CHAPTER 2

Nodal Analysis and Mesh Analysis

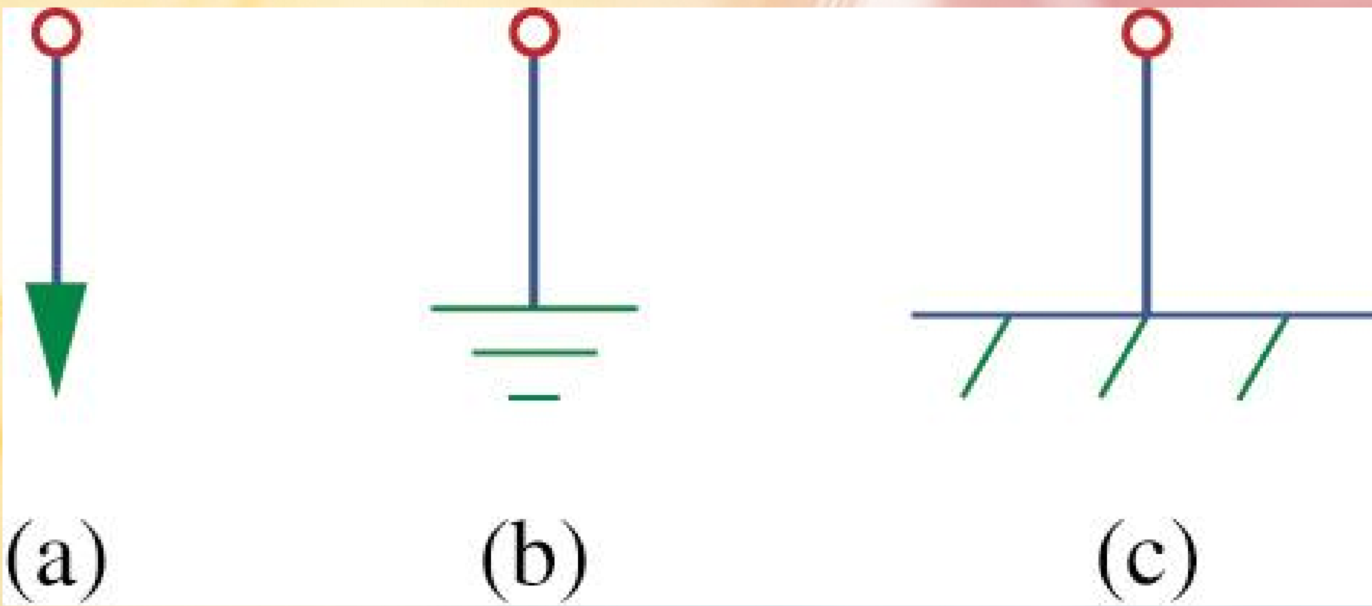
METHODS OF ANALYSIS

- Introduction
- Nodal analysis
- Nodal analysis with voltage source
- Mesh analysis
- Mesh analysis with current source
- Nodal and mesh analyses by inspection
- Nodal versus mesh analysis

STEPS OF NODAL ANALYSIS

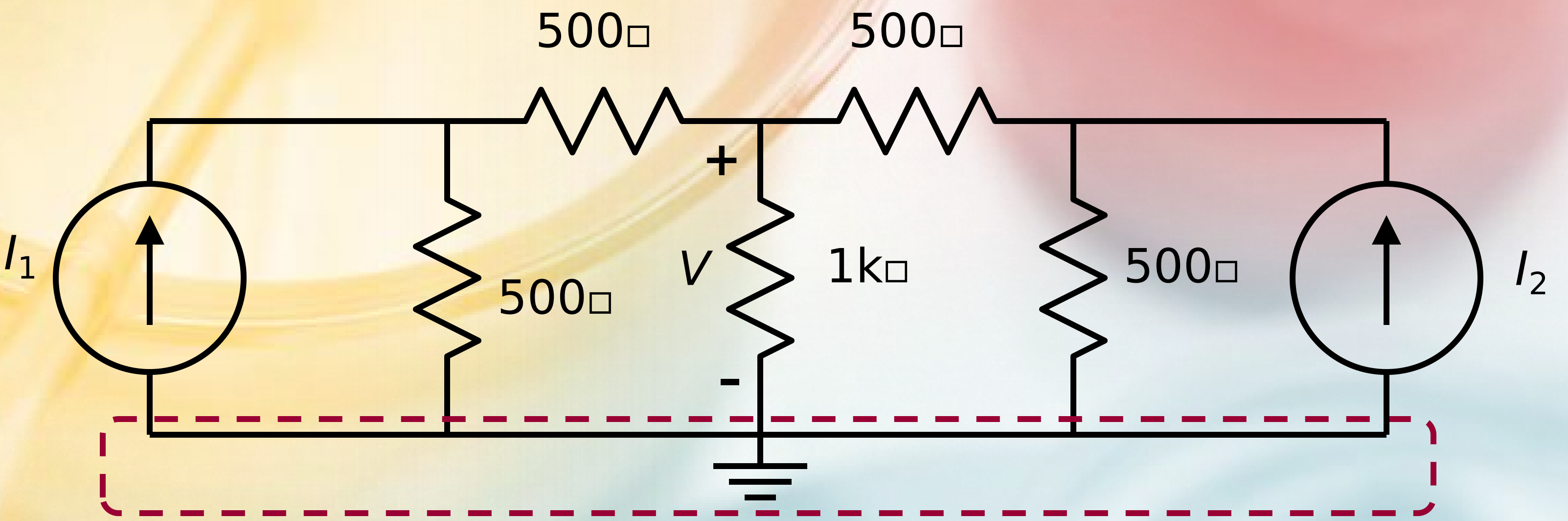
1. Choose a reference (ground) node.
2. Assign node voltages to the other nodes.
3. Apply KCL to each node other than the reference node; express currents in terms of node voltages.
4. Solve the resulting system of linear equations for the nodal voltages.

1.CHOOSE A REFERENCE (GROUND) NODE.



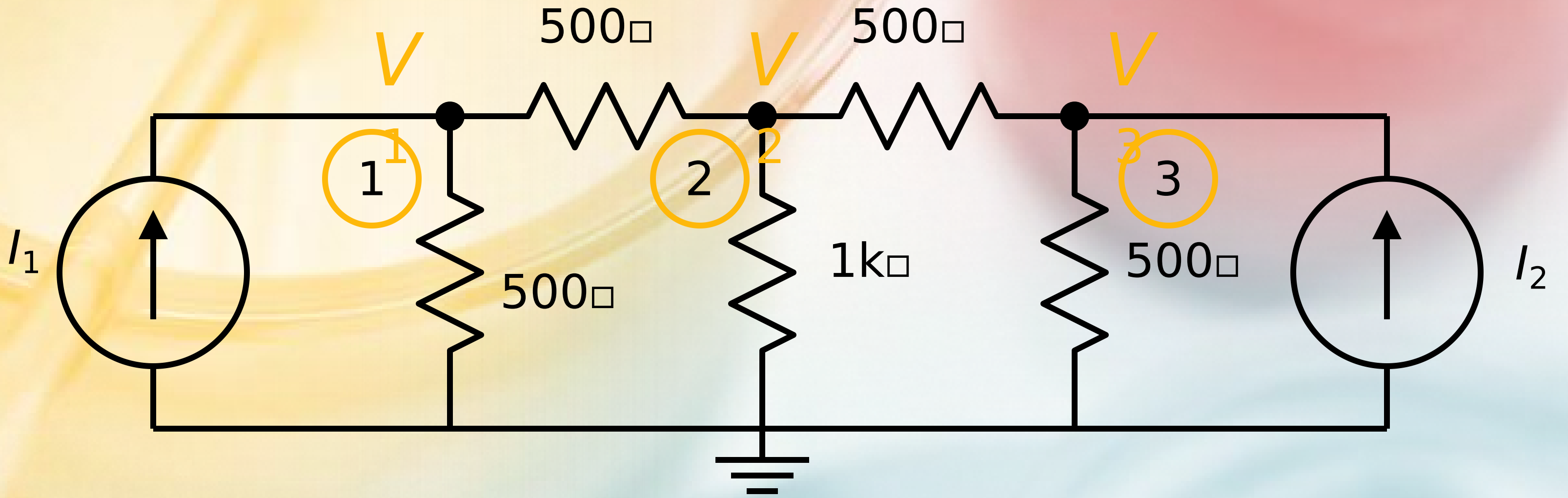
Common symbols for indicating a reference node, (a) common ground, (b) ground, (c) chassis.

1. REFERENCE NODE



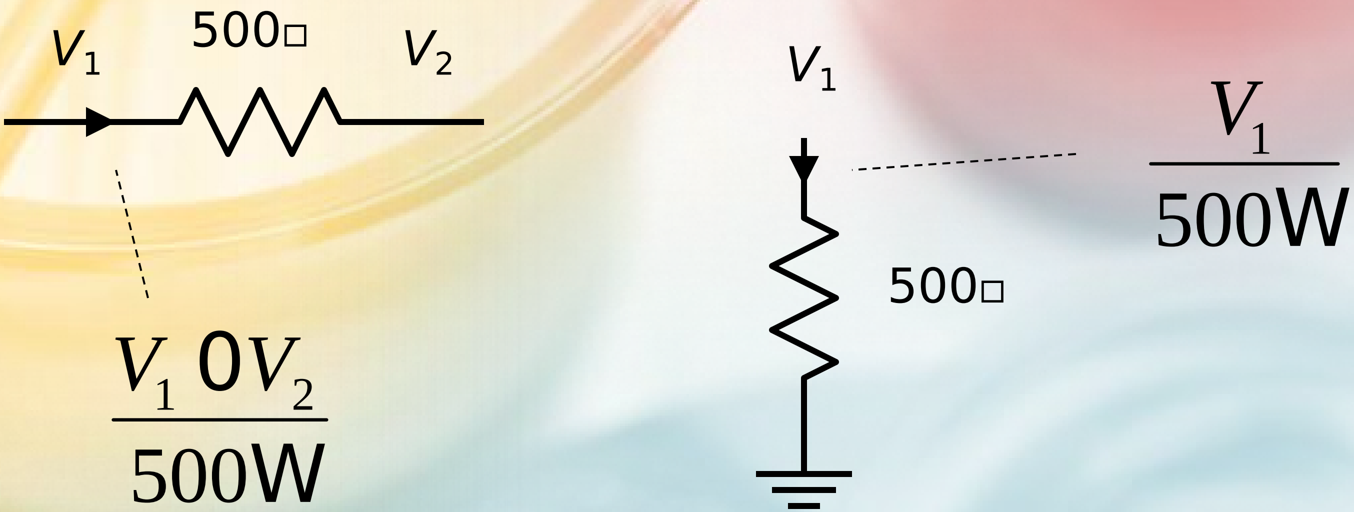
The reference node is called the *ground* node where $V = 0$

2. NODE VOLTAGES

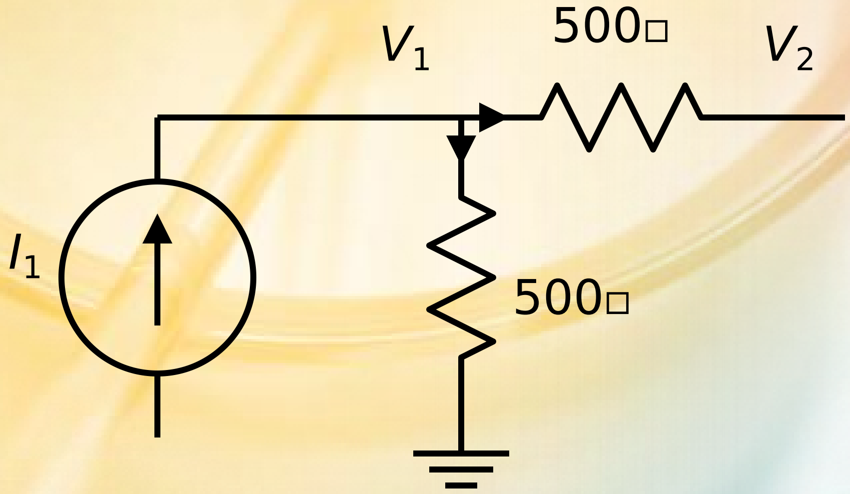


V_1, V_2, and V_3 are unknowns for which we solve using KCL

CURRENTS AND NODE VOLTAGES

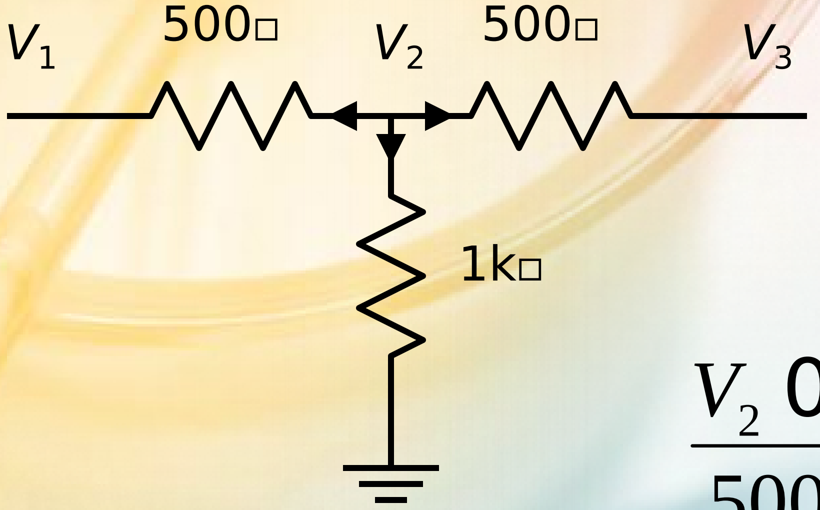


3. KCL AT NODE 1



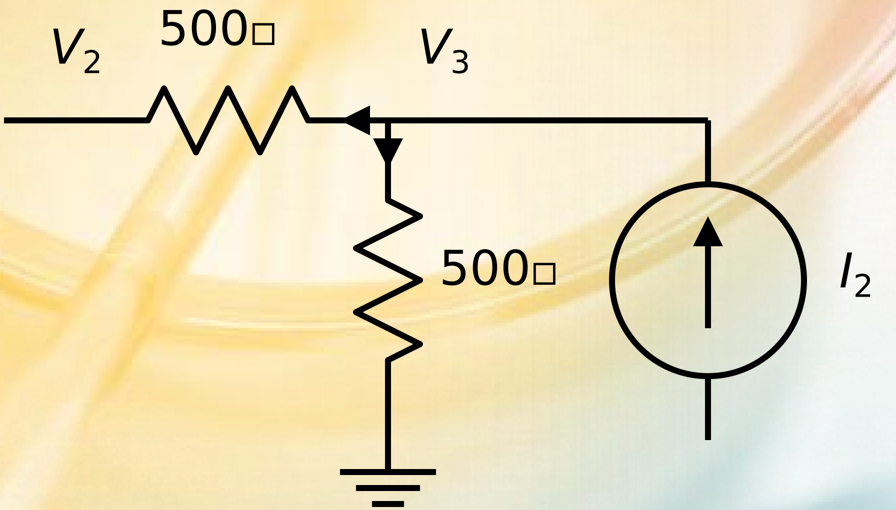
$$I_1 \; 6 \, \frac{V_1 \; 0 V_2}{500W} \; . \; \frac{V_1}{500W}$$

3. KCL AT NODE 2



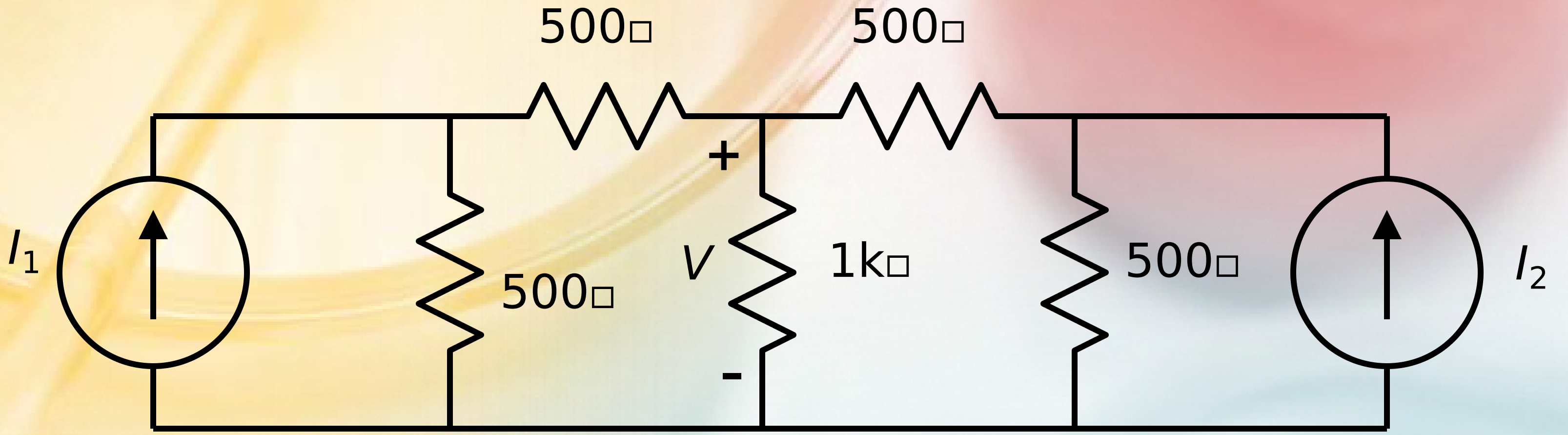
$$\frac{V_2\ 0 V_1}{500\text{W}} . \frac{V_2}{1\text{kW}} . \frac{V_2\ 0 V_3}{500\text{W}}\ 6\ 0$$

3. KCL AT NODE 3



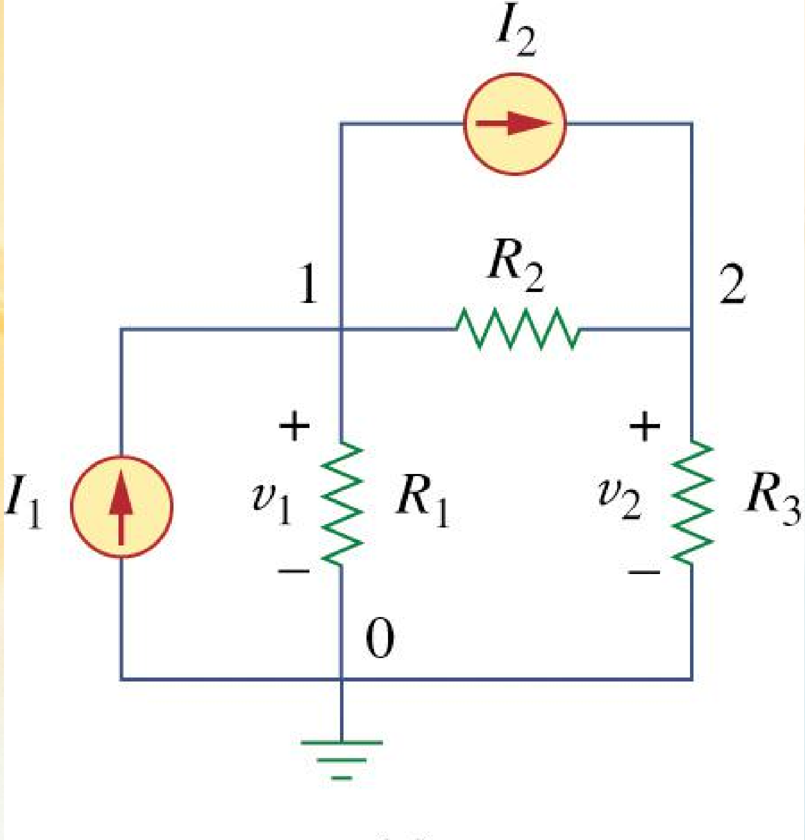
$$\frac{V_3 \; 0 V_2}{500W} \; . \; \frac{V_3}{500W} \; 6 \; I_2$$

4. SUMMING CIRCUIT SOLUTION

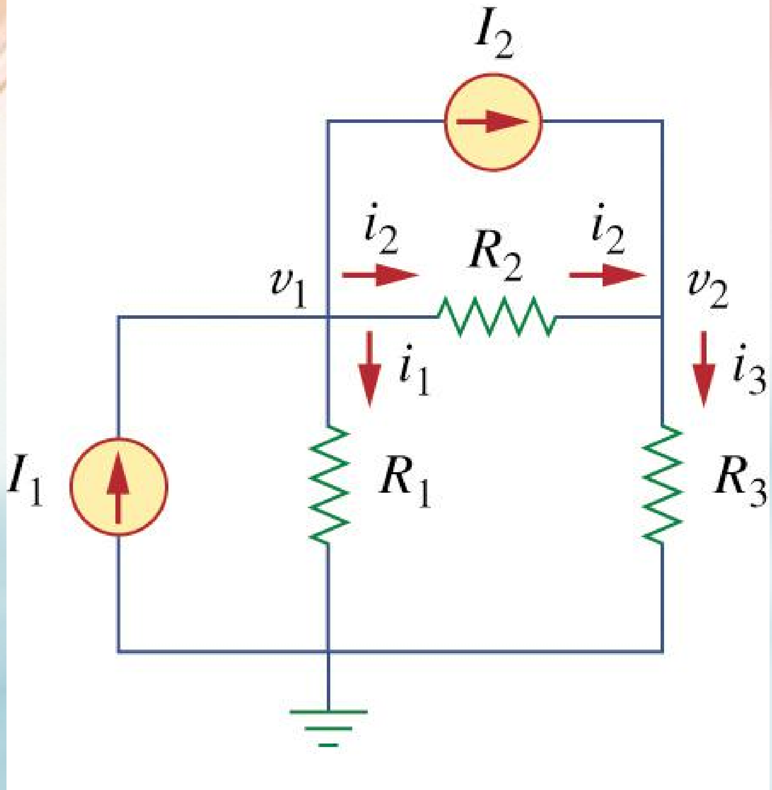


Solution: $V = 167 I_1 + 167 I_2$

TYPICAL CIRCUIT FOR NODAL ANALYSIS



(a)



(b)

NODAL ANALYSIS

$$I_1 \; 6 \; I_2 \; . \; i_1 \; . \; i_2$$

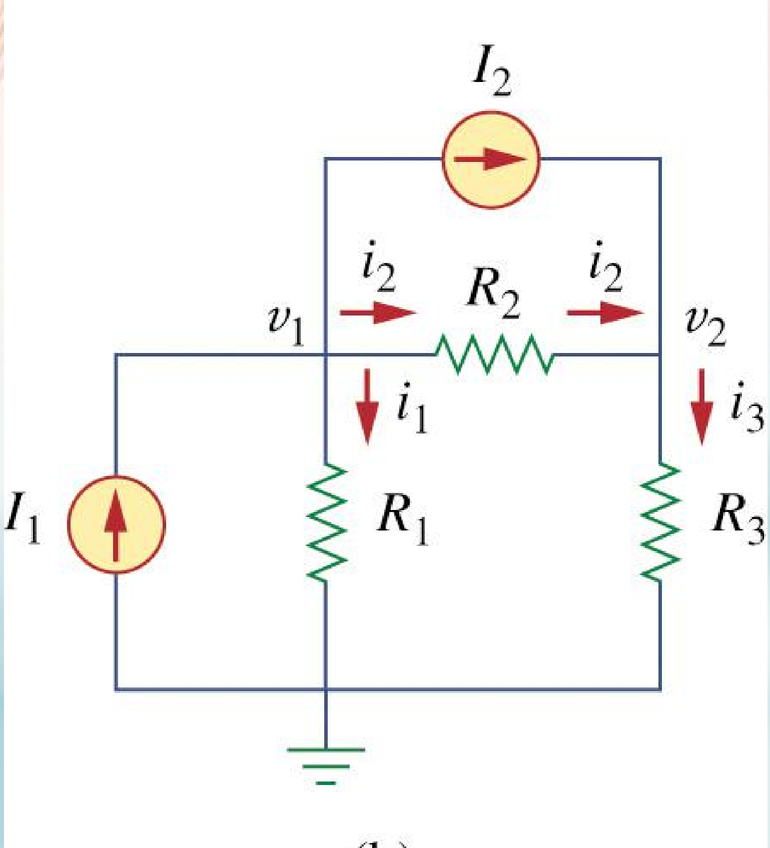
$$I_2 \; . \; i_2 \; 6 \; i_3$$

$$i \; 6 \; \frac{v_{\text{higher}} \; 0 \; v_{\text{lower}}}{R}$$

$$i_1 \; 6 \; \frac{v_1 \; 0 \; 0}{R_1} \quad \text{or} \quad i_1 \; 6 \; G_1 v_1$$

$$i_2 \; 6 \; \frac{v_1 \; 0 \; v_2}{R_2} \quad \text{or} \quad i_2 \; 6 \; G_2 (v_1 \; 0 \; v_2)$$

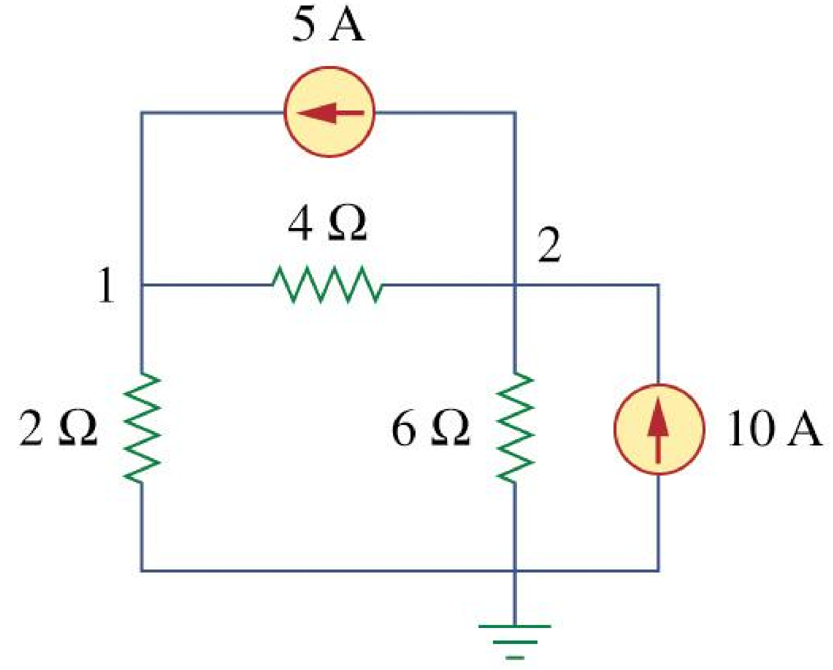
$$i_3 \; 6 \; \frac{v_2 \; 0 \; 0}{R_3} \quad \text{or} \quad i_3 \; 6 \; G_3 v_2$$



(b)

EXAMPLE

- Calculate the node voltage in the circuit shown in Fig.



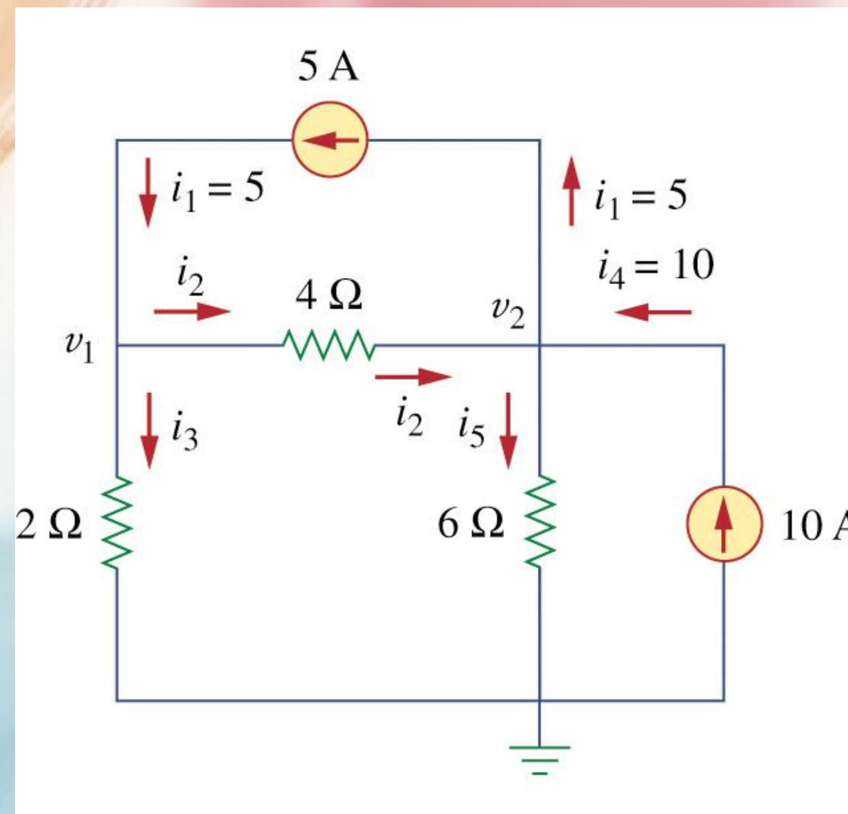
(a)

SOLUTION

- At node 1

$$i_1 \; 6 \; i_2 \; . \; i_3$$

$$Þ \;\; 5 \, 6 \, \frac{v_1 \, 0 \, v_2}{4} \, . \, \frac{v_1 \, 0 \, 0}{2}$$



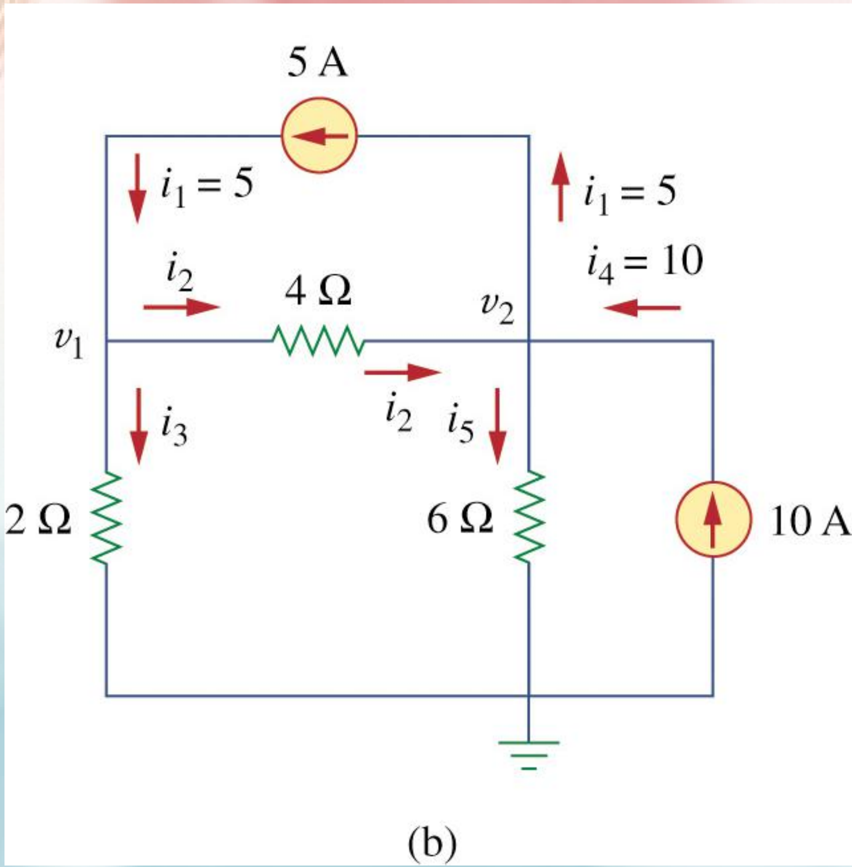
(b)

SOLUTION

- At node 2

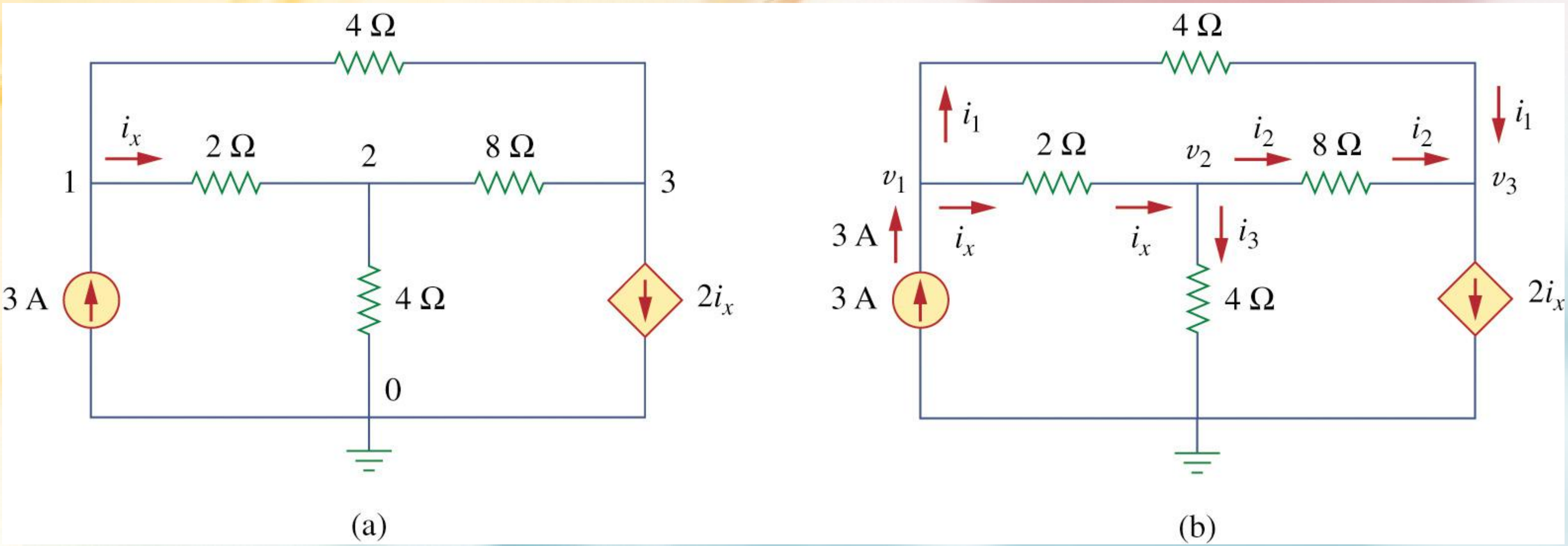
$$i_2 + i_4 = i_1 + i_5$$

$$\Rightarrow 5 = \frac{v_2 - v_1}{4} + \frac{v_2 - 0}{6}$$



EXAMPLE

- Determine the voltage at the nodes in Fig. below

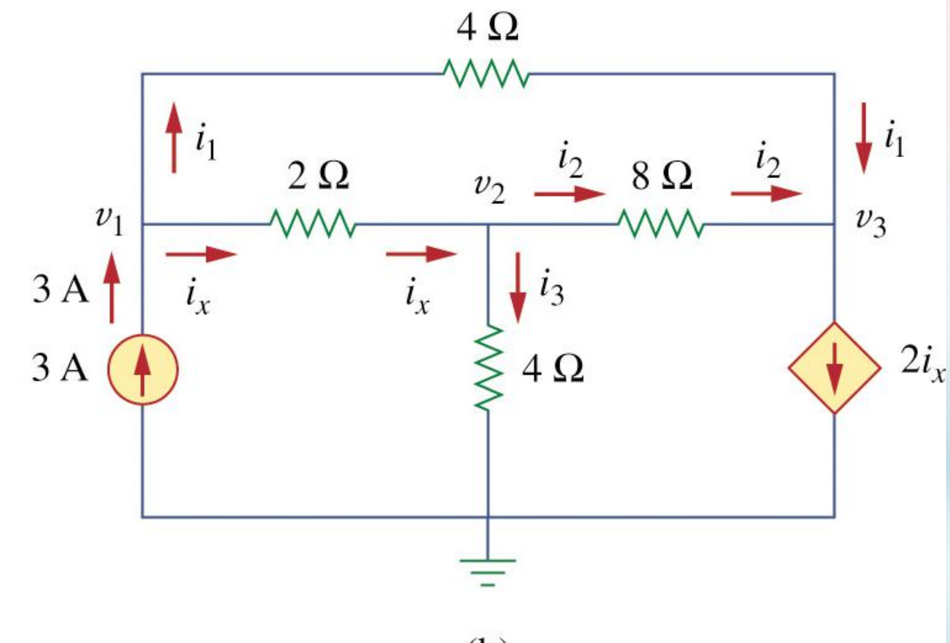


SOLUTION

- At node 1,

$$3\,6\,i_1\;.\;i_x$$

$$\text{Þ}\;\;3\,6\,\frac{v_1\,0\,v_3}{4}\,.\,\frac{v_1\,0\,v_2}{2}$$



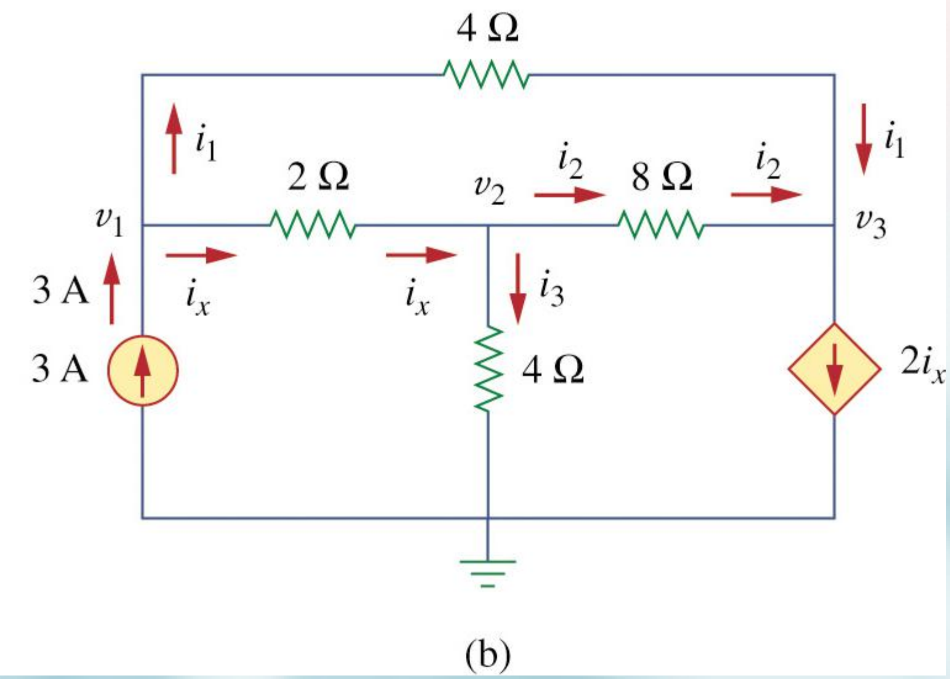
(b)

SOLUTION

- At node 2

$$i_x \; 6 \; i_2 \; . \; i_3$$

$$Þ \quad \frac{v_1 \, 0 \, v_2}{2} \; 6 \; \frac{v_2 \, 0 \, v_3}{8} \; . \; \frac{v_2 \, 0 \, 0}{4}$$

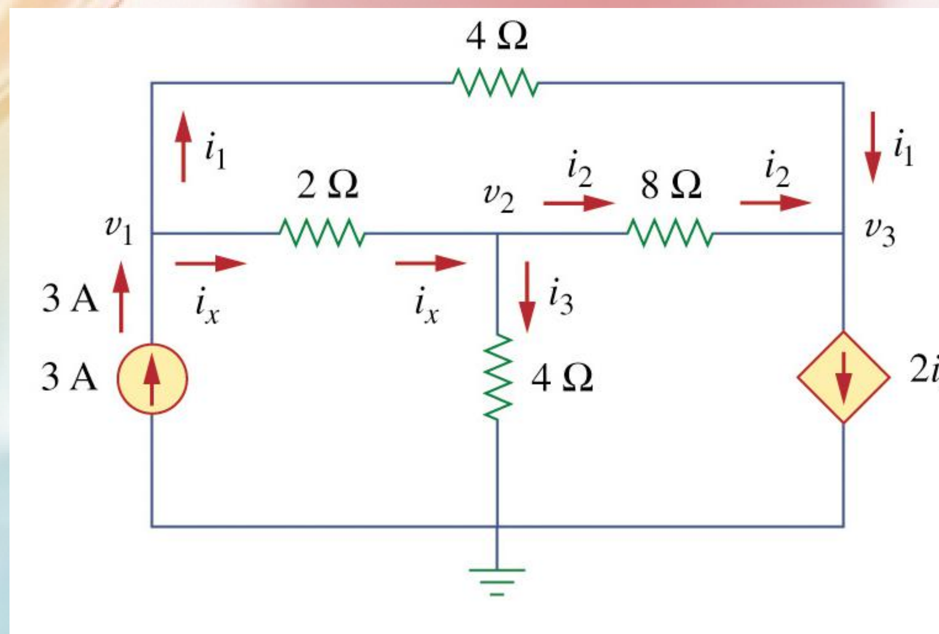


SOLUTION

- At node 3

$$i_1 \,.\, i_2 \;6\; 2i_x$$

$$Þ \;\frac{v_1 \,0\, v_3}{4} \,.\, \frac{v_2 \,0\, v_3}{8} \;6\; \frac{2(v_1 \,0\, v_2)}{2}$$



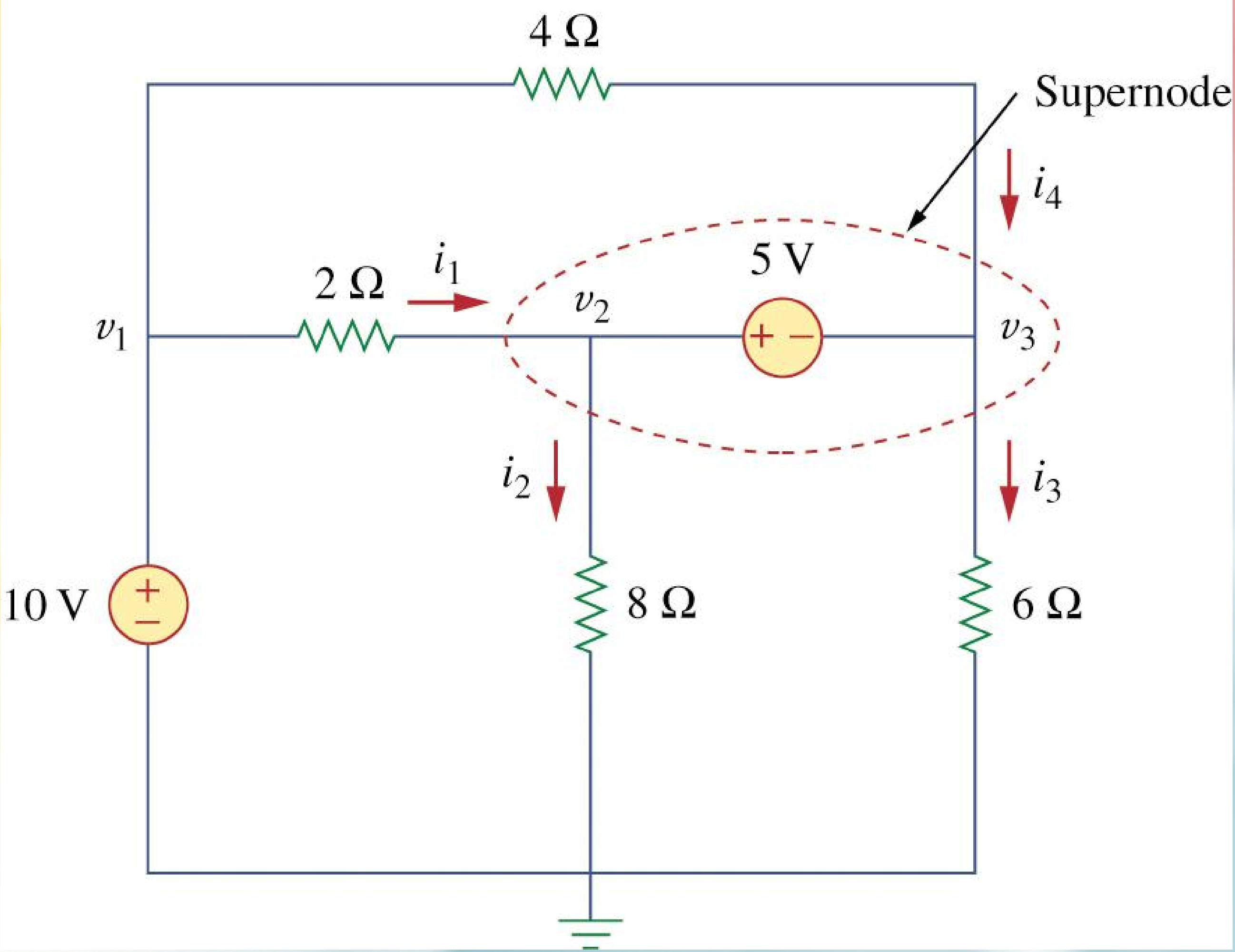
(b)

NODAL ANALYSIS WITH VOLTAGE SOURCES

- Case 1: The voltage source is connected between a nonreference node and the reference node: The nonreference node voltage is equal to the magnitude of voltage source and the number of unknown nonreference nodes is reduced by one.
- Case 2: The voltage source is connected between two nonreferenced nodes: a generalized node (supernode) is formed.

VOLTAGE SOURCES

A circuit with a supernode.

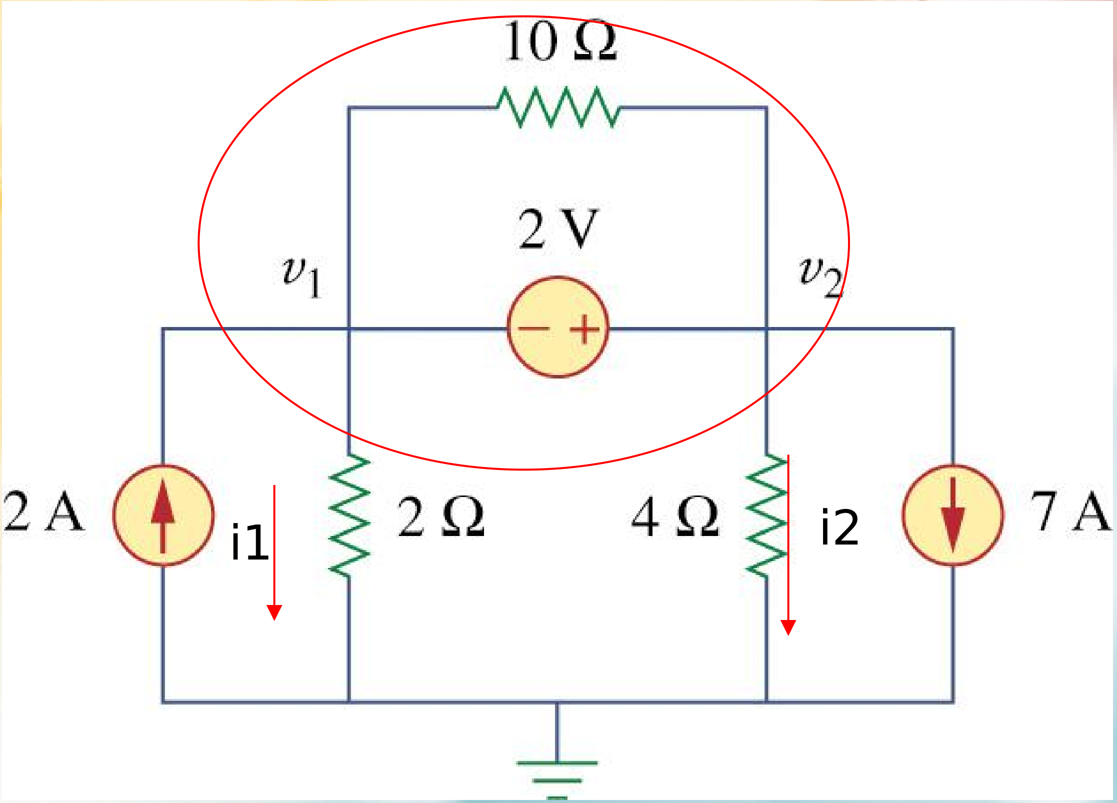


SUPERMODE

- A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.
- The required two equations for regulating the two nonreference node voltages are obtained by the KCL of the supernode and the relationship of node voltages due to the voltage source.

EXAMPLE

- For the circuit shown in Fig., find the node voltages.



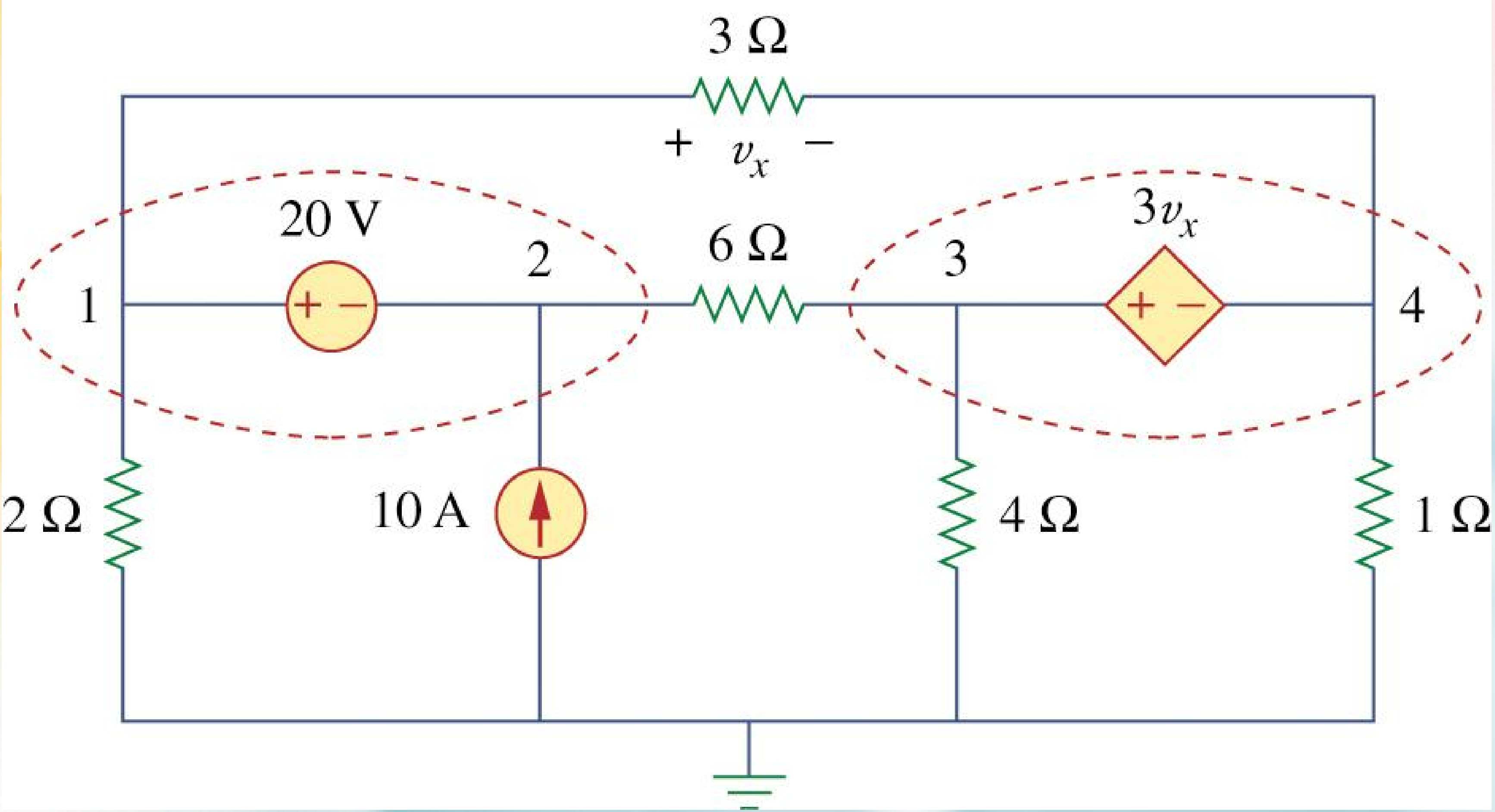
$$2\;0\;7\;0\;i1\;0\;i2\;6\;0$$

$$2\;0\;7\;0\;\frac{v_1}{2}\;0\;\frac{v_2}{4}\;6\;0$$

$$v_1\;0\;v_2\;6\;0\;2$$

EXAMPLE

Find the node voltages in the circuit below.

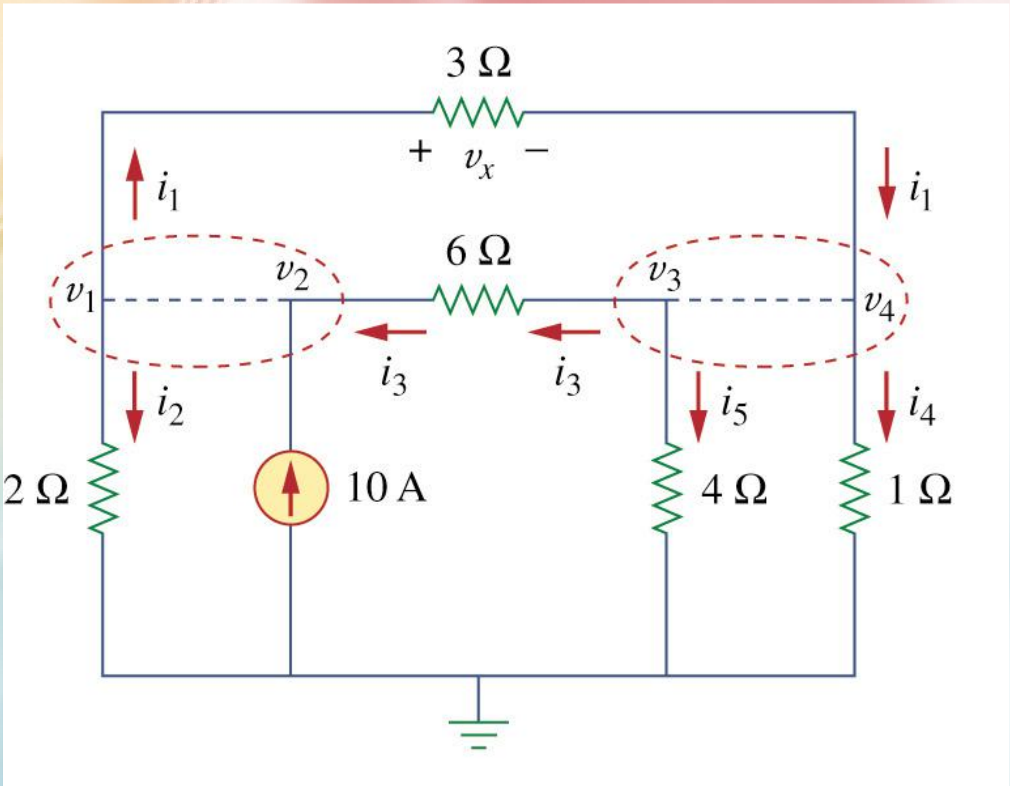


SOLUTION

- At suopernode 1–2,

$$\frac{v_3 \, 0 \, v_2}{6} \; . \; 10 \; 6 \; \frac{v_1 \, 0 \, v_4}{3} \; . \; \frac{v_1}{2}$$

$$v_1 \, 0 \, v_2 \; 6 \; 20$$



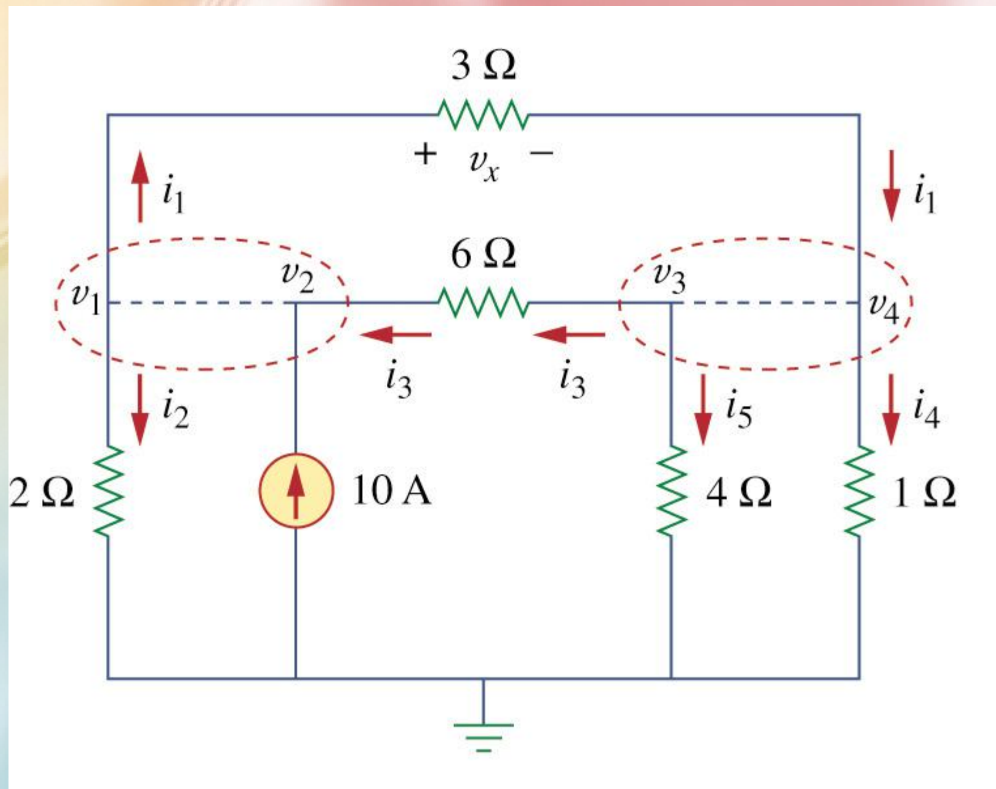
(a)

SOLUTION

- At supernode 3–4,

$$\frac{v_1 0 v_4}{3} 6 \frac{v_3 0 v_2}{6} . \frac{v_4}{1} . \frac{v_3}{4}$$

$$v_3 0 v_4 6 3(v_1 0 v_4)$$

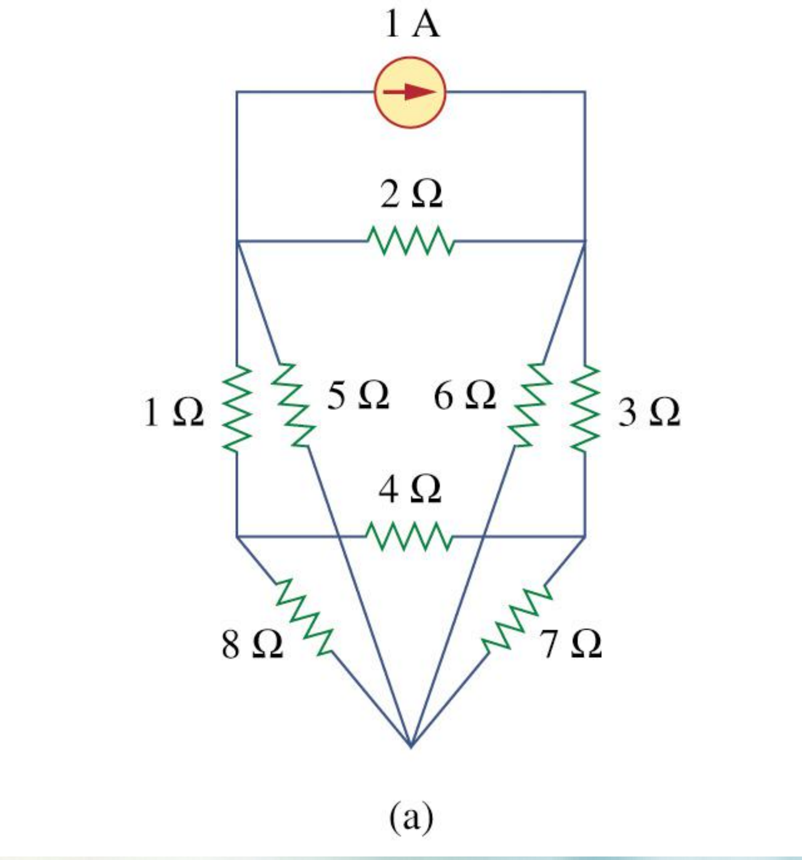


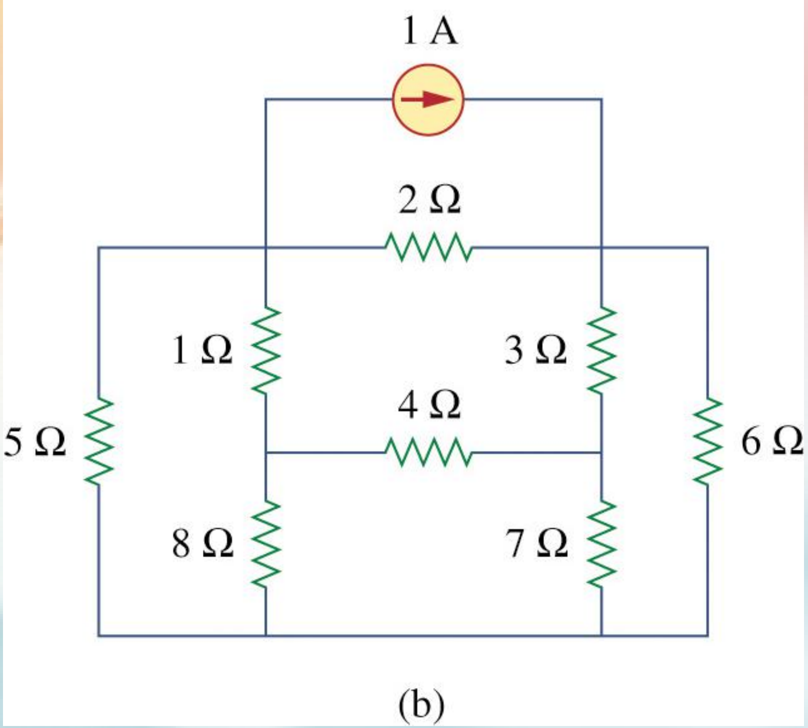
(a)

MESH ANALYSIS

- Mesh analysis: another procedure for analyzing circuits, applicable to planar circuit.
- A Mesh is a loop which does not contain any other loops within it

MESH ANALYSIS

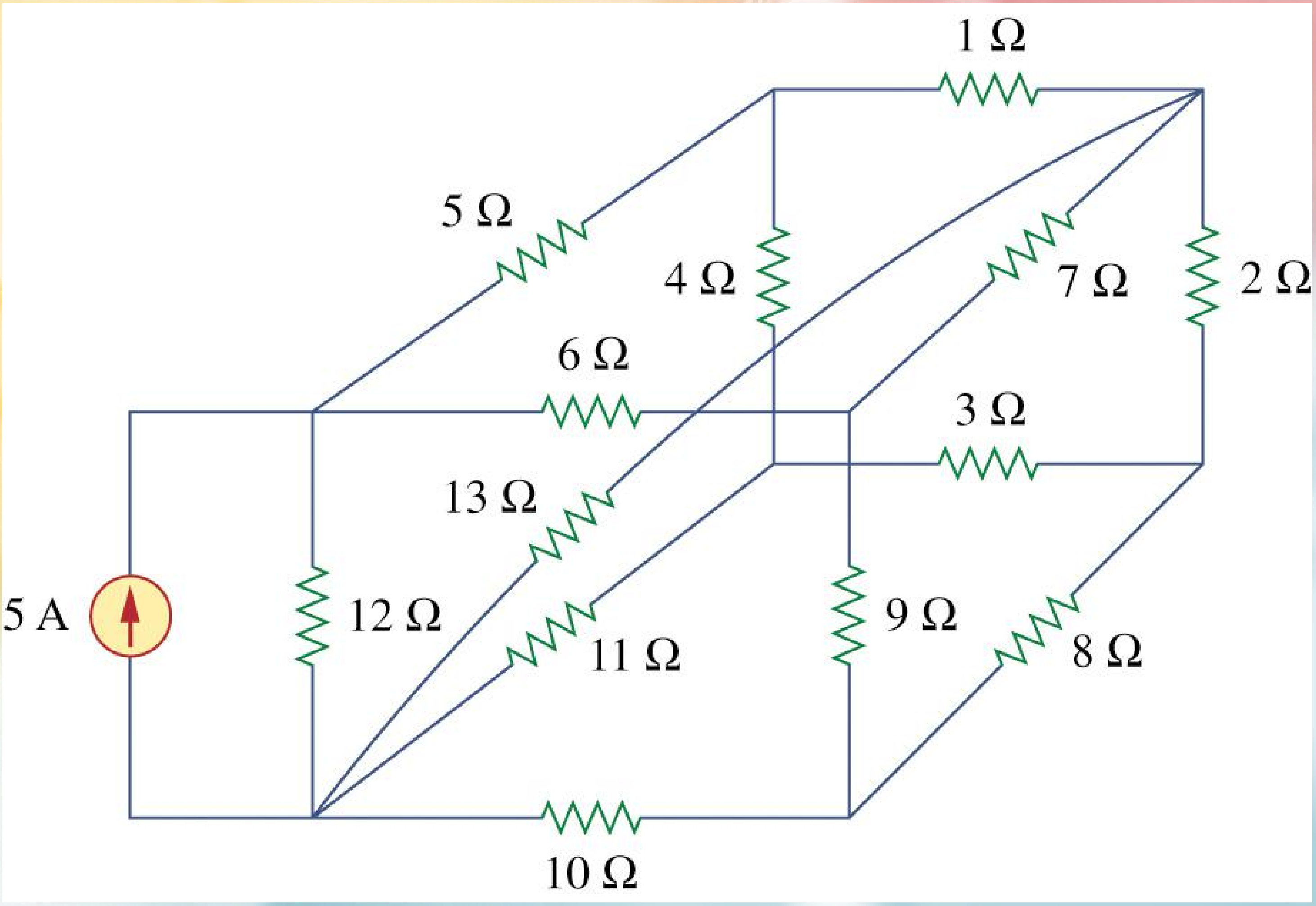




(a) A Planar circuit with crossing branches,
(b) The same circuit redrawn with no crossing branches.

MESH ANALYSIS

A nonplanar circuit.



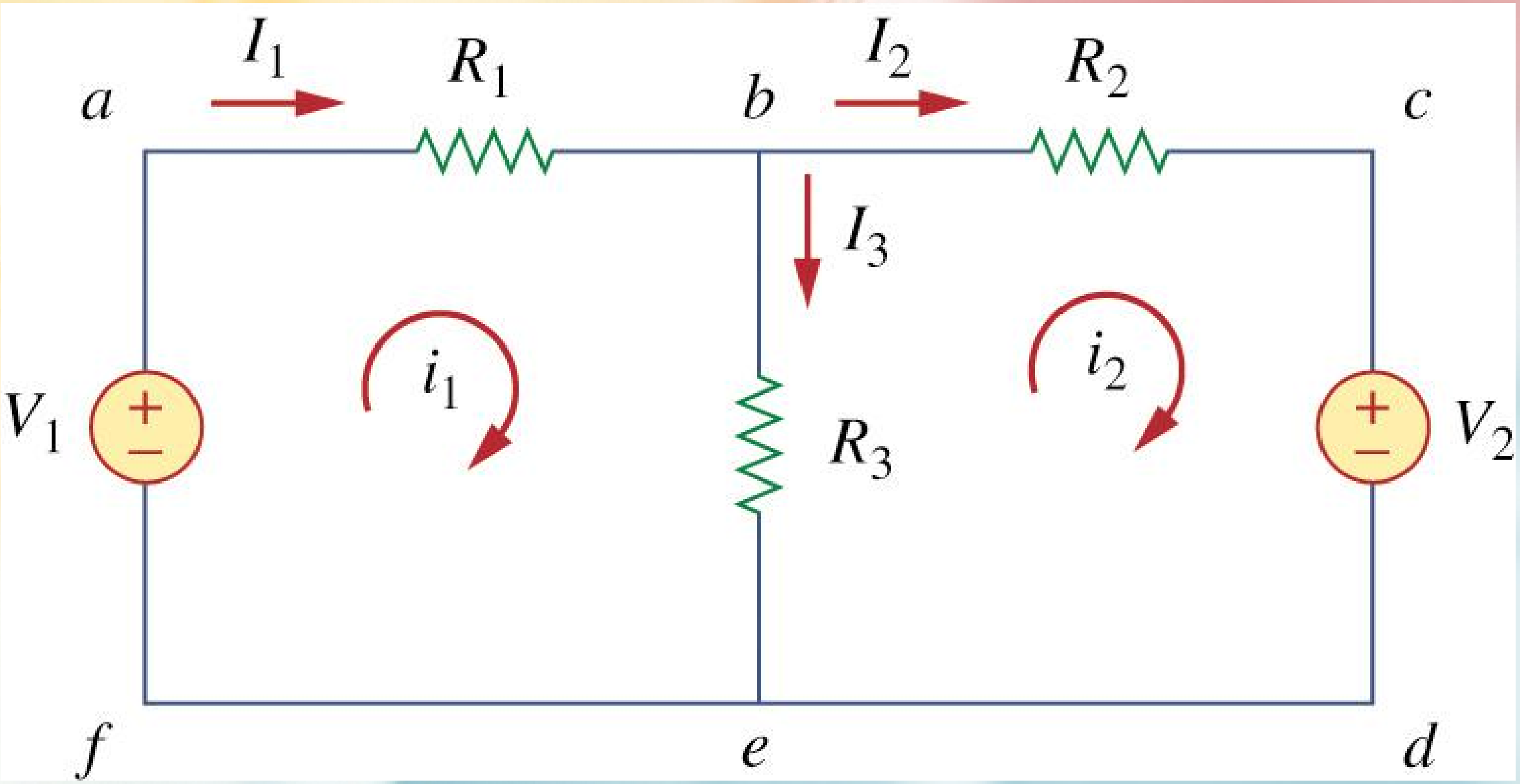
MESH ANALYSIS

- Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, .., i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting *n* simultaneous equations to get the mesh currents.

MESH ANALYSIS

A circuit with two meshes.



MESH ANALYSIS

- Apply KVL to each mesh. For mesh 1,

$$0V_1 \; . \; R_1 i_1 \; . \; R_3(i_1 \, 0 \, i_2) \, 6 \, 0$$

$$(R_1 \; . \; R_3) i_1 \, 0 \, R_3 i_2 \; 6 \, V_1$$

- For mesh 2,

$$R_2 i_2 \; . \; V_2 \; . \; R_3(i_2 \, 0 \, i_1) \, 6 \, 0$$

$$0 \, R_3 i_1 \; . \; (R_2 \; . \; R_3) i_2 \; 6 \, 0V_2$$

MESH ANALYSIS

- Solve for the mesh currents.

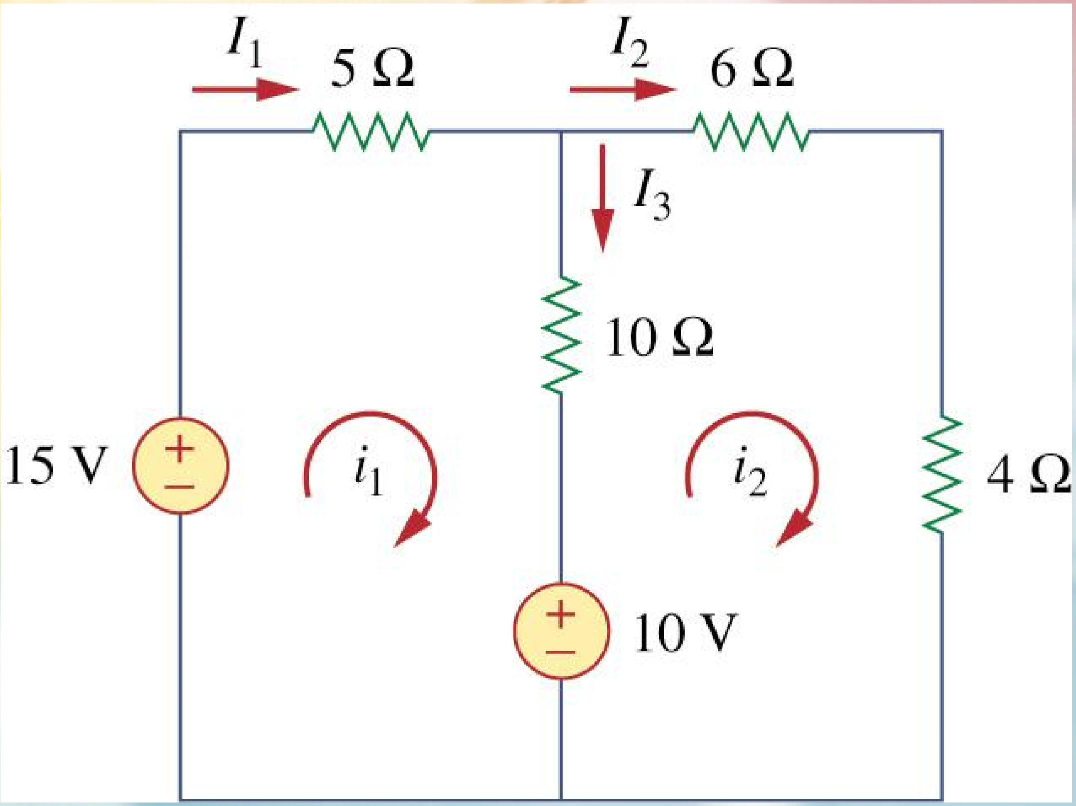
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

- Use *i* for a mesh current and *I* for a branch current. It's evident from Fig. 3.17 that

$$I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2$$

EXAMPLE

- Find the branch current I_1, I_2, and I_3 using mesh analysis.



SOLUTION

- For mesh 1,

$$015 .\ 5i_1 .\ 10(i_1\ 0\ i_2) .\ 10\ 6\ 0$$

$$3i_1\ 0\ 2i_2\ 6\ 1$$

- For mesh 2,

$$6i_2 .\ 4i_2 .\ 10(i_2\ 0\ i_1)\ 010\ 6\ 0$$

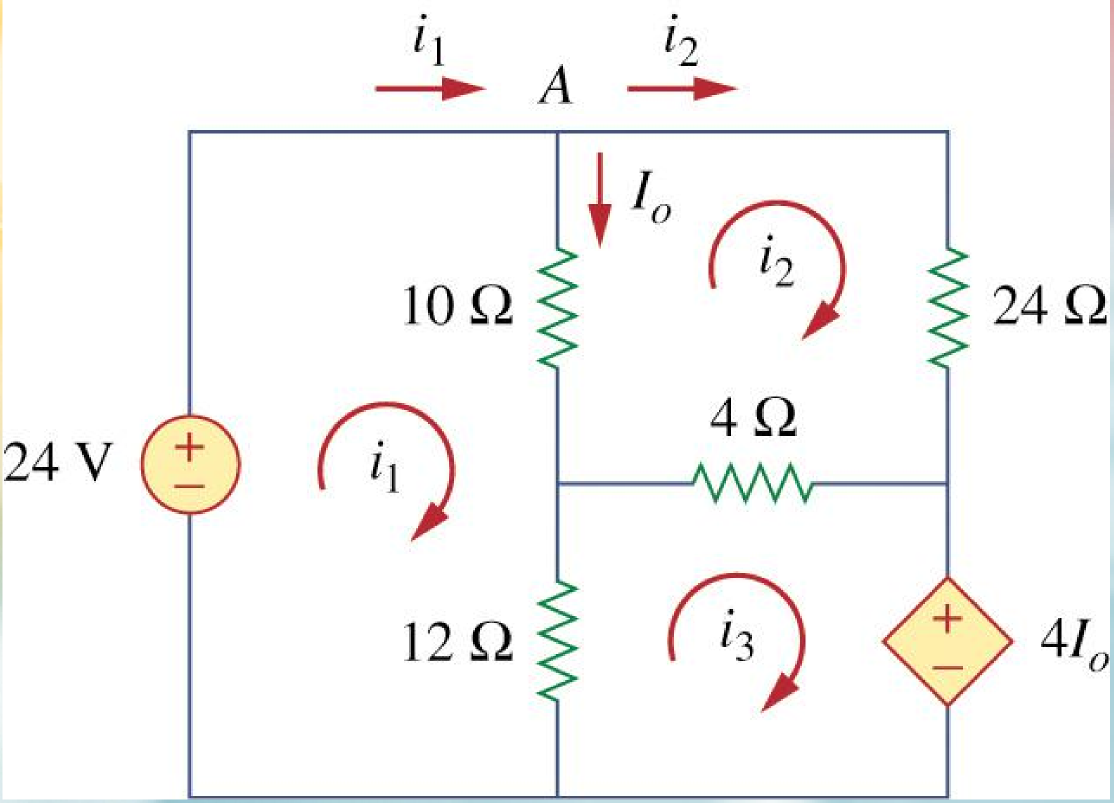
$$i_1\ 6\ 2i_2\ 01$$

- We can find i_1 and i_2 by substitution method or Cramer's rule. Then,

$$I_1\ 6\ i_1, \quad I_2\ 6\ i_2, \quad I_3\ 6\ i_1\ 0\ i_2$$

EXAMPLE

- Use mesh analysis to find the current I_0 in the circuit.



SOLUTION

- Apply KVL to each mesh. For mesh 1,

$$0\,24\;.\;10(i_1\;0\,i_2)\;.\;12(i_1\;0\,i_3)\;6\;0$$

$$11i_1\;0\,5i_2\;0\,6i_3\;6\,12$$

- For mesh 2,

$$24i_2\;.\;4(i_2\;0\,i_3)\;.\;10(i_2\;0\,i_1)\;6\;0$$

$$0\,5i_1\;.\;19i_2\;0\,2i_3\;6\;0$$

SOLUTION

- For mesh 3,

$$4I_0 \;.\; 12(i_3 \;0\; i_1) \;.\; 4(i_3 \;0\; i_2) \;6\; 0$$

At node A, $I_0 \;6\; I_1 \;0\; i_2$,

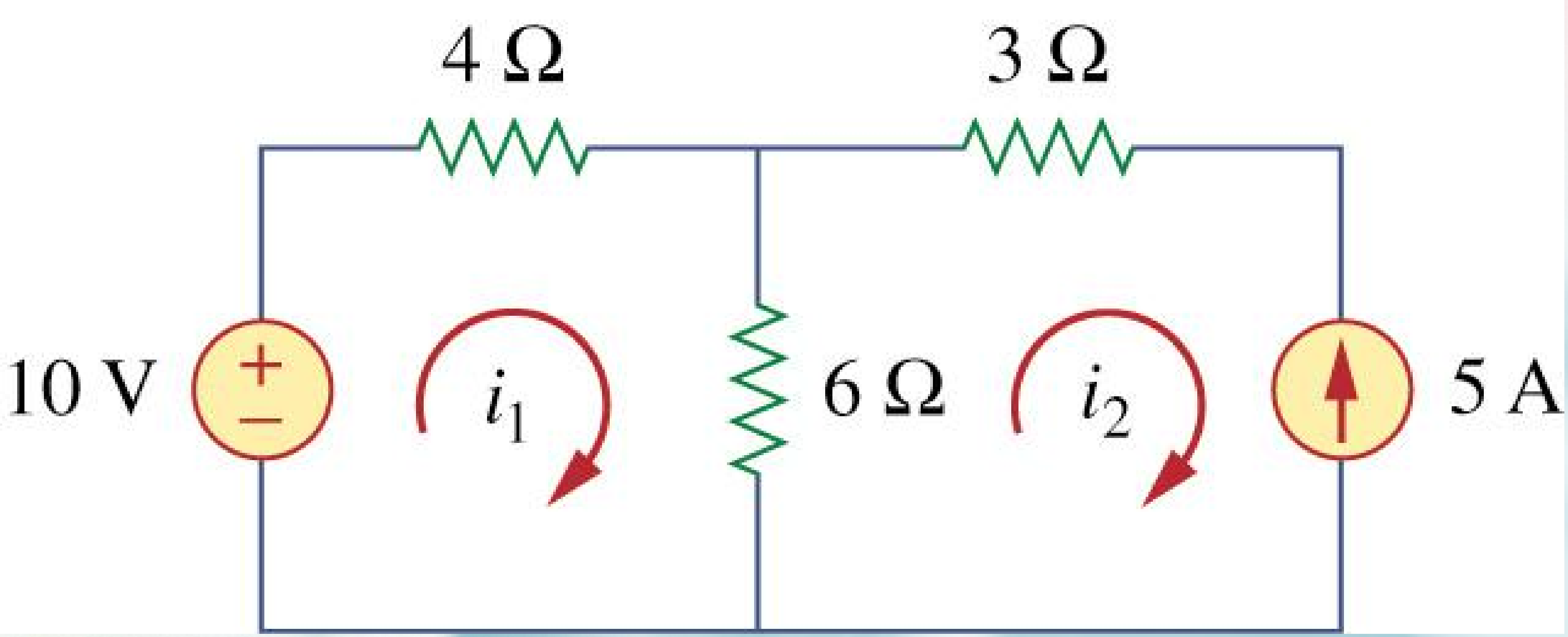
$$4(i_1 \;0\; i_2) \;.\; 12(i_3 \;0\; i_1) \;.\; 4(i_3 \;0\; i_2) \;6\; 0$$

- $$0 i_1 \;0\; i_2 \;.\; 2i_3 \;6\; 0$$

we can calculus i_1, i_2 and i_3 by Cramer’s rule, and find I_0.

MESH ANALYSIS WITH CURRENT SOURCES

A circuit with a current source.



MESH ANALYSIS WITH CURRENT SOURCES

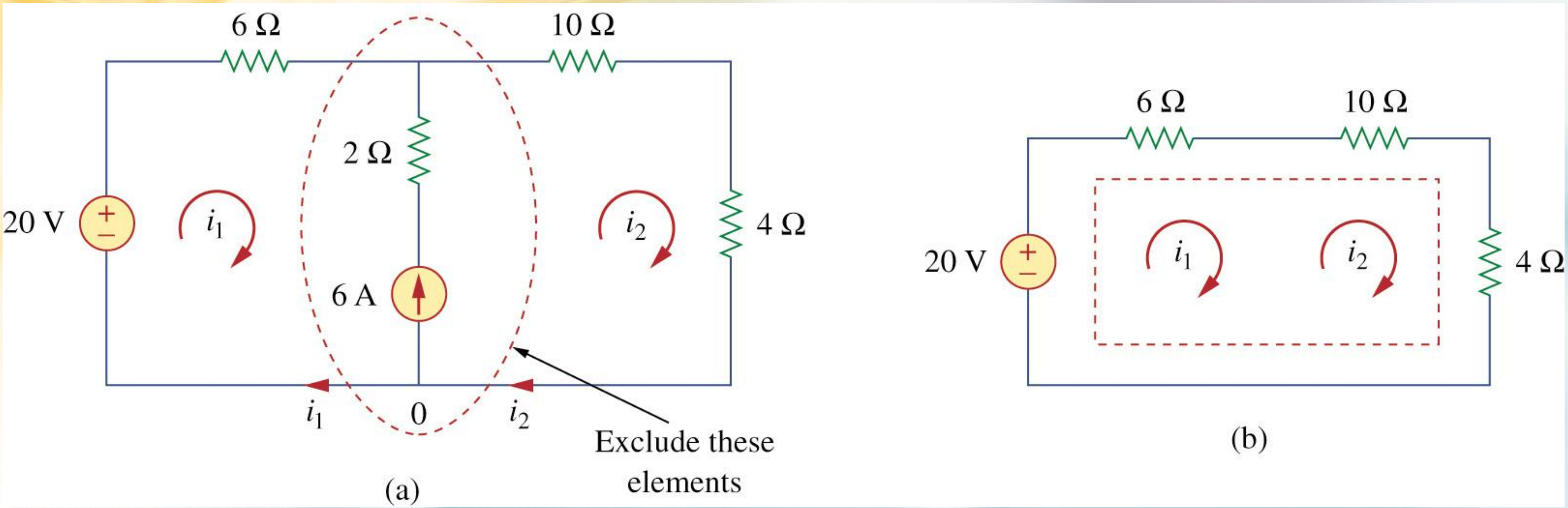
- Case 1
  - Current source exist only in one mesh

$$i_1 \ 6 \ 0 2 \, \text{A}$$

  - One mesh variable is reduced
- Case 2
  - Current source exists between two meshes, a super-mesh is obtained.

MESH ANALYSIS WITH CURRENT SOURCES

- a supermesh results when two meshes have a (dependent , independent) current source in common.



NODAL VERSUS MESH ANALYSIS

- Both nodal and mesh analyses provide a systematic way of analyzing a complex network.
- The choice of the better method dictated by two factors.
  - First factor : nature of the particular network. The key is to select the method that results in the smaller number of equations.
  - Second factor : information required.

SUMMERY

1. Nodal analysis: the application of KCL at the nonreference nodes
   – A circuit has fewer node equations
2. A supernode: two nonreference nodes
3. Mesh analysis: the application of KVL
   – A circuit has fewer mesh equations
4. A supermesh: two meshes