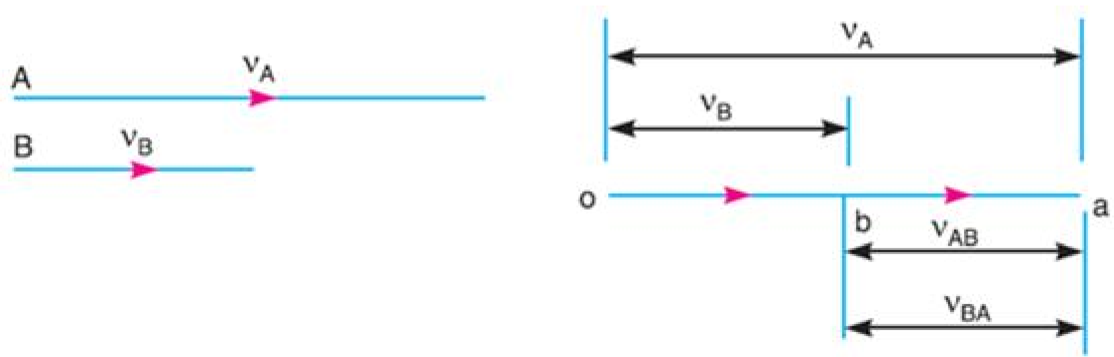
Velocity and Acceleration Analysis of Mechanisms (Graphical Methods)

Relative Velocity of Two Bodies Moving in Straight Lines

- Two bodies A and B moving along parallel lines in the same direction with absolute velocities V_A and V_B such that $V_A > V_B$, as shown in Fig. (a).
- The rela v_{AB} = Vector difference of v_A and v_B = $\overline{v_A} - \overline{v_B}$ is ....

$$\overline{ba} = \overline{oa} - \overline{ob}$$



(a) (b)

Fig. Relative velocity of two bodies moving along parallel lines.

Similarly, the relative velocity of B with respect to A, v_{BA} = Vector difference of v_B and $v_A = \overline{v_B} - \overline{v_A}$

$$\overline{ab} = \overline{ob} - \overline{oa}$$

• Motion of a link

Consider two points A and B on a rigid link AB, as shown in Fig. (*a*). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB.

Hence ***velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.***

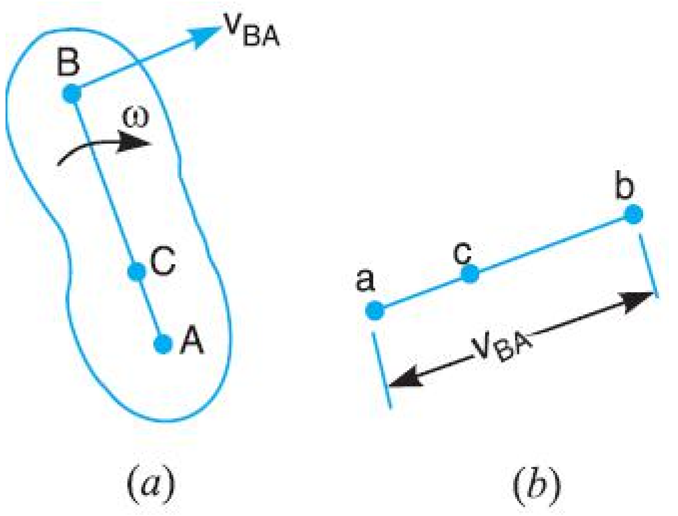


Fig. Motion of a Link.

The relative velocity of B with respect to A (*i.e.* v_{BA}) is represented by the vector ab and is perpendicular to the line AB as shown in Fig.

Let ω = Angular velocity of the link AB about A.

We know that the velocity of the point B with respect to A,

$$v_{BA} = \overline{ab} = \omega . AB \qquad ...(i)$$

Similarly, the velocity of any point C on AB with respect to A, $v_{CA} = \overline{ac} = \omega . AC$ ...(*ii*)

From equations (*i*) and (*ii*),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \qquad ...(iii)$$

Thus, we see from equation (*iii*), that the point c on the vector ab divides it in the same ratio as C divides the link AB.

Rubbing Velocity at pin joint

The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as **the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links OA and OB connected by a pin joint at O

Let ω_1 = Angular velocity of the link OA or the angular velocity of the point A with respect to O.

ω_2 = Angular velocity of the link OB or the angular velocity of the point B with respect to O, and

r = Radius of the pin.

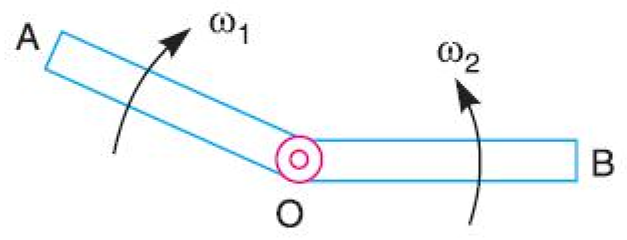


Fig. Links connected by pin joints.

According to the definition,

Rubbing velocity at the pin joint O

$= (\omega_1 - \omega_2)\, r$, if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$, if the links move in the opposite direction

Note : When the pin connects one sliding member and the other turning member, the angular velocity of the sliding member is zero. In such cases,

Rubbing velocity at the pin joint = $\omega.r$

where ω = Angular velocity of the turning member, and

r = Radius of the pin.

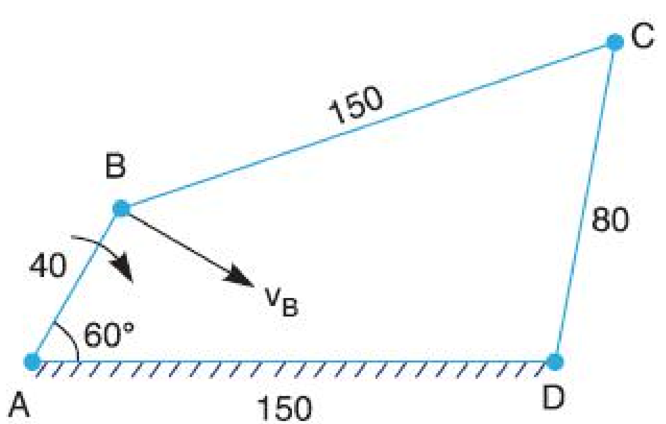
EX. In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.

Given Data :

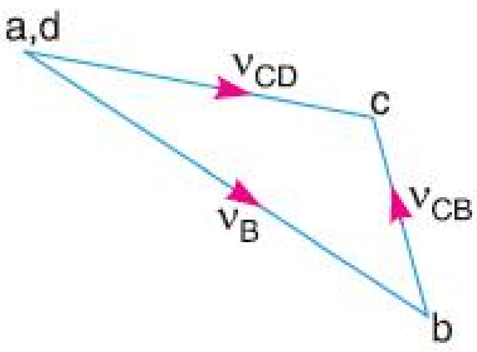
$$N_{BA} = 120 \text{ r.p.m. or } \omega_{BA} = 2\pi \times 120/60 = 12.568 \text{ rad/s}$$

Since the length of crank AB = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$



(a) Space diagram (All dimensions in mm).



(b) Velocity diagram.

First of all, draw the space diagram to some suitable scale, as shown in Fig. (*a*). Now the velocity diagram, as shown in Fig. (*b*), is drawn as discussed below :

1. Since the link *AD* is fixed, therefore points *a* and *d* are taken as one point in the velocity diagram. Draw vector *ab* perpendicular to *BA*, to some suitable scale, to represent the velocity of *B* with respect to *A* or simply velocity of *B* (*i.e.* v_{BA} or v_B) such that

$$\text{vector } ab = v_{BA} = v_B = 0.503 \text{ m/s}$$

2. Now from point *b*, draw vector *bc* perpendicular to *CB* to represent the velocity of *C* with respect to *B* (*i.e.* v_{CB}) and from point *d*, draw vector *dc* perpendicular to *CD* to represent the velocity of *C* with respect to *D* or simply velocity of *C* (*i.e.* v_{CD} or v_C). The vectors *bc* and *dc* intersect at *c*.

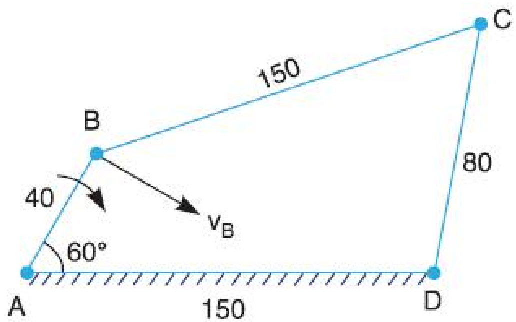
By measurement, we find that

$$v_{CD} = v_C = \text{vector } dc = 0.385 \text{ m/s}$$

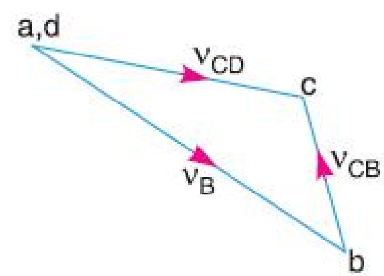
We know that $CD = 80 \text{ mm} = 0.08 \text{ m}$

$\therefore$ Angular velocity of link *CD*,

$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \text{ rad/s (clockwise about } D)$$ **Ans.**



(*a*) Space diagram (All dimensions in mm).

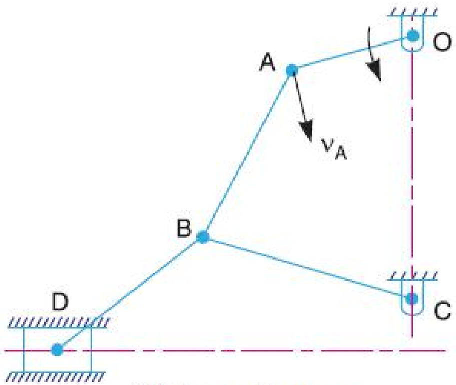


(*b*) Velocity diagram.

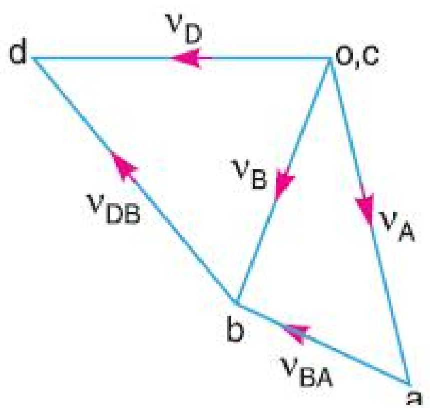
Example

- In Fig. the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical.

  The dimensions of various links are : OA = 28 mm ; AB = 44 mm ; BC 49 mm ; and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.



(*a*) Space diagram.



(*b*) Velocity diagram.

By measurement, we find that velocity of the slider *D*,

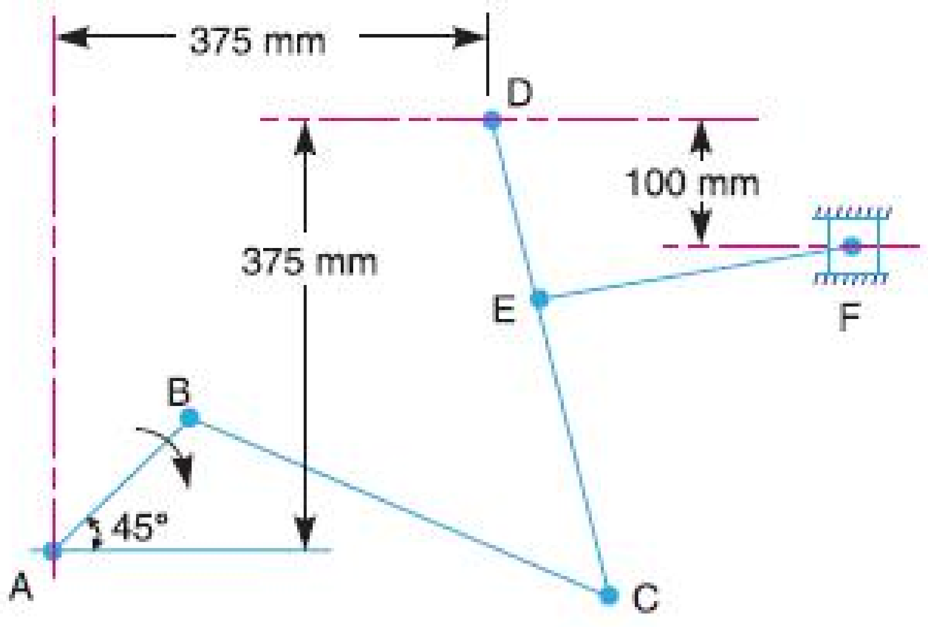
$$v_D = \text{vector } od = 1.6 \text{ m/s} \quad \textbf{Ans.}$$

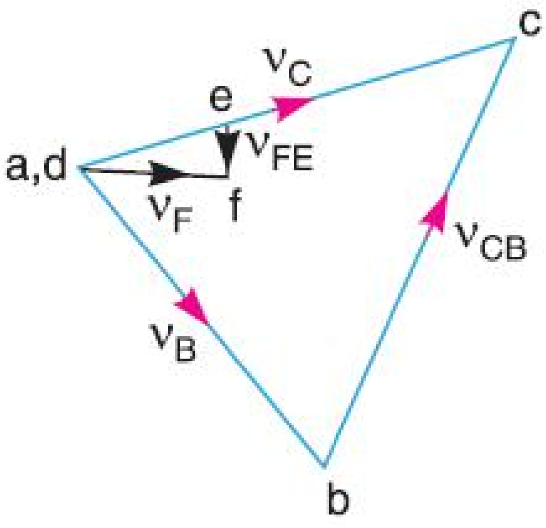
$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B\text{)}$$

Example:

The mechanism, as shown in Fig. , has the dimensions of various links as follows : AB = DE = 150 mm ; BC = CD = 450 mm ; EF = 375 mm.

The crank AB makes an angle of 45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m.





(b) Velocity diagram.

Acceleration Diagram for a Link

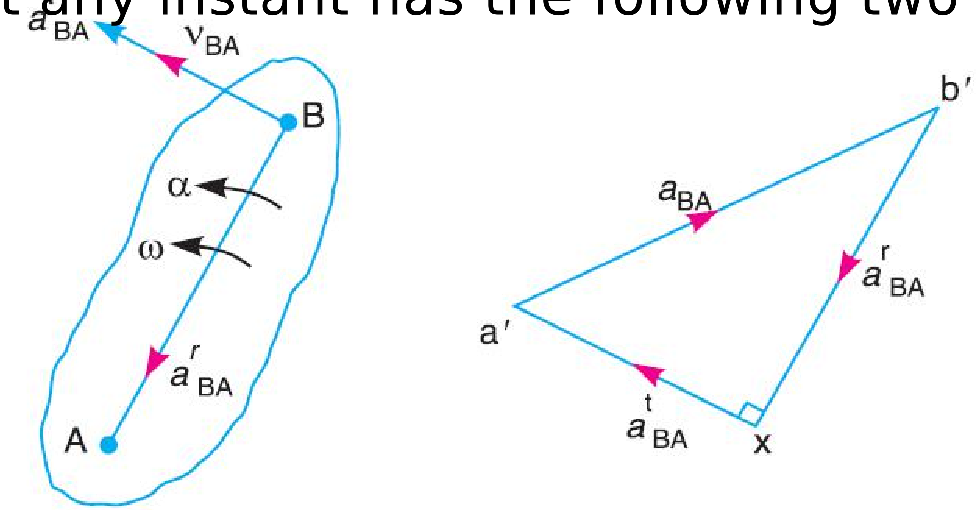
Consider two points *A and B on a rigid link as shown* in Fig.(*a).*

Let the point B moves with respect to A,

With an angular velocity of ω rad/s

And angular acceleration l α rad/s2 of the link *AB.*

Acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components :



(*a*) Link. (*b*) Acceleration diagram.

1. **The centripetal or radial component**

   Which is perpendicular to the velocity of the particle at the given instant.

1. **The tangential component**

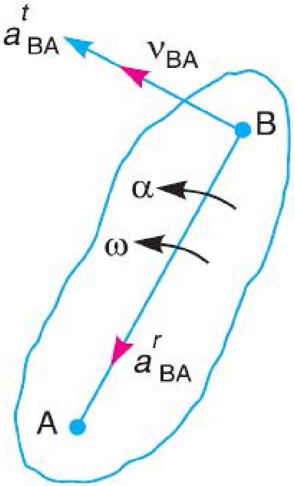
   Which is parallel to the velocity of the particle at the given instant.

- Velocity of point B with respect to A (i.e. V_{BA}) is perpendicular to the link AB as shown in Fig. (a).
- Centripetal or radial component of the acceleration of B with respect to A,

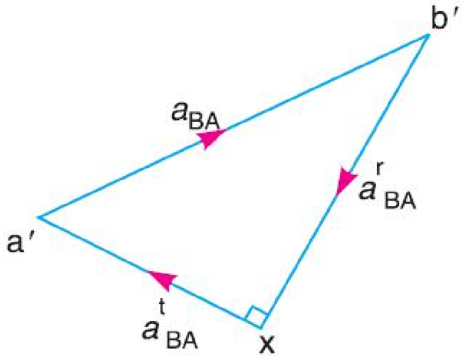
$$a^r_{BA} = \omega^2 \times \text{Length of link } AB$$

$$= \omega^2 \times AB = v^2_{BA} / AB \qquad \ldots \left(\because \omega = \frac{v_{BA}}{AB} \right)$$

Radial component of acceleration acts perpendicular to the velocity v_{BA}, In other words, it acts ***parallel to the link AB.***



(a) Link.



(b) Acceleration diagram.

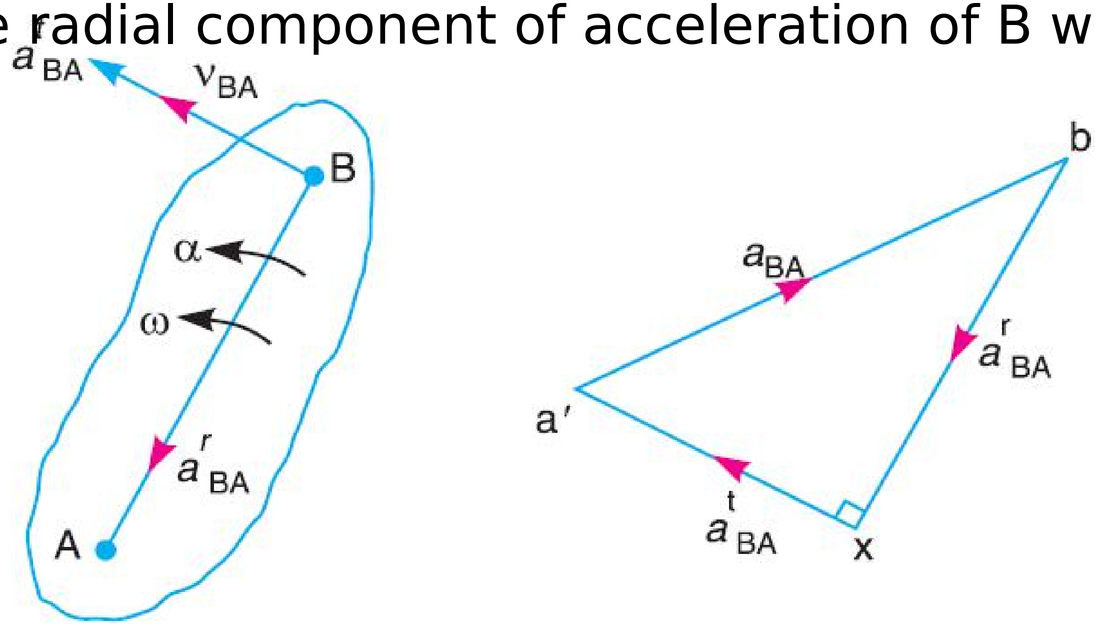
We know that tangential component of the acceleration of B with respect to A,

$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

Tangential component of acceleration acts parallel to the velocity V_{BA}.

In other words, it acts perpendicular to the link A B.

In order to draw the acceleration diagram for a link A B, as shown in Fig. From any point b', draw vector b' a^r_{BA} parallel to BA to represent the radial component of acceleration of B with respect to A i.e.

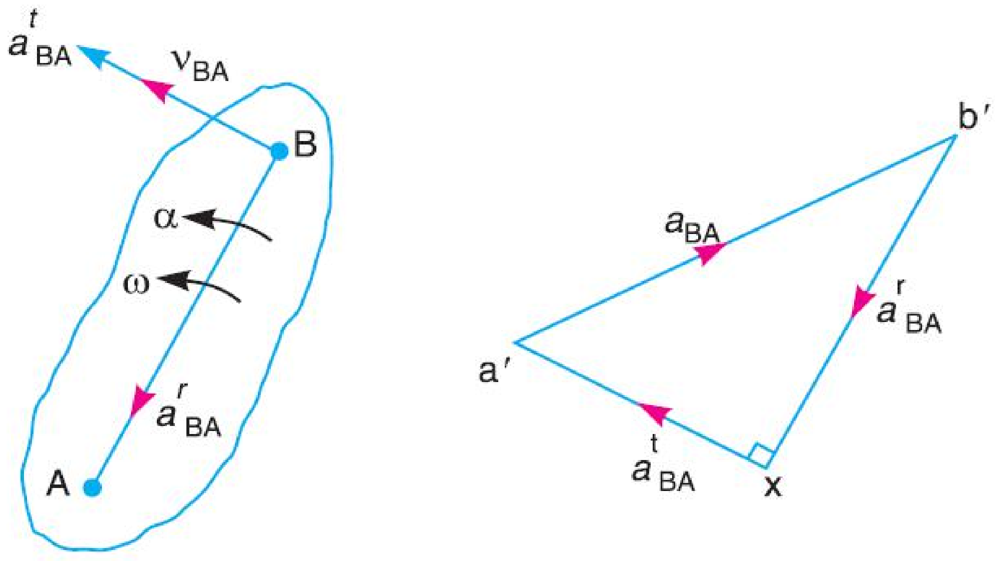


(*a*) Link. (*b*) Acceleration diagram.

From point x draw vector xa' perpendicular to B A to represent the tangential component of acceleration of B with respect to A .

Join b' a'. The vector b' a' (known as acceleration image of the link A B) represents the total acceleration of B with respect to A.

It is the vector sum of radial component and tangential component of acceleration.



(*a*) Link. (*b*) Acceleration diagram.

Ex. The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine :

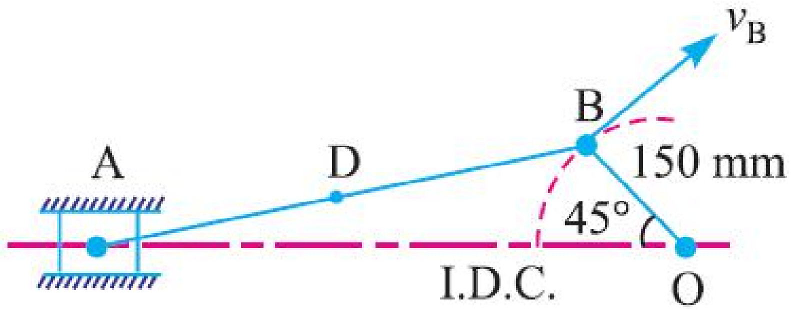
1. linear velocity and acceleration of the midpoint of the connecting rod, and
2. Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

Solution. Given : $N_{BO} = 300$ r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; $OB = 150$ mm = 0.15 m ; $BA = 600$ mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$

1. First of all draw the space diagram, to some suitable scale; as shown in Fig. (*a*).



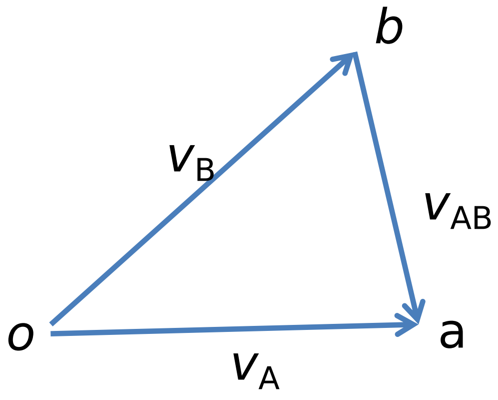
(*a*) Space diagram.

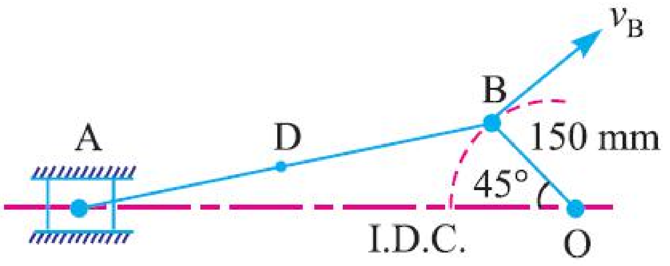
To Draw Velocity Vector polygon

1. Draw vector *ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B i.e.* v_{BO} or v_B, such that vector *ob* = $v_{BO} = v_B$ = 4.713 m/s
2. From point *b, draw vector ba perpendicular to BA to represent the velocity of A with* respect to *B i.e.* v_{AB} , and from point *o draw vector oa parallel to the motion of A (which is along AO)* to represent the velocity of *A i.e.* v_A. The vectors *ba and oa intersect at a.*
3. By measurement, we find that velocity of *A with respect to B,*

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$





(*a*) Space diagram.

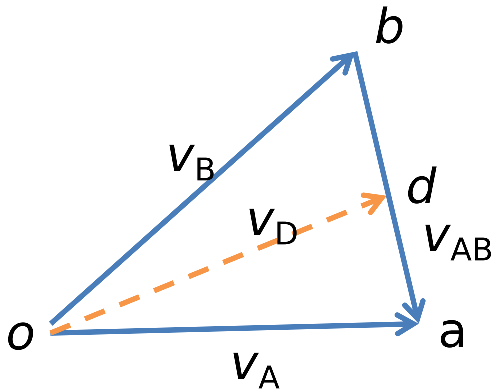
4. In order to find the velocity of the midpoint *D of the connecting rod AB, divide the vector ba at d in the same ratio as D divides AB, in the space diagram.*

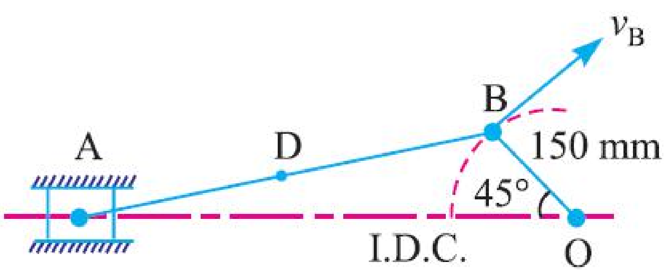
   In other words, bd / ba = BD/BA

Note: Since *D is the midpoint of AB, therefore d is also midpoint of vector ba.*

5. Join *od. Now the vector od represents the velocity of the midpoint D of the connecting* rod *i.e.* v_D.

   By measurement, we find that v_D = vector *od = 4.1 m/s*





(*a*) Space diagram.

Acceleration of the midpoint of the connecting rod

- We know that the radial component of the acceleration of B with respect to O or the acceleration o

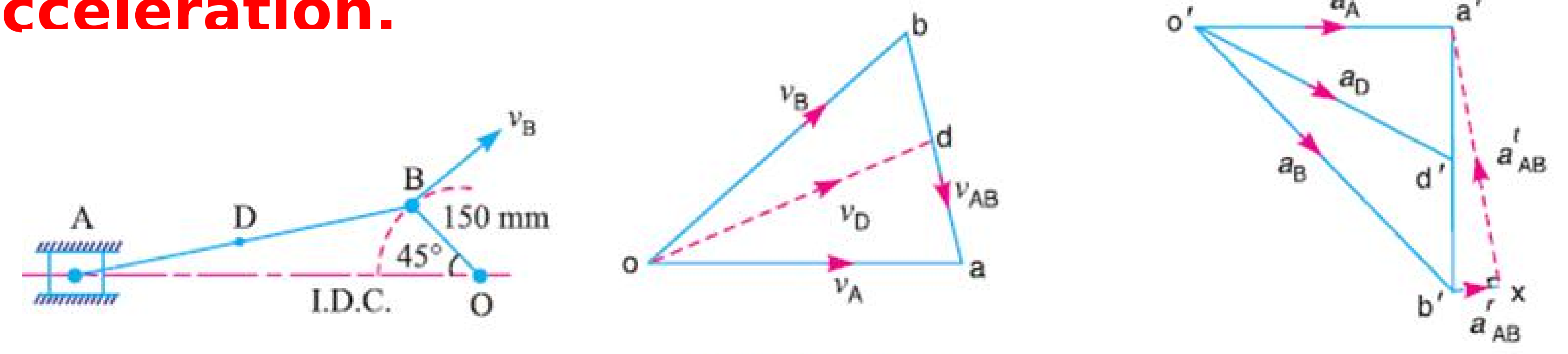
$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleration of *A with respect to B,*

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

NOTE: 1) A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.

2) When a point moves along a straight line, it has no centripetal or radial component of the acceleration.



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Methods for Determining the Velocity of a Point on a Link

1. Instantaneous centre method

2. Relative velocity method

The instantaneous centre method of analysing the motion in a mechanism is based upon the concept that any “Displacement of a body (or a rigid link) having motion in one plane, can be considered as a pure rotational motion of a rigid link as a whole about some centre”, known as instantaneous centre or virtual centre of rotation.

A body having motion in a plane can be considered to have a motion of rotation as well as translation, such as wheel of a car, a sphere rolling (but not slipping) on the ground. Such a motion will have the combined effect of rotation and translation.

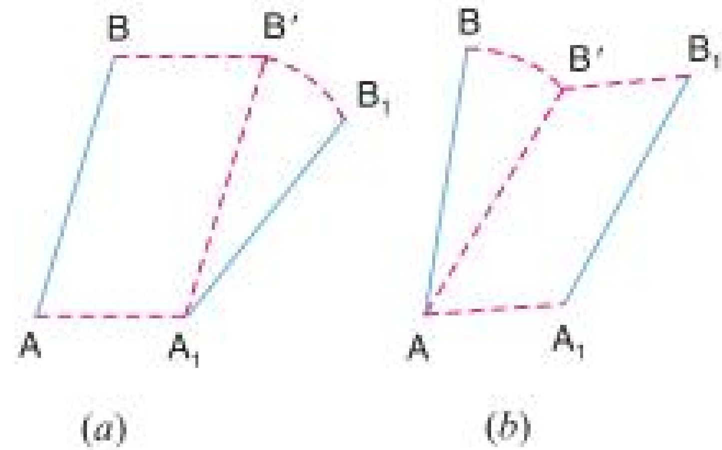


Fig. Motion of a link.

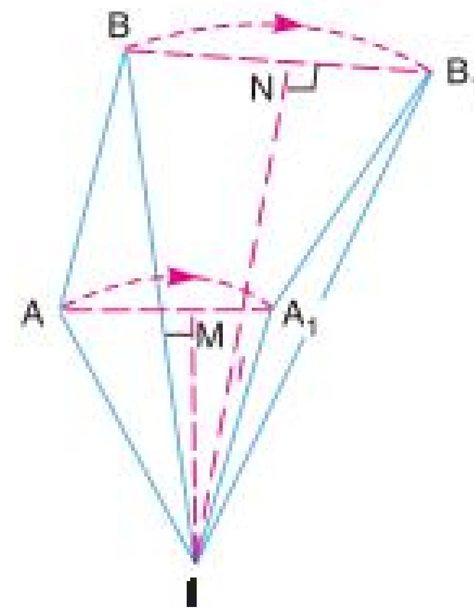


Fig. Instantaneous centre of rotation.

Number of Instantaneous Centres in a Mechanism

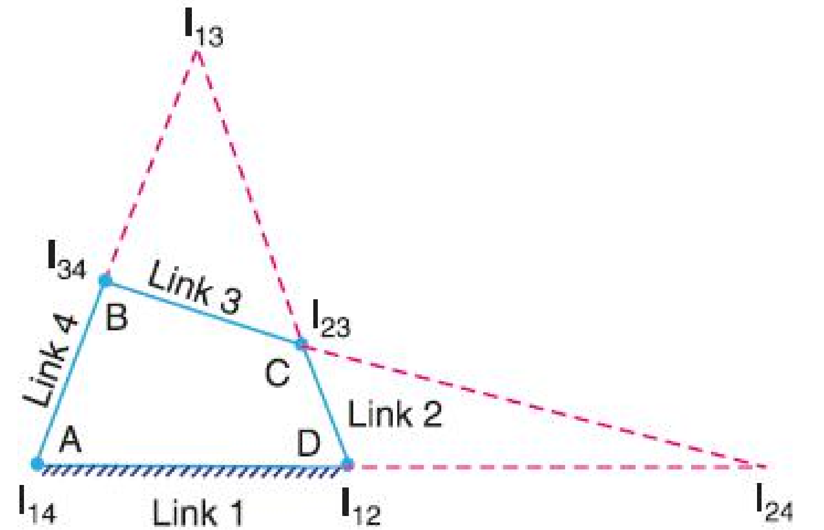
- The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links.
- Mathematically, number of instantaneous centers

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$

Types of Instantaneous Centres

1. Fixed instantaneous centres
2. Permanent instantaneous centres
3. Neither fixed nor permanent instantaneous centres.

The first two types i.e. fixed and permanent instantaneous centres are together known as primary instantaneous centres and the third type is known as secondary instantaneous centres.

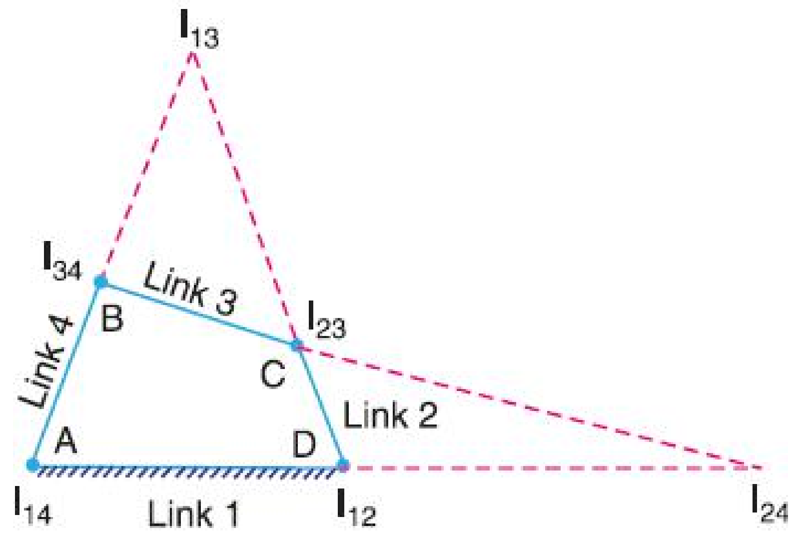


Types of instantaneous centres.

Types of Instantaneous Centres

- Consider a four bar mechanism *ABCD as* shown in Fig..
- The number of instantaneous centres (*N) in a four bar mechanism is given by*

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$



Types of instantaneous centres.

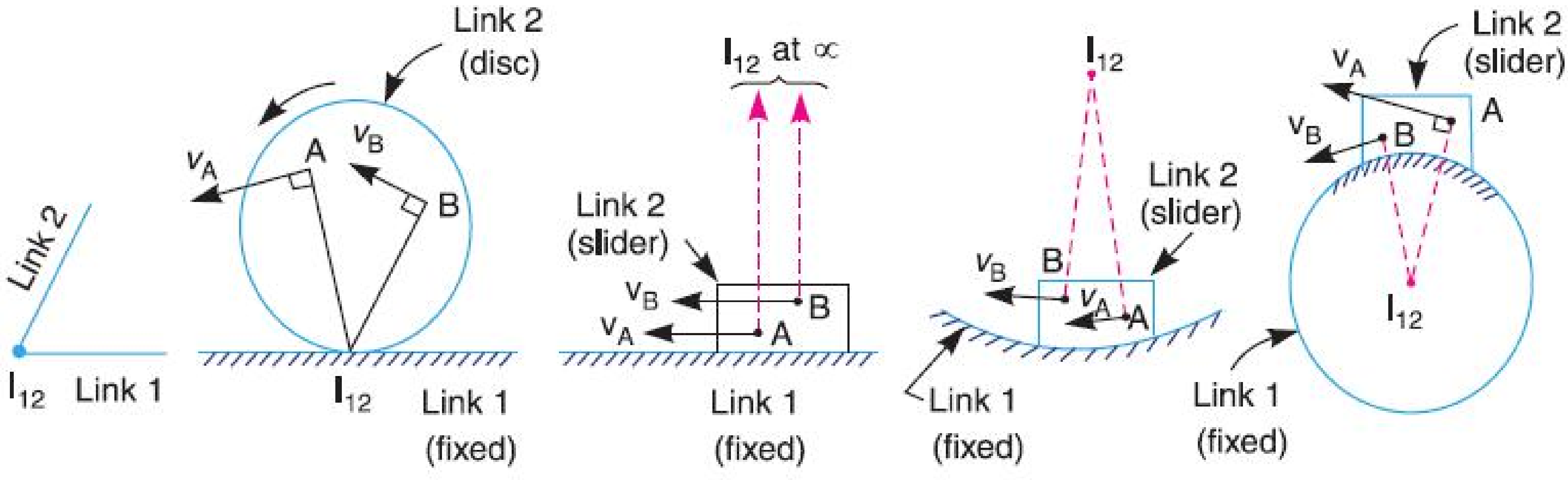
The instantaneous centres I_{12} and I_{14} are called the fixed instantaneous centres as they remain in the same place for all configurations of the mechanism.

The instantaneous centres I_{23} and I_{34} are the permanent instantaneous centres as they move when the mechanism moves, but the joints are of permanent nature.

The instantaneous centres I_{13} and I_{24} are neither fixed nor permanent instantaneous centres as they vary with the configuration of the mechanism.

Location of Instantaneous Centres

When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies on the centre of the pin as shown in **Fig.(a).** Such a instantaneous centre is of permanent nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.



(a) (b) (c) (d) (e)

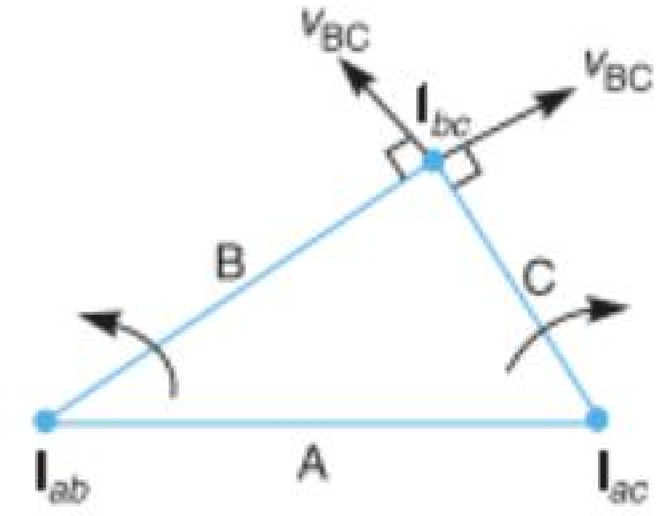
When the two links have a pure rolling contact (*i.e. link 2 rolls without slipping upon the* fixed link 1 which may be straight or curved), the instantaneous centre lies on their point of contact, as shown in **Fig.(*b*).** *The velocity of any point A on the link 2 relative to fixed link 1 will be* perpendicular to $I12A$ *and is proportional to* $I12A$.

When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact. We shall consider the following three cases :

- ***(a)*** *When the link 2 (slider) moves on fixed link 1 having straight surface as shown in* ***Fig.(c),*** *the instantaneous centre lies at infinity and each point on the slider have the same velocity.*
- **(*b*)** *When the link 2 (slider) moves on fixed link 1 having curved surface as shown in* **Fig.(*d*),***the instantaneous centre lies on the centre of curvature of the curvilinear path* in the configuration at that instant.
- **(*c*)** *When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as* shown in **Fig.(*e*),** *the instantaneous centre lies at the centre of curvature i.e. the* centre of the circle, for all configuration of the links.

Aronhold Kennedy (or Three Centres in Line) Theorem

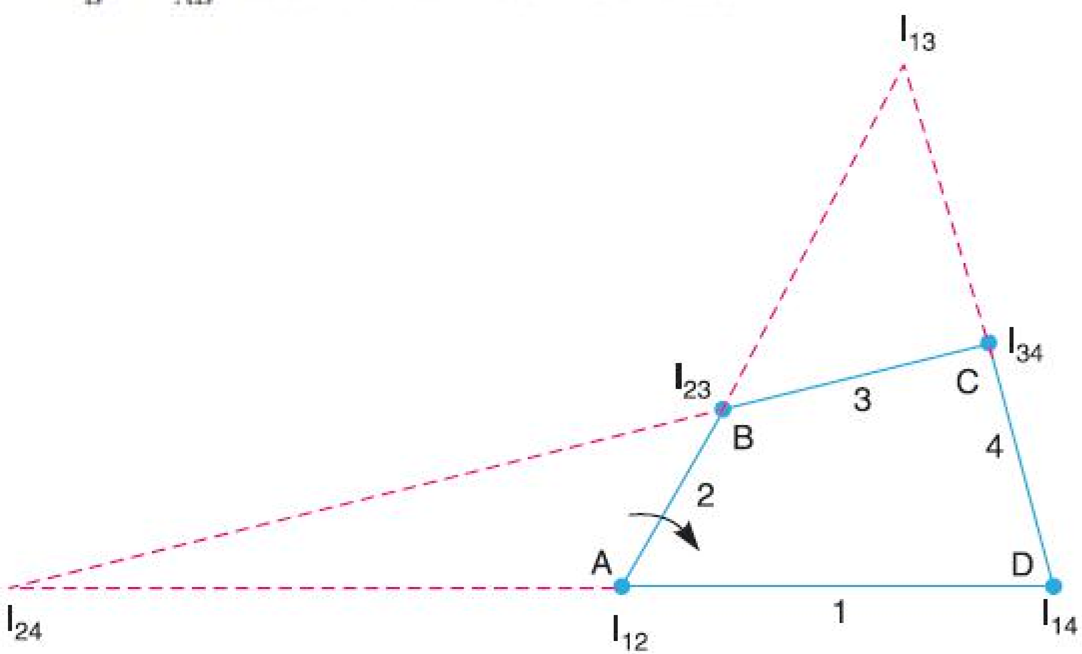
- The Aronhold Kennedy's theorem states that **if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.**

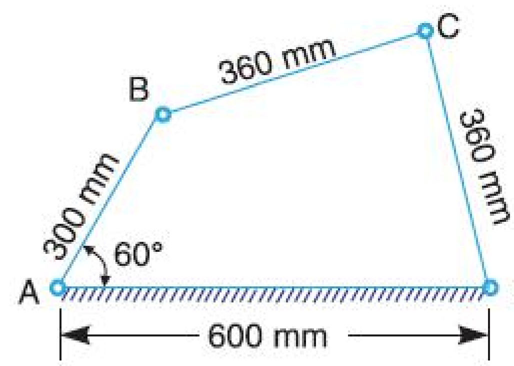


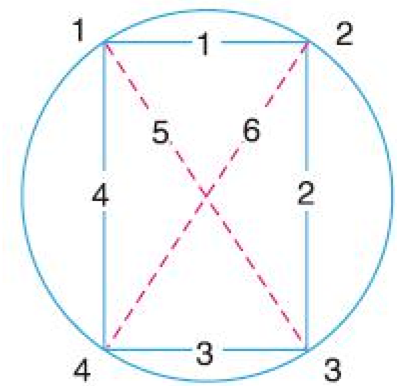
In a pin jointed four bar mechanism, as shown in Fig, AB = 300 mm, BC = CD = 360 mm, and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find angular velocity of link BC.

$\omega_{AB} = 2\,\pi \times 100/60 = 10.47$ rad/s

$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141$ m/s





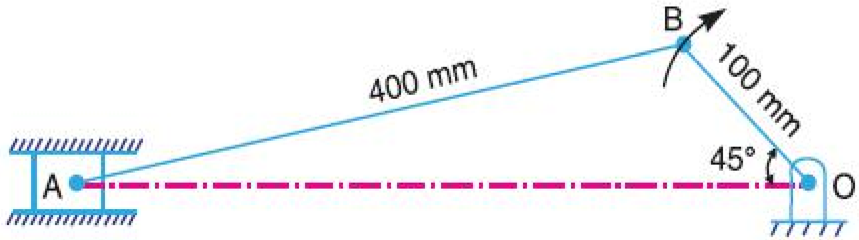


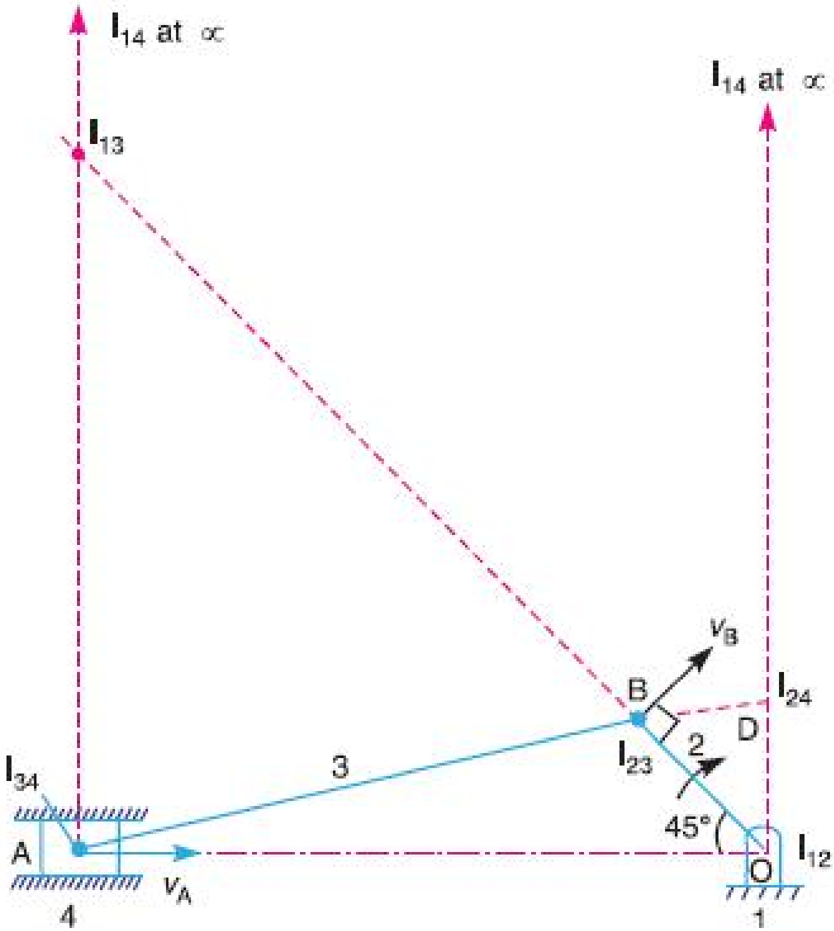
$$v_B = \omega_{BC} \times I_{13}B$$

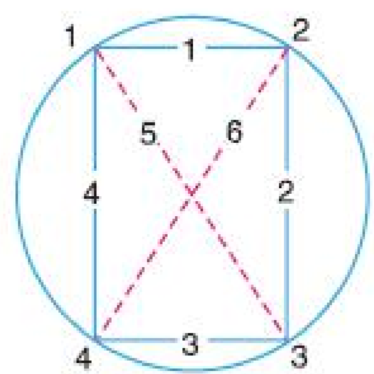
By measurement, we find that $I_{13}B$ = 500 mm = 0.5 m

$$\therefore \quad \omega_{BC} = \frac{v_B}{I_{13}B} = \frac{3.141}{0.5} = 6.282 \text{ rad/s} \quad \textbf{Ans.}$$

Locate all the instantaneous centres of the slider crank mechanism as shown in Fig. The lengths of crank OB and connecting rod AB are 100 n







A mechanism, as shown in Fig. has the following dimensions:

OA = 200 mm; AB = 1.5 m; BC = 600 mm; CD = 500 mm and BE = 400 mm. Locate all the instantaneous centres.

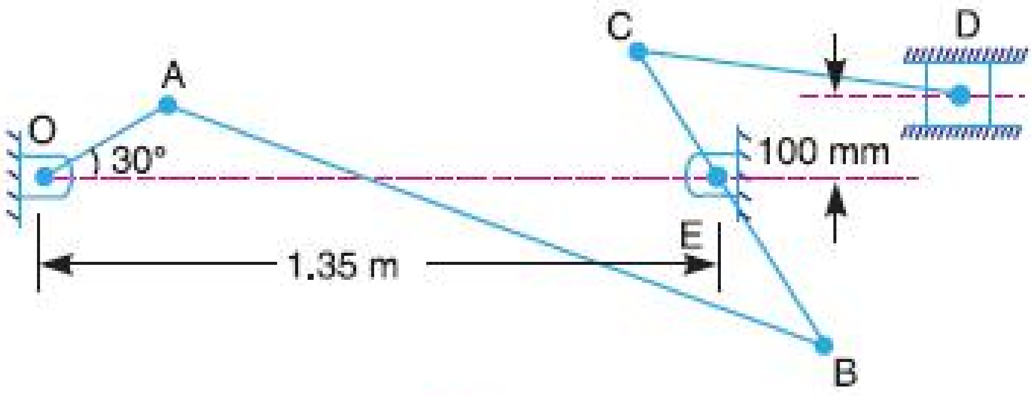
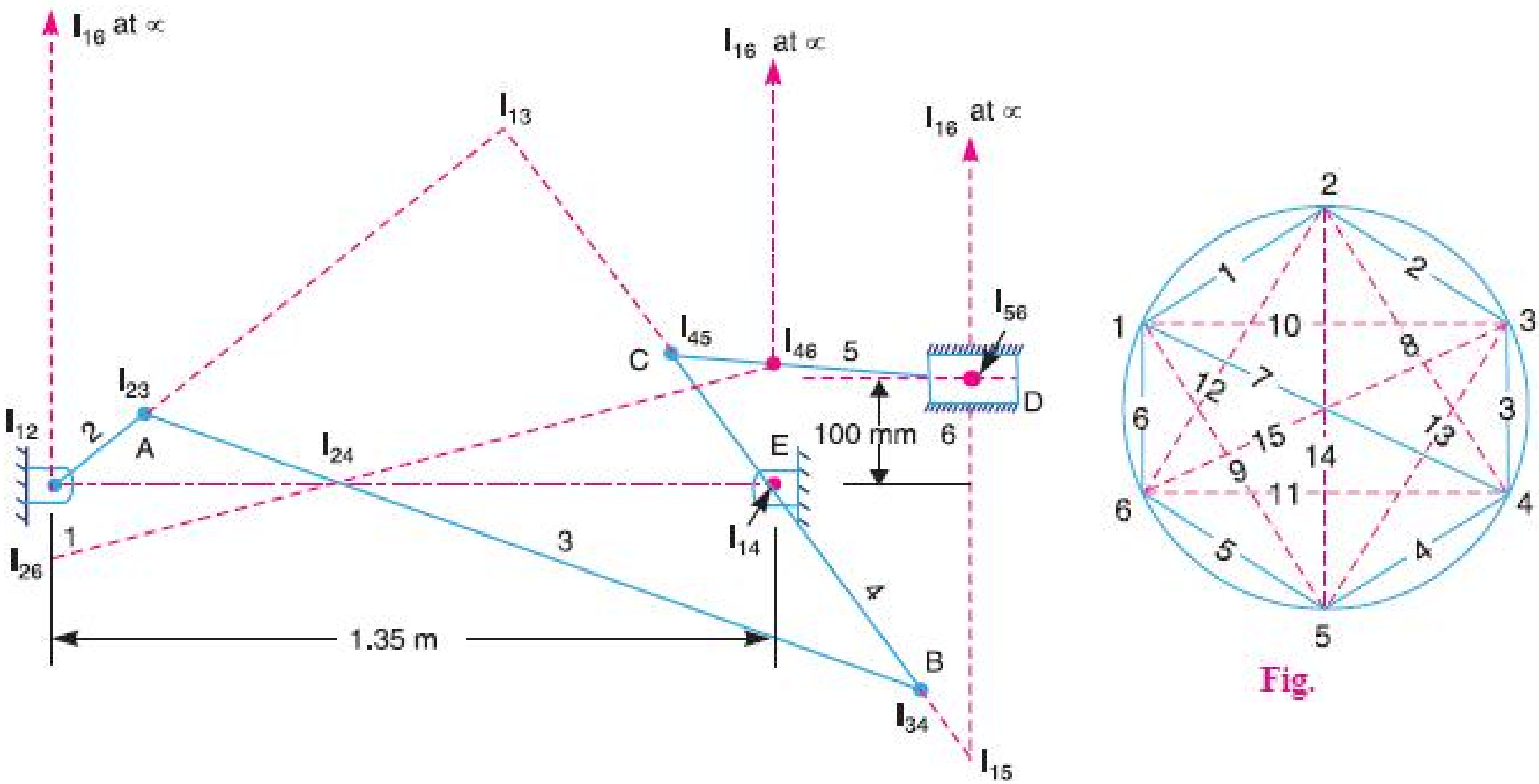


Fig.

Links	1	2	3	4	5	6
Instantaneous centres (15 in number)	12 13 14 15 16	23 24 25 26	34 35 36	45 46	56	



I16 at ∝
I16 at ∝
I16 at ∝
I13
I45
I46
5
I56
C
D
I23
I12
2
A
I24
E
100 mm
6
1
I26
3
I14
4
1.35 m
B
I34
I15
2
1
2
1
10
3
8
12
7
6
3
15
13
14
9
6
11
4
5
4
5

Fig.