Transient and steady state analysis – Unit 2

Contents

Transient response analysis

Standard test signals, First-order and second order systems, Higher order systems, Transient response of system, Steady-state error for unit , ramp and parabolic inputs.

Introduction

- In time-domain analysis the response of a dynamic system to an input is expressed as a function of time.
- It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.
- Usually, the input signals to control systems are not known fully ahead of time.
- For example, in a radar tracking system, the position and the speed of the target to be tracked may vary in a random fashion.
- It is therefore difficult to express the actual input signals mathematically by simple equations.

Standard Test Signals

- The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity, and constant acceleration.

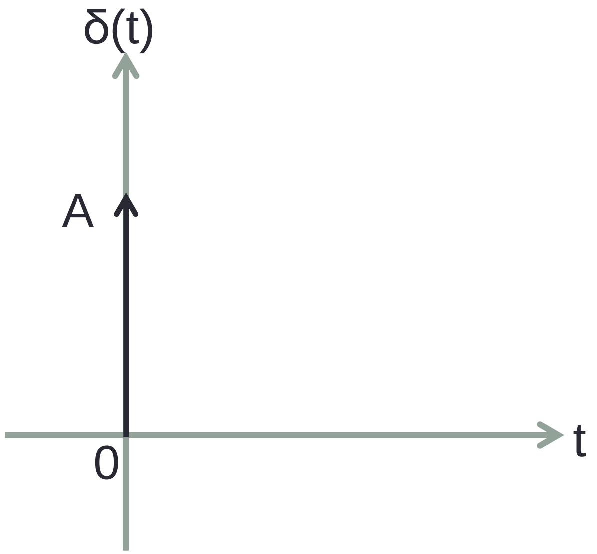
- The dynamic behavior of a system is therefore judged and compared under application of standard test signals – an impulse, a step, a constant velocity, and constant acceleration.

- Another standard signal of great importance is a sinusoidal signal.

Standard Test Signals

- Impulse signal
  - The impulse signal imitate the sudden shock characteristic of actual input signal.

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

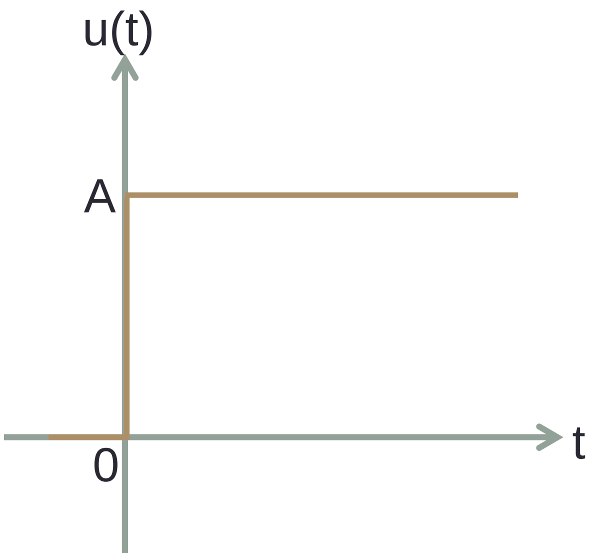


  - If A=1, the impulse signal is called unit impulse signal.

Standard Test Signals

- Step signal
  - The step signal imitate the sudden change characteristic of actual input signal.

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

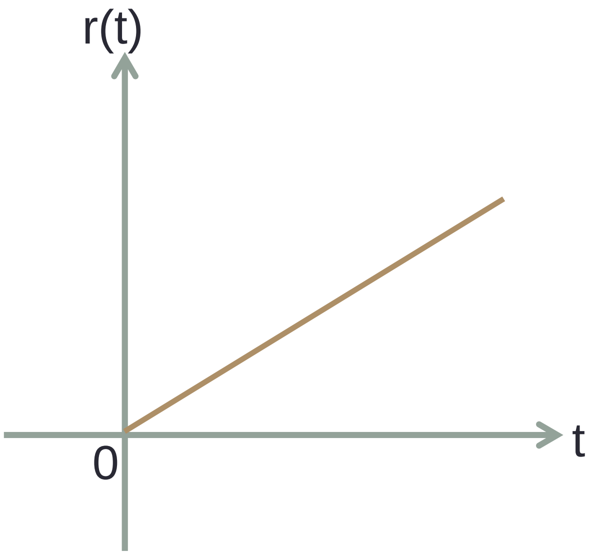


  - If A=1, the step signal is called unit step signal

Standard Test Signals

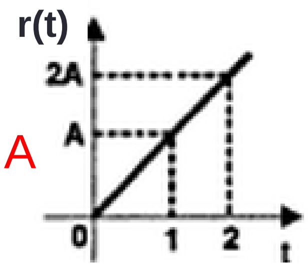
- Ramp signal
  - The ramp signal imitate the constant velocity characteristic of actual input signal.

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

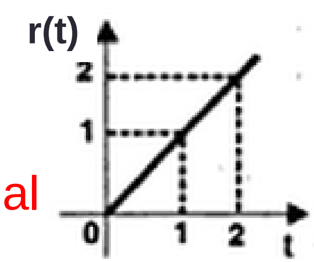


  - If *A=1*, the ramp signal is called unit ramp signal

ramp signal with slope A



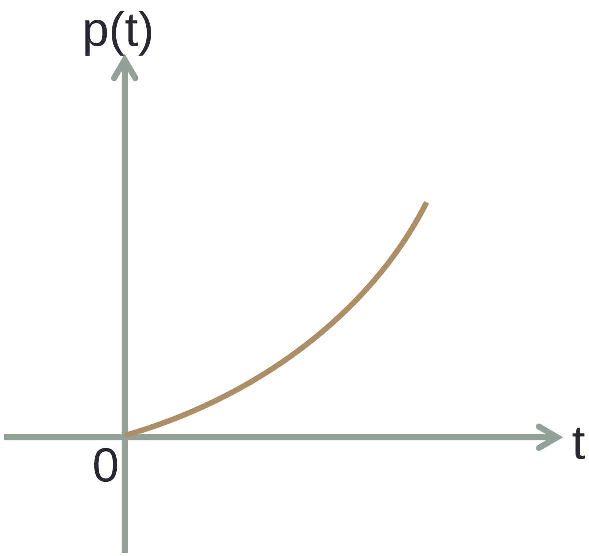
unit ramp signal



Standard Test Signals

- Parabolic signal
  - The parabolic signal imitate the constant acceleration characteristic of actual input signal.

$$p(t) = \begin{cases} \dfrac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$



  - If *A=1*, the parabolic signal is called unit parabolic signal.

Relation between standard Test Signals

- Impulse

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$\int$ $\qquad$ $\frac{d}{dt}$

- Step

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\int$ $\qquad$ $\frac{d}{dt}$

- Ramp

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\int$ $\qquad$ $\frac{d}{dt}$

- Parabolic

$$p(t) = \begin{cases} \dfrac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Laplace Transform of Test Signals

- Impulse

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$L\{\delta(t)\} = \delta(s) = A$$

- Step

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{u(t)\} = U(s) = \frac{A}{S}$$

Laplace Transform of Test Signals

- Ramp

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{r(t)\} = R(s) = \frac{A}{s^2}$$

- Parabolic

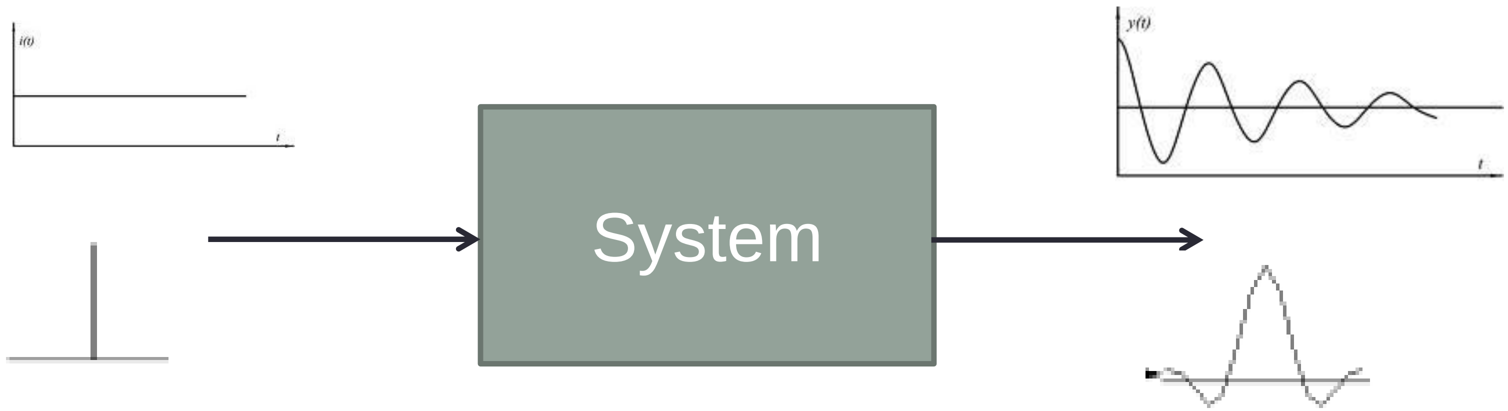
$$p(t) = \begin{cases} \frac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{p(t)\} = P(s) = \frac{A}{S^3}$$

Laplace transform of unit step, unit ramp, unit parabolic & unit impulse???

Time Response of Control Systems

- Time response of a dynamic system is response to an input expressed as a function of time.



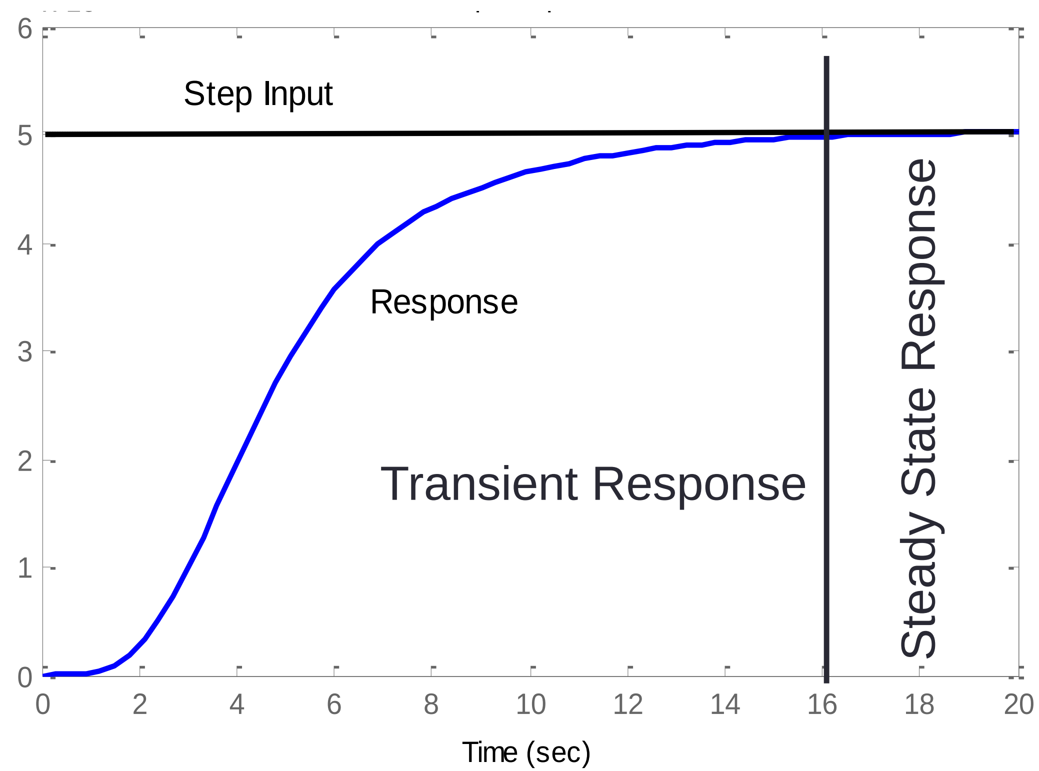
- The time response of any system has two components
  - Transient response
  - Steady-state response.

Time Response of Control Systems

- When the response of the system is changed form rest or equilibrium it takes some time to settle down.

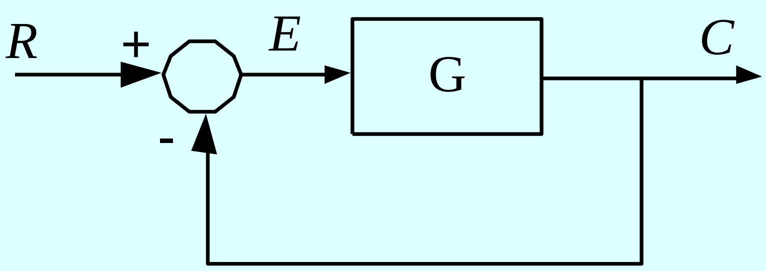
- Transient response is the response of a system from rest or equilibrium to steady state.

- The response of the system after the transient response is called steady state response.
- How far away actual output is reached from its desired value which is called as steady state error (ess) or its a difference between the actual output and the desired output.



Steady state Error

The same unity-feedback system



$$\frac{E}{R} = \frac{1}{1+G}$$

$$E(s) = \frac{1}{1+G(s)} R(s)$$

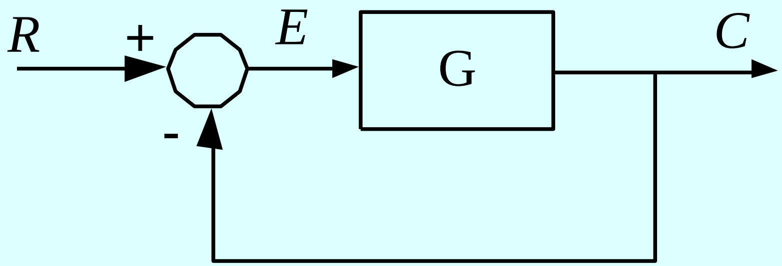
and the steady-state error is $e_{ss} = \lim_{t\to\infty} e(t)$

$$= \lim_{s\to 0} sE(s)$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

System Type

Consider the unity-feedback system



With $$G(s)=\frac{K(T_a s+1)(T_b s+1)\cdots(T_m s+1)}{s^N(T_1 s+1)(T_2 s+1)\cdots(T_p s+1)}$$ ... (2-1)

The parameter N associated with the term S^N in the denominator represents the "Type" of the system. Example: Type 0 if N=0, Type 1 if N=1 and so on.

The higher the type number, the better the steady-state accuracy of the closed-loop control system.

However, the higher the system type, the greater the problem with system stability.

Classification of Control Systems

- Consider the unity-feedback control system with the following open-loop transfer function

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N(T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

- It involves the term s^N in the denominator, representing N poles at the origin.
- A system is called type 0, type 1, type 2, ... , if N=0, N=1, N=2, ... , respectively.

$$E(s) = \frac{R(s)}{1 + G(s).H(s)}$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

$$e_{ss} = \frac{1}{1 + K_p}$$

$$e_{ss} = \frac{1}{K_v}$$

$$e_{ss} = \frac{1}{k_a}$$

		Type 0		Type 1		Type 2	
Input	**Laplace**	**Static Error Constant**	**e_{ss}**	**Static Error Constant**	**E_{ss}**	**Static Error Constant**	**e_{ss}**
Unit Step	1/s	K_p = K	1/(1+k)	K_p = Infinity	0	K_p = Infinity	0
Unit Ramp	$1/s^2$	K_v = 0	Infinity	K_v = K	1/k	K_v = Infinity	0
Unit parabola	$1/s^3$	K_a = 0	Infinity	K_a = 0	Infinity	K_a = k	1/k

An observation of the table gives the following information:

(1) For type 0 systems, the ramp and parabolic input are not acceptable.

(2) For type 1 systems, the parabolic input is not acceptable.

(3) For type 2 systems, all the three inputs are acceptable.

(4) Finite steady state error varies in inverse proportion to the forward path gain.

(5) All the steady state error are positional in nature.

Example

1) Determine the Kp , Kv and Ka and steady state error for a system given by

$$G(s).H(s) = \frac{100(s+2)(s+40)}{s^3(s^2+4s+200)}$$

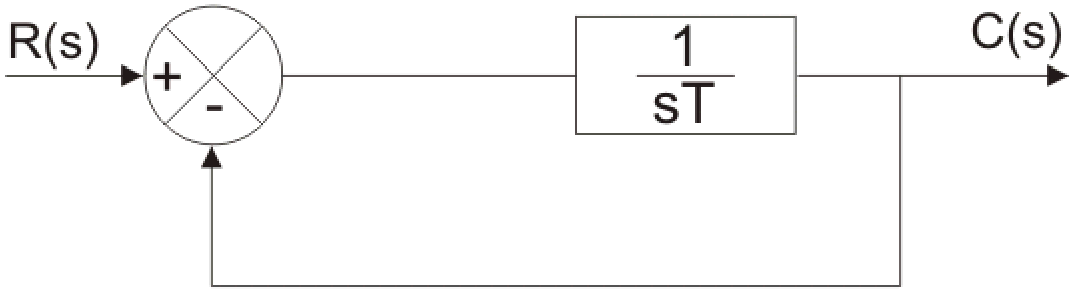
2) The open loop transfer function of a control system is given below:

$$G(s).H(s) = \frac{2(s^2+3s+20)}{s(s+2)(s^2+4s+10)}$$

Determine the static error coefficients and steady state error for the input given below.

(a) 5   (b) 4t   (c) $4t^2/2$.

Time Response of a First Order Control System



Test signal is Unit step function, R(s)=1/s

Continue…

- A first-order system without zeros can be represented by the following transfer function

$$\frac{C(s)}{R(s)} = \frac{1}{sT + 1}$$

- Given a step input, i.e., R(s) = 1/s , then the system output (called step response in this case) is

$$C(s) = \frac{1}{s} . \frac{1}{sT + 1}$$

Continue…

- Applying Partial fraction we get

$$C(s) = \frac{1}{s} - \frac{T}{sT+1}$$

$$C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

- Taking Inverse Laplace transform

$$L^{-1}C(s) = L^{-1}[\frac{1}{s} - \frac{1}{s+\frac{1}{T}}]$$

Continue…

$$c(t) = 1 - e^{\frac{-t}{T}}$$

The error signal is given by

$$e(t) = r(t) - c(t) = 1 - (1 - e^{\frac{-t}{T}})$$

$$= e^{\frac{-t}{T}}$$

So steady state error is zero.

Time response analysis of first order system subjected to unit ramp input function

- The output for the system expressed as

$$c(s) = R(s).\frac{1}{sT+1}$$

- As the input is unit ramp function
- $r(t) = t$ and $R(s) = 1/s^2$
- Therefore
- $$c(s) = \frac{1}{s^2}.\frac{1}{sT+1}$$

Continue..

- Applying partial fraction

$$C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{sT+1}$$

- Taking Inverse Laplace transform

$$L^{-1}[C(s)] = L^{-1}[\frac{1}{s^2} - \frac{T}{s} + \frac{T}{s+\frac{1}{T}}]$$

Continue..

- In Time Domain we get

$$C(t) = t - T + Te^{\frac{-t}{T}}$$

- Error is given as
-

$$e(t) = r(t) - c(t) = t - (t - T + Te^{\frac{-t}{T}})$$

$$= T - Te^{\frac{-t}{T}}$$

Time response of a first order control system subjected to unit impulse input function

- The output for the system expressed as

$$c(s) = R(s).\frac{1}{sT+1}$$

- As the input is unit Impulse function
- and R(s) = 1
- Therefore
- $$c(s) = \frac{1}{sT+1}$$

Continue..

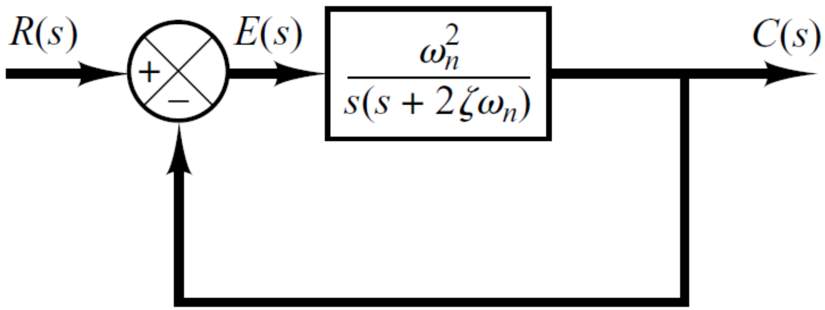
- Taking Inverse Laplace transform

$$L^{-1}c(s) = L^{-1}\frac{1}{T}[\frac{1}{s+\frac{1}{T}}]$$

$$C(t) = \frac{1}{T}e^{\frac{-t}{T}}$$

Time response analysis of Second order system

- A general second-order system is characterized by the following transfer function.



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Here, ζ and ω_n are damping ratio and natural frequency of the system respectively

Continue..

- Therefore, the output of the system is given as

$$C(s) = R(s) \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- If we consider a unit step function as the input of the system, then the output equation of the system can be rewritten as

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$Now,\ r(t) = 1\ or\ R(s) = \frac{1}{s}$$

$$\therefore C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2 + \omega_n^2 - \zeta^2\omega_n^2}$$

$$= \frac{1}{s} \cdot \frac{\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}$$

Continue..

- Applying partial fraction we get

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2\left(1 - \zeta^2\right)}$$

- Where $\varpi_d = \varpi_n\sqrt{(1-\zeta^2)}$, is the frequency of transient oscillations and is called damped natural frequency.

- The inverse Laplace transform of above equation can be obtained easily if C(s) is written in the following form:

$$C(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

Continue..

$$c(t) = 1 - e^{-\zeta\omega_n t}.\cos\omega_d t - \frac{\zeta\omega_n}{\omega_d}.e^{-\zeta\omega_n t}.\sin\omega_d t$$

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}[\sqrt{1-\zeta^2}.\cos\omega_d t + \zeta.\sin\omega_d t]$$

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}[\sin\phi.\cos\omega_d t + \cos\phi\sin\omega_d t]$$

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)$$

Continue..

$$\varpi_d = \varpi_n \sqrt{(1-\zeta^2)}$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$$

Error signal

- e(t) = r(t)-c(t)
- r(t) = 1

$$e(t) = 1 - \left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta})) \right]$$

$$e(t) = \left[\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta})) \right]$$

Steady state error

- The steady state error is given by

$$e_{ss} = \lim_{t \longrightarrow \infty} \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta}))$$

According the value of ζ, a second-order system can be set into one of the four categories:

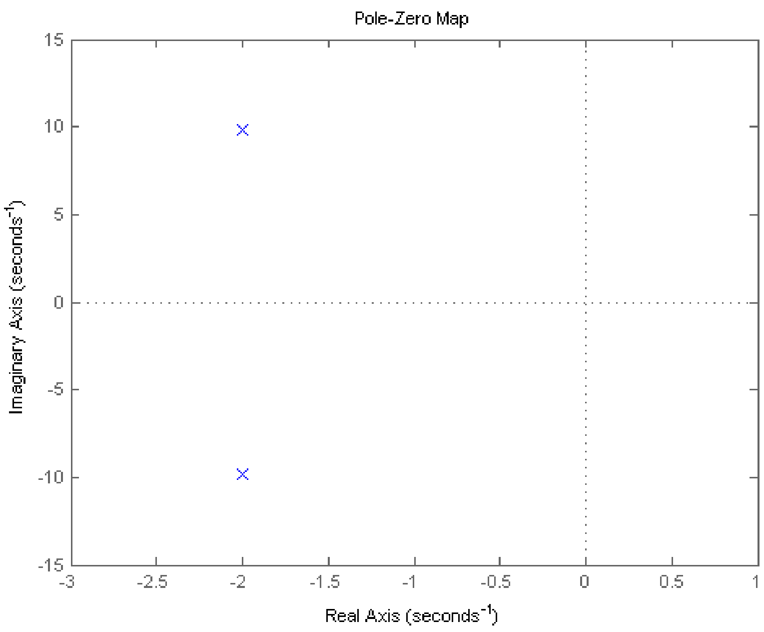
1. Overdamped - when the system has two real distinct poles (ζ >1).
2. *Underdamped* - when the system has two complex conjugate poles (0 <ζ <1) . Transient response is oscillatory
3. *Undamped* - when the system has two imaginary poles (ζ = 0). Transient response never dies.
4. *Critically damped* - when the system has two real but equal poles (ζ = 1).

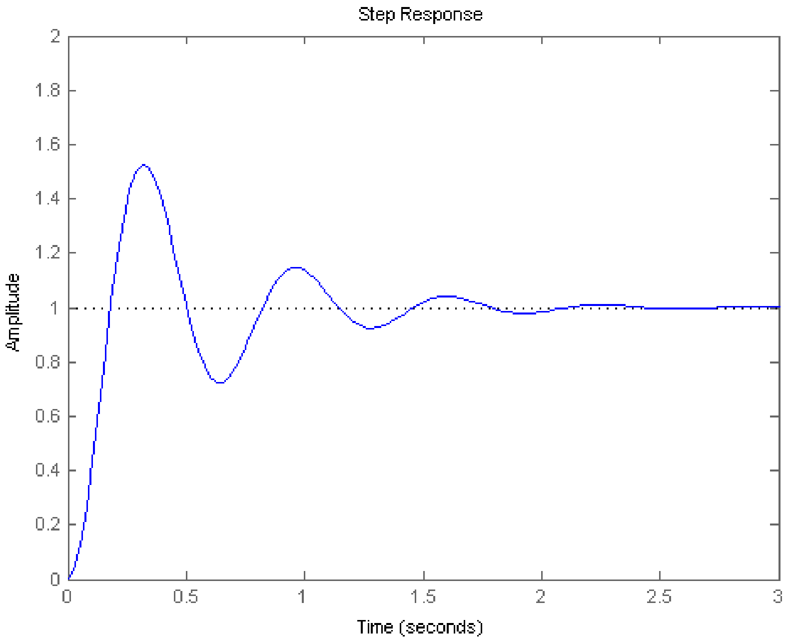
Under damped system

- If , zeta < 1 then the system is **under damped**. Both poles are complex valued with negative real parts; therefore the system is stable but oscillates while approaching the steady-state value.

$$G(s) = \frac{1}{ms^2 + bs + k} = \frac{k_{dc}\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Underdamped system





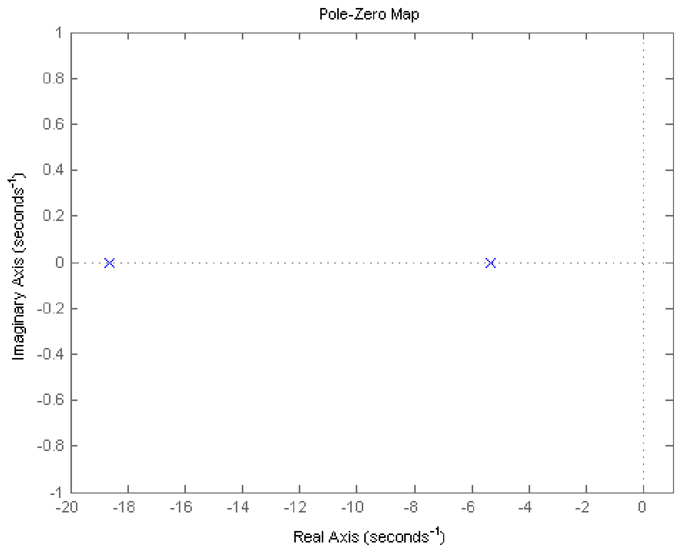

```
k_dc = 1;
w_n = 10;
 zeta = 0.2;
s = tf('s');
 G1 = k_dc*w_n^2/(s^2 + 2*zeta*w_n*s + w_n^2);
pzmap(G1) axis([-3 1 -15 15])
step(G1) axis([0 3 0 2])
```

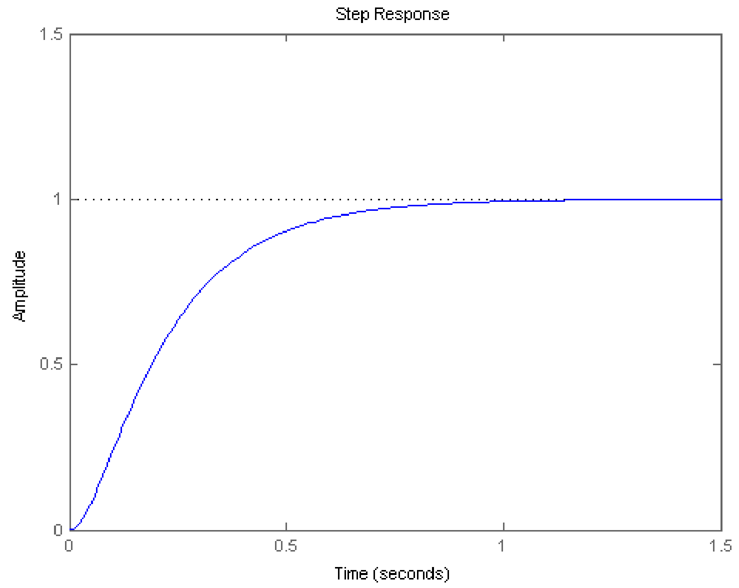
Overdamped system

- if , zeta > 1 then the system is **over damped**. Both poles are real and negative; therefore the system is stable and does not oscillate. The step response and a pole-zero map of an over damped system are calculated below:

Over damped

- zeta = 1.2;
- G2 = k_dc*w_n^2/(s^2 + 2*zeta*w_n*s + w_n^2); pzmap(G2) axis([-20 1 -1 1])
- step(G2) axis([0 1.5 0 1.5])

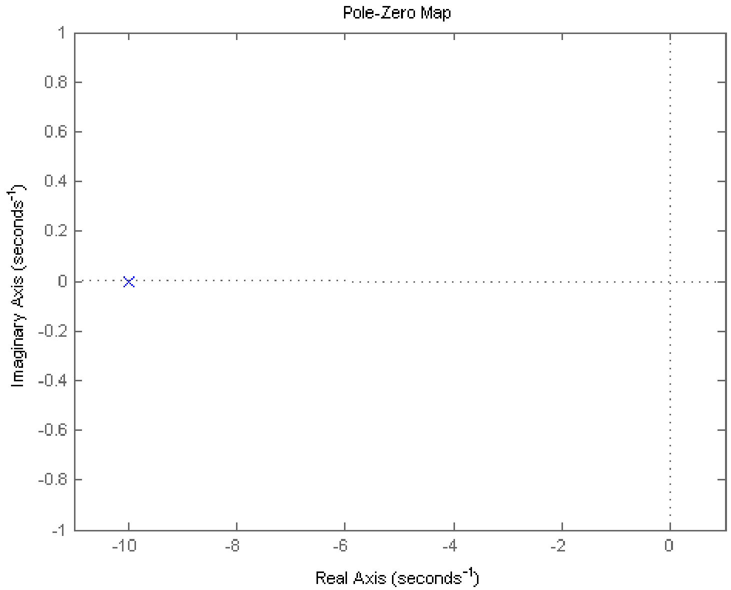


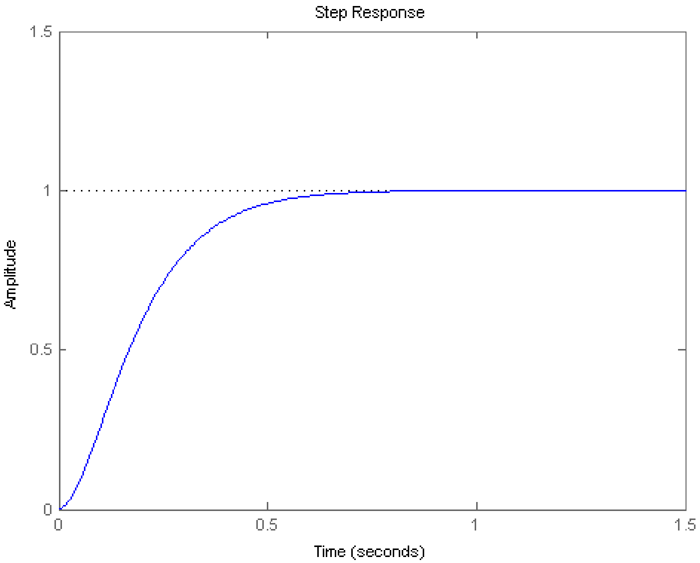


critically damped

- If , zeta = 1 then the system is **critically damped**. Both poles are real and have the same magnitude, . Critically damped systems approach steady-state quickest without oscillating. Now change the value of the damping to 1, and replot the step response and pole-zero map.

- zeta = 1;
- G3 = k_dc*w_n^2/(s^2 + 2*zeta*w_n*s + w_n^2); pzmap(G3) axis([-11 1 -1 1])
- step(G3) axis([0 1.5 0 1.5])

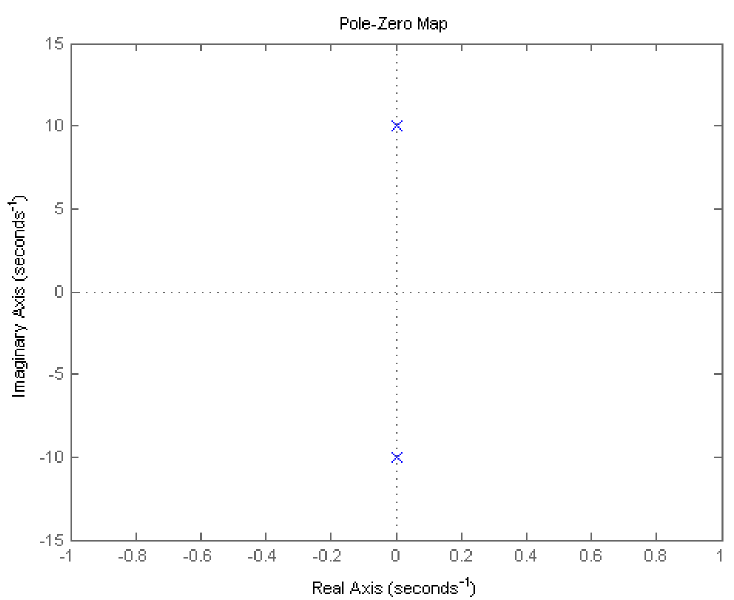


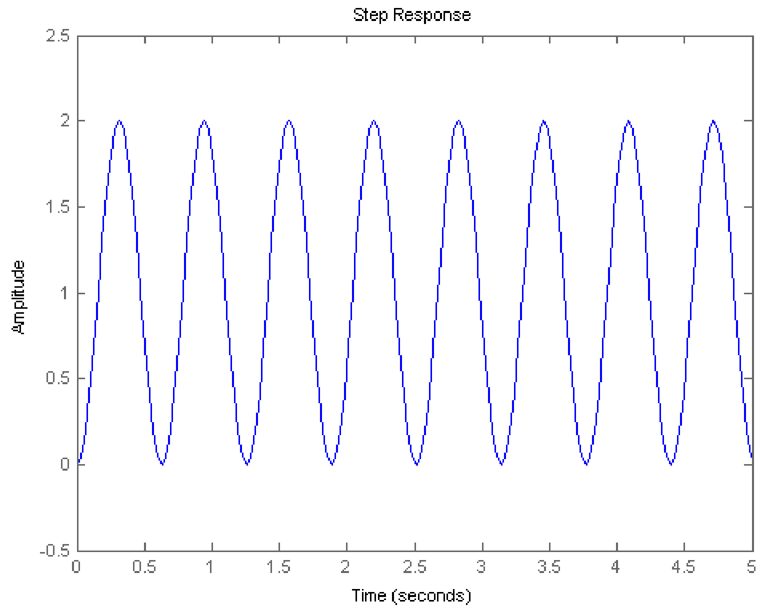


Undamped system

- If , zeta = 0 then the system is **undamped**. In this case, the poles are purely imaginary; therefore the system is marginally stable and oscillates indefinitely.

- zeta = 0;
- G4 = k_dc*w_n^2/(s^2 + 2*zeta*w_n*s + w_n^2); pzmap(G4) axis([-1 1 -15 15])
- step(G4) axis([0 5 -0.5 2.5])





1) An automatic door close is an example of an critically damped system.

2) A gun is made critically damped so that it returns to the neutral position in the shortest amount of time between firing.

- The shock absorber of a car is a spring which helps to reduce impact when the car goes over a corrugated surface.

- 1)If the shock absorber is over damped, it is faulty in the sense that it would be a hard and bumpy ride because the spring does not absorb the impact.

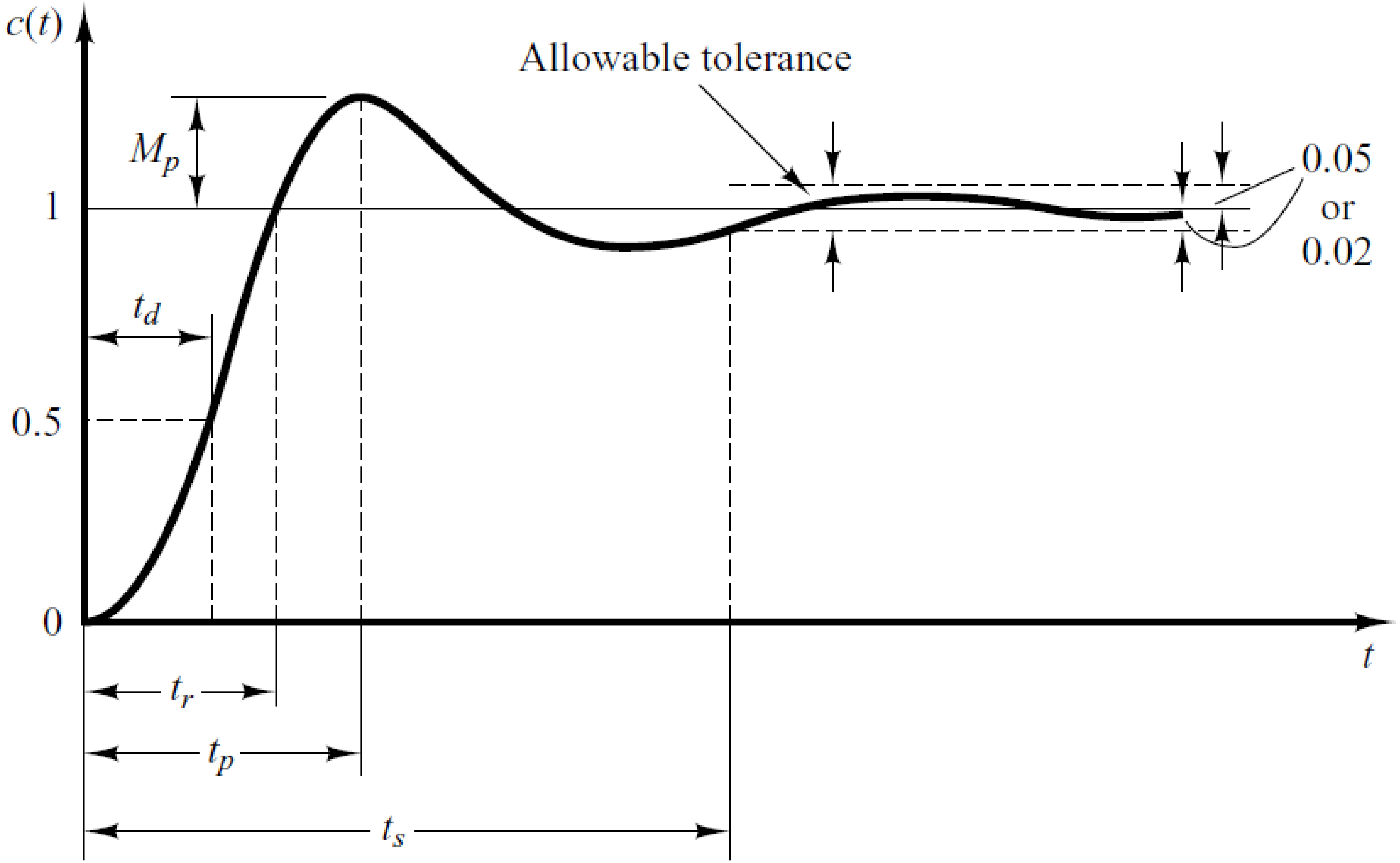
- 2)If the car's shock absorber is critically damped, you would experience a comfortable ride because any impact from the bumps the car goes over will be absorbed as the spring mechanism stretches and return to its equilibrium in no time.
- 3) If a car is very much under damped, the car will experience a vertical oscillation whenever it crosses a bump. You will be bouncing in the car at least several times although it's just a bump

Time Response specification of second order System

- In specifying the transient response characteristics of a control system to a unit step function , the following quantities are commonly specified.

1. Delay time
2. Rise time
3. Peak Time
4. Maximum overshoot
5. Settling time

Continue…



Derivation of time response specification

- **Calculation of Rise time (T_r)**

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta}))$$

- At C(t) = 1, t = T_r
- $$1 = 1 - \frac{e^{-\zeta\omega_n T_r}}{\sqrt{1-\zeta^2}} \sin(\omega_d T_r + \tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta}))$$

$$\frac{e^{-\zeta\omega_n T_r}}{\sqrt{1-\zeta^2}} \sin(\omega_d T_r + \tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta})) = 0$$

$$e^{-\zeta\omega_n T_r} \neq 0$$

$$\sin(\omega_d T_r + \tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta})) = \sin n\pi$$

$$\omega_d T_r + \tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta}) = \pi$$

$$T_r = \frac{(\pi - \phi)}{\omega_d}$$

$$T_r = \frac{\pi - \tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta})}{\omega_n\sqrt{1-\zeta^2}}$$

Calculation of Peak time (T_p)

- At peak time derivative of c(t) is zero .

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

$$\frac{d}{dt}c(t) = 0 + \frac{\zeta\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi) - \frac{\omega_d e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cos(\omega_d t + \phi)$$

$$\frac{d}{dt}c(t) = 0 \qquad t = T_p$$

$$\frac{\zeta\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi) - \frac{\omega_d e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cos(\omega_d t + \phi) = 0$$

Calculation of Peak time (T_p)

$$\left(\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\right)\left[\zeta\omega_n \sin(\omega_d t+\phi)-\omega_d \cos(\omega_d t+\phi)\right]=0$$

$$\zeta\omega_n \sin(\omega_d t+\phi)-\omega_n.\sqrt{1-\zeta^2}\cos(\omega_d t+\phi)=0$$

$$\left(\cos\phi \sin(\omega_d t+\phi)-\sin\phi \cos(\omega_d t+\phi)\right)=0$$

$$\sin(\omega_d t+\phi-\phi)=0$$

$$\sin(\omega_d t+\phi-\phi)=\sin n\pi$$

$$T_p=\frac{n\pi}{\omega_d}$$

Calculation of Peak overshoot

- The maximum overshoot occurs at the time t = Tp

$$M_p = c(t)\big|_{t=Tp} - 1$$

$$= \left(1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d T_p + \phi)\right) - 1$$

$$T_p \omega_d = \pi$$

$$M_p = -\frac{e^{-\zeta\omega_n T_p}}{\sqrt{1-\zeta^2}}\sin(\pi + \phi)$$

$$M_p = \frac{e^{-\zeta\omega_n T_p}}{\sqrt{1-\zeta^2}}\sin\phi$$

$$M_p = \frac{e^{-\zeta\omega_n T_p}}{\sqrt{1-\zeta^2}}\sqrt{1-\zeta^2}$$

$$M_p = e^{-\zeta\omega_n \frac{\pi}{\omega_d}}$$

$$M_p = e^{-\zeta\omega_n \frac{\pi}{\omega_n \cdot \sqrt{1-\zeta^2}}}$$

$$M_p = e^{-\zeta \frac{\pi}{\sqrt{1-\zeta^2}}}$$

Calculation of settling time

- In order to find the settling time , we must find the c(t) reaches and stays within $\pm 2\ \%$ of its final value.

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

$$0.98 = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}$$

$$\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} = 0.02$$

For lower value of zeta

$$e^{-\zeta\omega_n T_s} = 0.02$$

$$\zeta\omega_n T_s = \ln(0.02)$$

$$T_s = \frac{4}{\zeta\omega_n}$$

Time Response of a second – order system with unit impulse input

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.R(s)$$

For unit impulse input R(s) = 1.

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.1$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = (s + \zeta\omega_n)^2 + \omega_n{}^2(1 - \zeta^2)$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = (s + \zeta\omega_n)^2 + (\omega_n\sqrt{1 - \zeta^2})^2$$

Continue…

$$C(s) = \frac{\omega_n^2}{\omega_n.\sqrt{1-\zeta^2}} \cdot \frac{\omega_n.\sqrt{1-\zeta^2}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Taking Inverse Laplace of this we get

$$C(t) = \frac{\omega_n}{.\sqrt{1-\zeta^2}}.e^{-\zeta\omega_n t}\sin((\omega_n\sqrt{1-\zeta^2})t)$$

Time Response of a second – order system with unit ramp input

- Input is unit ramp we get R(s) = 1/s^2

$$C(s) = \frac{1}{s^2} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$A = -\frac{2\zeta}{\omega_n} \qquad B = 1 \qquad C = \frac{2\zeta}{\omega_n} \qquad D = -1 + 4\zeta^2$$

$$C(s) = \frac{-2\zeta}{\omega_n s} + \frac{1}{s^2} + \frac{\frac{2\zeta}{\omega_n} s - (1 - 4\zeta^2)}{s^2 + 2\zeta\omega_n s + \omega_n{}^2}$$

Time Response of a second – order system with unit ramp input

$$C(s)=\frac{-2\zeta}{\omega_n s}+\frac{1}{s^2}+\frac{\frac{2\zeta}{\omega_n}(s+\zeta\omega_n)-2\zeta^2-1+4\zeta^2}{s^2+2\zeta\omega_n s+\zeta^2\omega_n{}^2-\zeta^2\omega_n{}^2+\omega_n{}^2}$$

$$C(s)=\frac{-2\zeta}{\omega_n s}+\frac{1}{s^2}+\frac{2\zeta}{\omega_n}\frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2+\omega_n{}^2(1-\zeta^2)}-\frac{1}{\omega_n\sqrt{1-\zeta^2}}\frac{(1-2\zeta^2).\omega_n\sqrt{1-\zeta^2}}{(s+\zeta\omega_n)^2+\omega_n{}^2(1-\zeta^2)}$$

$$C(s)=t-\frac{2\zeta}{\omega_n}+e^{-\zeta\omega_n t}\left[\frac{2\zeta}{\omega_n}\cos(\omega_d t)-\frac{1-2\zeta^2}{\omega_d}\sin(\omega_d t)\right]$$

Example

- The Transfer functions of certain second order system are given below. Determine the type of Damping in the systems.

$$\frac{C(s)}{R(s)} = \frac{8}{s^2 + 3s + 8}$$

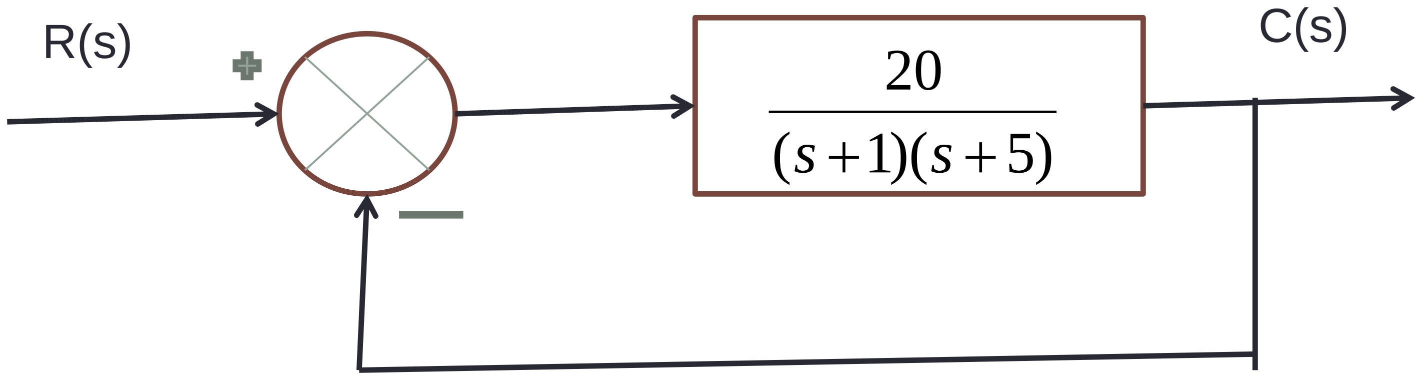
$$\frac{C(s)}{R(s)} = \frac{2}{s^2 + 4s + 2}$$

$$\frac{C(s)}{R(s)} = \frac{2}{s^2 + 2s + 1}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 16}$$

Example

- The block diagram of unity feedback system is shown in figure. Find the value of wn,zeta,wd,tp and Mp.



$$\frac{20}{(s+1)(s+5)}$$

Wn = 5 rad/sec
Zeta = 0.6
Wd = 4 rad/sec
Tp = 0.78 sec
Mp = 9.4%

Example

- The open loop transfer function of a unity feedback control system is given by

G(s) = 25/s(s+5)

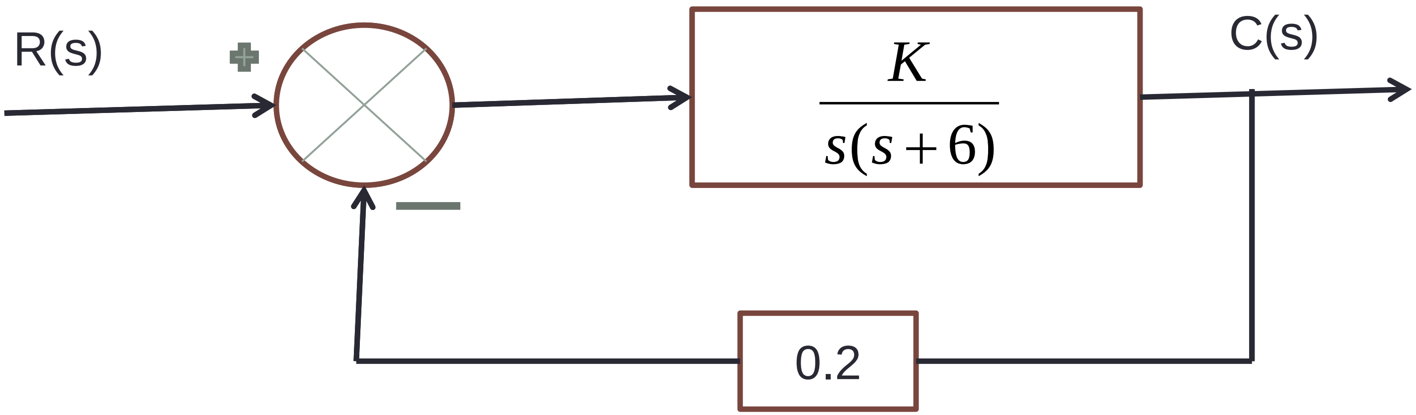
- Calculate
  1. Natural frequency of oscillations
  2. Damped frequency of oscillations
  3. Damping factor
  4. Damping ratio
  5. Maximum overshoot

Wn = 5 rad/sec
Zeta = 0.5
Damping factor = 2.5
Wd = 4.3 rad/sec
Mp = 16.3%

consider Zeta is the damping ratio & damping factor is Zeta*wn, ideally. Practically we can consider both quantities same.

Example

- A close loop control system is shown in the figure. The system is to have a damping ratio of 0.7. Determine the value of K to satisfy this condition. Calculate the settling time , peak time and maximum overshoot.



$$\frac{K}{s(s+6)}$$

K = 91.8

Wn = 4.28 rad/sec

Ts = 1.33 sec

Tp = 1.03 sec

Mp = 4.59%