_{s1} - \Phi_{w1}) = \Phi_{w2} + x_2(\Phi_{s2} - \Phi_{w2})$

i.e., $2{\cdot}1792 + 0{\cdot}97\,(6{\cdot}5536 - 2{\cdot}1792) = 1{\cdot}9566 + x_2\,(6{\cdot}7383 - 1{\cdot}9566)$

$\therefore\ 2{\cdot}1792 + 4{\cdot}2432 = 1{\cdot}9566 + 4{\cdot}7817 \times x_2$

$\therefore\ x_2$ (dryness fraction at throat) = 0·934

Enthalpy, $H_1 = h_1 + x_1L_1 = 781{\cdot}34 + 0{\cdot}97 \times 2{,}000{\cdot}4 = 2{,}722{\cdot}09$ kJ/kg

Enthalpy, $H_2 = h_2 + x_2L_2 = 681{\cdot}6 + 0{\cdot}934 \times 2{,}078 = 2{,}622{\cdot}45$ kJ/kg.

Enthalpy drop from inlet to throat, $H_t = H_1 - H_2 = 2{,}722{\cdot}09 - 2{,}622{\cdot}45 = 99{\cdot}64$ kJ/kg.

Velocity at throat, $V_2 = 44{\cdot}72\sqrt{H_t} = 44{\cdot}72\sqrt{99{\cdot}64} = 446{\cdot}3$ m/sec.

Specific volume at throat, $v_2 = x_2 \times V_{s2} = 0{\cdot}934 \times 0{\cdot}297 = 0{\cdot}2774$ m^3/kg.

For mass continuity, $m = \dfrac{A_2V_2}{v_2} = \dfrac{5 \times 446{\cdot}3}{10^4 \times 0{\cdot}2774} =$ **0·8044 kg/sec.**

(b) For exit :

$$\Phi_1 = \Phi_3$$

$\Phi_{w1} + x_1(\Phi_{s1} - \Phi_{w1}) = \Phi_{w3} + x_3(\Phi_{s3} - \Phi_{w3})$

i.e., $2{\cdot}1792 + 0{\cdot}97\,(6{\cdot}5536 - 2{\cdot}1792) = 0{\cdot}9439 + x_3\,(7{\cdot}7686 - 0{\cdot}9439)$

$\therefore\ 2{\cdot}1792 + 4{\cdot}2432 = 0{\cdot}9439 + 6{\cdot}8247 \times x_3$

$\therefore\ x_3 = \dfrac{6{\cdot}4224 - 0{\cdot}9439}{6{\cdot}8247} = 0{\cdot}8028$ (dryness fraction at exit)

Enthalpy, $H_1 = 2{,}722{\cdot}09$ kJ/kg.

Enthalpy, $H_3 = h_3 + x_3L_3 = 289{\cdot}23 + 0{\cdot}8028 \times 2{,}336{\cdot}1 = 2{,}164{\cdot}23$ kJ/kg

Enthalpy drop from inlet to exit.

$H_e = H_1 - H_3 = 2{,}722{\cdot}09 - 2{,}164{\cdot}23 = 557{\cdot}86$ kJ/kg.

Enthalpy drop after considering *friction* in the divergent part

$= 0{\cdot}88 \times 557{\cdot}86 = 490{\cdot}92$ kJ/kg.

Velocity at exit, $V_3 = 44{\cdot}72\sqrt{490{\cdot}92} = 990$ m/sec.

Specific volume at exit, $v_3 = x_3 \times v_{s_3} = 0{\cdot}8028 \times 5{\cdot}229$ m^3.kg

For mass continuity, $m = \dfrac{A_3V_3}{v_3}$

$$\therefore A_3 = \frac{m \times v_3}{V_3} = \frac{10^4 \times 0{\cdot}8044 \times (0{\cdot}8028 \times 5{\cdot}229)}{990} = \mathbf{34.13\ cm^2} \text{ (outlet area).}$$

Problem – 11 : *Steam enters a group of convergent-divergent nozzles at 21 bar and 270°C, the discharge pressure being 0.07 bar. The expansion is in equilibrium throughout and the loss of friction in the converging portion of the nozzle is negligible, but the loss by friction in the divergent section of the nozzle is equivalent to 10 per cent of the enthalpy drop available in that section (i.e., enthalpy drop available in the divergent section).*

Calculate the total throat and exit areas in cm², to discharge 14 kg of steam per second. Sketch enthalpy-entropy (H – Φ) chart and show on it the various stages of expansion.

Let suffixes 1, 2 and 3 represent conditions at entry, throat and exit of the nozzle respectively as shown in fig. 8-10.

As the steam is initially superheated, critical or throat pressure,

$p_2 = 0{\cdot}546 \times p_1 = 0{\cdot}546 \times 21 = 11{\cdot}47$ bar.

A sketch (fig 8-10) of the readings taken from the $H - \Phi$ chart is given.

Isentropic enthalpy drop from throat to exit = 770 kJ/kg (from $H = \Phi$ chart).

The actual (useful) enthalpy drop from throat to exit is 90% of the isentropic enthalpy drop.

$\therefore$ Actual enthalpy drop after allowing for friction in the divergent section

$= 0{\cdot}9 \times 770 = 693$ kJ/kg.

For throat : Enthalpy drop,

$H_t = H_1 - H_2 = 140$ kJ/kg (from $H - \Phi$ chart).

Temperature of steam at throat, $t_2 = 194$°C (from $H - \Phi$ chart), i.e., steam is superheated at the throat.

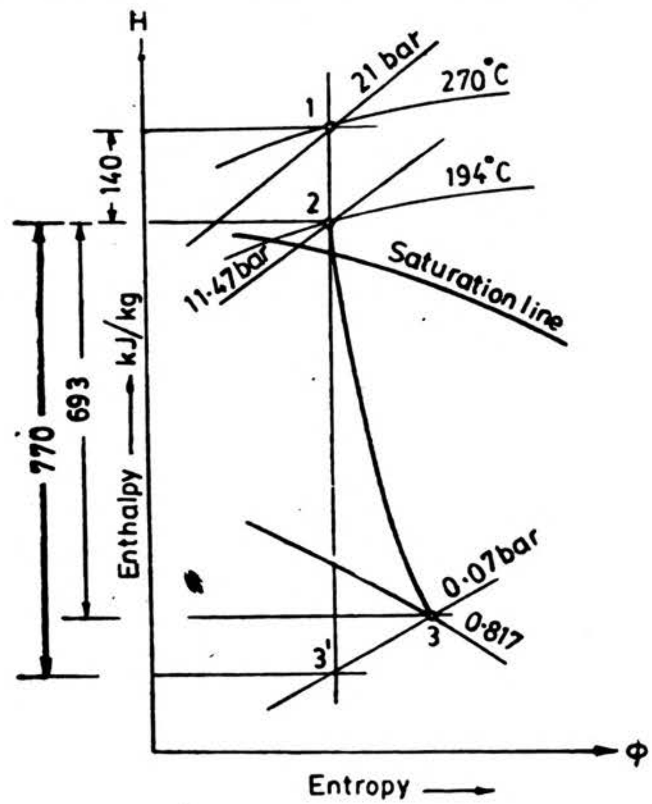


Fig. 8 10. H - φ diagram.

At 11.47 bar (from steam tables by arithmetical interpolation), $t_{s2} = 186$°C and

$v_{s2} = 0.17$ m³/kg.

Specific volume at throat,

$$v_2 = v_{s2} \times \frac{T_{sup2}}{T_{s2}}$$

$$= 0{\cdot}17 \times \frac{194 + 273}{186 + 273}$$

$$= 0{\cdot}173 \text{ m}^3/\text{kg}.$$

Velocity at throat,

$V_2 = 44{\cdot}72\sqrt{H_t}$

$= 44{\cdot}72\sqrt{140} = 529$ m/sec.

For mass continuity,

$$m = \frac{A_2 V_2}{v_2}$$

$$\text{i.e., } 14 = \frac{A_2 \times 529}{0{\cdot}173 \times 10^4}$$

$\therefore$ Total area of nozzles at throat, $A_2 = \dfrac{14 \times 0{\cdot}173 \times 10^4}{529} = \mathbf{45.78\ cm^2}$

For exit :

Actual enthalpy drop from entry to exit, $He = H_1 - H_3 = 833$ kJ/kg, and dryness fraction after reheating, $x_3 = 0{\cdot}817$ (from $H - \Phi$ chart).

At 0.07 bar, $vs_3 = 20.53$ m^3/kg (from steam tables).

Specific volume at exit, $v_3 = x_3 \times v_{s3} = 0.817 \times 20.53$ m^3/kg.

Velocity at exit, $V_3 = 44{\cdot}72\sqrt{H_e} = 44{\cdot}72\sqrt{833} = 1{,}291$ m/sec.

For mass continuity, $m = \dfrac{A_3 V_3}{v_3}$

i.e., $14 = \dfrac{A_3 \times 1291}{(0{\cdot}817 \times 20{\cdot}53) \times 10^4}$

$\therefore$ Total area of nozzles at exit, $A_3 = \dfrac{14 \times 0{\cdot}817 \times 20{\cdot}53 \times 10^4}{1{,}291} = \mathbf{1819\ cm^2}$

Problem – 12 : *A convergent divergent nozzle is required to pass 1.8 kg of steam per sec. At inlet the steam pressure and actual temperature are 7 bar and 186.2°C respectively and the speed is 75 m/sec. Expansion is stable throughout to the exit pressure of 1.1 bar. There is no loss by friction in the converging section of the nozzle but loss by friction between throat and outlet is equivalent to 70 kJ/kg of steam. Calculate, assuming throat pressure of 4 bar :*

*(a) the required area of throat in cm², *

(b) the required area of outlet in cm², and

(c) the overall efficiency of the nozzle, based on the heat drop between the actual inlet pressure and temperature and the outlet pressure.

Let suffixes, 1, 2 and 3 represent conditions at inlet, throat and exit (outlet) of the nozzle respectively as shown in fig. 8-11.

(1) Referring to fig. 8-11.

For throat :

Throat pressure, $p_2 = 4$ bar (given) and velocity of steam at entry,

$V_1 = 75$ m/sec. (given).

Enthalpy drop from inlet to throat,

$H_t = H_1 - H_2 = 110$ kJ/kg

and dryness fraction,

$x_2 = 0{\cdot}985$ (from $H - \Phi$ chart).

Now, $\dfrac{(V_2)^2 - (V_1)^2}{2 \times 1{,}000} = H$

where V_1 = velocity of steam at inlet,

V_2 = velocity of steam at throat, and

H = enthalpy drop from entry to throat.

i.e., $\dfrac{(V_2)^2 - (75)^2}{2 \times 1000} = 110$.. From which, velocity at throat, $V_2 = 475$ m/sec.

At 4 bar, $v_{s2} = 0{\cdot}4625$ m^3/kg (from steam tables).

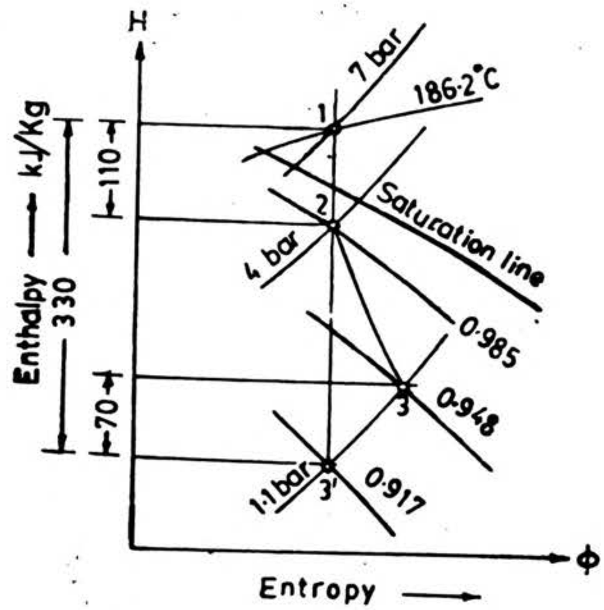


Fig. 8-11. H. φ diagram.

Specific volume at throat,

$v_2 = x_2 \times v_{s2} = 0{\cdot}985 \times 0{\cdot}4625$ m³.kg.

For mass continuity, $m = \dfrac{A_2 V_2}{v_2}$ i.e.,

$$1{\cdot}8 = \frac{A_2 \times 475}{(0{\cdot}985 \times 0{\cdot}4625) \times 10^4}$$

$\therefore$ Area of throat,

$$A_2 = \frac{1{\cdot}8 \times 0{\cdot}985 \times 0{\cdot}4625 \times 10^4}{475}$$

$$= \mathbf{17.3\ cm^2}$$

(b) *For exit :*

Isentropic enthalpy drop from inlet to exit

$= H_1 - H_3 = 330$ kJ/kg.
(from $H - \Phi$ chart).

$\therefore$ Actual (useful) enthalpy drop ($H_1 - H_3$) after allowing for friction in the divergent part $= 330 - 70 = 260$ kJ/kg.

Now, $\dfrac{(V_3)^2 - (V_1)^2}{2 \times 1{,}000}$ = Enthalpy drop, H i.e., $\dfrac{(V_3)^2 - (75)^2}{2 \times 1{,}000} = 260$

From which, velocity of steam at exit from the nozzle, $V_3 = 725$ m/sec.

Reheated condition at exit or dryness fraction at exit, $x_3 = 0.948$ (from $H - \Phi$ chart).

At 1.1 bar, $v_{s3} = 1{\cdot}5495$ m³/kg (from steam tables).

Specific volume at exit, $v_3 = x_3 \times v_{s3} = 0.948 \times 1{\cdot}5495$ m³/kg.

For mass continuity, $m = \dfrac{A_3 V_3}{v_3}$ i.e., $1{\cdot}8 = \dfrac{A_3 \times 725}{0{\cdot}948 \times 1{\cdot}5495 \times 10^4}$

$\therefore$ Area of outlet (exit), $A_3 = \dfrac{1{\cdot}8 \times 0{\cdot}948 \times 1{\cdot}5495 \times 10^4}{725} = \mathbf{36.47\ cm^2}$

(c) Overall efficiency of nozzle = $\dfrac{\text{Actual enthalpy drop}}{\text{Insentropicenthalpy drop}} = \dfrac{260}{330} = 0{\cdot}788$, i.e., **78.8/%**

Problem – 13 : *The nozzles in the stage of an impulse turbine receive steam at 17 bar with 80°C superheat and the pressure in the wheel is 9.5 bar. If there are 16 nozzles, find the cross-sectional area at exit of each nozzle for the total discharge of 280 kg per minute. Assume nozzle efficiency of 88 per cent.*

If the steam had a velocity of 120 m/sec. at entry to the nozzles, by how much would the discharge be increased ?

Let suffixes 1 and 2 represent conditions at inlet and exit of the nozzle.

As the steam is initially superheated,

critical or throat pressure $= 0{\cdot}546 \times p_1 = 0{\cdot}546 \times 17 = 9{\cdot}28$ bar.

Since the exit pressure (p_2) is greater than the critical pressure, the nozzles are convergent.

From Mollier chart, isentropic enthalpy drop from inlet to exit,

$H_e = H_1 - H_2 = 131{\cdot}0$ kJ/kg.

$\therefore$ Actual enthalpy drop after allowing for friction $0{\cdot}88 \times 131 = 115{\cdot}3$ kJ/kg.

Neglecting the velocity of approach :

Velocity of steam at exit, $V_2 = 44{\cdot}72\sqrt{115{\cdot}3} = 480{\cdot}0$ m/sec.

From Mollier chart, steam at the end of expansion (at 9·5 bar) has a temperature of 222°C (taking friction into account).

At 9·5 bar, $v_{s2} = 0{\cdot}2042$ m^3/kg, $t_{s2} = 177{\cdot}69$°C (from steam tables).

$\therefore$ Specific volume of steam at 9·5 bar and 222°C,

$$v_2 = 0{\cdot}2042 \times \left(\frac{222 + 273}{177{\cdot}69 + 273}\right) = 0{\cdot}224 \text{ m}^3\text{/kg}$$

For mass continuity, $m = \dfrac{A_2 V_2}{v_2}$

$\therefore$ Area at exit for one nozzle, $A_2 = \dfrac{m \times v_2}{V_2 \times \text{No. of nozzle}}$

$$= \frac{280 \times 0{\cdot}224 \times 10^4}{60 \times 480{\cdot}0 \times 16} = \mathbf{13{\cdot}6 \text{ cm}^2}$$

Considering velocity at inlet or velocity of approach (V_1) of 120 m/sec. :

Now, $\dfrac{(V_2')^2 - (V_1)^2}{2 \times 1{,}000}$ = actual enthalpy drop, H

i.e., $\dfrac{(V_2')^2 - (120)^2}{2 \times 1{,}000} = 115{\cdot}3$ $\therefore (V_2')^2 = 2{,}45{,}000$

$\therefore V_2' = 495$ m/sec. (Velocity of steam at exit)

Since there is no increase in specific volume, the discharge is directly proportional to velocity.

$\therefore$ Increase in discharge $= \dfrac{V_2' - V_2}{V_2} \times 100 = \dfrac{495 - 480{\cdot}0}{480{\cdot}0} \times = \mathbf{3{\cdot}13\%}$.

Problem – 14 : *Steam expands through a nozzle under supersaturated adiabatic conditions, from an initial pressure of 8 bar and a temperature of 210°C, to a final pressure of 2 bar, Determine :*

(i) the final condition of the steam, (ii) the exit velocity of the steam, (iii) the degree of undercooling, and (iv) the degree of supersaturation at the end of expansion.

Compare the mass flow through the above nozzle with one in which the expansion takes place under conditions of thermal equilibrium.

For the supersaturated state you may use the following relationship :

$$v = \frac{0{\cdot}233\,(H - 1940)}{p}; \quad pv^{1\cdot3} = \text{constant;} \quad \text{and} \quad \frac{p}{(T)^{\frac{13}{3}}} = \text{constant.}$$

where, v is the specific volume in m^3/kg, H is the enthalpy in kJ/kg, p is the pressure in kPa and T is the absolute temperature.

Take kp of superheated steam as 2·3 kJ/kg K.

Expansion under supersaturated conditions (metastable flow) :

From Steam tables :

p bar	t_s °C	v_s m^3/kg	h kJ/kg	L kJ/kg	H kJ/kg	Φ_w kJ/kgK	Φ_s kJ/kg K
8	170·43	0·2404	721·11	2048	2769·1	2·0411	6·6628
2	120·23	0·8857	504·7	2201·9	2706·7	1·5301	7·1271

(i) Let suffixes 1 and 2 represent the initial and final conditions respectively.

From steam tables, steam enthalpy, $H_1 = 2769·1 + 2·3\,(210 - 170·43) = 2,860$ kJ/kg.

$$v_1 = \frac{0·233\,(H_1 - 1940)}{p_1} = \frac{0·233\,(2,860 - 1940)}{800} = 0·27 \text{ m}^3\text{/kg}.$$

From given equation $p_1 v_1^{1.3} = p_2 v_2^{1.3}$

$$\therefore\; v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{1·3}} = 0·27 \left(\frac{8}{2}\right)^{\frac{1}{1·3}} = 0·7843 \text{ m}^3\text{/kg (specific volume at exit).}$$

$$\text{But, } v_2 = \frac{0·233\,(H_2 - 1,940)}{p_2} \qquad \text{i.e., } 0·7843 = \frac{0·233\,(H_2 - 1,940)}{200}$$

From which, steam enthalpy, $H_2 = 2,613$ kJ/kg.

At 2 bar, $h = 504·7$ kJ/kg, $L = 2,201·9$ kJ/kg (from steam tables).

$$\text{Dryness fraction at exit, } x_2 = \frac{H_2 - h_2}{L_2} = \frac{2,613 - 504·7}{2,201·9} = \mathbf{0·9575}$$

(ii) Actual enthalpy drop from inlet to exit,

$H_e = H_1 - H_2 = 2,860 - 2,613 = 247$ kJ/kg

Then, velocity at exit, $V_2 = 44·72\sqrt{H_1 - H_2} = 44·72\sqrt{247}$ = **703 m/sec.**

(iii) Now, $T_1 = 273 + 210 = 483$ K.

$$\text{From given equation } \frac{p_1}{(T_1)^{\frac{13}{3}}} = \frac{p_2}{(T_2)^{\frac{13}{3}}} \quad \text{i.e., } \frac{8}{(483)^{\frac{13}{3}}} = \frac{2}{(T_2)^{\frac{13}{3}}}$$

From which temperature after supersaturation, $T_2 = 350·8$ K,

then, $t_2 = 350·8 - 273 = 77·8$°C (actual temperature).

Saturation temperature at 2 bar = 120·23°C (from steam tables)

Degree of undercooling is the difference between the normal saturation temperature corresponding to the pressure and the actual temperature.

$\therefore$ Degree of undercooling = 120·23 − 77·8 = **42.43°C**

(iv) Saturation pressure corresponding to 77·8°C = 0·43 bar (from steam tables by arithmetical interpolation).

Degree of supersaturation

$$= \frac{\text{Pressure after supersaturation}}{\text{Saturation pressure corresponding to the undercooled temp.}} = \frac{2}{0·43} = \mathbf{4·65}$$

Expansion under supersaturated condition :

For mass continuity, $m_2 = \dfrac{A_2 V_2}{v_2}$ (where A_2 is the exit area)

$$= \frac{A_2 \times 703}{0.7843} = 896\ A_2 \text{ kg/sec.}$$

Expansion under conditions of the thermal equilibrium (stable flow) :

For adiabatic flow,

Entropy before expansion, Φ_1 = Entropy after expansion, Φ_2

$$\Phi_{s1} + k_p \log_e \left(\frac{T_{sup1}}{T_{sat1}}\right) = \Phi_{w2} + x_2 (\Phi_{s2} - \Phi_{w2})$$

$$\text{i.e., } 6.6628 + 2.3 \log_e \frac{210 + 273}{170.43 + 273} = 1.5301 + x_2 (7.1271 - 1.5301)$$

$$6.6628 + 2.3 \times 0.0855 = 1.5301 + x_2 \times 5.597$$

$$\therefore x_2 = \frac{6.8595 - 1.5301}{5.597} = 0.952 \quad \text{(dryness fraction at exit)}$$

Steam enthalpy, $H_2 = h_2 + x_2 L_2 = 504.7 + 0.952 \times 2{,}201.9 = 2{,}603$ kJ/kg.

Steam enthalpy, $H_1 = 2{,}769.1 + 2.3 (210 - 170.43) = 2{,}860$ kJ/kg.

Insentropic enthalpy drop from inlet to exit, $H_1 - H_2 = 2{,}860 - 2{,}603 = 257$ kJ/kg

Velocity at exit, $V_2 = 44.72 \sqrt{H_1 - H_2} = 44.72 \sqrt{257} = 717$ m/sec.

From steam tables at 2 bar, $v_{s2} = 0.8857$ m^3/kg.

$v_2 = x_2 \times v_{s2} = 0.952 \times 0.8857$ m^3/kg

For mass community, $m_2 = \frac{A_2 V_2}{v_2} = \frac{A_2 V_2}{x_2 \times v_{s2}}$ (where A_2 is the exit area)

$$= \frac{A_2 \times 717}{0.952 \times 0.8857} = 850\ A_2 \text{ kg/sec.}$$

Increase of discharge due to supersaturated flow = $896\ A_2 - 850\ A_2 = 46\ A_2$ kg/sec.

$\therefore$ Increase in discharge due to supersaturated flow $= \frac{46\ A_2}{850\ A_2} \times 100 = $ **5·41%**

Thus, supersaturated flow increases the discharge by 5·41%.

It should be noted that the effect of supersaturated flow (compared with flow under the conditions of thermal equilibrium) is :

Reduction in the enthalpy drop, which will cause corresponding reduction in velocity,

the dryness fraction at exit is higher, and

the rate of discharge is increased.

8.6 Steam Injector

A steam injector utilises the kinetic energy of a steam jet for increasing the pressure and velocity of corresponding quantity of water; they are frequently used for forcing the water into steam boilers under pressure. The action of the injector is illustrated diagrammatically in fig. 8-12.

Steam from the boiler is supplied to the convergent nozzle A, and this steam issuing there from with a high velocity into the mixing cone, is condensed by the cold water flowing from the feed water tank E.

This tank may be either above or below the level of the injector. Owing to conversion of enthalpy of evaporation (latent heat) of steam to kinetic energy, the mixture of water and condensate has a high velocity at B. Thus, mixture issuing from the nozzle B, then flows through the divergent delivery nozzle or diffuser C, in which its kinetic energy is

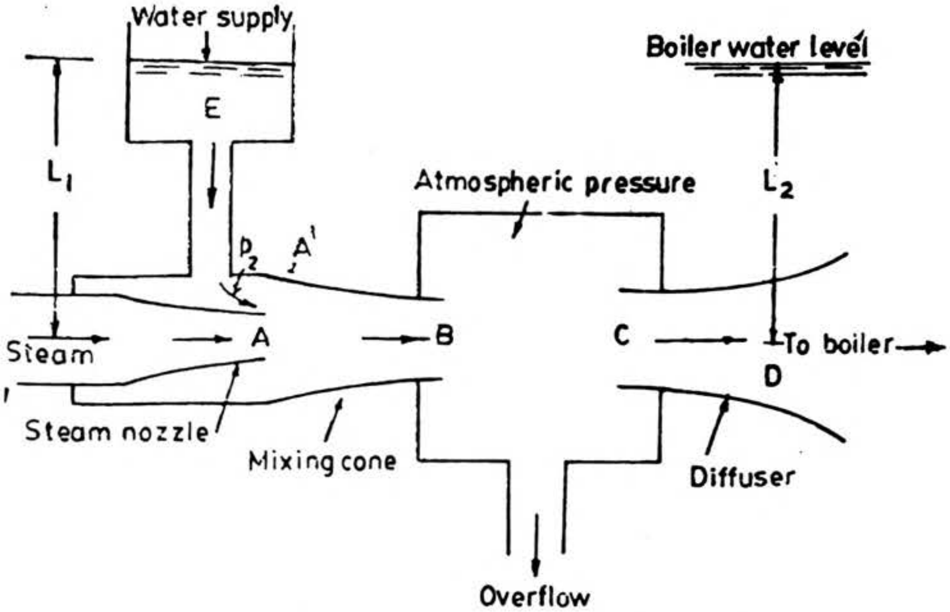


Fig. 8-12. Principle of steam injector.

reduced and converted to pressure energy, until on leaving at D, this pressure energy is sufficient to overcome boiler pressure and to lift the water through the height L_2, and the water enters the boiler. The pressure of water at D must be about 25 per cent higher than the boiler pressure in order to overcome all resistances. Because of the gap between the nozzles B and C, provided for excess water which may overflow during the starting of the injector, the pressure in the gap is nearly atmospheric.

Let M_w = Mass of the water per kg of steam entering at A' in kg/sec.,

V_s = Velocity of steam leaving the nozzle at A in m/sec,

V_w = Velocity of water entering at A' in m/sec., and

V_m = Velocity of mixture leaving nozzle at B in m/sec.

Applying the principle of conversion of momentum to the mixing of the steam jet and water supply, per kilogram of steam supplied to the nozzle. Then,

[Momentum of steam entering combining nozzle] + [Momentum of water entering combining nozzle] = [Momentum of mixture leaving combining nozzle]

H.E. 15

i.e., $1 \times V_s + M_w V_w = (1 + M_w) V_m$

or $V_s + M_w V_w = (1 + M_w) V_m$

If the water level in the tank E is below the level of the injector, then

$V_s - M_w V_w = (1 + M_w) V_m$

$$\text{Hence, } M_w = \frac{V_s - V_m}{V_m \pm V_w} \quad \text{... (8.13)}$$

according to whether the water supply level is below or above the injector level. This formula gives the amount of water injected per kilogram of steam if the velocities are known.

The velocity of steam V_s from the nozzle may be found by assuming that the steam expands isentropically (frictionless adiabatic) from the initial condition to the back pressure p_2. Using enthalpy-entropy (Mollier) chart or by calculation, the enthalpy drop, H in the nozzle can be found. Then, $V_s = 44{\cdot}72\sqrt{H}$

The velocity of water, V_w entering the annular space A' will be given by the equation, $V_w = \sqrt{2g.L_1}$

To find the velocity of mixture, V_m leaving nozzle B :

Let p_m = pressure at B in kPa, and

w = density of warm water at B in kg/m^3.

Due to the presence of the gap between nozzles B and C, the pressure p_m of the water in the throat of B may be taken as atmospheric, say 101·33 kPa, and as the water is warm its density may be taken as 995 kg/m^3.

Then, the total energy, kJ per kilogram of water at $B = \frac{p_m}{w} + \frac{V_m^2}{2,000}$

This energy must be enough to lift the water through a height L_2 metres at the delivery end and inject it into the boiler. The final pressure on leaving at D must therefore be some what greater than this height (L_2) plus the boiler pressure.

At D the pressure energy of 1 kg of water is $\frac{p}{w} + \frac{g}{1,000}L_2$

where p is the absolute pressure of steam in the boiler in kPa.

Then, $\frac{p_m}{w} + \frac{V_m^2}{2,000} = \frac{p}{w} + \frac{g}{1,000}L_2 + \frac{V^2}{2,000}$

The water ultimately comes to rest in the boiler and the kinetic energy $\frac{V^2}{2,000}$ may be taken equal to the pressure energy due to addition of, say, 9·3 metres to the lift L_2, which approximately corresponds to V = 13·5 m/sec. and to an addition of about 90 kPa to the boiler pressure.

Hence, $\frac{V_m^2}{2,000} + \frac{101{\cdot}33}{955} = \frac{p}{995} + \frac{g}{1,000} \times L_2 + \frac{g}{1,000} \times 9{\cdot}3$

$$\therefore\ V_m = \sqrt{2,000\left[\frac{(p - 101{\cdot}33)}{995} + \frac{g}{1,000} \times L_2 + \frac{g}{1,000} \times 9{\cdot}3\right]} \qquad (8.14)$$

If the actual delivery is to be M kilogram of water per sec. and M_s kg of condensed steam per second,

Then, $M + M_s = M\left(1 + \frac{1}{M_w}\right)$

Let a_b = area of throat of B in cm^2, and d_b = diameter of throat of B in cm,

Then, $M\left(1 + \frac{1}{M_w}\right) = \frac{a_b V_m}{10^4\, v}$ where $v = \frac{1}{w} = \frac{1}{995}$

$$\therefore\ a_b = \frac{10^4\, M\left(1 + \frac{1}{M_w}\right)}{995\, V_m} \quad \text{and } d_b = \sqrt{\frac{a_b \times 4}{\pi}} \qquad \ldots(8.15)$$

Let a_a = area of throat of A in cm^2,

d_a = diameter of throat of A in cm, and

v_s = volume of wet steam after expansion in the nozzle A in m^3/kg, then, $M_s = \frac{a_a V_s}{10^4 v_s} = \frac{M}{M_w}$

$$\therefore a_a = \frac{10^4 M \times v_s}{M_w \times V_s} \quad \text{and } d_a = \sqrt{\frac{a_a \times 4}{\pi}} \quad ....(8.16)$$

The heat balance per kilogram of steam may be determined as follows :

Let H_s = enthalpy per kilogram of steam entering the injector,

h_w = enthalpy per kilogram of water supplied to injector,

h_m = enthalpy per kilogram of water leaving at B.

Then,

Heat supplied in steam + Heat supplied in water ± Kinetic energy of water at A'

= Heat in mixture at B + Kinetic energy of mixture at B

$$\text{i.e., } 1 \times H_s + M_w h_w \pm \frac{M_w V_w^2}{2{,}000} = (M_w + 1)h_m + \frac{(M_w + 1)}{2{,}000} V_m^2 \quad ...(8.17)$$

according to whether, the water supply level is below the injector level.

From this equation the temperature of the mixture may be found by taking h_m equal to temperature of mixture multiplied by specific heat of water, i.e., 4·187 kJ/kg K.

Problem – 15 : *An injector is required to deliver 100 kg of water per minute from a tank whose constant water level is 1·2 metres below the level of the injector, into a boiler in which the steam pressure is 14 bar. The water level in the boiler is 1·5 metres above the level of the injector. The steam for the injector is to be taken from the same boiler and it is assumed as dry saturated. The temperature of the water in the supply tank is 15°C. Find : (a) the mass of water taken from the supply tank per kg of steam, (b) the diameter of the throat of the mixing nozzle (c) the diameter of the throat of steam nozzle, and (d) the temperature of water leaving the injector. Neglect the radiation losses.*

(a) Referring to fig. 8-12, throat pressure, $p_1 = 0{\cdot}578 \times 14 = 8{\cdot}08$ bar.

At 14 bar, H_s = 2,790 kJ/kg and Φ_s = 6·4693 (from steam tables)

At 8·08 bar, (by arithmetical interpolation),

Φ_{w1} = 2·05 , Φ_{s1} = 6·66, h_1 = 723 kJ/kg.

L_1 = 2,047 kJ/kg, v_{s1} = 0·237 m^3/kg (from steam tables).

$\Phi_s = \Phi_{w1} + x_1 (\Phi_{s1} - \Phi_{w1})$

$6{\cdot}4693 = 2{\cdot}05 + x_1 (6{\cdot}66 - 2{\cdot}05) \qquad \therefore x_1 = 0{\cdot}96$

$H_1 = h_1 + x_1 L_1 = 723 + 0{\cdot}96 \times 2{,}047 = 2{,}688$ kJ/kg

$\therefore V_s = 44{\cdot}72 \sqrt{2{,}790 - 2{,}688} = 44{\cdot}72 \sqrt{102} = 452$ m/sec. (velocity of steam)

Now, $V_w = \sqrt{2g L_1}$

$= \sqrt{2 \times 9{\cdot}81 \times 1{\cdot}2}$

$= \sqrt{23{\cdot}55} = 4{\cdot}85$ m/sec. (velocity of water entering injector)

Using eqn. (8.14), velocity of mixture leaving nozzle,

$$V_m = \sqrt{2{,}000 \left[\frac{(p - 101{\cdot}33)}{955} + \frac{g}{1{,}000} \times L_2 + \frac{g}{1{,}000} \times 9{\cdot}3\right]}$$

$$V_m = \sqrt{2{,}000\left[\frac{(1{,}400 - 101{\cdot}33)}{995} + \frac{9{\cdot}81}{1{,}000} \times 1{\cdot}5 + \frac{9{\cdot}81}{1{,}000} \times 9{\cdot}3\right]}$$

$$= \sqrt{2{,}822} = 53{\cdot}1 \text{ m/sec.}$$

Using eqn. (8.13), mass of water,

$$M_w = \frac{V_s - V_m}{V_m + V_w} = \frac{452 - 53{\cdot}1}{53{\cdot}1 + 4{\cdot}85} = \textbf{6·88 kg/kg of steam.}$$

(b) Using eqn. (8.15), area of the throat of mixing nozzle,

$$a_b = 10^4 M \frac{\left(1 + \frac{1}{M_w}\right)}{995\ V_m} = \frac{10^4 \times \frac{100}{60}\left(1 + \frac{1}{6{\cdot}88}\right)}{995 \times 53{\cdot}1} = 0{\cdot}361 \text{ cm}^2$$

$$\therefore\ d_b = \sqrt{\frac{0{\cdot}361 \times 4}{\pi}} = \sqrt{0{\cdot}46} = \textbf{0·678 cm} \text{ (dia. of throat of mixing nozzle)}$$

(c) Specific volume at throat of steam nozzle, $v_s = x_1 \times v_{s1} = 0{\cdot}96 \times 0{\cdot}237$ $= 0{\cdot}228 \text{ m}^3\text{/kg}$.

Using eqn. (8.16), area of the throat of steam nozzle,

$$a_a = \frac{10^4 M \times v_s}{M_w \times V_s} = \frac{10^4 \times \frac{100}{60} \times 0{\cdot}228}{6{\cdot}88 \times 452} = 1{\cdot}22 \text{ cm}^2$$

$$\therefore\ d_a = \sqrt{\frac{1{\cdot}22 \times 4}{\pi}} = \sqrt{1{\cdot}553} = \textbf{1·246 cm} \text{ (dia. of throat of steam nozzle).}$$

(d) Using eqn. (8.17),

$$1 \times H_s + M_w h_w - \frac{M_w V_w^2}{2{,}000} = (M_w + 1) h_m + \frac{(M_w + 1)}{2{,}000} V_m^2$$

$$1 \times 2790 + 6{\cdot}88 \times 4{\cdot}187 \times 15 - \frac{6{\cdot}88\,(4{\cdot}85)^2}{2{,}000} = (6{\cdot}88 + 1)\, h_m + \frac{(6{\cdot}88 + 1)\,(53{\cdot}1)^2}{2{,}000}$$

$$2{,}790 + 432{\cdot}1 - 0{\cdot}081 = 7{\cdot}88\ h_m + 11{\cdot}1$$

$$\therefore\ h_m = \frac{3{,}210{\cdot}9}{7{\cdot}88} = 407{\cdot}5 \text{ kJ/kg}$$

Let t_m be the temperature of water leaving the injector. Then,

$$407{\cdot}5 = 4{\cdot}187\ (t_m - 0)$$

$\therefore\ t_m =$ **97·3°C** (Temp. of water leaving the injector).

Tutorial – 8

1 Delete the phrase which is not applicable in the following statements :

(i) When the steam flows through a correctly shaped nozzle, its velocity and specific volume both will decrease/increase.

(ii) The flow of steam in the convergent portion of the steam nozzle is subsonic/supersonic.

(iii) Friction is of negligible magnitude between entry and throat/between throat and exit of the nozzle.

(iv) In a steam nozzle, as the pressure of steam decreases, velocity of steam decreases/increases.

(v) The length of the converging part of a convergent-divergent steam nozzle is short/long as compared with the length of its diverging part.

(vi) The effect of friction is to reduce/increase the available enthalpy drop for conversion into kinetic energy.

(vii) The nozzle critical pressure ratio for initially dry saturated steam is 0·578/0·582

[Delete : (i) decrease, (ii) supersonic, (iii) between throat and exit, (iv) decreases, (v) long, (vi) to increase, (vii) 0·582]

2 Fill in the blanks in the following statements :

(i) During an adiabatic process with friction, ______ does not remain constant.

(ii) The smallest section of the convergent-divergent nozzle is known as the ______.

(iii) Discharge through a nozzle will be ______ when the ratio of pressure at throat to pressure at entry reaches the ______ value.

(iv) The effect of friction in nozzles is to reduce the ______.

(v) ______ ratio depends on the value of index, n for isentropic expansion of steam through the nozzle.

(vi) The value of index n for isentropic expansion of initially superheated steam is ______.

(vii) The critical pressure ratio, $\frac{p_2}{p_1}$ = ______.

(viii) The pressure of steam at which the area of the nozzle is minimum and the discharge per unit area is maximum is termed as ______.

(ix) A nozzle which first converges to throat and then diverges is termed as ______ nozzle.

(x) The flow of steam in the convergent portion of the nozzle is ______.

(xi) Friction reduces the enthalpy drop in a steam nozzle by ______ per cent.

(xii) Friction is of ______ magnitude between entry and throat as most of the friction occurs between the throat and exit of the nozzle.

(xiii) The supersaturated flow is also called the ______ flow.

(xiv) A steam injector utilises the kinetic energy of steam jet for ______ the pressure and velocity of corresponding quantity of water.

(xv) Steam injectors are frequently used for forcing the ______ into steam boilers under pressure.

[(i) entropy, (ii) throat, (iii) maximum, critical, (iv) enthalpy drop, (v) critical pressure, (vi) 1·3, (vii) $\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$, (viii) critical pressure, (ix) convergent-divergent nozzle, (x) subsonic, (xi) 4 to 15, (xii) negligible, (xiii) metastable, (xiv) increasing, (xv) feed water]

3 Indicate the correct answer by selecting correct phrases from each of the following :

(i) The value of index n for isentropic expansion of superheated steam through the nozzle is

(a) 1·4, (b) 1·3, (c) 1·135, (d) 1·113

(ii) Critical pressure ratio $\frac{p_2}{p_1}$ for steam nozzle in terms of index n for isentropic expansion is given by :

(a) $\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$ (b) $\left(\frac{n+1}{2}\right)^{\frac{n-1}{n}}$ (c) $\left(\frac{n}{n-1}\right)^{\frac{2}{n+1}}$ (d) $\left(\frac{n-1}{n}\right)^{\frac{n+1}{2}}$

(iii) For a convergent-divergent nozzle, the mass flow rate remains constant, if the ratio of exit pressure and inlet pressure is

(a) more than critical pressure ratio, (b) less than critical pressure ratio, (c) unity, (d) infinity.

(iv) For a convergent-divergent nozzle, critical pressure ratio occurs when

(a) nozzle efficiency is maximum,

(b) friction is zero,

(c) decrease in ratio of exit pressure and inlet pressure does not increase steam flow rate.

(v) The kinetic energy lost in friction is transformed into heat which tends to

(a) dry or superheat the steam,

(b) cool or condense the steam,

(c) increase the pressure of the steam,

(d) decrease the specific volume of steam.

(vi) The velocity of steam in the divergent portion of a convergent-divergent nozzle is

(a) subsonic, (b) sonic, (c) supersonic.

(vii) Semi-cone angle of the divergent part of the convergent-divergent steam nozzle is of the order of

(a) 3° to 10°, (b) 13° to 20°, (c) 23° to 30°, (d) 33° to 40°.

[(i) b, (ii) a, (iii) b, (iv) c, (v) a, (vi) c, (vii) a]

4 What is the function of a steam nozzle ? Mention the types of nozzles, you know.

Steam flows through a properly designed nozzle and the pressure drops from 12 bar to 0·15 bar. Assuming frictionless adiabatic flow, calculate the dryness fraction and velocity of steam as it leaves the nozzle, when the steam at the higher pressure is (a) dry saturated, and (b) superheated by 60°C.

[(a) 0·7952 dry, 1,159 m/sec; (b) 0.834 dry, 1,197·5 m/sec.]

5 What do you understand by the term critical pressure as applied to steam nozzles ?

Dry saturated steam at pressure of 700 kPa is expanded in a convergent-divergent nozzle to 100 kPa. The mass of steam passing through the nozzle is 270 kg per hour. Assuming the flow to be frictionless adiabatic, determine the throat and exit diameters .[0·966 cm; 1·332 cm]

6 Dry saturated steam at a pressure of 12 bar is supplied to a convergent-divergent nozzle and is delivered at a pressure of 0·15 bar. Determine the diameters at the throat and exit of the nozzle if the delivery of steam is 18 kg per minute. Assume frictionless adiabatic flow. [1·486 cm; 5·125 cm]

7 Calculate the diameters at the throat and exit of a nozzle which is to discharge 150 kg of steam per hour. The steam supply to the nozzle is dry saturated at a pressure of 5 bar and the pressure at exit is 0·6 bar. Assume index of expansion for steam as 1·135. Neglect effect of friction

[0·849 cm; 1·24 cm]

8 Explain the term "critical pressure" as applied to steam nozzles. Why are the turbine nozzles made divergent after the throat ?

Steam is supplied at a dryness fraction of 0·95 and pressure of 14 bar to a convergent-divergent nozzle and expands down to a back pressure of 0·4 bar. The outlet area of the nozzle is 10 cm^2. Assuming frictionless adiabtic flow through the nozzle, determine :

(a) the steam flow in kg per hour, and

(b) the diameter of the nozzle at throat. [(a) 1,196.3 kg; (b) 1·438 cm]

9 A convergent-divergent nozzle is required to discharge 300 kg of steam per hour. The nozzle is supplied with steam at a pressure of 10 bar and 90 per cent dry and discharges against a back pressure of 0·3 bar. Assuming frictionless adiabtic flow, determine the throat and exit diameters.

[0· 842 cm; 2· 047 cm]

10. Steam is supplied at a dryness fraction of 0.97 and 10 bar to a convengent-divergent nozzle and expands down to a back pressure of 0.3 bar. The throat area is 5 cm^2. Assuming frictionless diabetic flow, determine:

(a) The steam flow in kg. per minute, and

(b) The nozzle outlet area. [(a) 43.37 kg.; (b) 29.28 cm^2]

11 A nozzle is required to discharge 8 kg of steam per minute. The nozzle is supplied with steam at 11 bar and 200°C and discharges against a back pressure of 0·7 bar. Assuming frictionless adiabtic flow, determine:

(a) the throat area, (b) the exit velocity and (c) the exit area. Take K_p of superheated steam as 2·1 kJ/kgK. [(a) 0·848 cm^2; (b) 963·6 m/sec; (c)2·875 cm^2]

12 Steam expands from 13 bar and 10°C superheat to 1·4 bar in a convergent-divergent nozzle. The mass of steam passing through the nozzle is 1,800 kg per hour. Assuming the flow to be frictionless adiabatic, determine the condition of steam and the diameters of the nozzle at the throat and exit. Assume that for maximum discharge the throat pressure is 7 bar. Take k_p of superheated steam as 2·1 kJ/kg K.

[(a) 0·964 dry, 1·854 cm; 0·879 dry, 2·802 cm]

13 Show that the maximum discharge of steam per unit area, through a nozzle, takes place, when the ratio of the steam pressure at the throat to the inlet steam pressure is

$$\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$ where n is the index of adiabatic expansion.

Calculate the discharge in kg/m^2 at the throat of a nozzle, supplied with dry saturated steam at 700 kPa. [1020·9 kg/m^2]

14 From first principles, prove that maximum discharge in a steam nozzle per unit area at the throat is given by

$$M_{max} = \sqrt{1{,}000\, n\, \frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}}$$

where, p_1 = Initial pressure of steam in kPa,

v_1 = Volume of steam in m^3/kg at the initial pressure, and

n = Index of expansion.

15 A convergent-divergent nozzle is supplied with dry saturated steam at 1,200 kPa. If the divergent portion of the nozzle is 11 cm long and the throat diameter is 1·2 cm, determine the semi-vertical angle of the cone so that steam may leave the nozzle at 15 kPa. Assume frictionless adiabatic flow. [7°–33′]

16 (a) Draw the "discharge versus ratio of pressures at outlet to inlet" curve for a convergent steam nozzle. Discuss the physical significance of critical pressure ratio.

(b) Dry saturated steam at a pressure of 8 bar enters a convergent-divergent nozzle and leaves it at a pressure of 1·5 bar. If the flow is isentropic and the corresponding expansion index is 1·135, find the ratio of cross-sectional areas at exit and throat for maximum discharge. [1·592]

17 Dry saturated steam at 1·8 bar is allowed to discharge through a long convergent nozzle into the atmosphere. Taking atmospheric pressure as 1 bar, calculate the mass of steam which should be discharged per second if the exit diameter of the nozzle is 1·2 cm. Neglect friction in the nozzle.

If the mass of steam actually discharged be 94% of the calculated mass, estimate the percentage of enthalpy drop which is wasted in friction. [0·0309 kg/sec; 11·7%]

18 Explain the term "Nozzle efficiency".

A convergent-divergent nozzle is to pas 4,000 kg of steam per hour. Initially the steam is 0·98 dry at 21 bar and finally it is at 0·7 bar. Assuming that the friction loss in the divergent part is 18 per cent of the total isentropic enthalpy drop, determine the required areas of the throat and outlet.

[3·688 cm^2; 23·453 cm^2]

19 A convergent-divergent nozzle is required to pass 360 kg of steam per hour with a pressure drop from 13 bar to 0·15 bar. The steam at the higher pressure is dry saturated. Assuming that the frictional resistance occurs only between throat and exit and is equivalent to 13 per cent of the total isentropic enthalpy drop, determine the diameters at the throat and exit. [0·827 cm; 3·114 cm]

20 A convergent-divergent nozzle is to be designed to discharge 0·075 kg of steam per second into a vessel in which the pressure is 1·4 bar, when nozzle is supplied with steam at 7 bar and also superheated to 200°C. Find the throat and exit diameters on the assumption that the friction loss in the divergent part is 10% of the total insentropic enthalpy drop. Take k_p of superheated steam as 2·3 kJ/kg K.

[0·971 cm; 1·245 cm]

21 Steam at a pressure of 10 bar and a dryness friction of 0·97 is to be discharged through a convergent-divergent nozzle to a back pressure of 0·15 bar. The mass flow rate through the nozzle is at a rate of 8 kg/kW-hr. If the turbine develops 150 kW, determine : (i) the throat pressure, (ii) the number of nozzle required, the diameter of nozzle at throat being 6·5 mm, and (iii) suitable exit diameter of the nozzle, assuming that 10% of the overall isentropic enthalpy drop reheats the steam in the divergent portion of the nozzle.

[(i) 5·82 bar; (ii) 7; (iii) 2·175 cm]

22 (a) State what is meant by expansion of steam (i) under stable adiabatic conditions, and (ii) under conditions of supersaturation.

(b) Discuss the causes of supersaturated flow in nozzles.

(c) Explain what is meant by the supersaturated expansion of steam and give some idea of the limits within which this condition is possible.

(d) Steam is expanded in a nozzle from an initial pressure of 10 bar and a temperature 200°C, to a final pressure of 2·5 bar. The expansion is supersaturated.

Determine : (a) the final condition of steam, (b) the exit velocity of steam, (c) the degree of undercooling, (d) the degree of supersaturation, (e) the actual enthalpy drop, and (f) the insentropic enthalpy drop.

Compare the mass flow through the above nozzle with one in which expansion takes place under conditions of the thermal equilibrium.

For supersaturated conditions use the following relationship :

$$V = \frac{0{\cdot}233\,(H - 1{,}940)}{p}; \quad pv^{1\cdot3} = \text{constant}; \quad \text{and} \quad \frac{p}{(T)^{\frac{13}{3}}} = \text{constant}.$$

where v is the specific volume in m^3/kg, H is the steam enthalpy in kJ per kg, p is the pressure in kPa, and T is the absolute temperature.

Take kp of superheated steam as 2·3 kJ/kg K.

[(a) 0·938 dry; (b) 696·77 m/sec; (c) 56·69°C
(d) 7·764, (e) 242·76 kJ/kg; (f) 254·71 kJ/kg; 9·48%]

23 Describe with a neat sketch the working of a steam injector used for a locomotive boiler.

Derive the formula for the amount of water injected into the boiler per kilogram of steam.

24 An injector is required to deliver 100 kg of water per minute from a tank whose constant water level is

1·2 metres below the level of the injector into a boiler in which the steam pressure is 4 bar. The water level in the boiler is 1·5 metres above the level of the injector. The steam for the injector is to be taken from the same boiler and it is to be assumed as dry saturated. The temperature of the water in the supply tank is 15°C. Find : (a) the mass of water taken from the supply tank per kg of steam, (b) the diameter of the throat of the mixing nozzle, (c) the diameter of the throat of the steam nozzle, and (d) the temperature of the water leaving the injector. Neglect the radiation losses.

[(a) 12·395 kg; (b) 0·9 cm ; (c) 1·708 cm; (d) 62·6°C]

25 Write short notes on the following, illustrating your answers with neat sketches wherever necessary :

(a) Types of steam nozzles,

(b) Effect of friction on the flow of steam through convergent-divergent steam nozzles.

(c) Effect of supersaturated flow in steam nozzles, and

(d) Steam injector.