Belt, Rope and Chain Drive

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Introduction

The belt or ropes are used to **Transmit power** from one shaft to another by means of pulley which rotate at the same speed or at different Speeds.

Belt

A **Belt** is a looped strip of flexible material, used to mechanically link two or more rotating shafts.

Belts are looped over pulleys.

Belt Drive

When a belt is used for power transmission it is called a **belt drive**

Belts are the cheapest utility for **power transmission** between shafts that may not be parallel.

Power transmission is achieved by specially designed belts and pulleys.

Belts run smoothly and with little noise, and cushion motor and bearings against load changes, but has less strength than gears or chains.

Power Transmitted depend upon

- The velocity of the belt.
- The tension under which the belt is placed on the pulley.
- The arc of contact between the belt and the smaller pulley.
- The condition under which the Belt is used.

Selection of Belt Drive

Speed of the Driving and Driven shaft.
Speed reduction ratio.
Power to be Transmitted.
Center Distance between the shaft.
Positive Drive requirement.
Shaft layout.
Space available.
Service Condition.

Types of Belt Drive

Light Drive:- Used to Transmit small power at the speeds upto about 10 m/s. Agricultural machines & small machine tool.

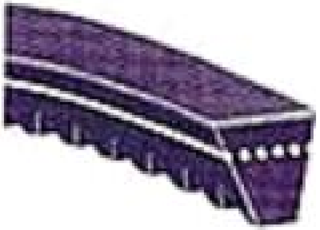
Medium Drive:- Used to Transmit at the speeds Over 10 m/s but upto 22m/s. Machine Tools.

Heavy Drive:-Used to Transmit Large power at the speeds upto Above 22 m/s. Compressor and generators

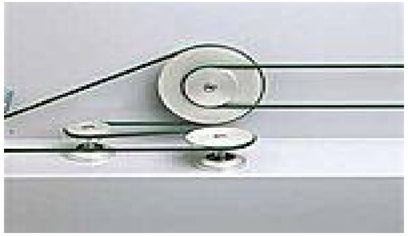
Types of Belts

Flat Belt

V- Belt



Circular Belt.



Flat belt

The flat belt mostly used in the factories and workshop.

where a moderate amount of power to be transmitted, from one pulley to another when the two pulley are not more than 8 meters apart.

V–Belt

The V- Belt mostly used in the factories and workshop,

where a moderate amount of power to be transmitted, from one pulley to another when the two pulley are Very near to Each other.

Circular Belt or Rope

The Circular belt mostly used in the factories and workshop.

where a great moderate amount of power to be transmitted, from one pulley to another when the two pulley are more than 8 meters apart.

Material used for belts must be strong, durable and flexible. It must have high coefficient of friction, μ. The material used are;

1. Leather belt
2. *Rubber belt*
3. *Cotton / fabric belts*

The coefficient of friction of belts depends upon factors such as belt material, pulley material, slip of belt and speed of belt. Table shows some of μ value for belt and pulley material;

Belt material	*Pulley material*						
	Cast iron, steel			*Wood*	*Compressed paper*	*Leather face*	*Rubber face*
	Dry	*Wet*	*Greasy*				
1. Leather oak tanned	0.25	0.2	0.15	0.3	0.33	0.38	0.40
2. Leather chrome tanned	0.35	0.32	0.22	0.4	0.45	0.48	0.50
3. Convass-stitched	0.20	0.15	0.12	0.23	0.25	0.27	0.30
4. Cotton woven	0.22	0.15	0.12	0.25	0.28	0.27	0.30
5. Rubber	0.30	0.18	—	0.32	0.35	0.40	0.42
6. Balata	0.32	0.20	—	0.35	0.38	0.40	0.42

Types of Belt Drives.

Open belt drive.

Crossed or Twisted belt drive.

Quarter turn belt drive.

Belt Drive with Idler pulley.

Compound Belt Drive.

Stepped or cone Pulley Drive.

Fast and loose pulley Drive.

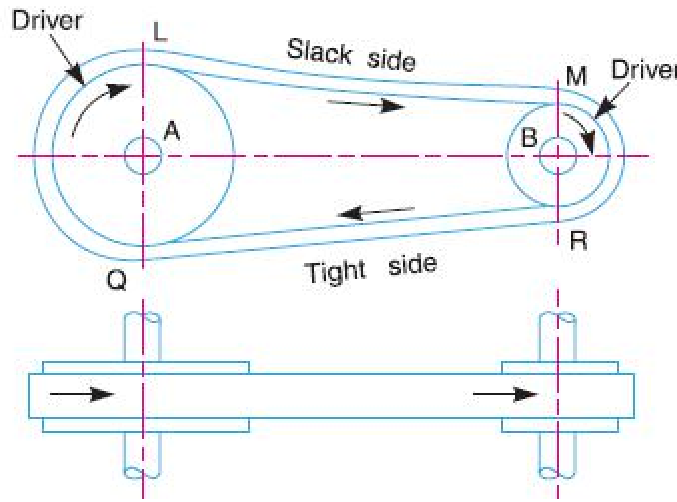
Open belt drive.

Open belt drive is used with shafts arranged parallel and rotating in the same direction.

The driver A pulls the belt from one side (i.e. lower side RQ) and delivers it to the other side (i.e. upper side LM).

Thus the tension in the lower side belt will be more than that in the upper side belt.

The lower side belt (because of more tension) is known as tight side whereas the upper side belt (because of less tension) is known as slack side.



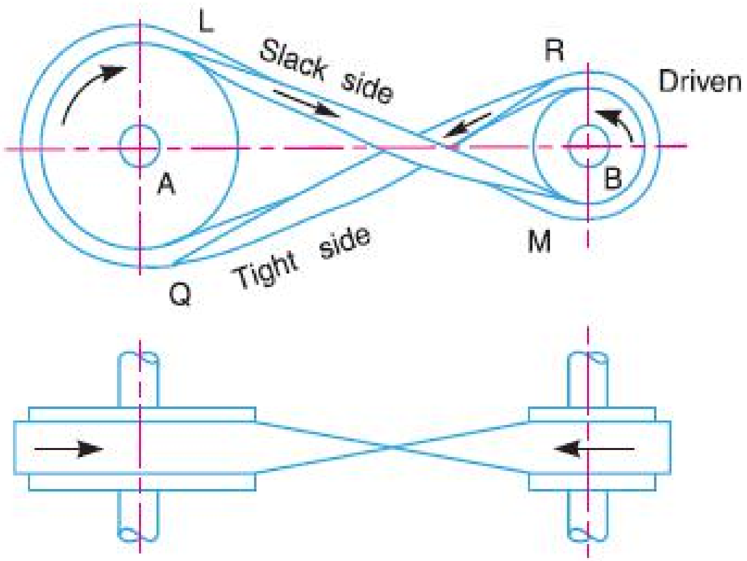
Crossed or twist belt drive.

Cross belt drive is used with shafts arranged parallel and rotating in the opposite direction.

The driver pulls the belt from one side (i.e. RQ) and delivers it to the other side (i.e. LM).

Thus the tension in the belt RQ will be more than that in the belt LM.

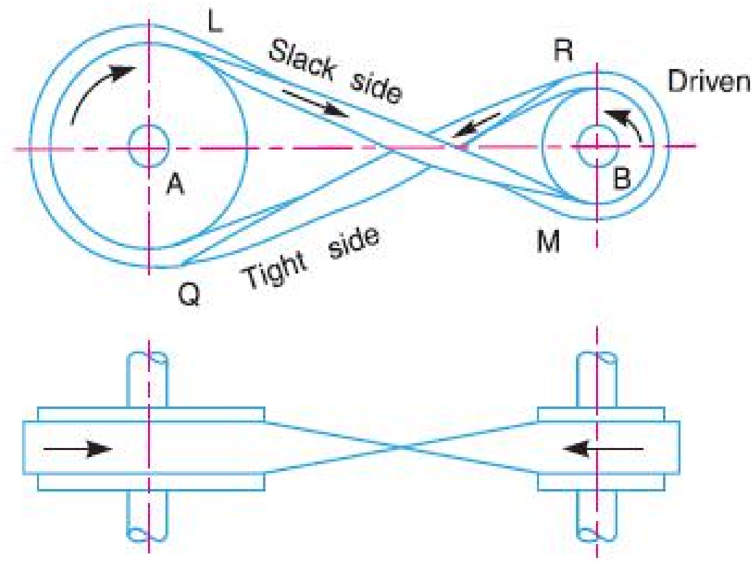
The belt RQ (because of more tension) is known as tight side, whereas the belt LM (because of less tension) is known as slack side.



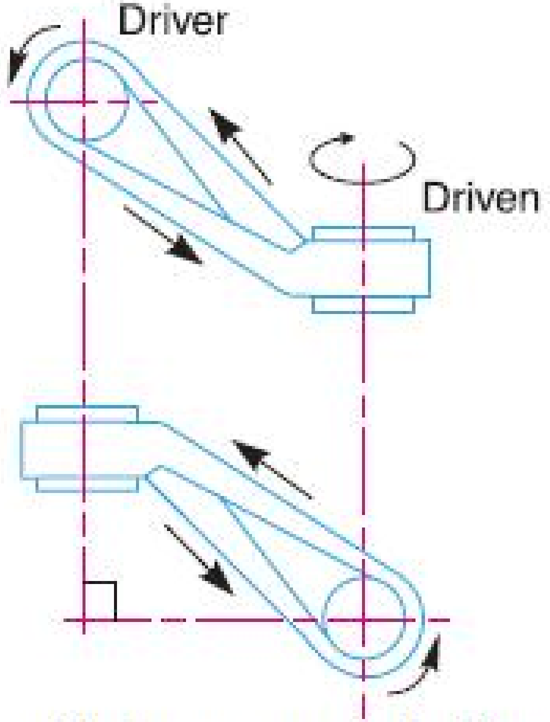
Crossed or twist belt drive.

At a point where the belt crosses, it rubs against each other and there will be excessive wear and tear.

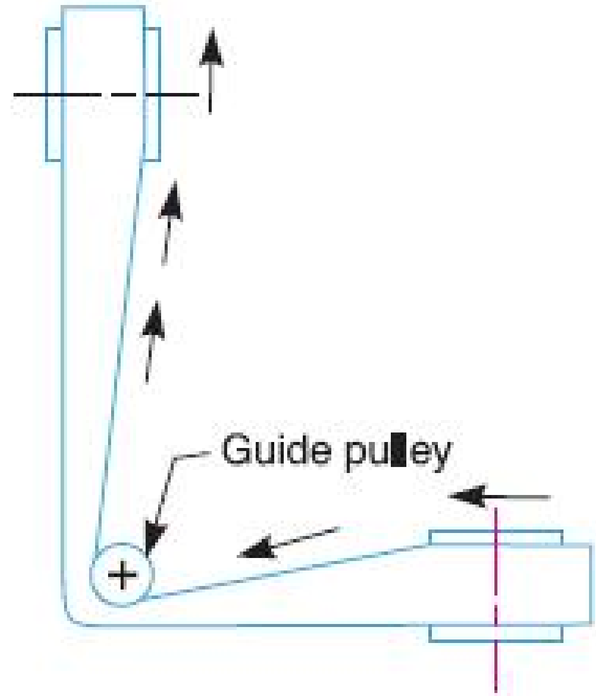
In order to avoid this, the shafts should be placed at a maximum distance of 20 b, where b is the width of belt and the speed of the belt should be less than 15 m/s.



Quarter turn belt drive.



(*a*) Quarter turn belt drive.



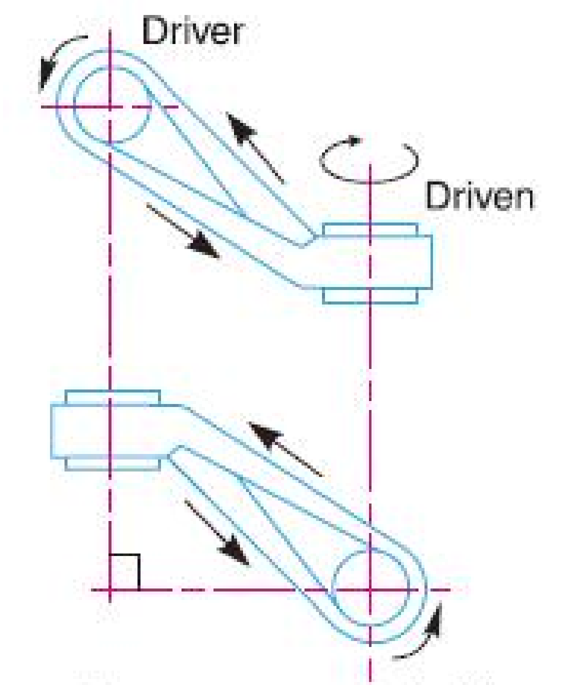
(*b*) Quarter turn belt drive with guide pulley.

Quarter turn belt drive.

The quarter turn belt drive also known as **right angle belt drive.**

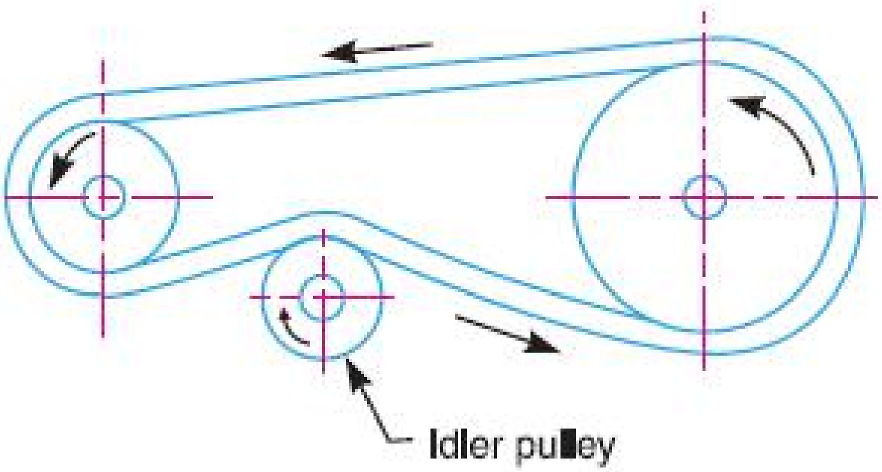
It is used with shafts arranged at right angles and rotating in one definite direction.

In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 b, where b is the width of belt.

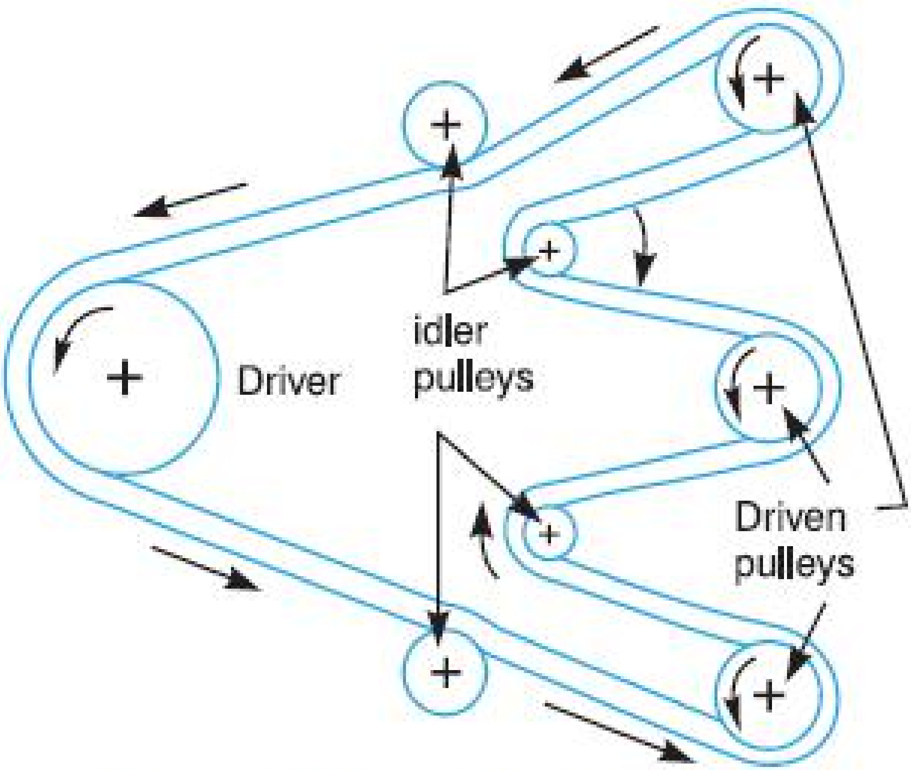


(*a*) Quarter turn belt drive.

Belt drive with idler pulleys.



(*a*) Belt drive with single idler pulley.



(*b*) Belt drive with many idler pulleys.

Belt drive with idler pulleys.

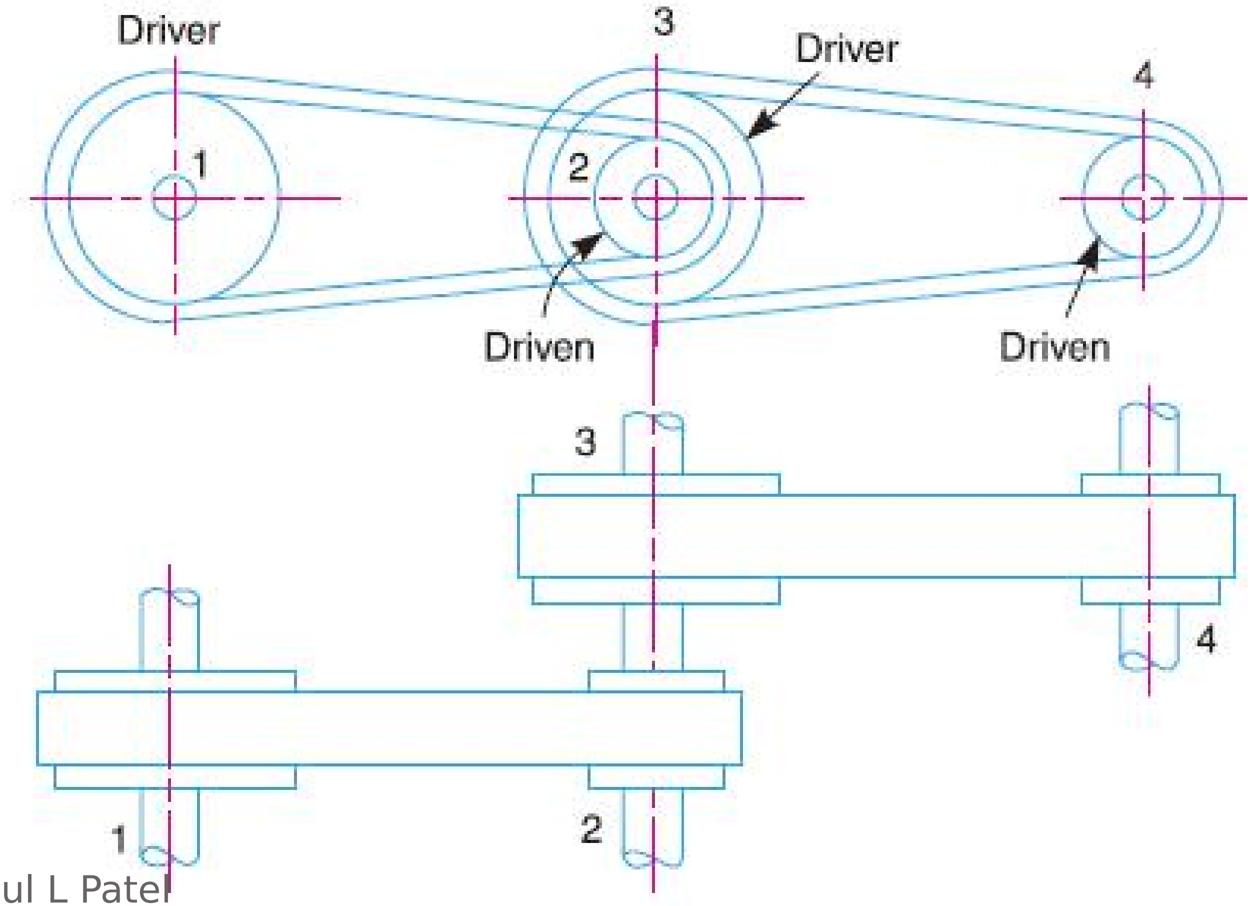
Idler pulley is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley.

This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means.

When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel and they are called belt drive with many idler pulleys

Compound belt drive.

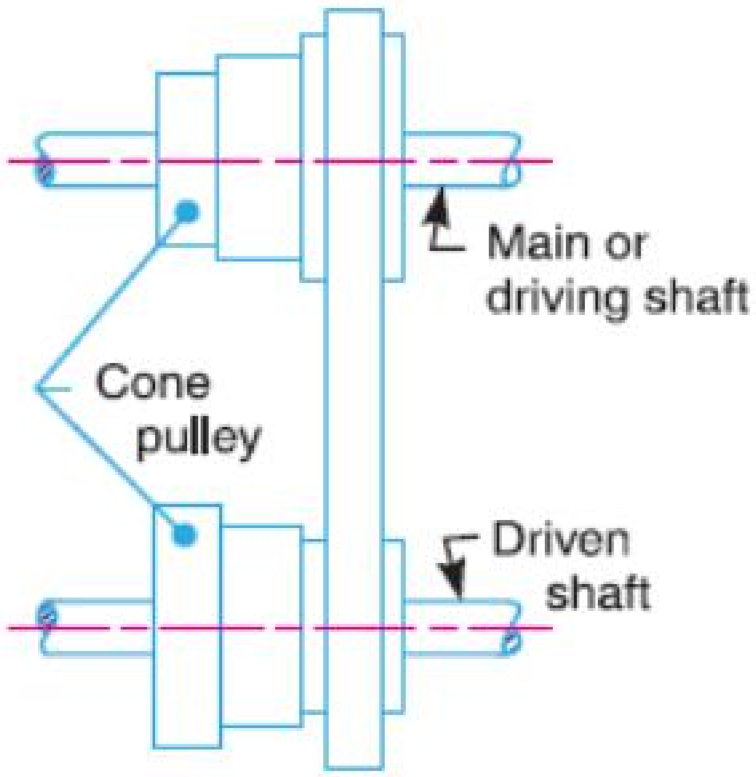
A compound belt drive is used when power is transmitted from one shaft to another through a number of pulleys.



Stepped or cone pulley drive.

A stepped or cone pulley drive is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed.

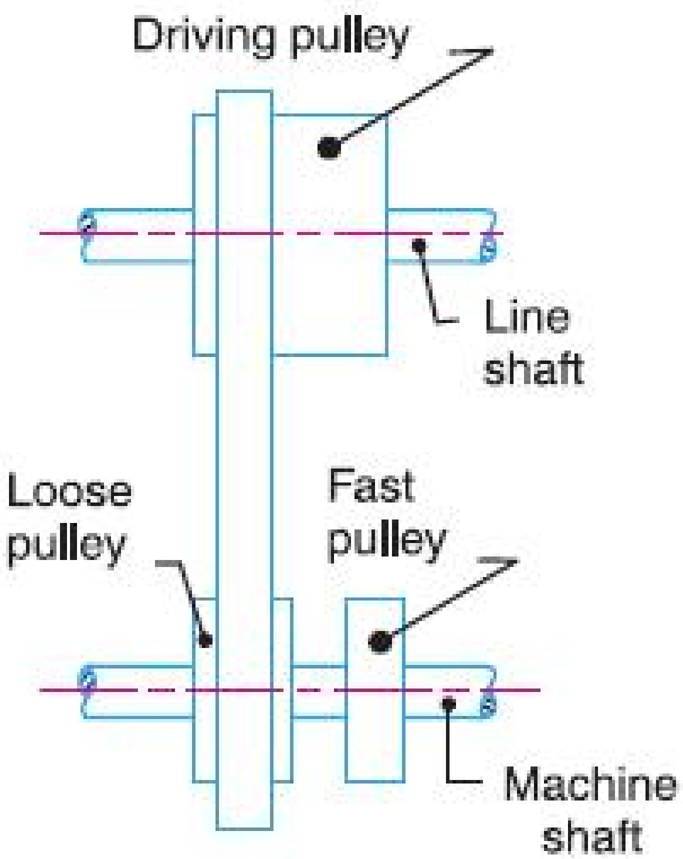
This is accomplished by shifting the belt from one part of the steps to the other.



Fast and loose pulley drive.

A fast and loose pulley drive is used when the driven or machine shaft is to be started or stopped when ever desired without interfering with the driving shaft.

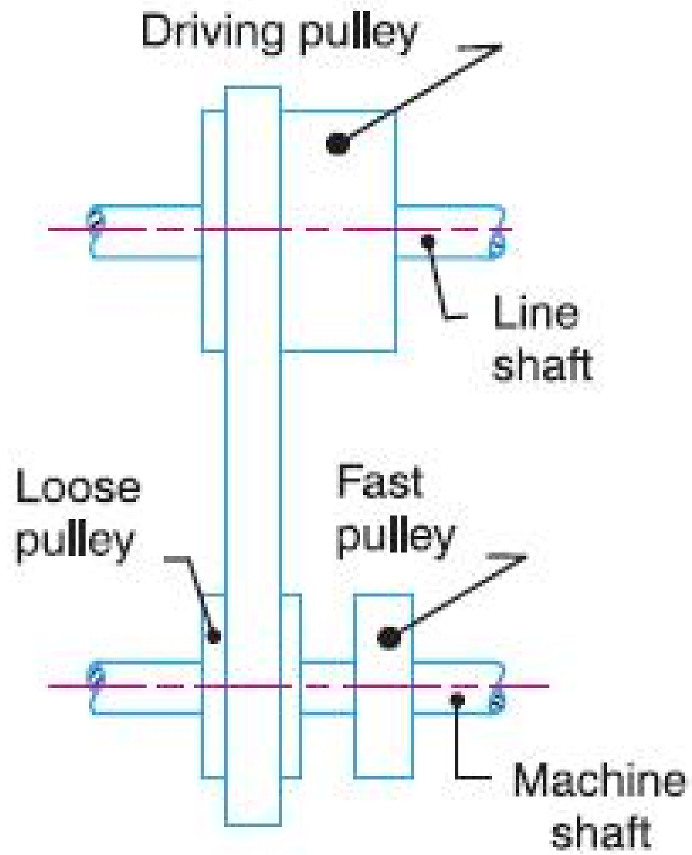
A pulley which is keyed to the machine shaft is called fast pulley and runs at the same speed as that of machine shaft.



Fast and loose pulley drive.

A loose pulley runs freely over the machine shaft and is incapable of transmitting any power.

When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.



Velocity Ratio of Belt Drive

It is the ratio between the velocities of the driver and the follower or driven. It may be

d_1 = Diameter of the driver

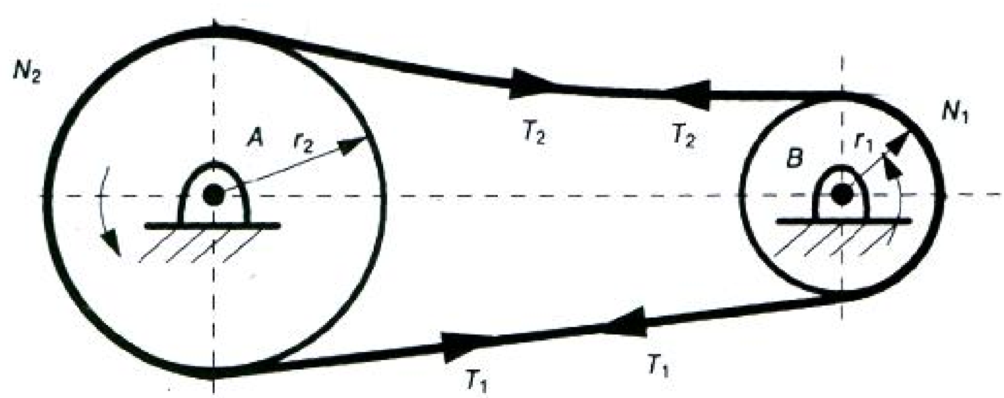
d_2 = Diameter of the follower

N_1 = Speed of the driver in r.p.m., and

N_2 = Speed of the follower in r.p.m.

Length of the belt that passes over the driver, in one minute= $\pi\ d_1.N_1$

length of the belt that passes over the follower, in one minute = $\pi\ d_2\ .\ N_2$



Velocity Ratio of Belt Drive

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\pi\, d_1 . N_1 = \pi\, d_2 \, . \, N_2$$

$\therefore$ Velocity ratio, $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

When the thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Slip of Belt

The motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it.

This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called ***slip of the belt and is generally expressed*** as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of hour, minute and second arms in a watch).

Let, $s1\% =$ *Slip between the* driver and the belt, and

$s2\% =$ *Slip between the belt and the follower.*

∴ Velocity of the belt passing over the driver per second

$$v = \frac{\pi d_1 . N_1}{60} - \frac{\pi d_1 . N_1}{60} \times \frac{s_1}{100} = \frac{\pi d_1 . N_1}{60}\left(1 - \frac{s_1}{100}\right) \quad \text{...(i)}$$

and velocity of the belt passing over the follower per second,

$$\frac{\pi d_2 . N_2}{60} = v - v \times \frac{s_2}{100} = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (i),

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \quad \ldots\left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

$$= \frac{d_1}{d_2}\left(1 - \frac{s_1 + s_2}{100}\right) = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

... (where $s = s_1 + s_2$, *i.e.* total percentage of slip)

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Creep of Belt

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as ***creep.***

The total effect of creep is to reduce slightly the speed of the driven pulley or follower.

Power Transmitted by a Belt

T_1 and T_2 = Tensions in the tight and slack side of the belt respectively in newtons

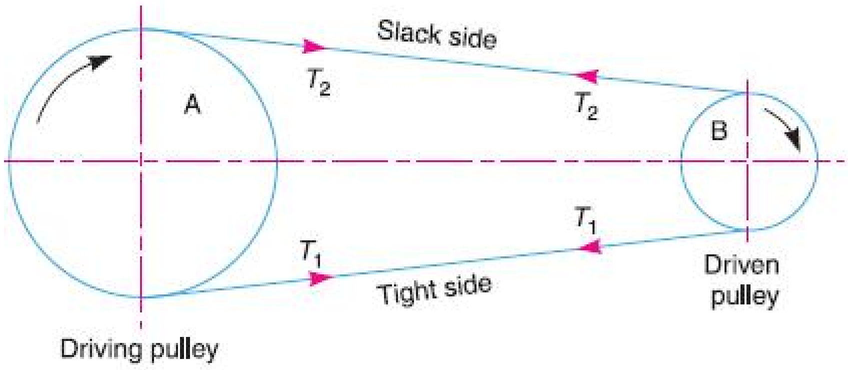
r_1 and r_2 = Radii of the driver and follower respectively, and

v = Velocity of the belt in m/s.

The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (i.e. $T_1 – T_2$).

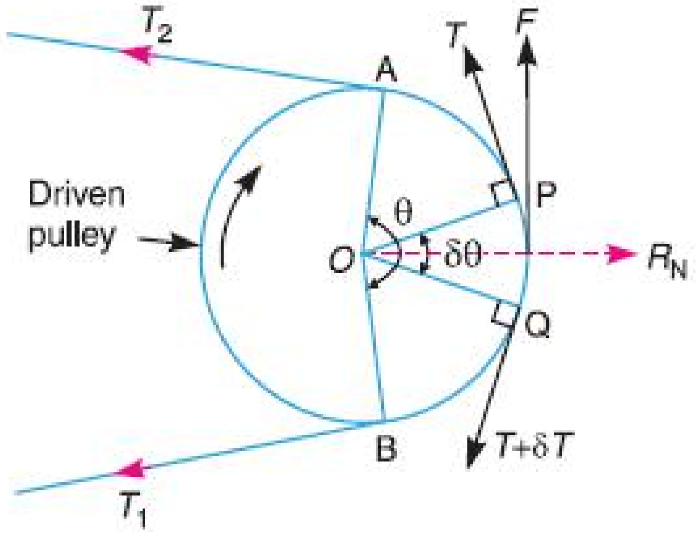
Work done per second = $(T_1 – T_2).\, v$

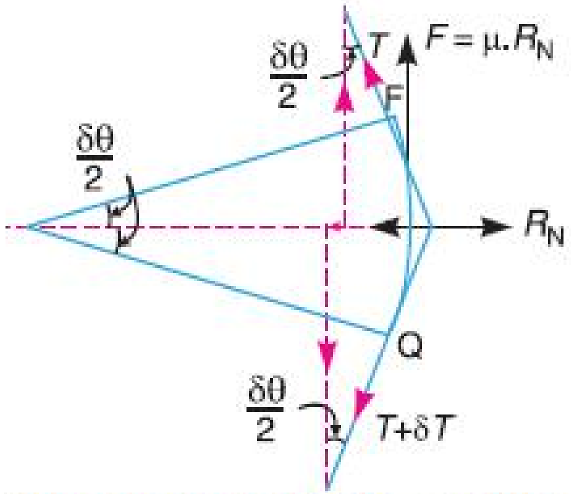
power transmitted, $P = (T_1 – T_2)v$



Power transmitted by a belt.

Ratio of Driving Tensions For Flat Belt Drive





Now consider a small portion of the belt PQ, subtending an angle $\delta\theta$

T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

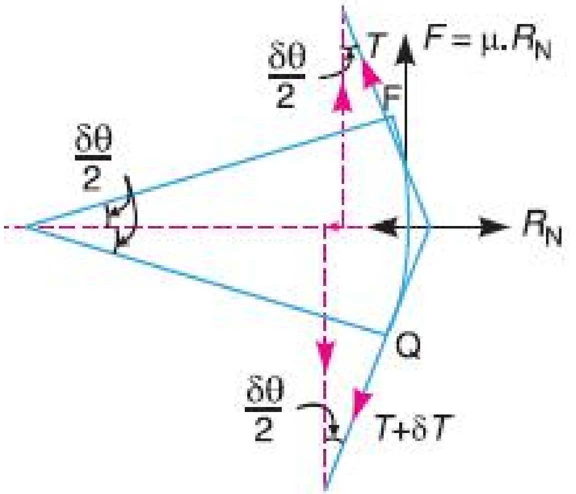
θ = Angle of contact in radians

1. Tension T in the belt at P,
2. Tension $(T + \delta T)$ in the belt at Q,
3. Normal reaction R_N, and
4. Frictional force, $F = \mu \times R_N$

Ratio of Driving Tensions For Flat Belt Drive

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T) \sin \frac{\delta\theta}{2} + T \sin \frac{\delta\theta}{2}$$



Since the angle δθ is very small, therefore putting sin δ θ / 2 = δθ / 2 in equation

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2}$$

$$= \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} \quad \ldots\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

$$= T.\delta\theta$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

Since the angle δ θ is very small, therefore putting cos δ θ / 2 = 1 in equation

$$\mu \times R_N = T + \delta T - T = \delta T$$

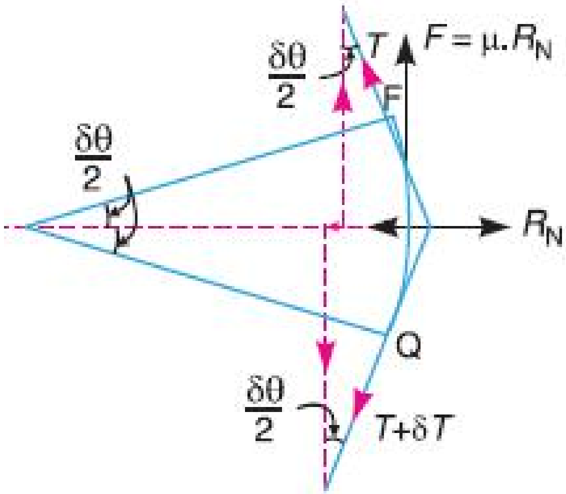
$$R_N = \frac{\delta T}{\mu}$$

Ratio of Driving Tensions For Flat Belt Drive

Equating the values of R_N from equations

$$T.\delta\theta = \frac{\delta T}{\mu} \qquad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,



$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta \qquad \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \qquad \frac{T_1}{T_2} = e^{\mu.\theta}$$

Equation can be expressed in terms of corresponding logarithm to the base 10,

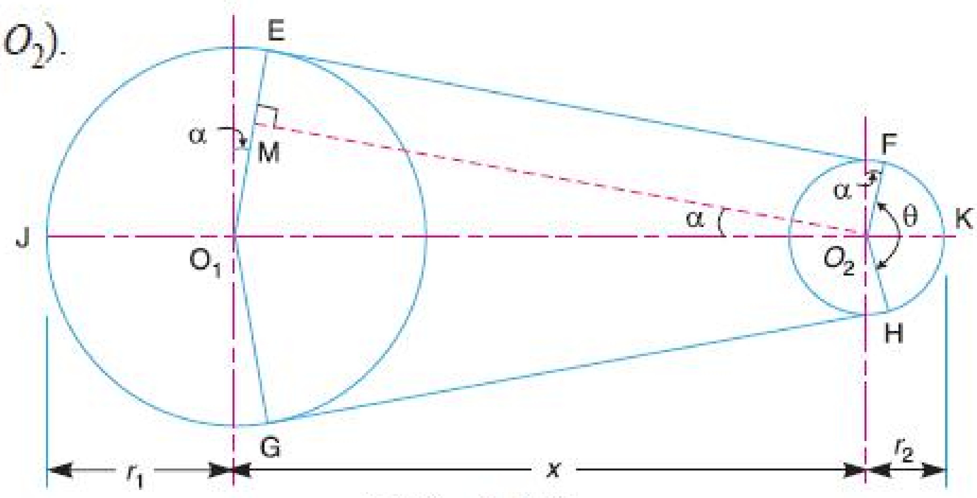
$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

Determination of Angle of Contact

r_1 = Radius of larger pulley,

r_2 = Radius of smaller pulley, and

x = Distance between centres of two pulleys (*i.e.* $O_1 O_2$).



(*a*) Open belt drive.

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - ME}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

Angle of contact or lap,

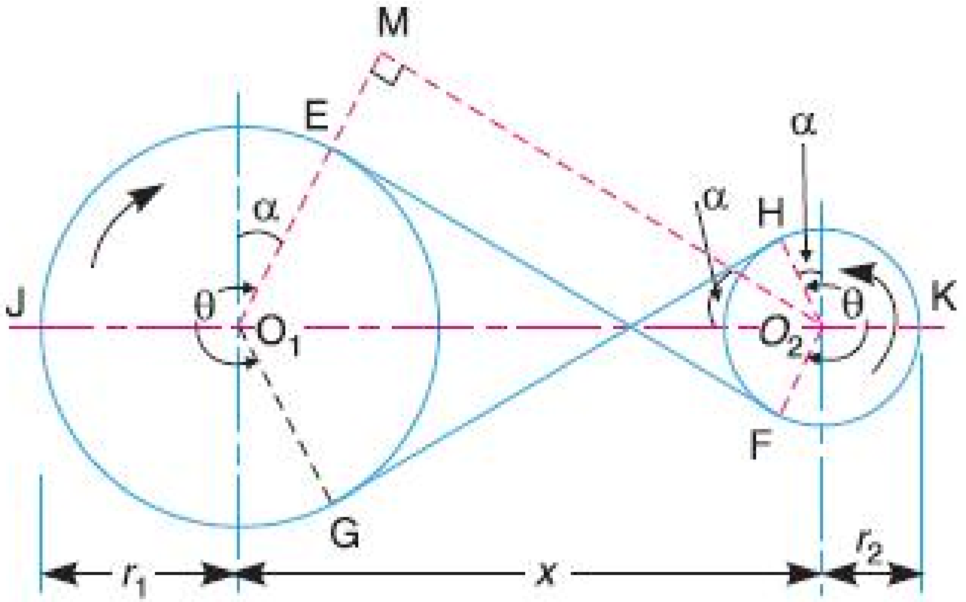
$$\theta = (180° - 2\alpha)\frac{\pi}{180}\text{ rad}$$

Determination of Angle of Contact

$$\sin\alpha = \frac{O_1 M}{O_1 O_2}$$

$$= \frac{O_1 E + ME}{O_1 O_2}$$

$$= \frac{r_1 + r_2}{x}$$



Angle of contact or lap, $\theta = (180° + 2\alpha)\dfrac{\pi}{180}$ rad

Centrifugal Tension

Belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides.

The tension caused by centrifugal force is called centrifugal tension.

At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

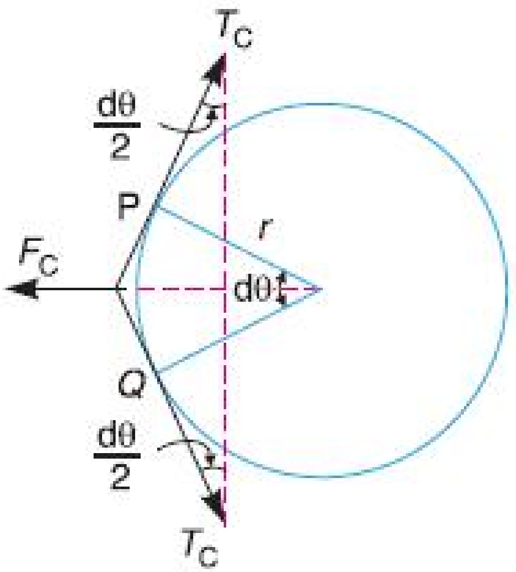
Centrifugal Tension

m = Mass of the belt per unit length in kg,

v = Linear velocity of the belt in m/s,

r = Radius of the pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at P and Q in newtons.



length of the belt $PQ = r.\,d\theta$

mass of the belt $PQ = m.\,r.\,d\theta$

Centrifugal force acting on the belt PQ,

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

Now resolving the forces *horizontally and* equating the same

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C \quad = m.d\theta.v^2$$

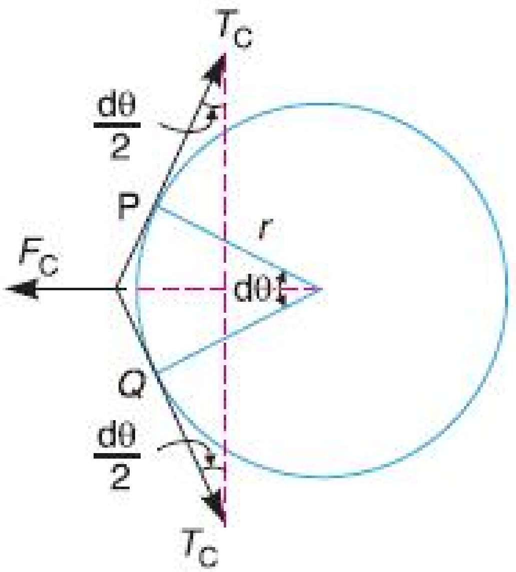
Centrifugal Tension

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C \quad = m.d\theta.v^2$$

Since the angle $d\theta$ is very small

$$\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$$

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \qquad T_C = m\,.\,v^2$$



When the centrifugal tension is taken into account,

Then total tension in the tight side, $T_{t1} = T_1 + T_C$

Total tension in the slack side, $T_{t2} = T_2 + T_C$

$$P = (T_{t1} - T_{t2})\,v$$
$$= [(T_1 + T_C) - (T_2 + T_C)]\,v = (T_1 - T_2)\,v$$

The ratio of driving tensions may also be written as

$$2.3\log\left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

Maximum Tension in the Belt

Maximum tension in the belt (*T) is equal to the total* tension in the tight side of the belt (T_{t1}).

σ = Maximum safe stress in N/mm^2,

b = Width of the belt in mm, and

t = Thickness of the belt in mm.

maximum tension in the belt,

$$T = \text{Maximum stress} \times \text{cross-sectional area of belt}$$

$$= \sigma . b . t$$

centrifugal tension is neglected, then

$$T \text{ (or } T_{t1}) = T_1, \text{ i.e. Tension in the tight side of the belt}$$

centrifugal tension is considered, then

$$T \text{ (or } T_{t1}) = T_1 + T_C$$

Condition For the Transmission of Maximum Power

power transmitted by a belt,

$$P = (T_1 - T_2)\,v$$

T_1 = Tension in the tight side of the belt in newtons,

T_2 = Tension in the slack side of the belt in newtons, and

v = Velocity of the belt in m/s.

Ratio of driving tensions is $\frac{T_1}{T_2} = e^{\mu.\theta}$ or $T_2 = \frac{T_1}{e^{\mu.\theta}}$

Substituting the value of T_2 *in equat* $P = \left(T_1 - \frac{T_1}{e^{\mu.\theta}}\right)v = T_1\left(1 - \frac{1}{e^{\mu.\theta}}\right)v = T_1.v.C$

Where value of C is consider by $C = 1 - \frac{1}{e^{\mu.\theta}}$

We know that

$$T_1 = T - T_C$$

T = Maximum tension to which the belt can be subjected in newtons, and

T_C = Centrifugal tension in newtons.

Substituting the value of *T_1 in equation*

$$P = (T - T_C)\,v.C$$

$$= (T - m.v^2)\,v.C = (T.v - m\,v^3)\,C$$

Substituting $T_C = m.\,v^2$

Differentiate the above expression with respect to *v and equate to zero*

$$\frac{dP}{dv} = 0 \quad \text{or} \quad \frac{d}{dv}(T.v - mv^3)\,C = 0$$

$$T - 3\,m\,.\,v^2 = 0$$

$$T - 3\,T_C = 0 \text{ or } T = 3\,T_C$$

We know that $T_1 = T - T_C$ and for maximum power, $T_C = \frac{T}{3}$.

$$T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

Velocity of the belt for the maximum power frc $T - 3\,m\,.\,v^2 = 0$ $\quad v = \sqrt{\frac{T}{3m}}$

Initial Tension in the Belt

T_0 = Initial tension in the belt,

T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

Increase of tension in the tight s $= T_1 - T_0$

Increase in the length of the belt on the tigh $= \alpha (T_1 - T_0)$
side

Decrease in tension in the slack s $= T_0 - T_2$

Decrease in the length of the belt on the slack side $= \alpha (T_0 - T_2)$

Initial Tension in the Belt

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion,

Therefore increase in length on the tight side is equal to decrease

$$\alpha(T_1 - T_0) = \alpha(T_0 - T_2) \quad \text{or} \quad T_1 - T_0 = T_0 - T_2$$

$$T_0 = \frac{T_1 + T_2}{2}$$ ...Neglecting centrifugal tension

$$= \frac{T_1 + T_2 + 2T_C}{2}$$ ...Considering centrifugal tension

Chain Drives

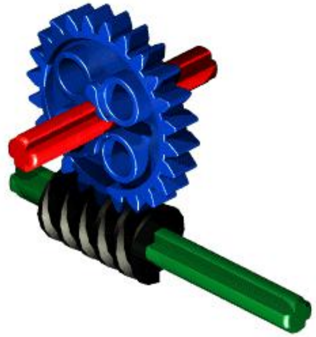
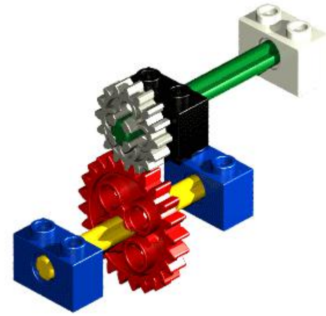
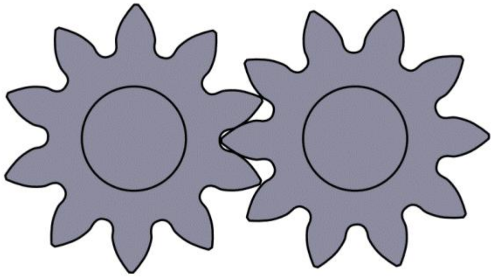
In belt and rope drives that slipping may occur.

In order to avoid slipping, steel chains are used.

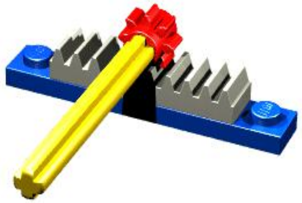
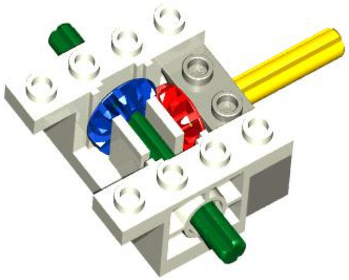
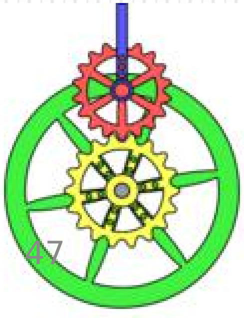
The chains are made up of rigid links which are hinged together in order to provide the necessary flexibility for warping around the driving and driven wheels.

The wheels have projecting teeth and fit into the corresponding recesses, in the links of the chain.

The wheels and the chain are thus constrained to move together without slipping and ensures **perfect velocity ratio.** The toothed wheels are known as ***sprocket wheels*** or simply ***sprockets***.

Gears and Gear train

TYPES OF GEARS

1. According to the position of axes of the shafts.

a. Parallel

    1.Spur Gear

    2.Helical Gear

    3.Rack and Pinion

b. Intersecting

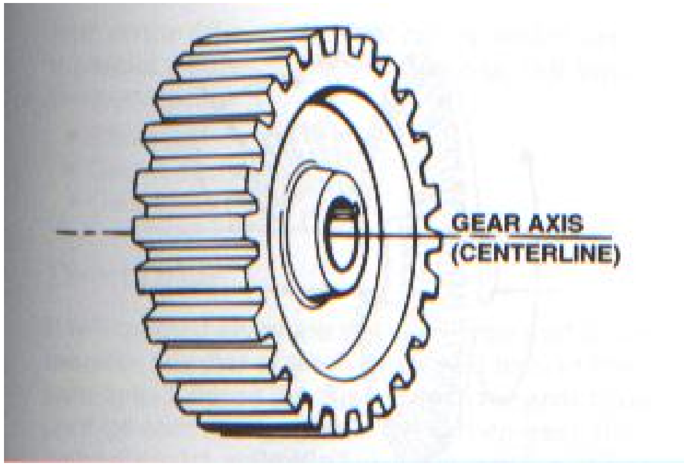
    Bevel Gear

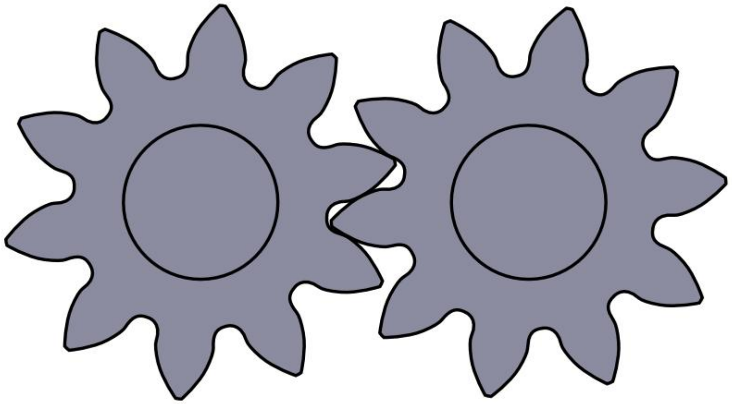
c. Non-intersecting and Non-parallel

    worm and worm gears

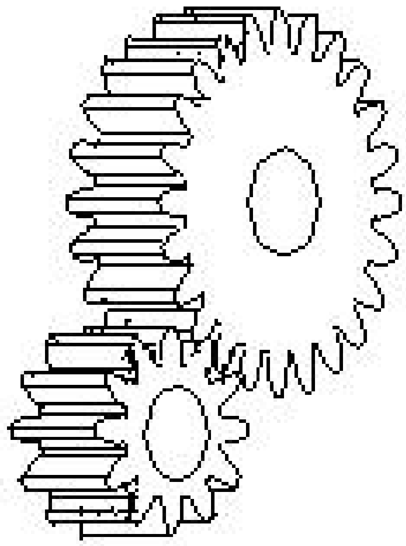
SPUR GEAR

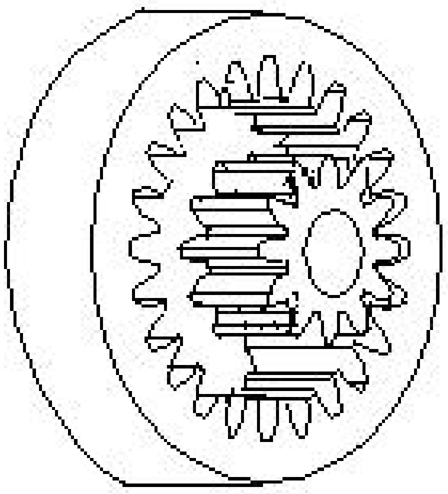
- Teeth is parallel to axis of rotation
- Transmit power from one shaft to another parallel shaft
- Used in Electric screwdriver, oscillating sprinkler, windup alarm clock, washing machine and clothes dryer





External and Internal spur Gear…



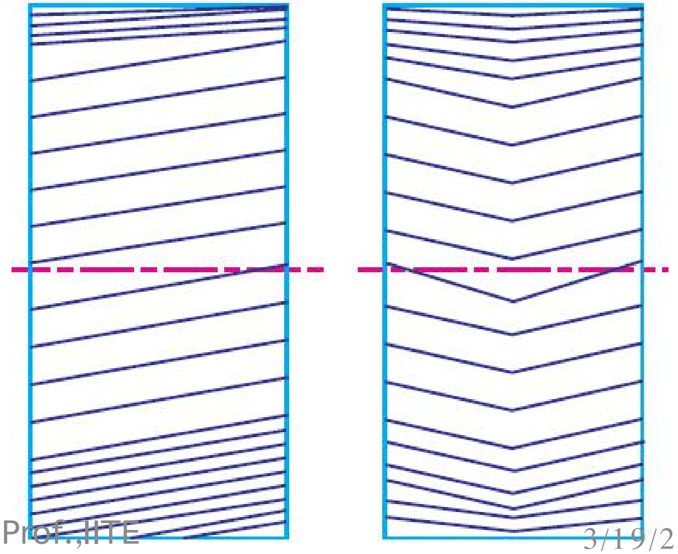


Helical Gear

The teeth on helical gears are cut at an angle to the face of the gear

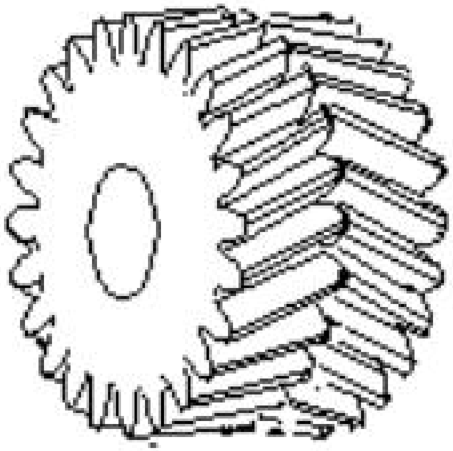
This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears

One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle by 90 degrees



Herringbone gears

To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces

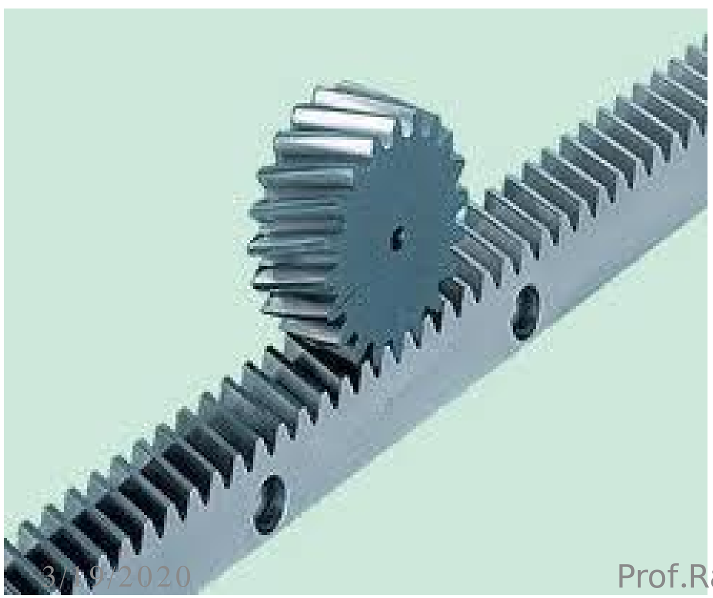


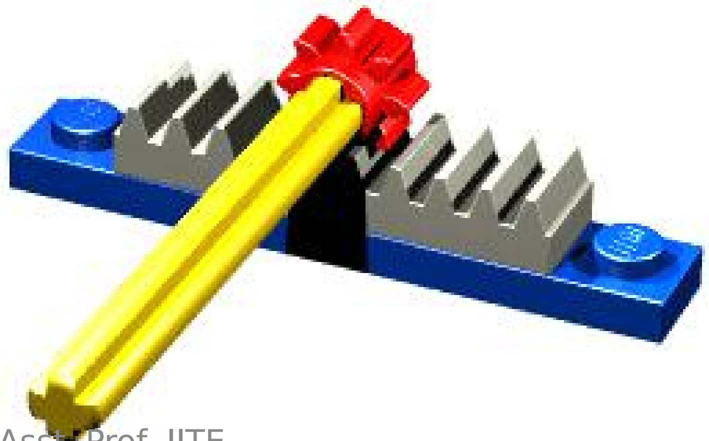
Herringbone gears are mostly used on heavy machinery.

Rack and pinion

Rack and pinion gears are used to convert rotation (From the pinion) into linear motion (of the rack)

A perfect example of this is the steering system on many cars





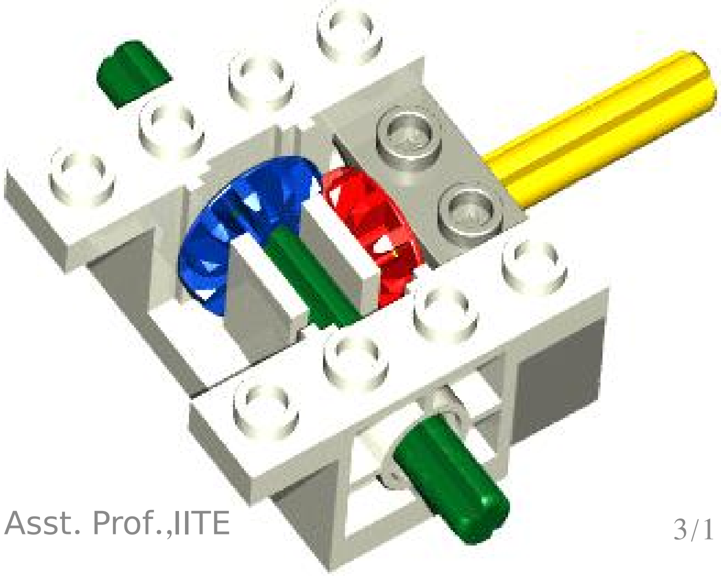
Bevel gears

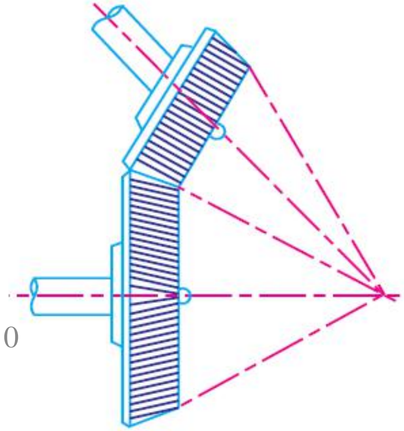
Bevel gears are useful when the direction of a shaft's rotation needs to be changed

They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well

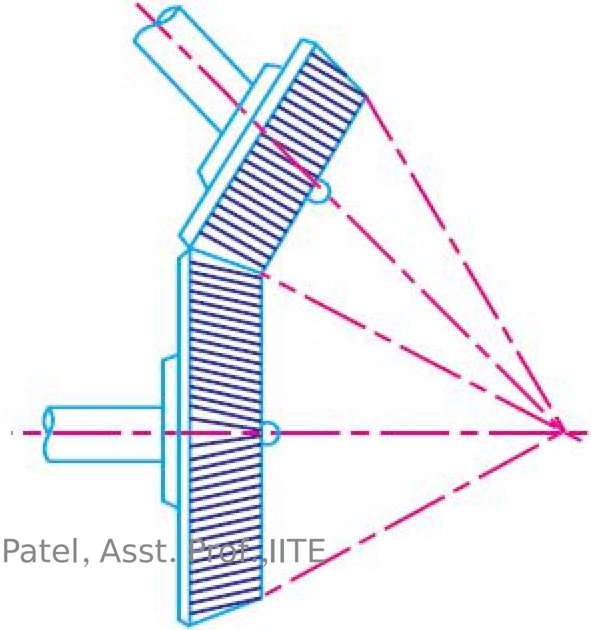
The teeth on bevel gears can be **straight**, **spiral** or **hypoid**

locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants, railway track inspection machines, etc.





Straight and Spiral Bevel Gears

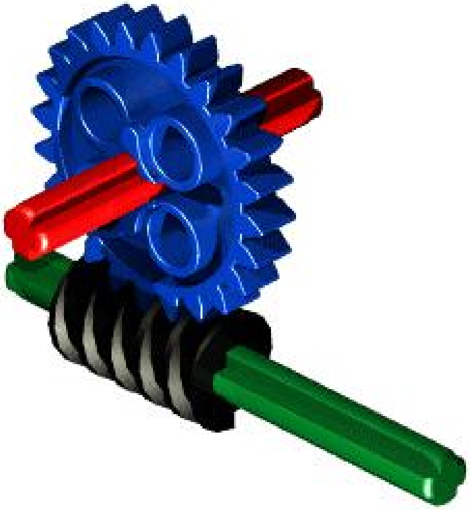


WORM AND WORM GEAR

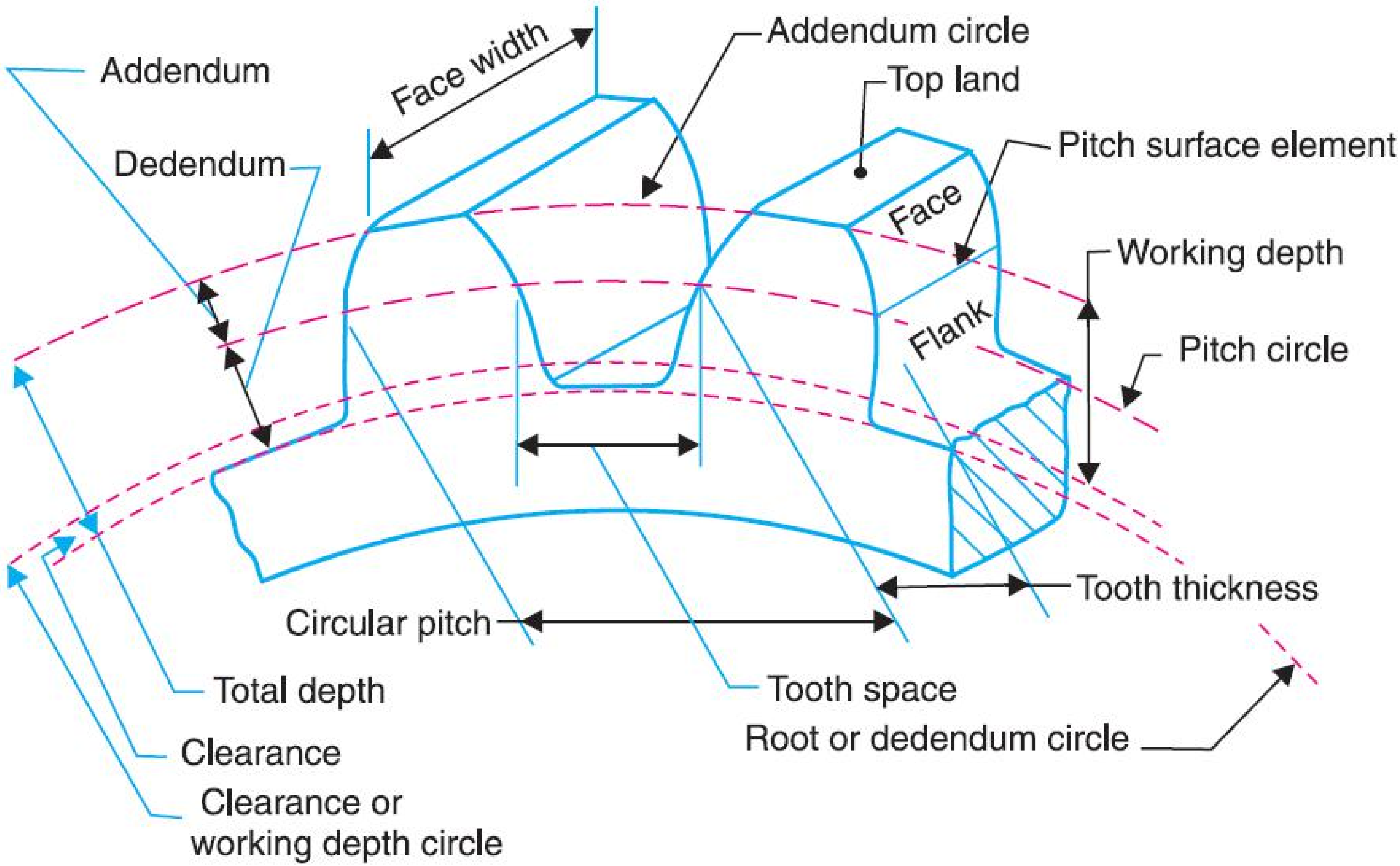
Worm gears are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater

Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm

Worm gears are used widely in material handling and transportation machinery, machine tools, automobiles etc



NOMENCLATURE OF SPUR GEARS



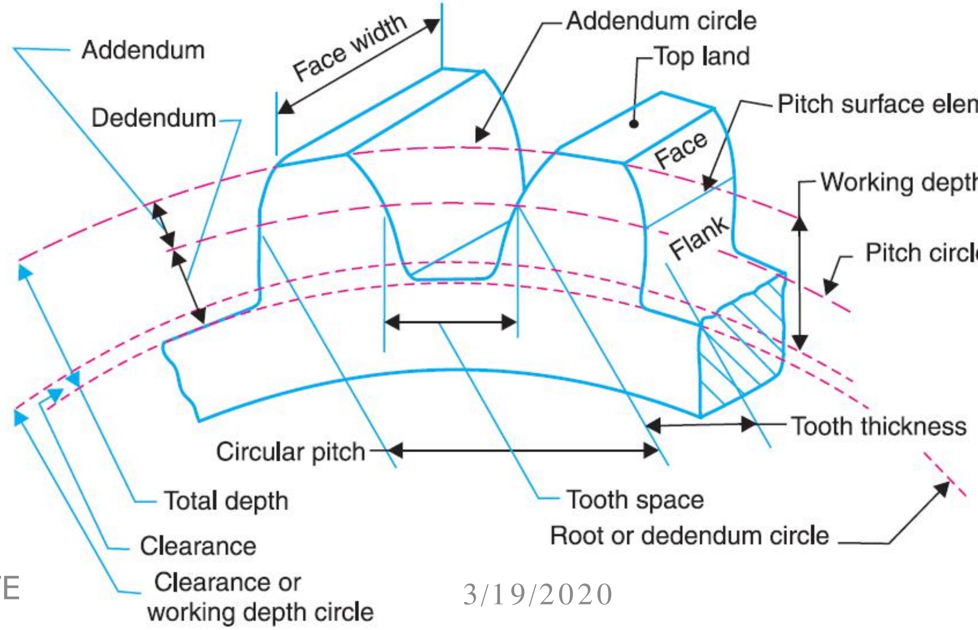
Prof.Rahul L. Patel, Asst. Prof.,IITE

Pitch circle. *It is an imaginary circle which by pure rolling action would give the same* motion as the actual gear.

Pitch circle diameter. *It is the diameter of the pitch circle. The size of the gear is usually* specified by the pitch circle diameter. It is also known as *pitch diameter.*

Pitch point. *It is a common point of contact between two pitch circles.*

Pressure angle or angle of obliquity. *It is the angle between the common normal to two* gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are $14\ 1/2°$ and $20°$.



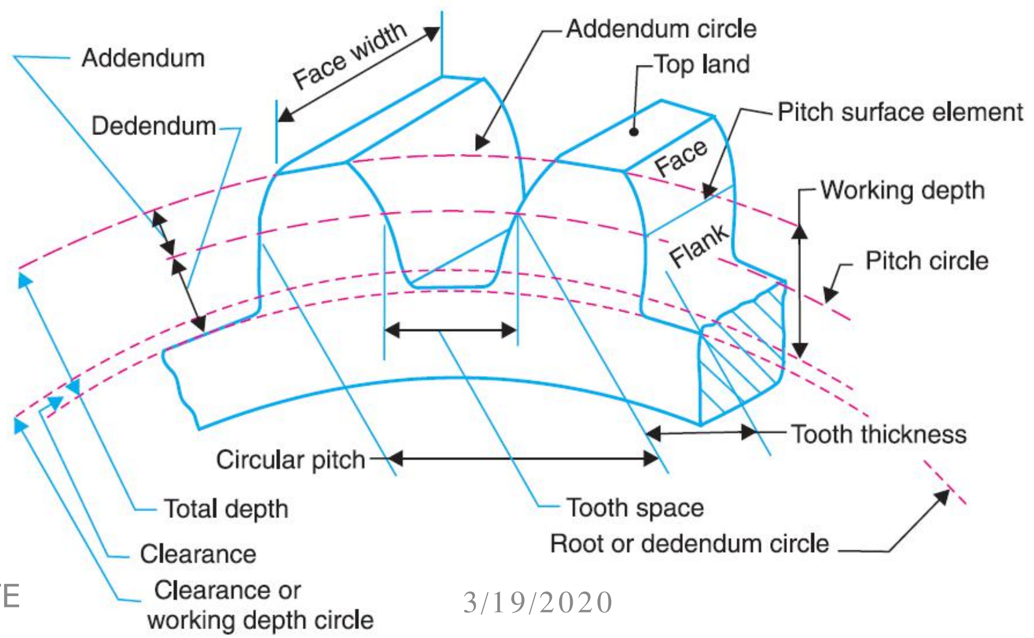
Addendum. *It is the radial distance of a tooth from the pitch circle to the top of the tooth.*

Dedendum. *It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.*

Addendum circle. *It is the circle drawn through the top of the teeth and is concentric with* the pitch circle.

Dedendum circle. *It is the circle drawn through the bottom of the teeth. It is also called* root circle.

Note : Root circle diameter = Pitch circle diameter × cos φ , where φ is the pressure angle.



Circular pitch. *It is the distance measured on the circumference of the pitch circle from* a point of one tooth to the corresponding point on the next tooth. It is usually denoted by P_c ,Mathematically,

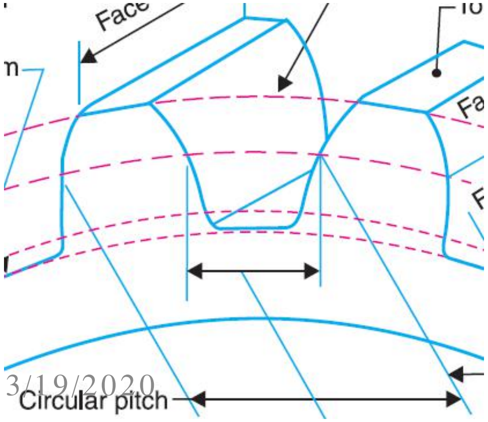
Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

The two gears will mesh together correctly, if the two wheels have the same circular pitch.

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$



Diametral pitch. *It is the ratio of number of teeth to the pitch diameter in millimetres.* It is denoted by p_d. Mathematically,

$$\text{Diametral pitch,} \quad p_d = \frac{T}{D} = \frac{\pi}{p_c} \quad \ldots \left(\because \; p_c = \frac{\pi D}{T} \right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

Module. *It is the ratio of the pitch circle diameter in millimeters to the number of teeth.* It is usually denoted by *m. Mathematically,*

$$\text{Module, } m = D/T$$

Clearance. *It is the radial distance from the top of the tooth to the bottom of the tooth, in* a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle.***

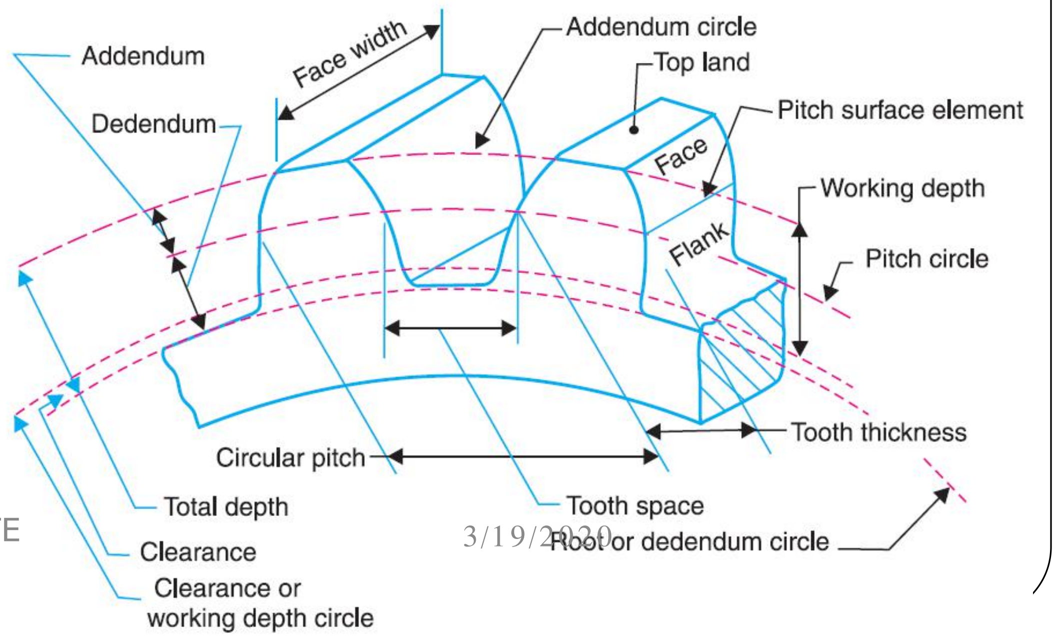
Total depth. *It is the radial distance between the addendum and the dedendum circles of* a gear. It is equal to the sum of the addendum and dedendum.

Working depth. *It is the radial distance from the addendum circle to the clearance circle.* It is equal to the sum of the addendum of the two meshing gears.

Tooth thickness. *It is the width of the tooth measured along the pitch circle.*

Tooth space . *It is the width of space between the two adjacent teeth measured along the pitch* circle.

Backlash. *It is the difference between the tooth space and the tooth thickness, as measured* along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.



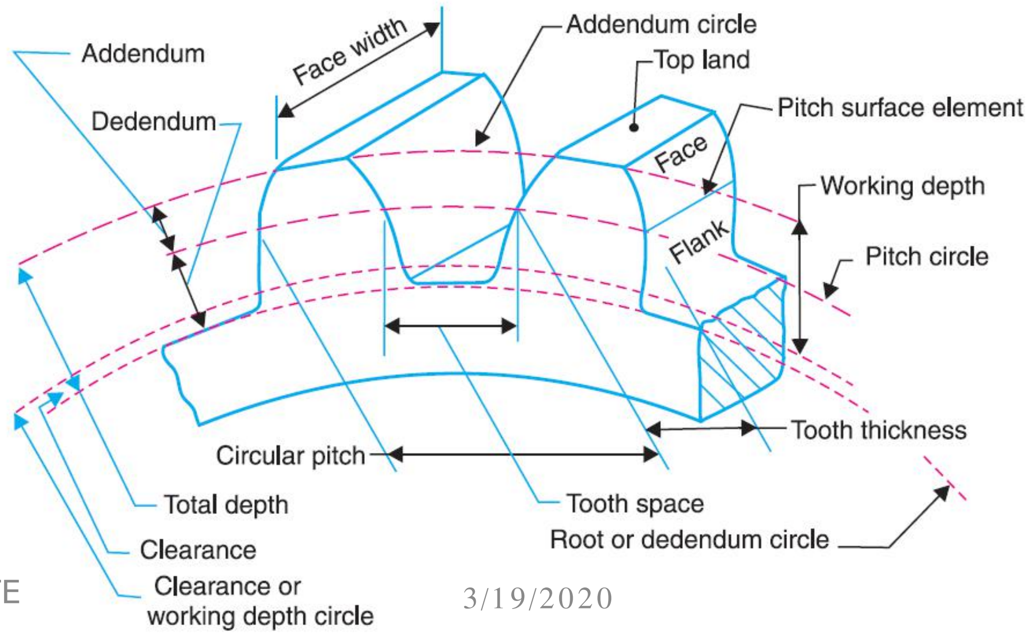
Face of tooth. *It is the surface of the gear tooth above pitch surface.*

Flank of tooth. *It is the surface of the gear tooth below the pitch surface.*

Top land. *It is the surface of the top of the tooth.*

Face width. *It is the width of the gear tooth measured parallel to its axis.*

Profile. *It is the curve formed by the face and flank of the tooth.*



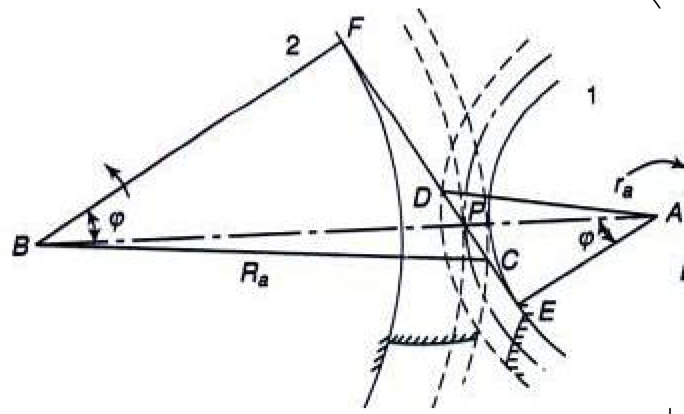
Path of contact. *It is the path traced by the point of contact of two teeth from the* beginning to the end of engagement.

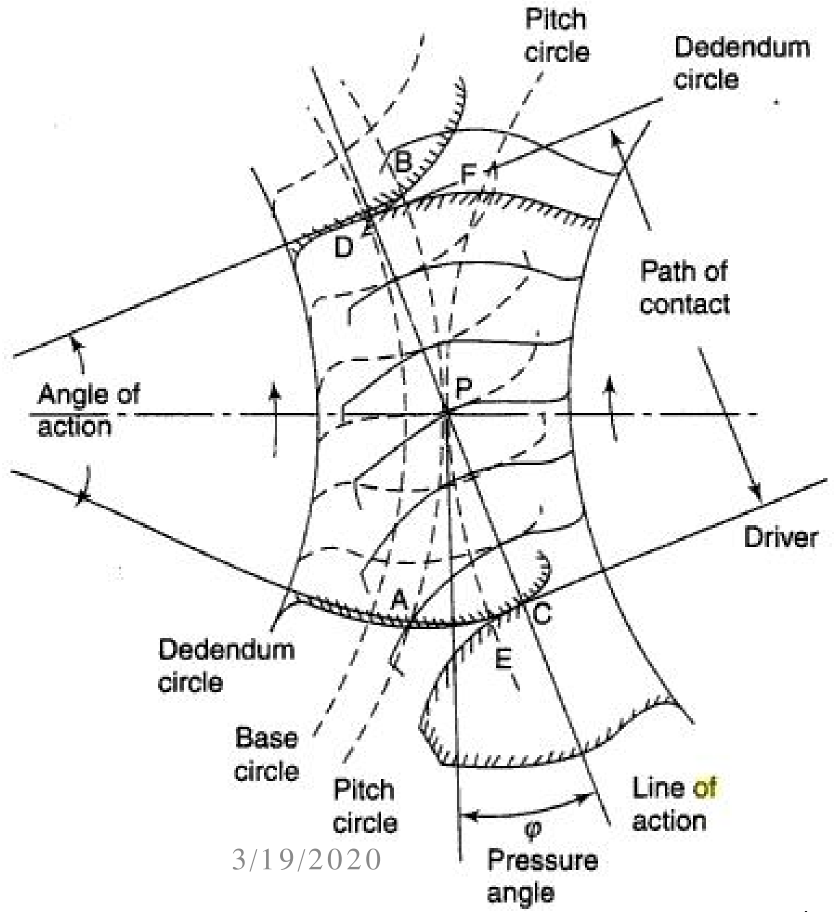
Length of the path of contact*. It is the length of the common normal cut-off by the* addendum circles of the wheel and pinion.

Arc of contact. *It is the path traced by a point on the pitch circle from the beginning* to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

(***a***) ***Arc of approach.*** *It is the portion of the path of contact from the beginning of the* engagement to the pitch point.

(***b***) ***Arc of recess.*** *It is the portion of the path of contact from the pitch point to the end of the* engagement of a pair of teeth.





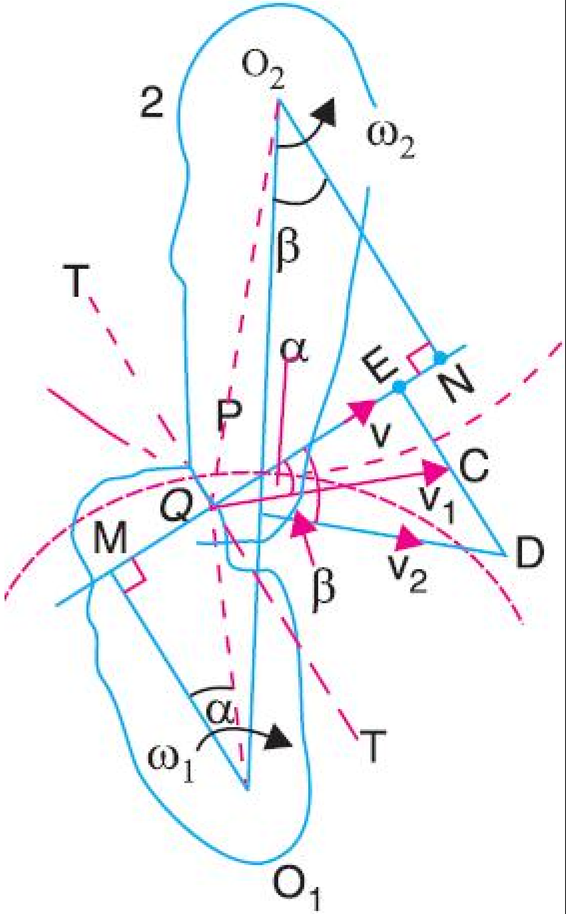
Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig.

Let the two teeth come in contact at point *Q, and the wheels rotate in the directions as* shown in the figure.

Let *TT be the common tangent and MN be the* common normal to the curves at the point of contact *Q. From the* centres O_1 and O_2, draw *O_1M and O_2N perpendicular to MN.*

Let v_1 and v_2 be the velocities of the point *Q on the wheels* 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal *MN must* be equal.



$$v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 \times O_1 Q)\cos\alpha = (\omega_2 \times O_2 Q)\cos\beta$$

$$(\omega_1 \times O_1 Q)\frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q)\frac{O_2 N}{O_2 Q} \quad \text{or} \quad \omega_1 \times O_1 M = \omega_2 \times O_2 N$$

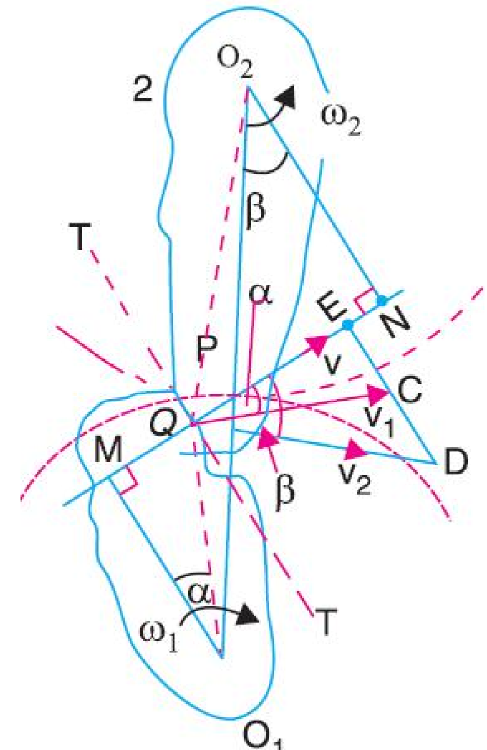
$$\therefore \quad \frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}$$

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}$$

Combining equations *(i)* and *(ii)*, we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}$$



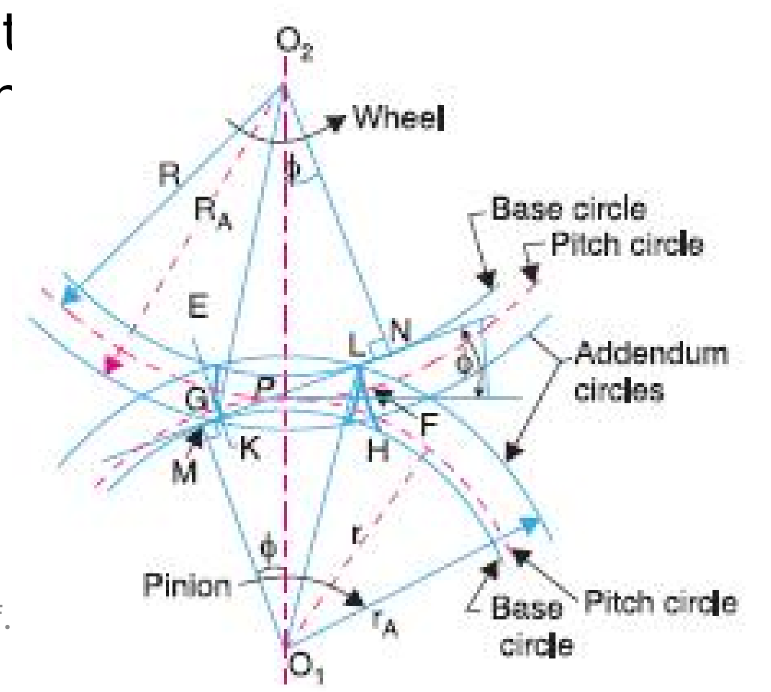
- To have a constant angular velocity ratio for all positions of the wheels, the point *P must be the fixed point (called pitch point) for the two wheels. In other words,* ***the common normal at the point of contact between a pair of teeth must always pass through the pitch point***.
- This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as *law of gearing.*

Length of Path of Contact

Consider a pinion driving the wheel as shown in Fig. When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at *K (on the flank near the* base circle of pinion or the outer end of the tooth face on the wheel) and ends at *L (outer end of the* tooth face on the pinion or on the flank near the base circle of wheel).

MN is the common normal at the point of contacts and the common tangent to the base circles.

The point *K is the intersection of the* addendum circle of wheel and the common tangent *ion of the*
addendum circle of pinior



Length of path of contact is KL = *Parts of Path of contact*

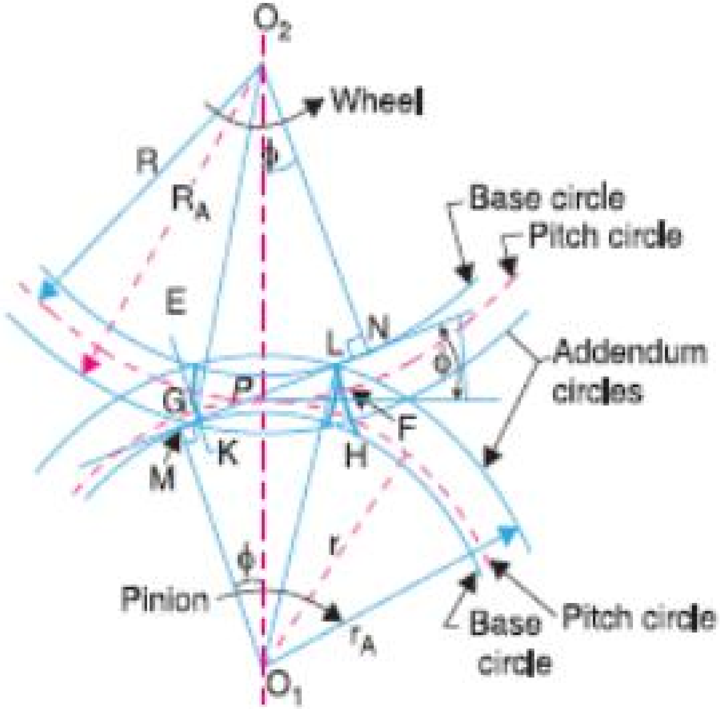
= *P*ath of approach *KP* + *path of recess*

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P$ = Radius of pitch circle of wheel.



radius of the base circle of pinion,

$O_1M = O_1P \cos Ø = r \cos Ø$

and radius of the base circle of wheel,

$O_2N = O_2P \cos Ø = R \cos Ø$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2 \cos^2 \phi}$$

and $$PN = O_2P \sin \phi = R \sin \phi$$

∴ Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

Similarly from right angled triangle O_1ML,

and $$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2 \cos^2 \phi}$$

$$MP = O_1P \sin \phi = r \sin \phi$$

∴ Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

∴ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} + \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi$$

Length of Arc of Contact

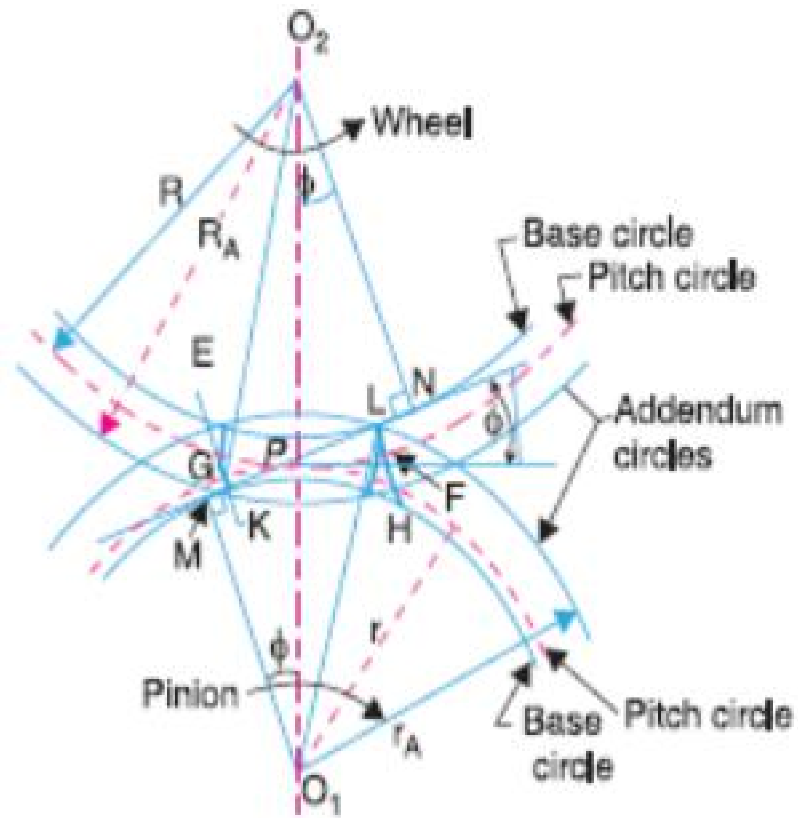
The arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth.

In Fig. ,the arc of contact is EPF or GPH.

Considering the arc of contact GPH, it is divided into two parts i.e. arc GP and arc PH.

The arc GP is known as arc of approach and the arc PH is called arc of recess.

The angles subtended by these arcs at O_1 are called angle of approach and angle of recess respectively.



We know that the length of the arc of approach (arc GP)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

and the length of the arc of recess (arc PH)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

Since the length of the arc of contact GPH is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact

$$= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the **ratio of the length of the arc of contact to the circular pitch.** Mathematically,

Contact ratio or number of pairs of teeth in contact

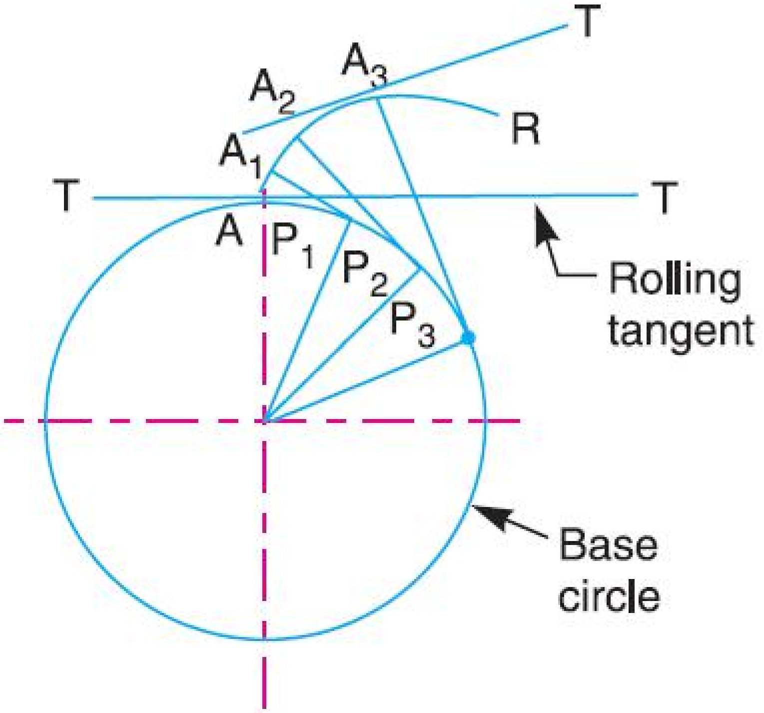
$$= \frac{\text{Length of the arc of contact}}{p_c}$$

where p_c = Circular pitch = πm, and

m = Module.

Involute Teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig.



Advantages of involute gears

The most important advantage of the involute gears is that **the centre distance for a pair of involute gears can be varied within limits** without changing the velocity ratio. This is not true for cycloidal gears which requires exact centre distance to be maintained.

In involute gears, the pressure angle, from the start of the engagement of teeth to the end of the engagement, remains constant. It is necessary for smooth running and less wear of gears. But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts decreasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.

Advantages of cycloidal gears

Following are the advantages of cycloidal gears :

Since the cycloidal teeth have wider flanks, therefore the **cycloidal gears are stronger than the involute gears**, for the same pitch. Due to this reason, the cycloidal teeth are preferred specially for cast teeth.

In cycloidal gears, the contact takes place between a convex flank and concave surface, whereas in involute gears, the convex surfaces are in contact. This condition results in **less wear in cycloidal gears as compared to involute gears**. However the difference in wear is negligible.

In cycloidal gears, the interference does not occur at all. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

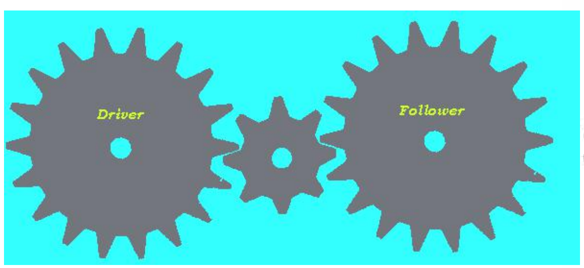
GEAR TRAINS

Introduction

- Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train or train of toothed wheels***.
- The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts.
- A gear train may consist of spur, bevel or spiral gears.

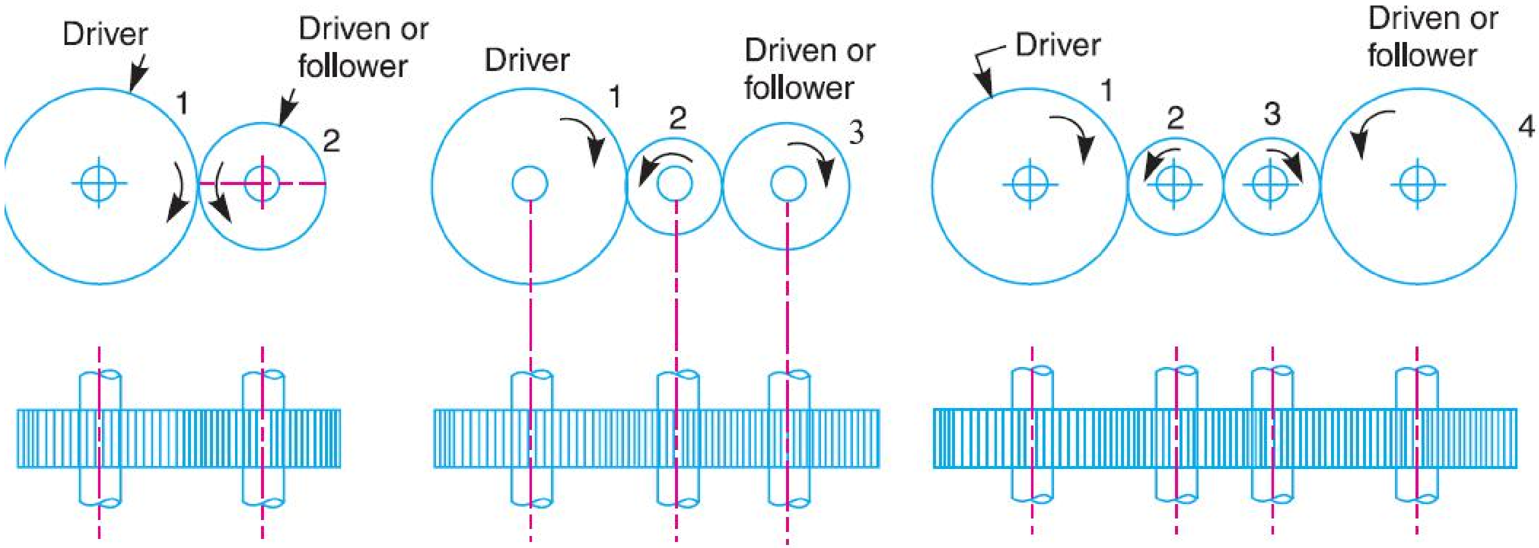
Types of Gear Trains

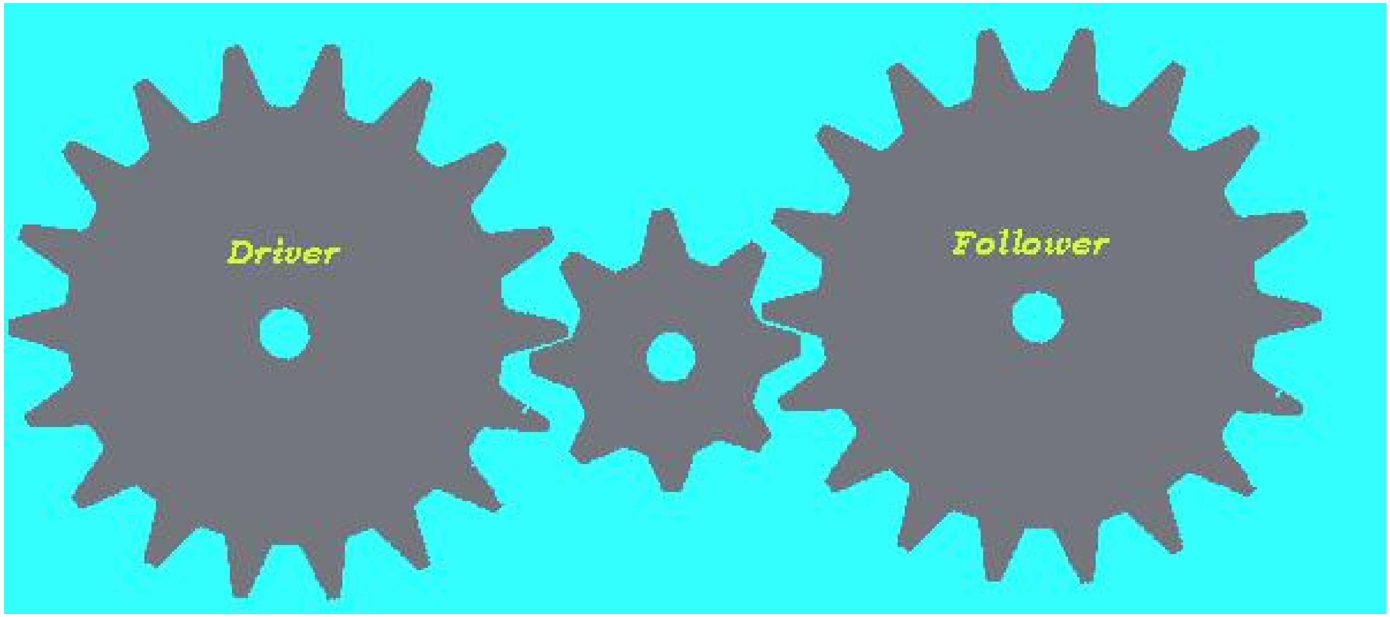
- Following are the different types of gear trains, depending upon The arrangement of wheels :
  1. Simple gear train
  2. Compound gear train
  3. Reverted gear train
  4. Epicyclic gear train
- In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other.
- But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

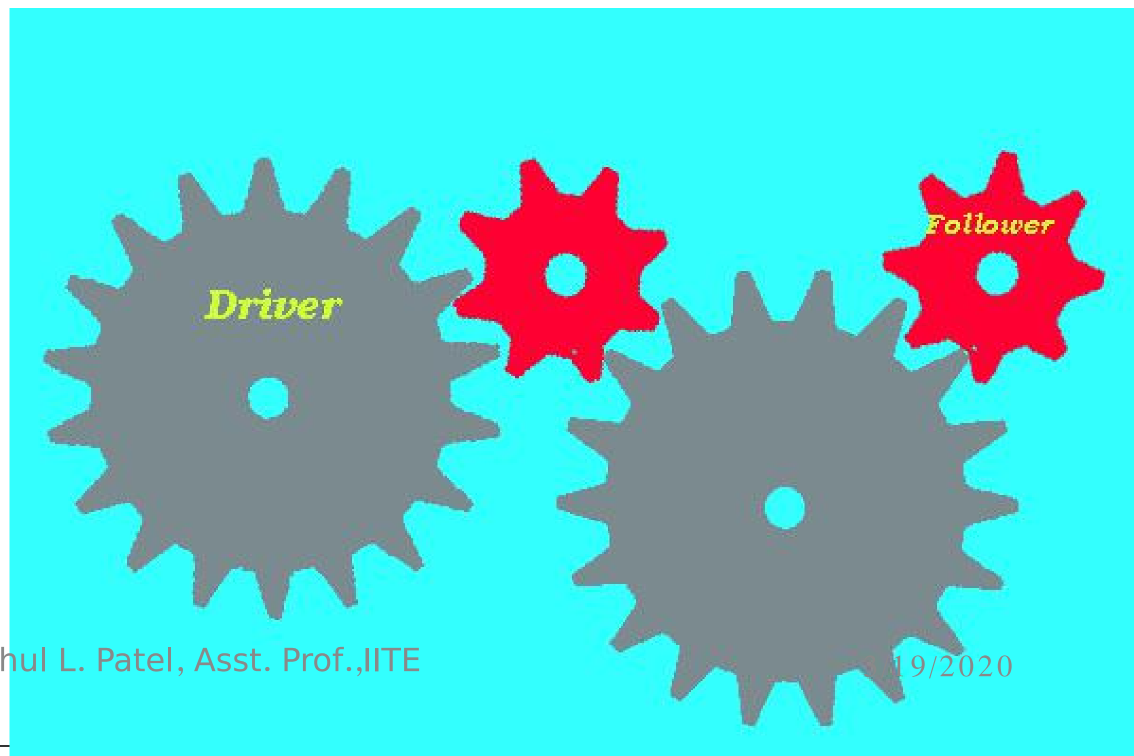


Simple Gear Train

- When there is only one gear on each shaft, as shown in Fig., it is known as *simple gear train.*
- *The gears are* represented by their pitch circles.
- When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. (*a*).




Driver
Follower


Driver
Follower

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From above, we see that the train value is the reciprocal of speed ratio.

Now consider a simple train of gears with one intermediate gear as shown in Fig. 13.1 (*b*).

Let N_1 = Speed of driver in r.p.m.,

N_2 = Speed of intermediate gear in r.p.m.,

N_3 = Speed of driven or follower in r.p.m.,

T_1 = Number of teeth on driver,

T_2 = Number of teeth on intermediate gear, and

T_3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{...}(i)$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \quad \text{...}(ii)$$

The speed ratio of the gear train as shown in Fig. 13.1 (*b*) is obtained by multiplying the equations (*i*) and (*ii*).

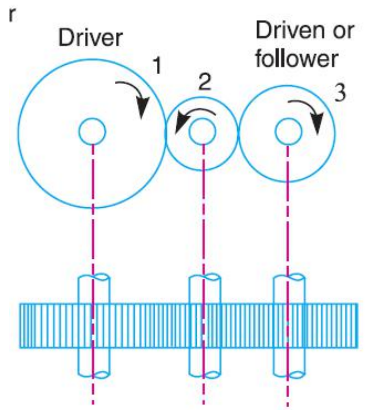
$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

and

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

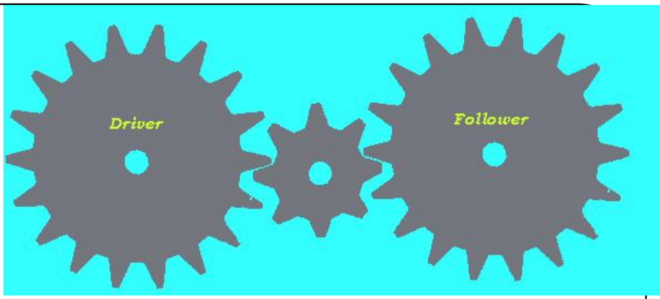


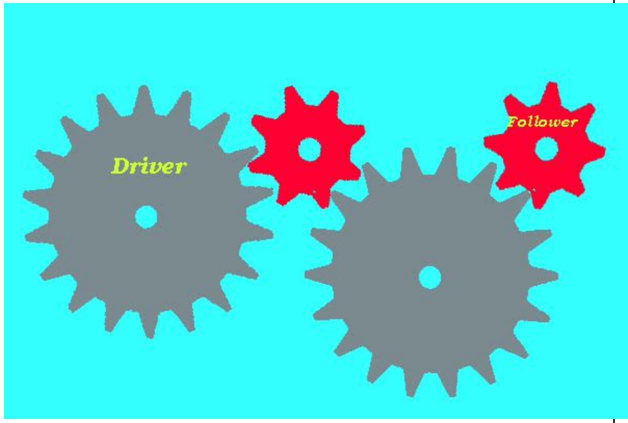
(*b*)

Similarly, it can be proved that the above equation holds good even if there are any number of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called ***idle gears***, as they do not effect the speed ratio or train value of the system. The idle gears are used for the following two purposes :

1. To connect gears where a large centre distance is required, and

2. To obtain the desired direction of motion of the driven gear (*i.e.* clockwise or anticlockwise).





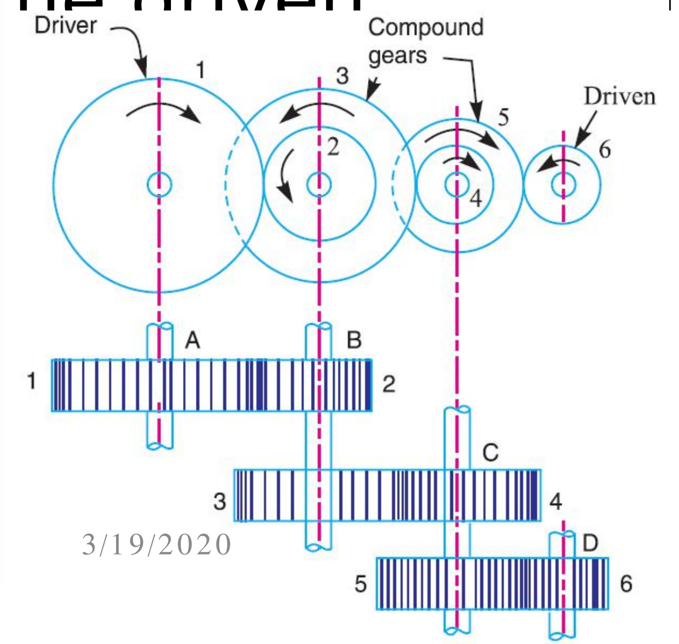
Compound Gear Train

When there are more than one gear on a shaft, as shown in Fig. , it is called a *compound train of gear.*

We have seen that the idle gears, in a simple train of gears do not effect the speed ratio of the system.

But these gears are useful in bridging over the

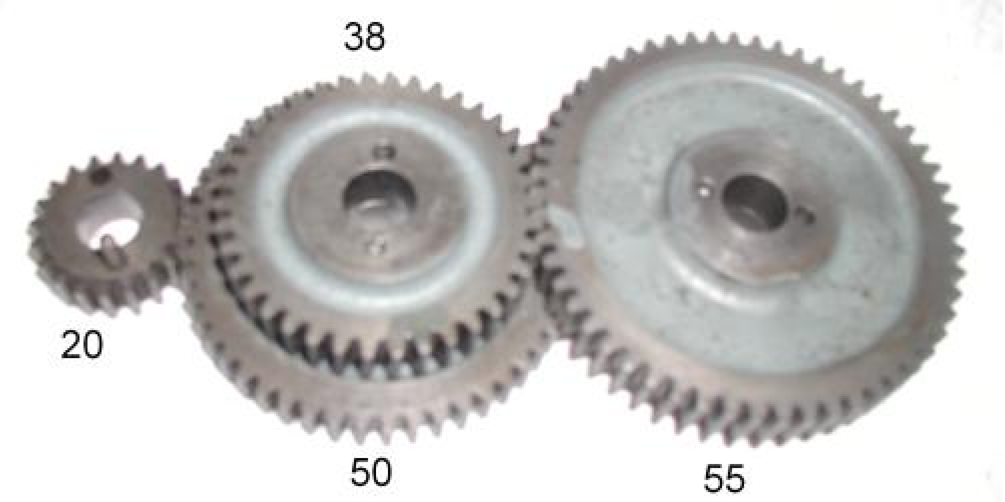
he driven



Compound Gear Train (Continued)

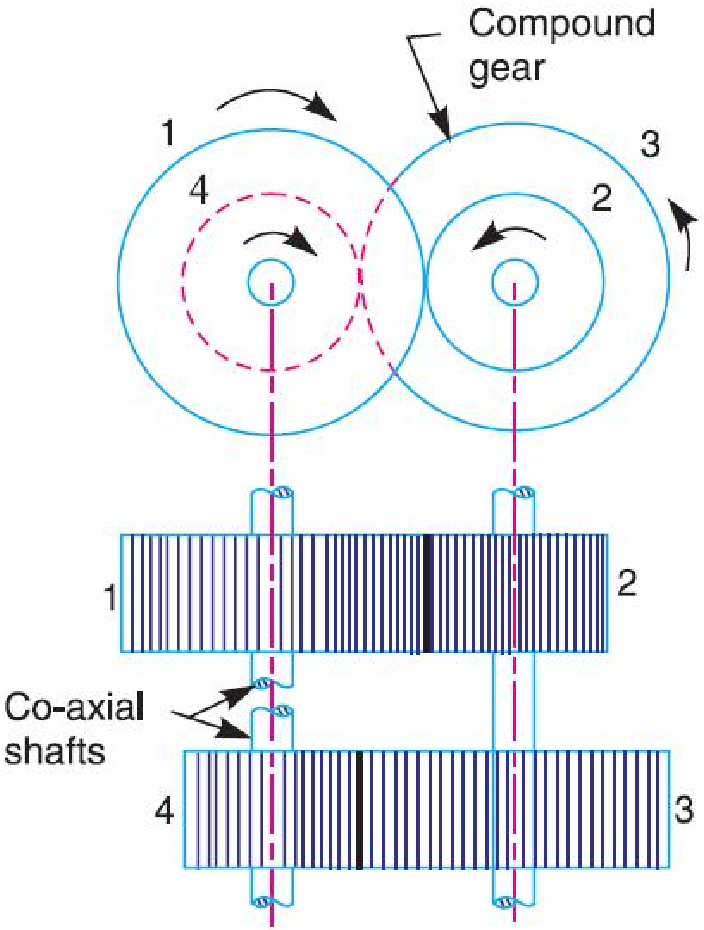
But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is increased by providing compound gears on intermediate shafts.

In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed.



Reverted Gear Train

- When the axes of the first gear (*i.e. first driver*) and the last gear (*i.e. last driven or follower*) *are co-axial*, then the gear train is known as ***reverted gear train as*** shown in Fig.
- gear 1 (*i.e. first driver*) *drives the* gear 2 (*i.e. first driven or follower*) *in the opposite direction.*
- Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2.
- The gear 3 (which is now the second driver) drives the gear 4 (*i.e. the last driven or follower*) *in the same direction as* that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is ***like.***



Let T_1 = Number of teeth on gear 1,

r_1 = Pitch circle radius of gear 1, and

N_1 = Speed of gear 1 in r.p.m.

Similarly,

T_2, T_3, T_4 = Number of teeth on respective gears,

r_2, r_3, r_4 = Pitch circle radii of respective gears, and

N_2, N_3, N_4 = Speed of respective gears in r.p.m.

Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4 \quad \text{...}(i)$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

$$\therefore \quad {}^*T_1 + T_2 = T_3 + T_4 \quad \text{...}(ii)$$

and

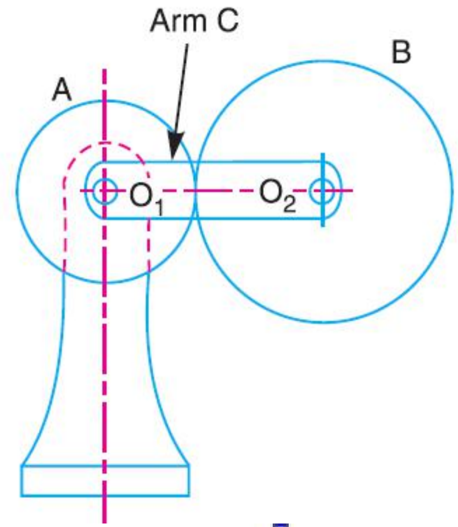
$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

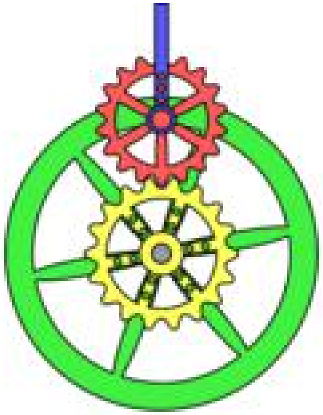
or

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \quad \text{...}(iii)$$

Epicyclic Gear Train

- In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.
- A simple epicyclic gear train is shown in Fig., where a gear *A and the arm C have a common axis at O*1 about which they can rotate.
- The gear *B meshes with gear A and has its axis on the arm at O*2, about which the gear *B can rotate.*





- If the arm is fixed, the gear train is simple and gear A can drive gear B or vice- versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate upon and around gear A.
- Such a motion is called epicyclic and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as epicyclic gear trains (epi. means upon and cyclic means around).
- The epicyclic gear trains may be simple or compound.
- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space.
- The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.