CONTROL THEORY

Block Diagram Reduction

Syllabus of Unit-1

Introduction, Open-loop system and its examples, Closed-loop system and its examples, Open-loop vs Closed-loop

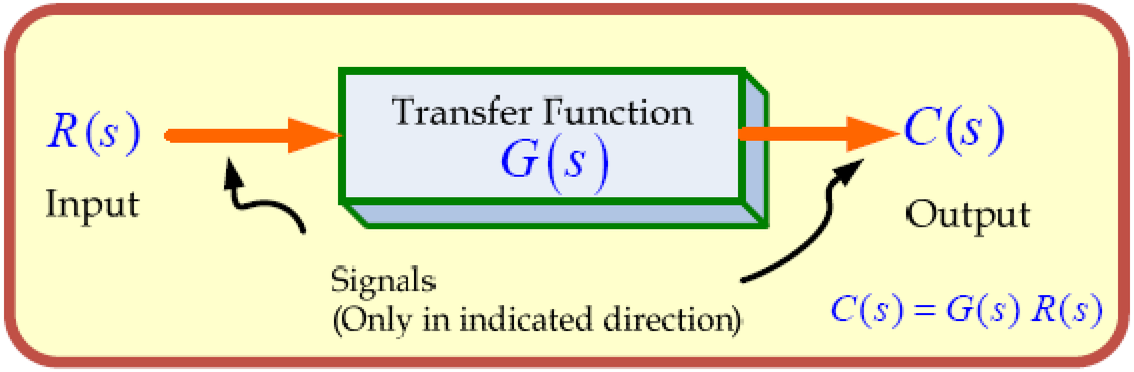
Mathematical Modeling

Modeling of Mechanical system, Modeling of Electronic and electrical system, Modeling of Liquid-level system, Transfer function of system, Modeling in state-space

Block diagram reduction techniques & signal flow graph

Block Diagram

- Pictorial Representation of functions performed by each component of a system and that of flow of signals.

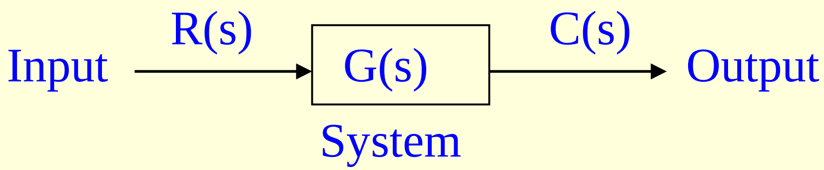


Single block diagram representation

Block Diagram Representation

A block diagram is a graphical tool can help us to ***visualize the model*** of a system and ***evaluate the mathematical relationships between their elements***, using their transfer functions.

The Transfer Function Block

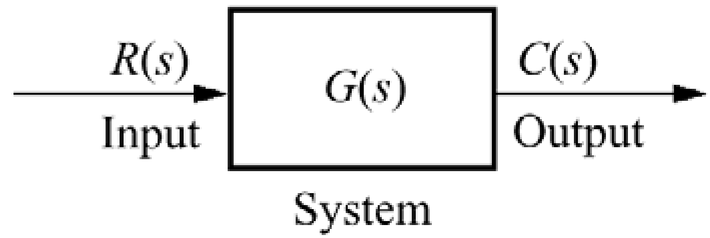


$$G(s) = \frac{C(s)}{R(s)}$$

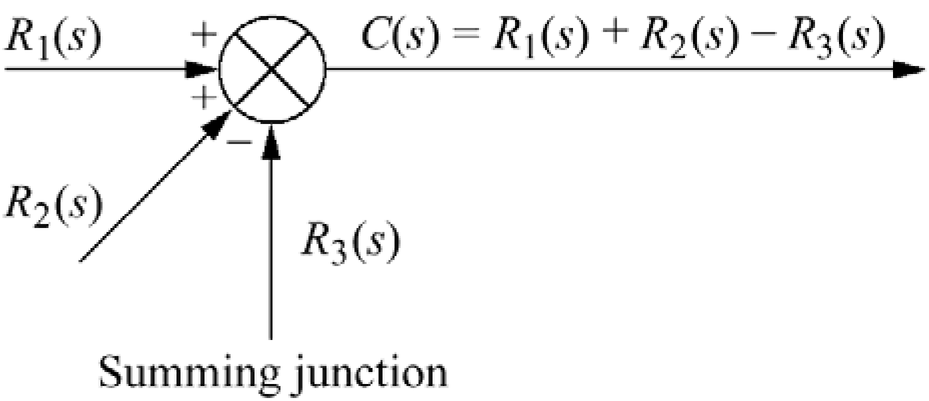
The transfer function G(s) is

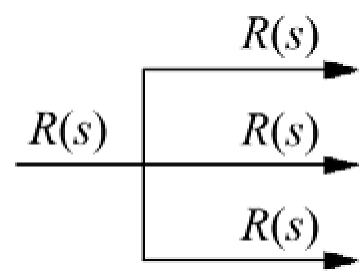
- defined only for a linear time-invariant system and not for nonlinear systems.
- Is a property of the system and is independent of the input to the system.
- Commutative $G_1 G_2 = G_2 G_1$
- Associative $G_1 + G_2 = G_2 + G_1$

Components of a system



Signals

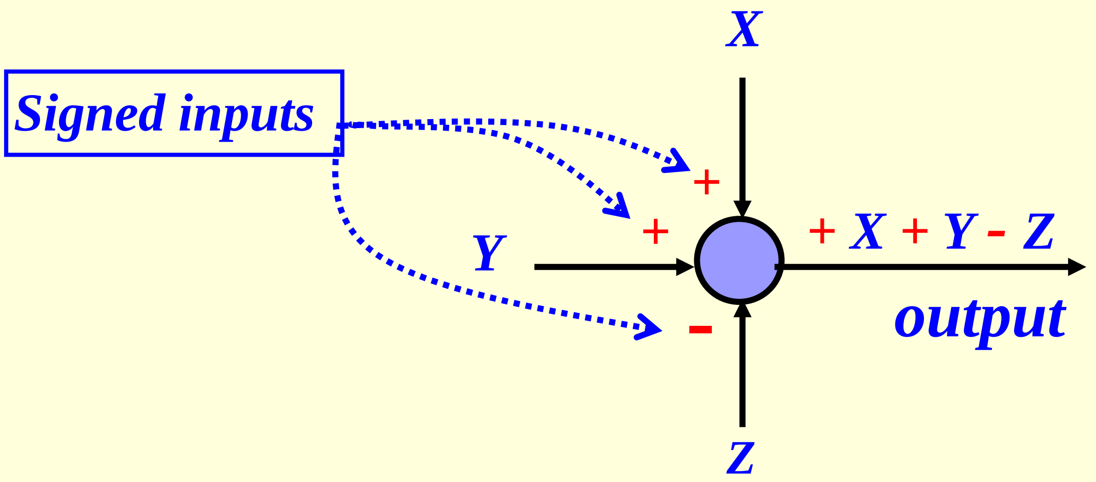




Take off Point

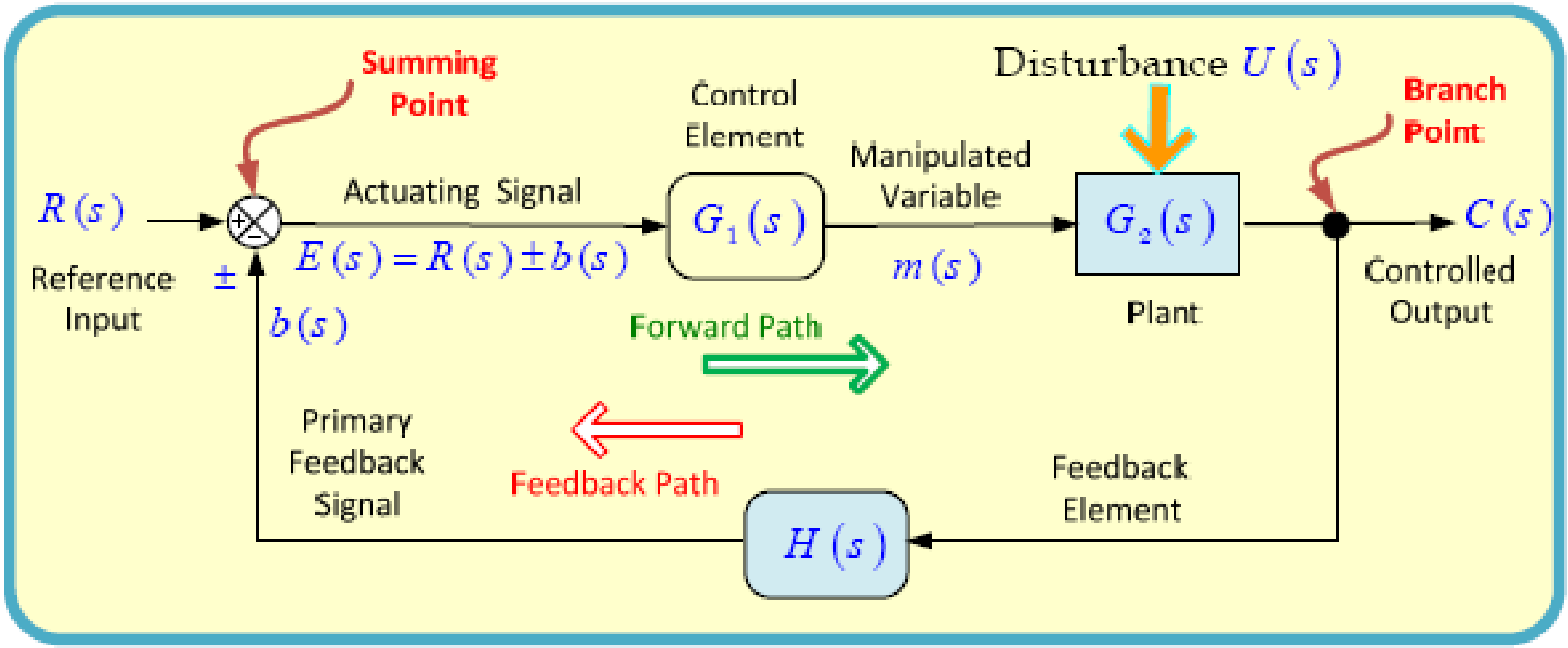
Block Diagram Elements

The Summing Point



- Any number of inputs. Only one output

Terminology



Terminology

- Plant: Physical object to be controlled. G2(S)
- Control Element: G1(s) , also called the controller required to generate the appropriate control signal applied to the plant.
- Feedback Element: H(S) is the component required to establish the functional relationship between the primary feedback signal B (s) and the controlled output C(s).
- Reference Input: R (s) is an external signal applied to a feedback control system in order to command a specified action of the plant.
- The Controlled Output C(s) is that quantity or condition of the plant which is controlled.

Terminology

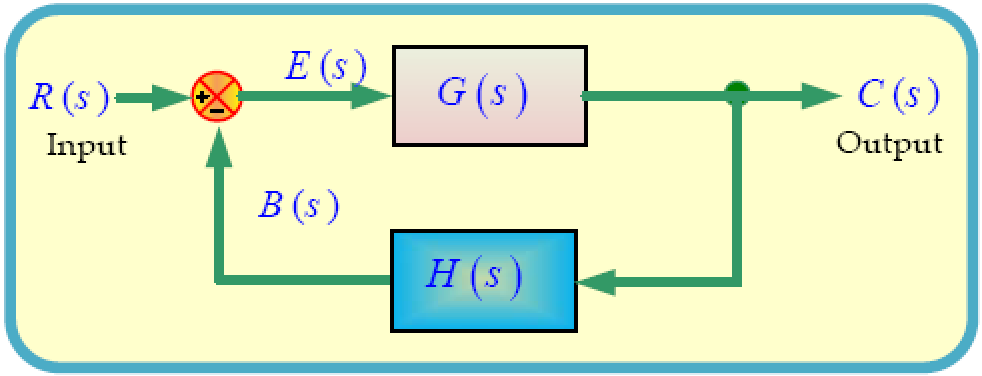
- Actuating Signal E(s) , also called the error or control action, is the algebraic sum consisting of the reference input R (s) plus or minus (usually minus) the primary feedback B (s) .
- Manipulated Variable M(s) (control signal) is that quantity or condition which the control elements G 1(s) apply to the plant G2(s) .
- Disturbance U (s) is an undesired input signal which affects the value of the controlled output C(s). It may enter the plant by summation with M (s) , or via an intermediate point, as shown in the block diagram.

Terminology

- Forward Path is the transmission path from the actuating signal E(s) to the output C(s).
- Feedback Path is the transmission path from the output C(s) to the feedback signal B (s).
- Summing Point: A circle with a cross is the symbol that indicates a summing point. The (+) or (–) sign at each arrowhead indicates whether that signal is to be added or subtracted.

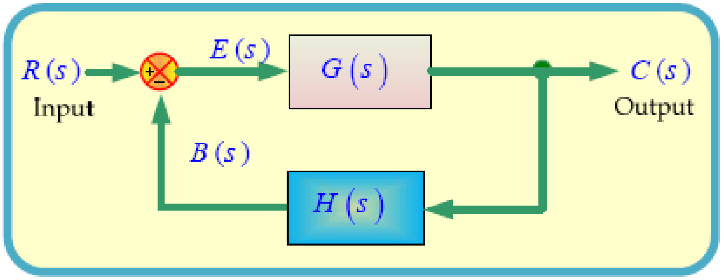
Definitions

- *G(s) = Direct transfer function = Forward transfer function.*
- *H (s) = Feedback transfer function.*
- *C(s) / R(s) = Closed-loop transfer function = Control ratio*



Closed loop transfer function

- the output *C(s) and input R(s) are related as follows* $C(s) = G(s)E(s)$



where

$$E(s)=R(s)-B(s)=R(s)-H(s)C(s)$$

Eliminating *E(s) from these equations gives*

$$C(s)=G(s)\,[R(s)-H(s)C(s)]$$

This can be written in the form

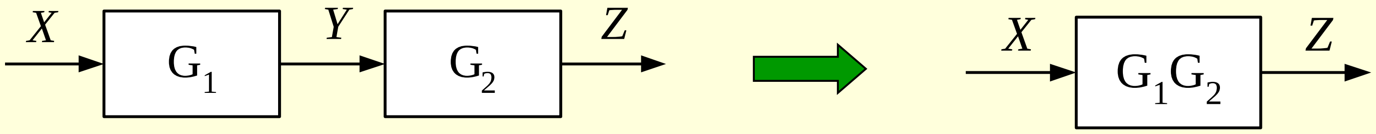
$$[1+G(s)H(s)]C(s)=G(s)R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Block diagram and Simplifications

When manipulating block diagrams, the original relationships, or equations, relating the various variables must remain the same.

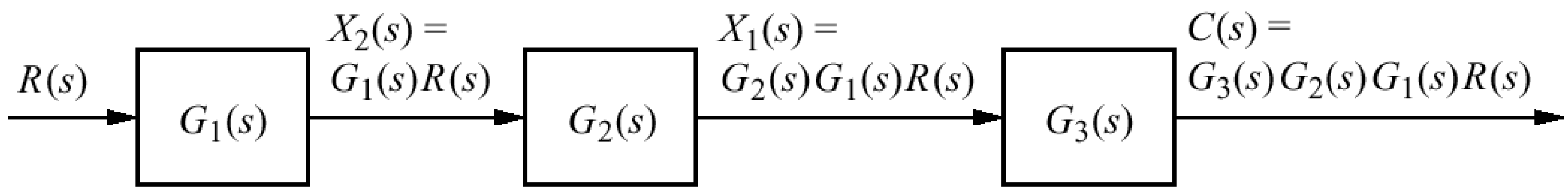
Blocks in series or cascaded blocks

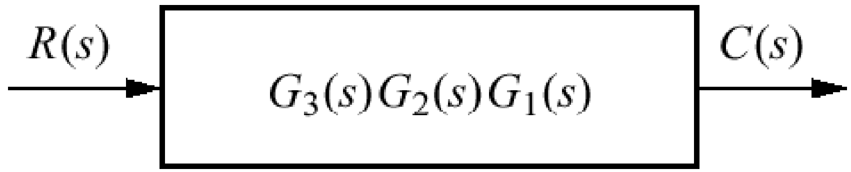


- When blocks are connected in series, there must be no loading effect.

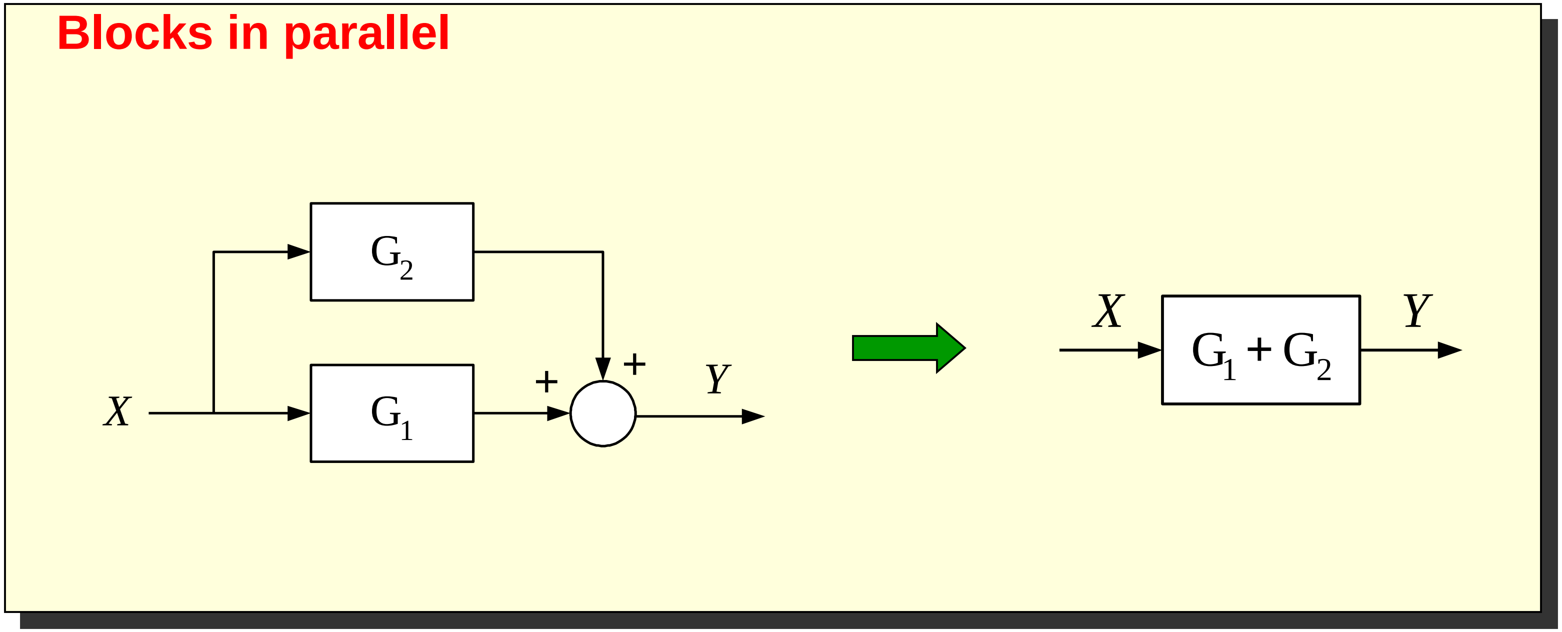
Block diagram and Simplifications

- Cascade Connections

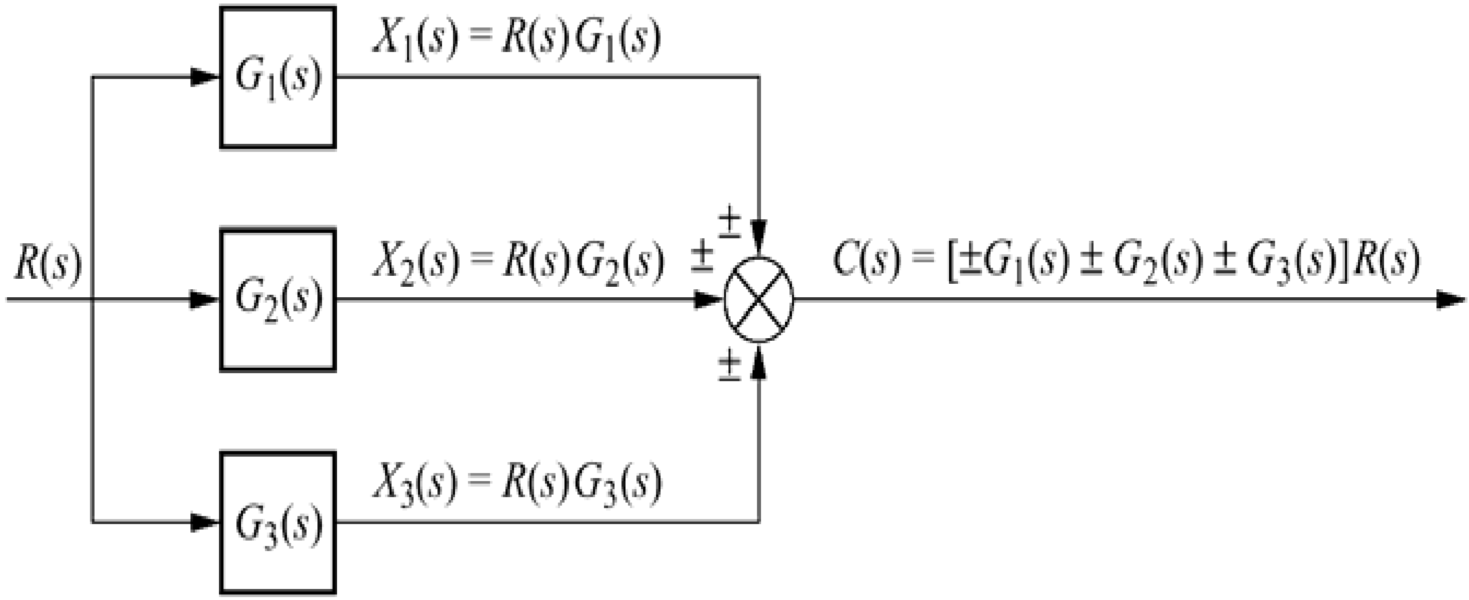


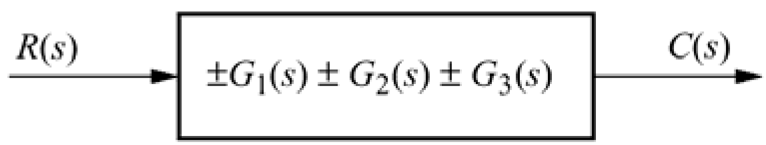


Parallel Connections

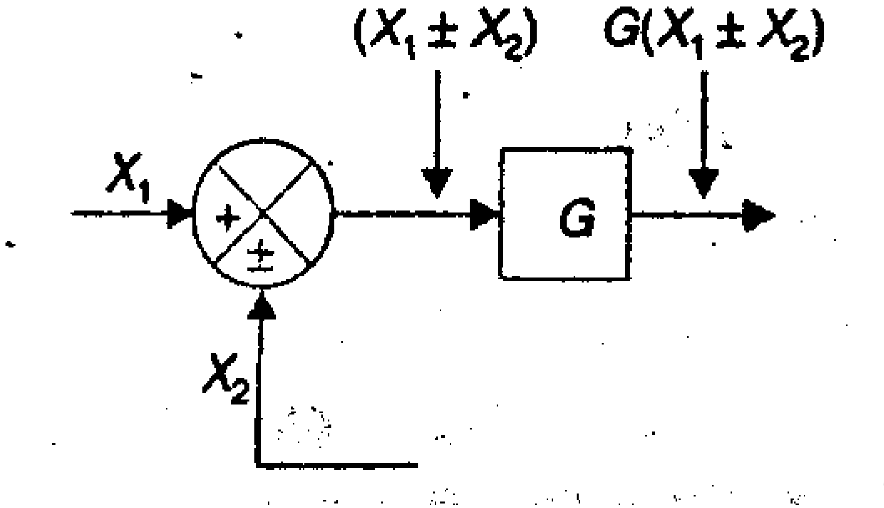


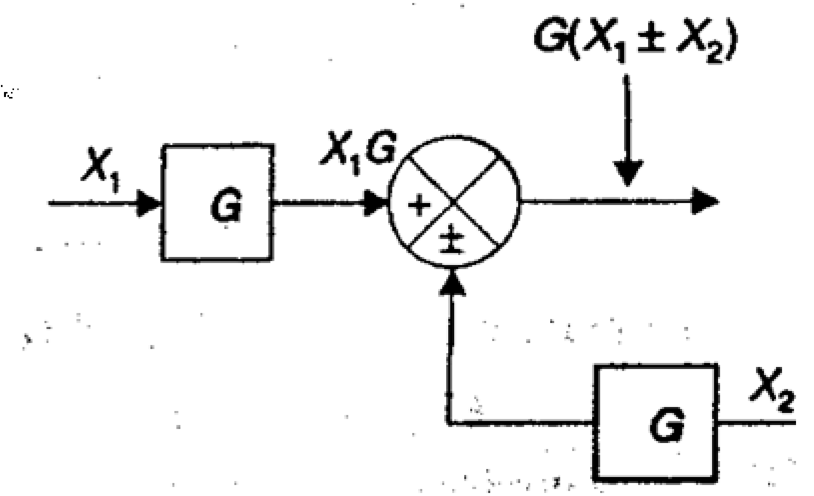
Parallel Connections



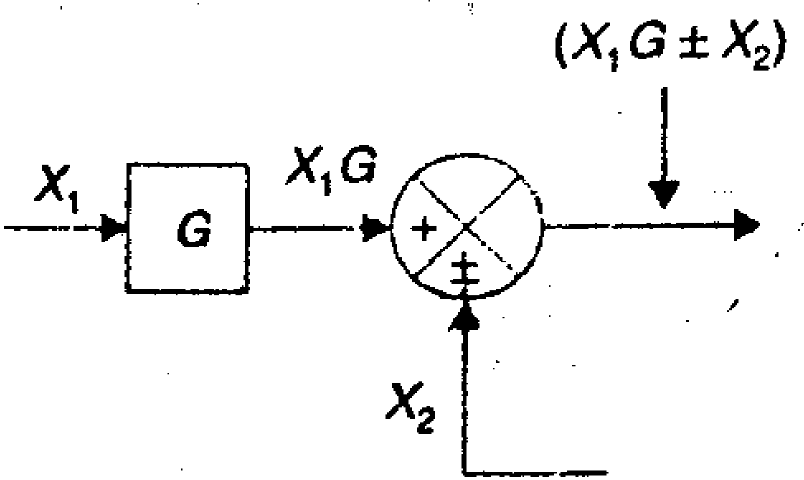


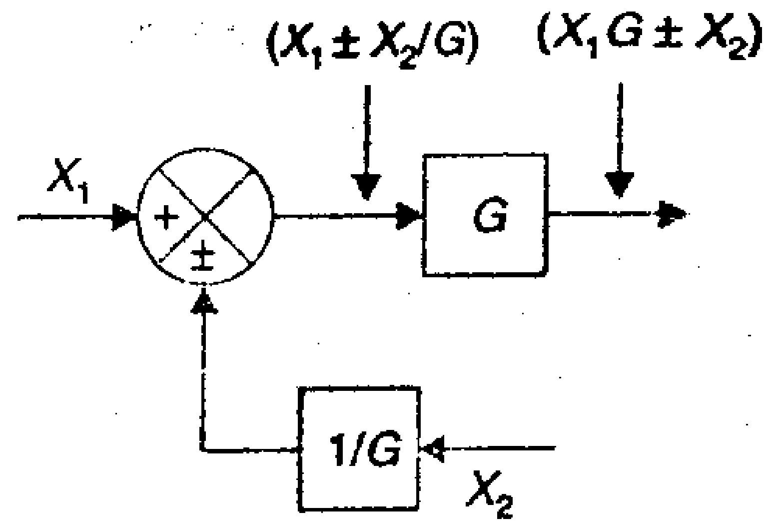
Moving a summing point after a block



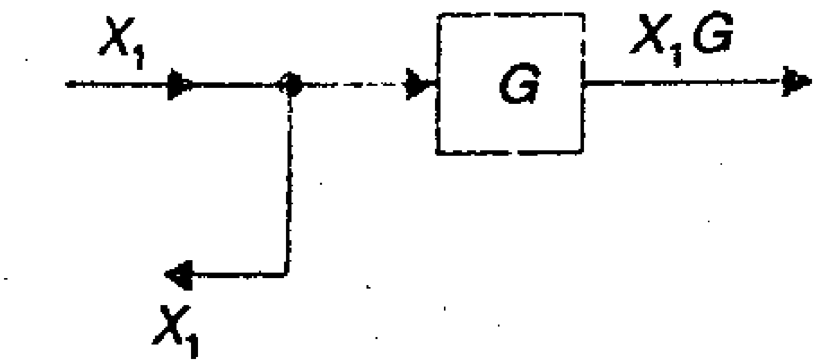


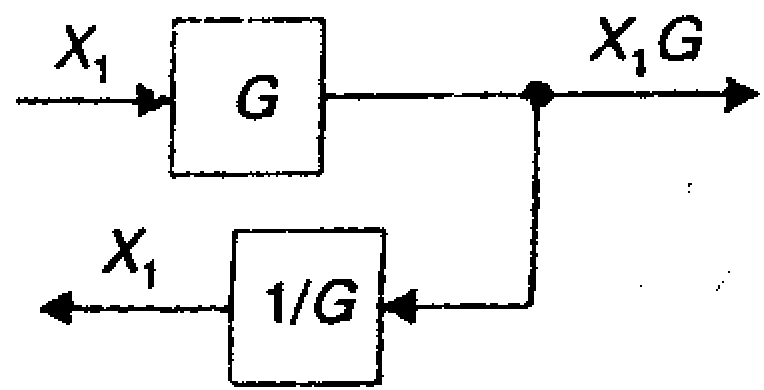
Moving a summing point ahead of block



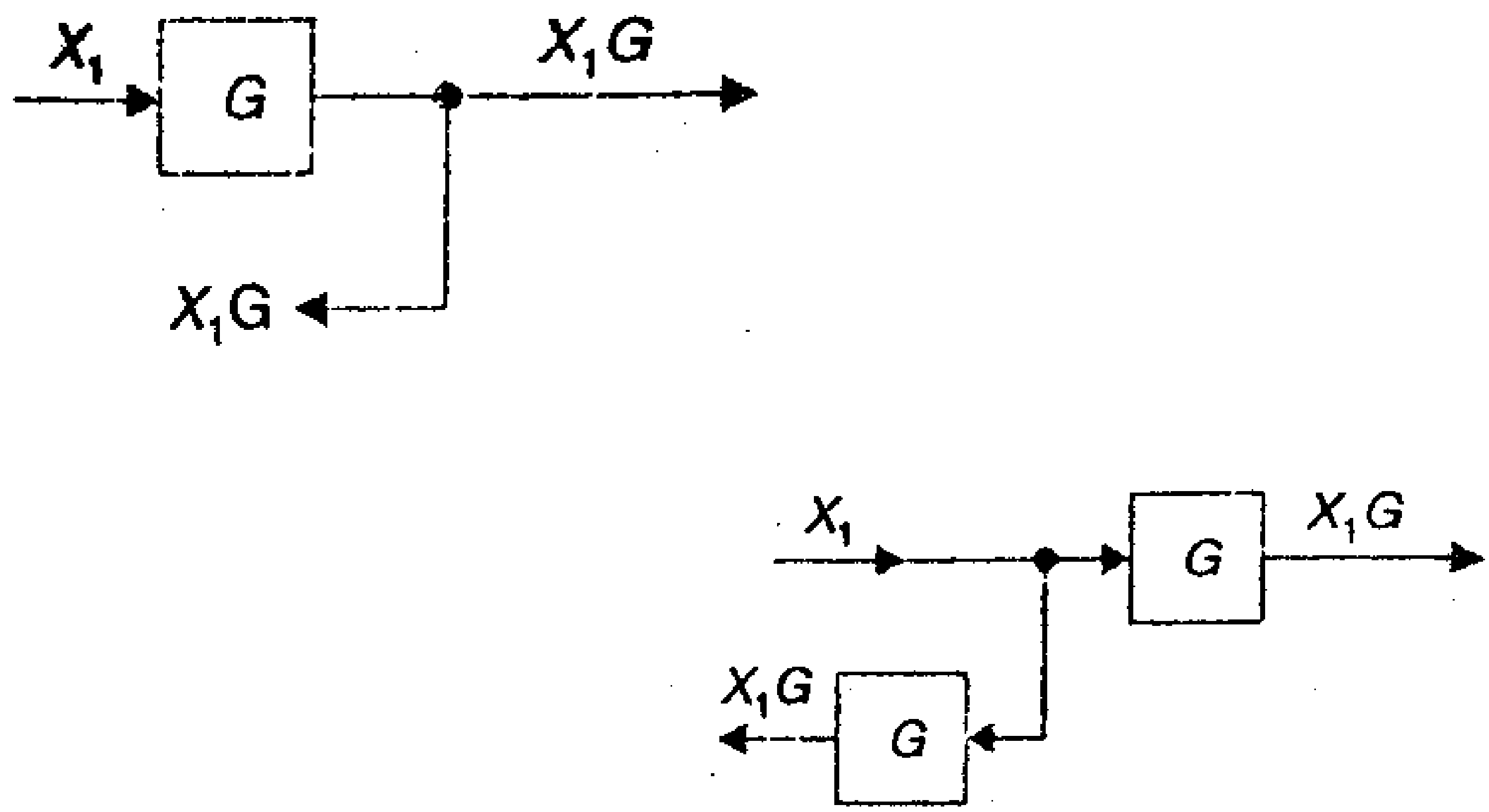


Moving a take of point after a block

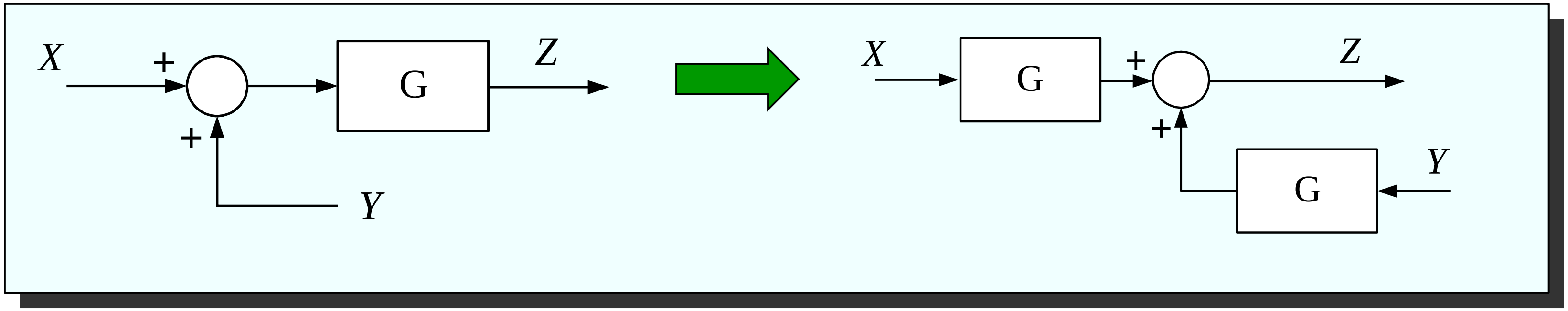


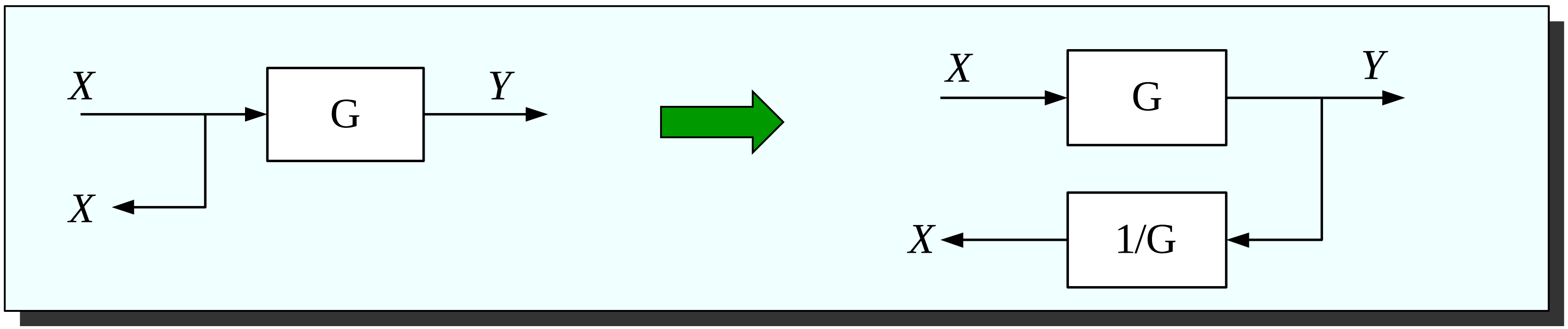


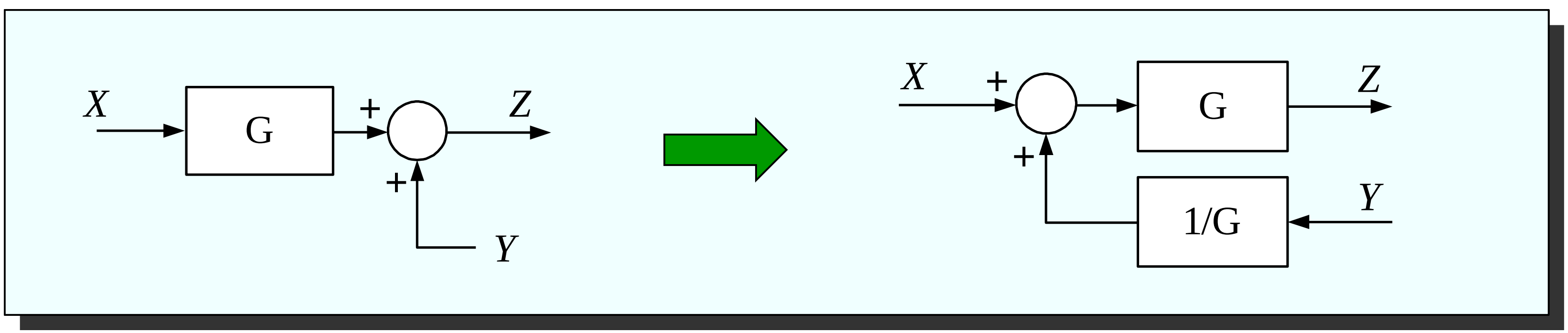
Moving a take of point before a block



Block Diagram







Procedure to solve Block diagram reduction problems

Step-1: Reduce the blocks connected in series

Step-2: Reduce the blocks connected in Parallel.

Step-3: Reduce the minor internal feedback loops.

Step-4: as far as possible try to shift take of points towards right and summing points towards left.

Step-5: repeat step-1 to 4 till simple form obtained.

Step-6: obtain closed loop transfer function using standard method.

Example - 1

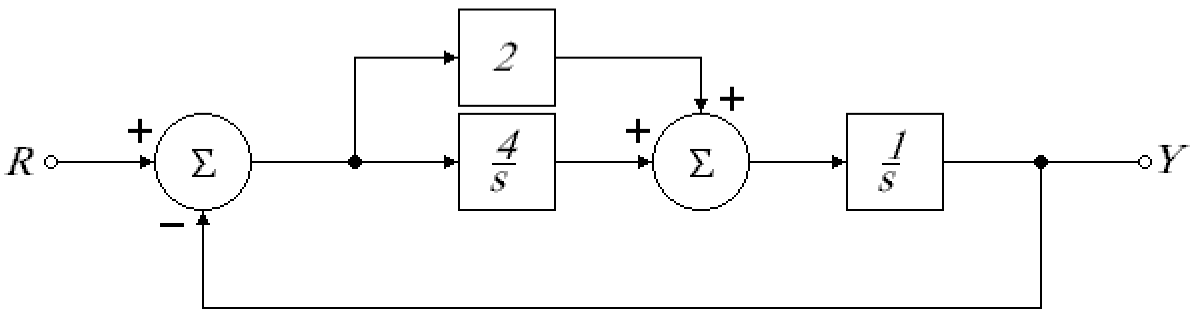


fig. (a)

$$T(s) = \frac{Y(s)}{R(s)}$$

$$T(s) = \frac{\frac{2s+4}{s^2}}{1+\frac{2s+4}{s^2}}$$

$$T(s) = \frac{2s+4}{s^2+2s+4}$$

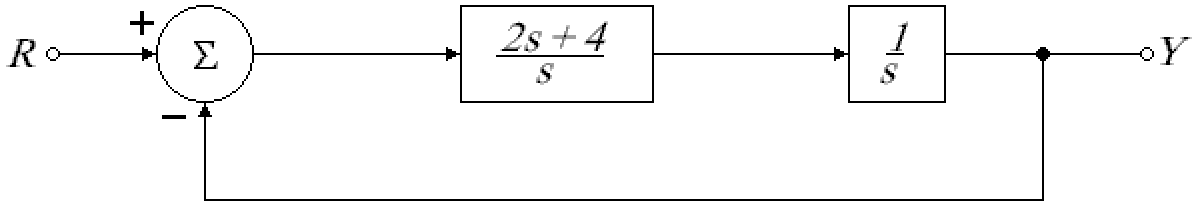
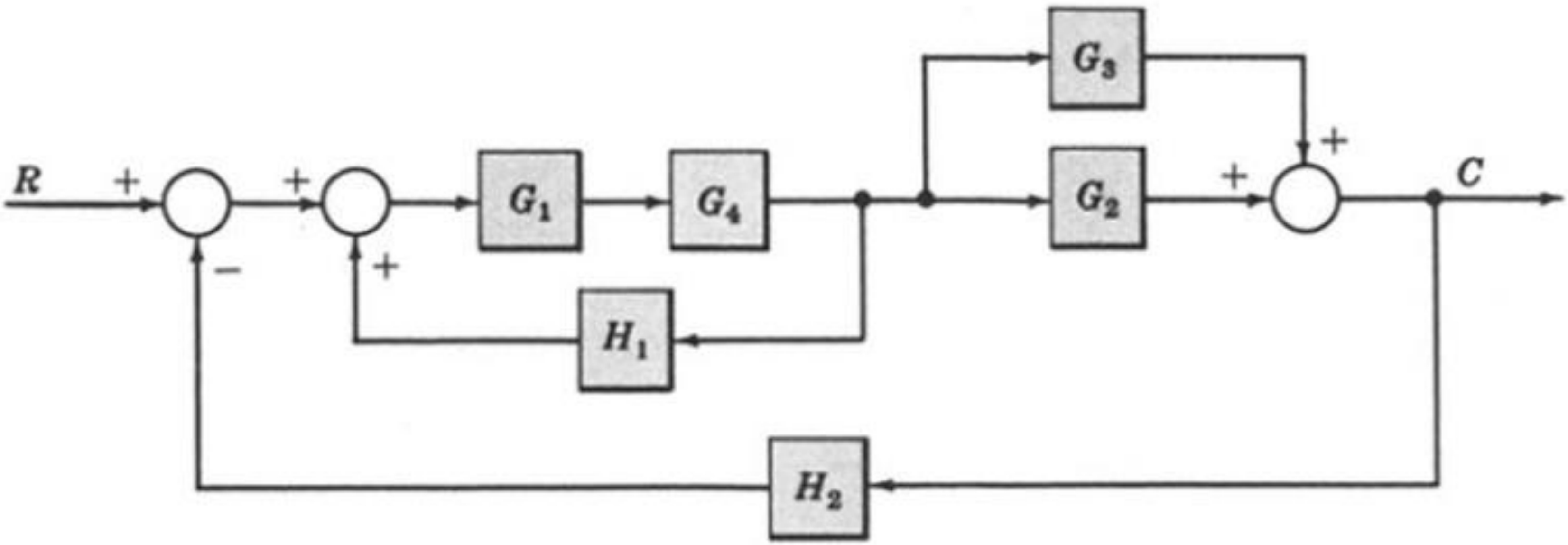


fig. (b)

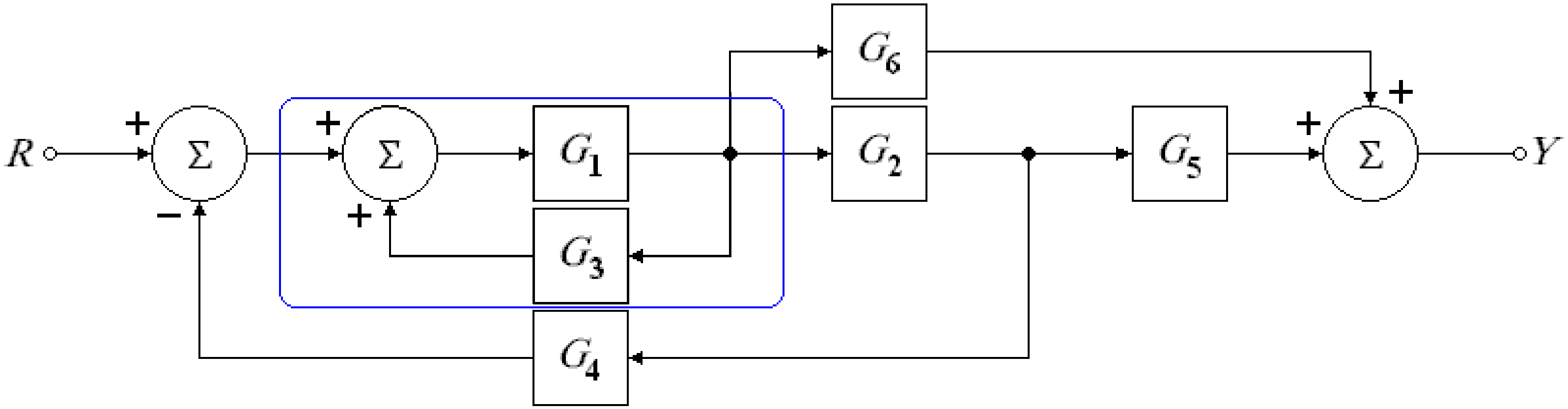
Example - 2

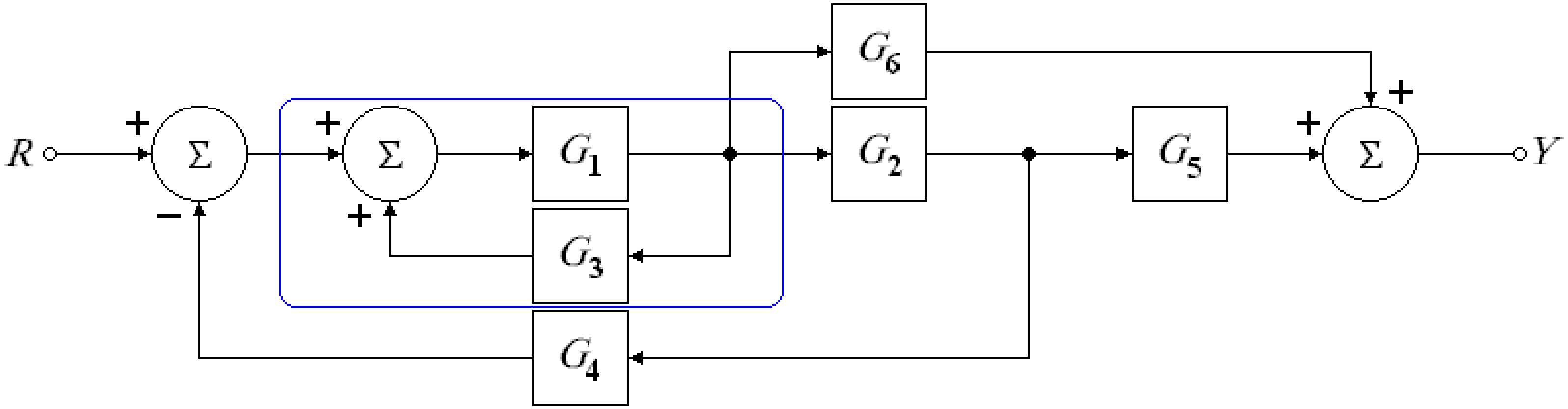
To reduce the block diagram to simple form.

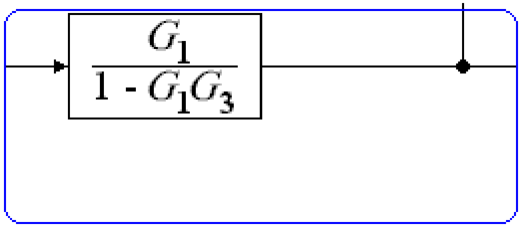


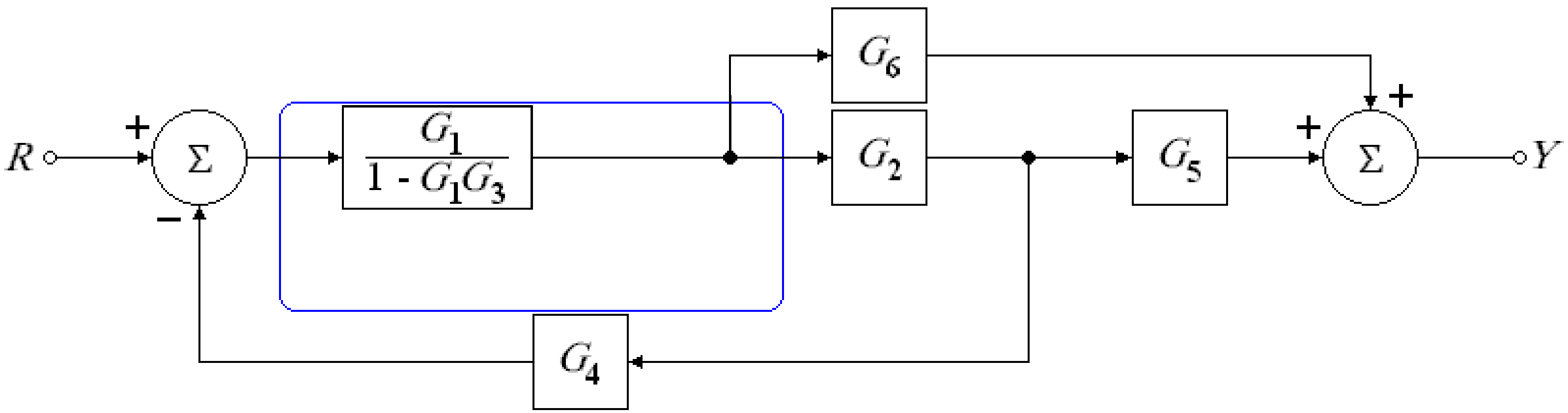
$$\frac{C}{R} = \frac{(G1G4G2 + G1G4G3)}{1 - G1G4H1 + G1G4G2H2 + G1G4G3H2}$$

Example 3

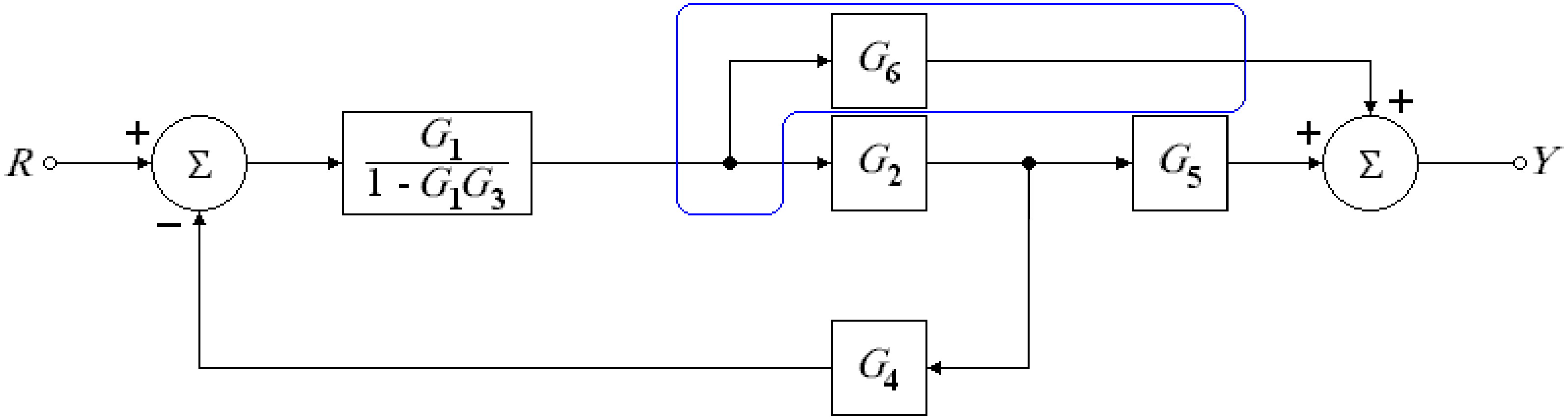



R
+
Σ
−
+
Σ
+
G1
G3
G4
G6
G2
G5
+
+
Σ
Y

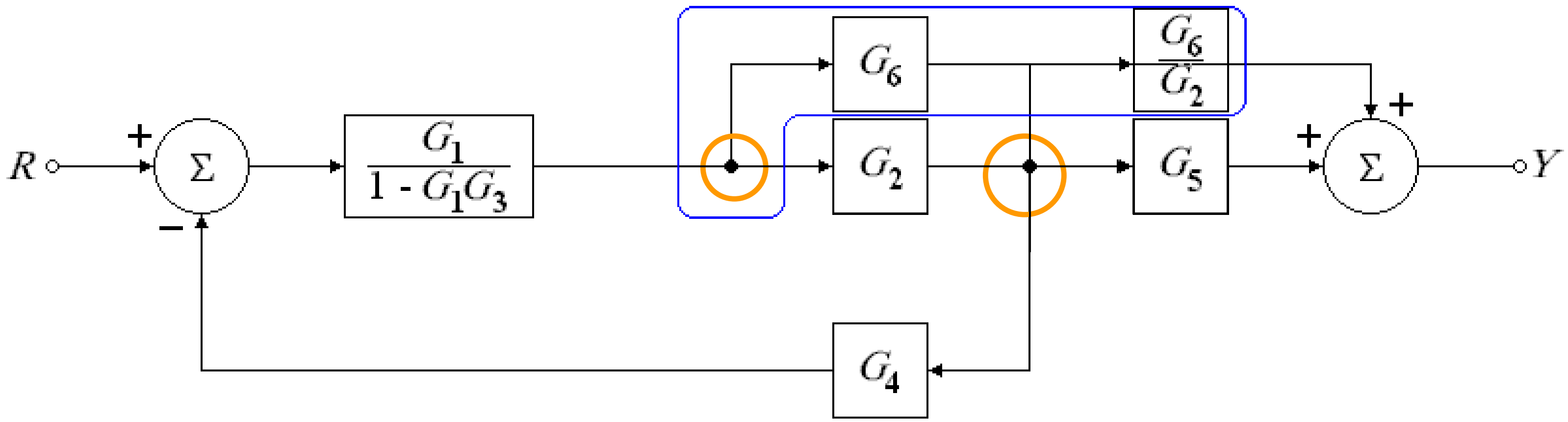

G1
1 - G1G3

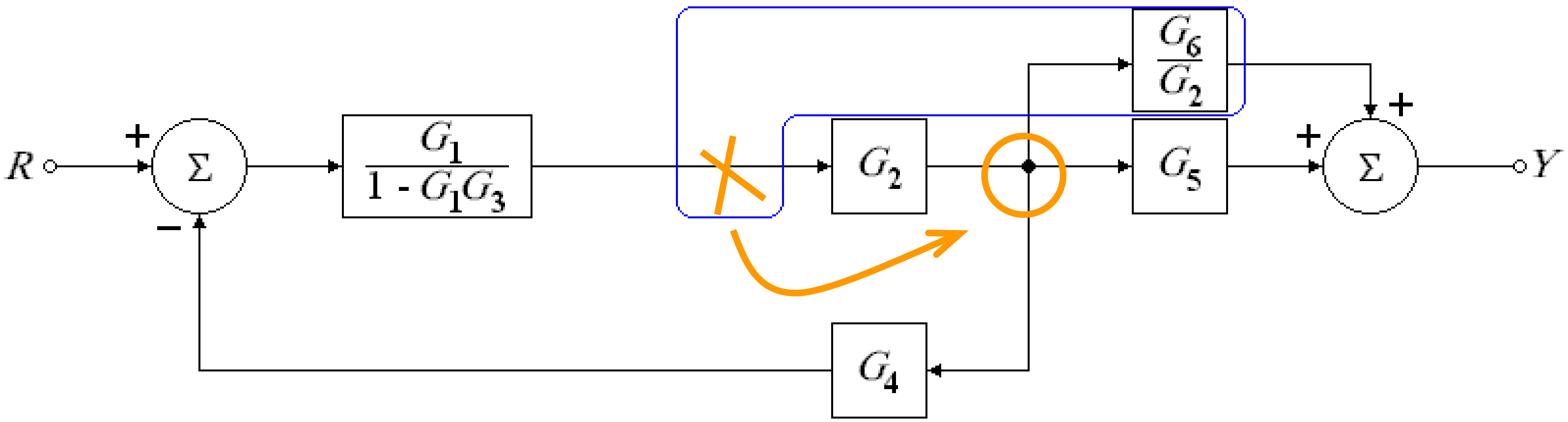


R
+
Σ
−
$\frac{G_1}{1 - G_1G_3}$
G_6
G_2
G_5
G_4
+
+
Σ
Y

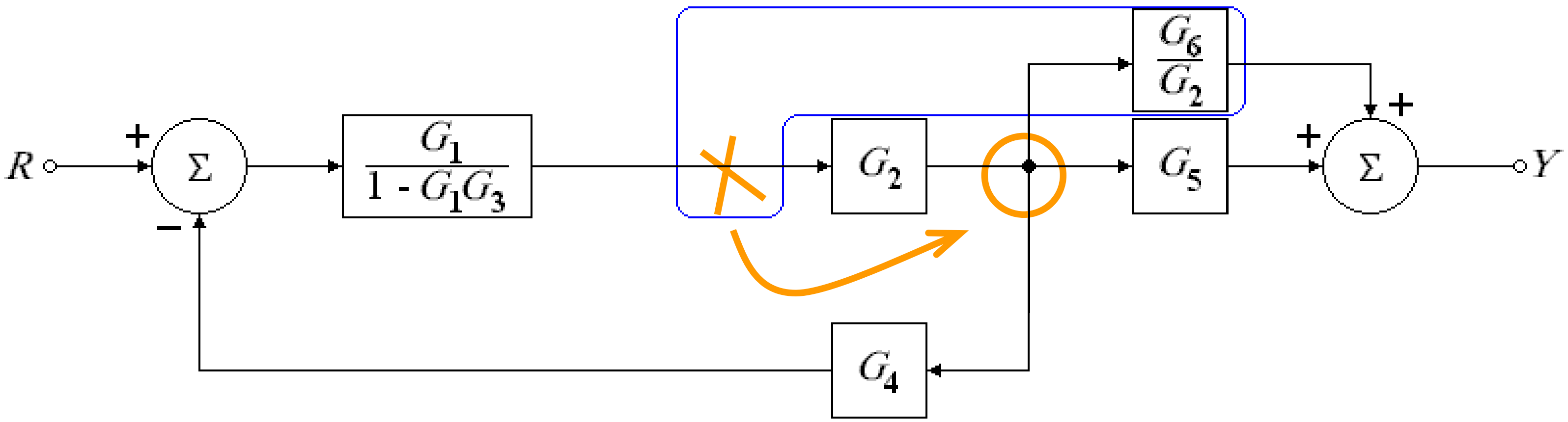


R
+
Σ
−
$\frac{G_1}{1 - G_1G_3}$
G_6
G_2
G_5
G_4
+
+
Σ
Y


R
+
−
Σ
G1
1 - G1G3
G6
G2
G6
G2
G5
G4
+
+
Σ
Y


R
+
Σ
−
$\frac{G_1}{1 - G_1G_3}$
G_2
$\frac{G_6}{G_2}$
G_5
+
+
Σ
Y
G_4

TF from the Block Diagram



$$T(s) = \frac{G_1G_2G_5 + G_1G_6}{1 - G_1G_3 + G_1G_2G_4}$$

Block diagram reduction

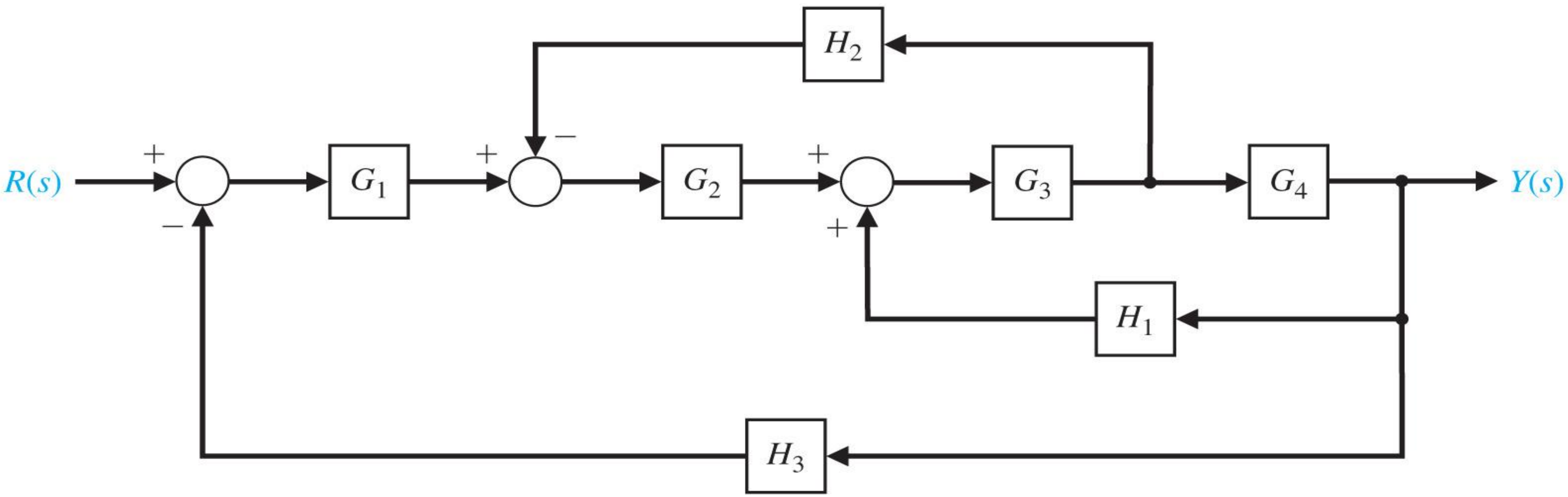


Figure: 02-26

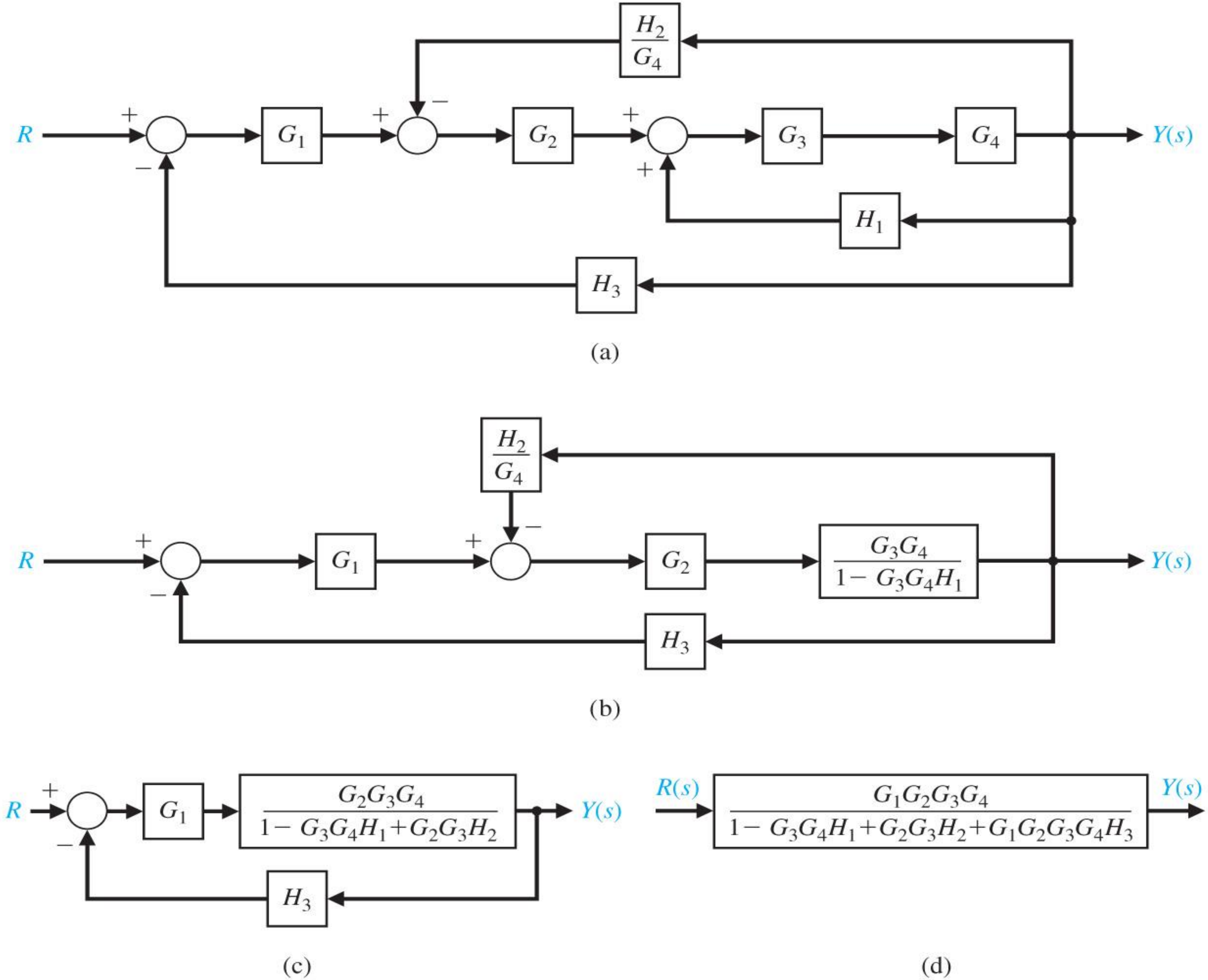
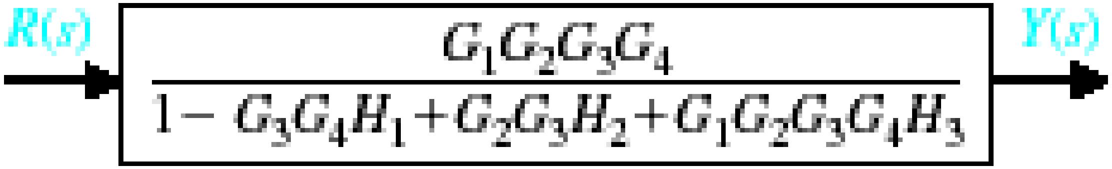
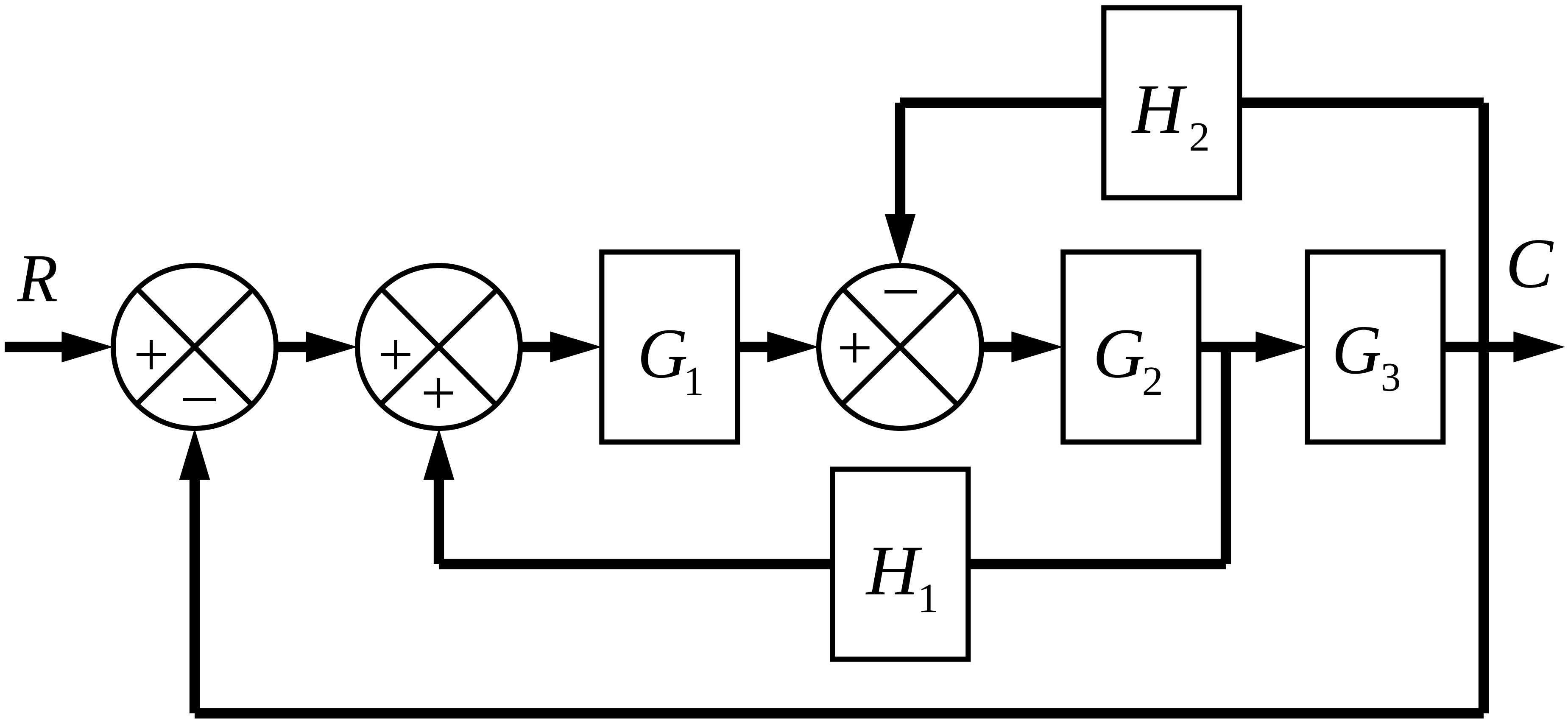

R
+
−
G_1
+
$\frac{H_2}{G_4}$
−
G_2
+
+
G_3
G_4
$Y(s)$
H_1
H_3
(a)
R
+
−
G_1
+
$\frac{H_2}{G_4}$
−
G_2
$\frac{G_3G_4}{1 - G_3G_4H_1}$
$Y(s)$
H_3
(b)
R
+
−
G_1
$\frac{G_2G_3G_4}{1 - G_3G_4H_1 + G_2G_3H_2}$
$Y(s)$
H_3
(c)
$R(s)$
$\frac{G_1G_2G_3G_4}{1 - G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4H_3}$
$Y(s)$
(d)

Figure: 02-27

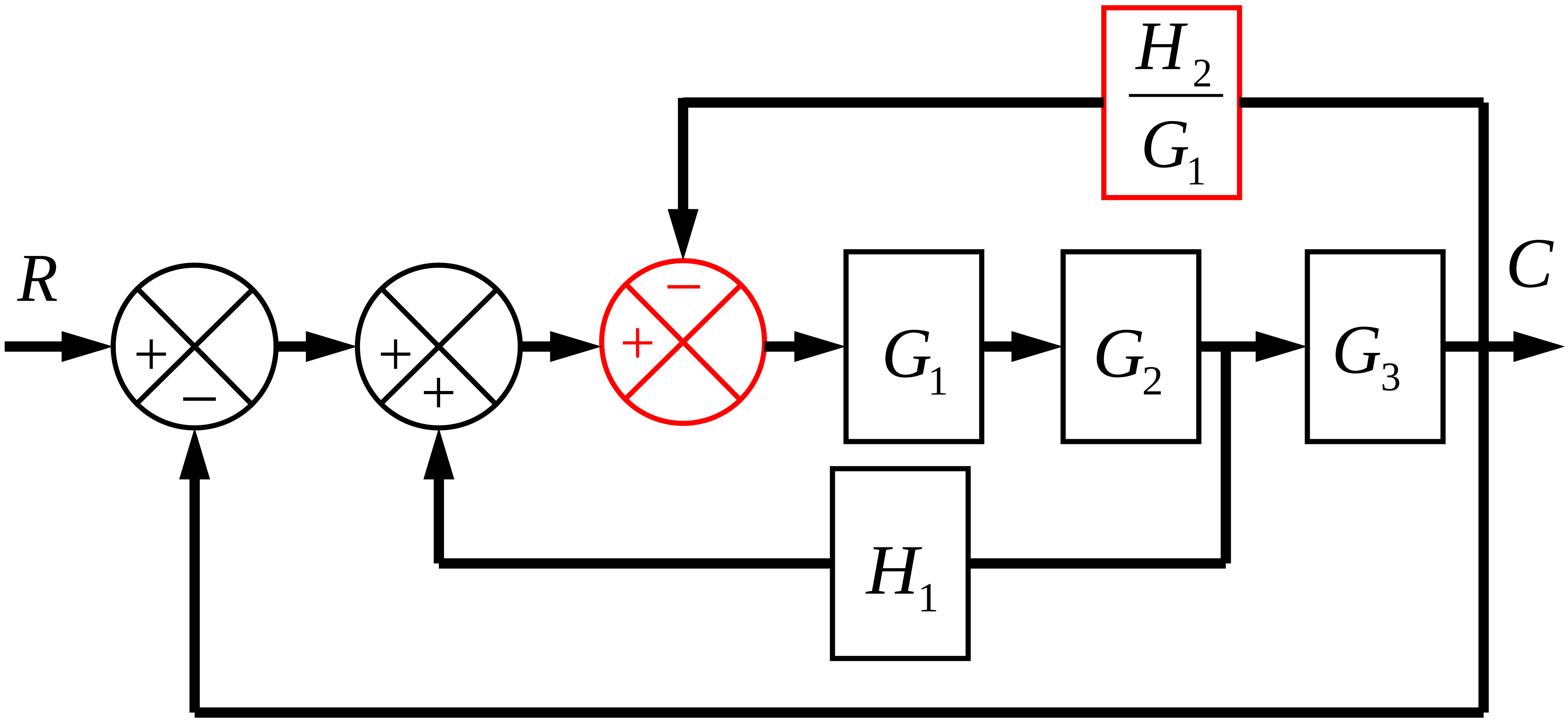


(d)

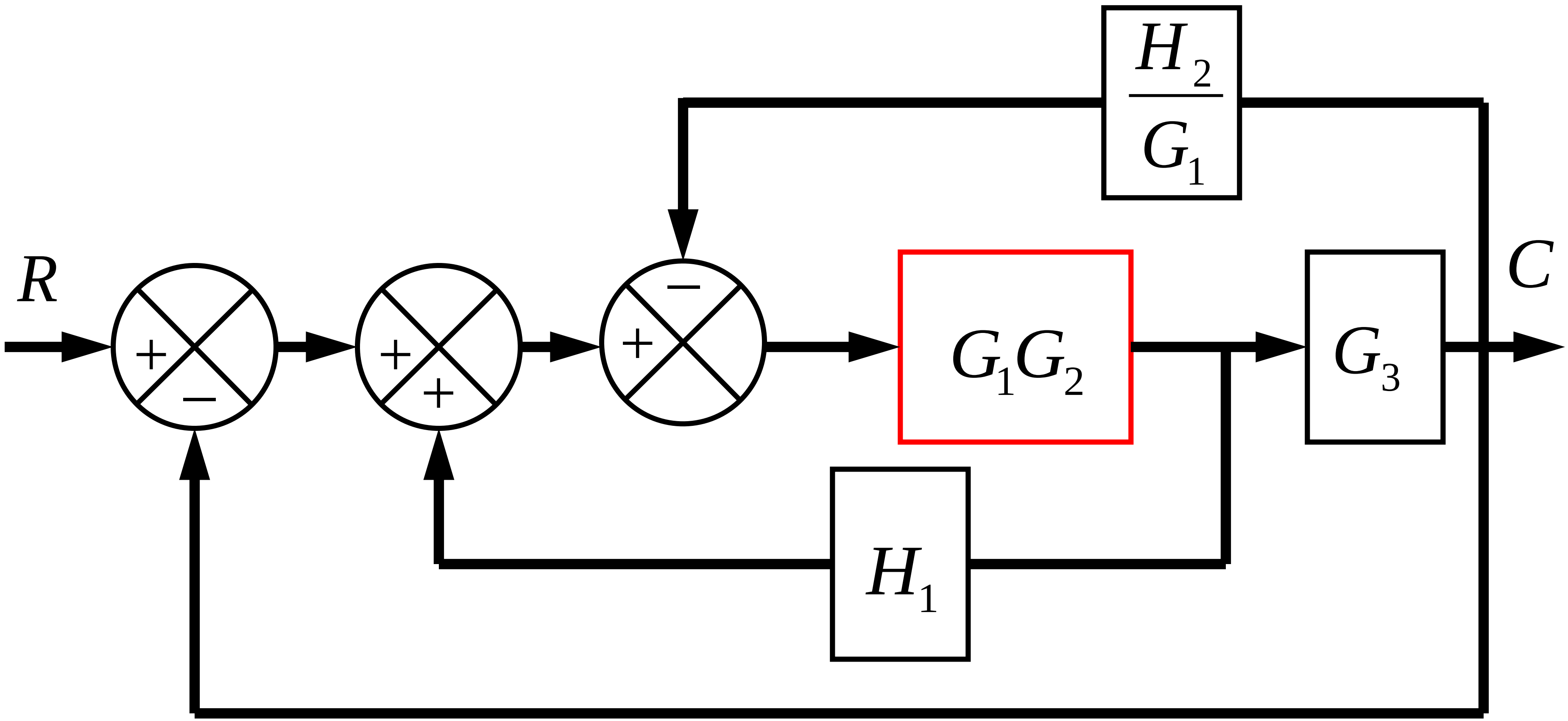
block diagram reduction example



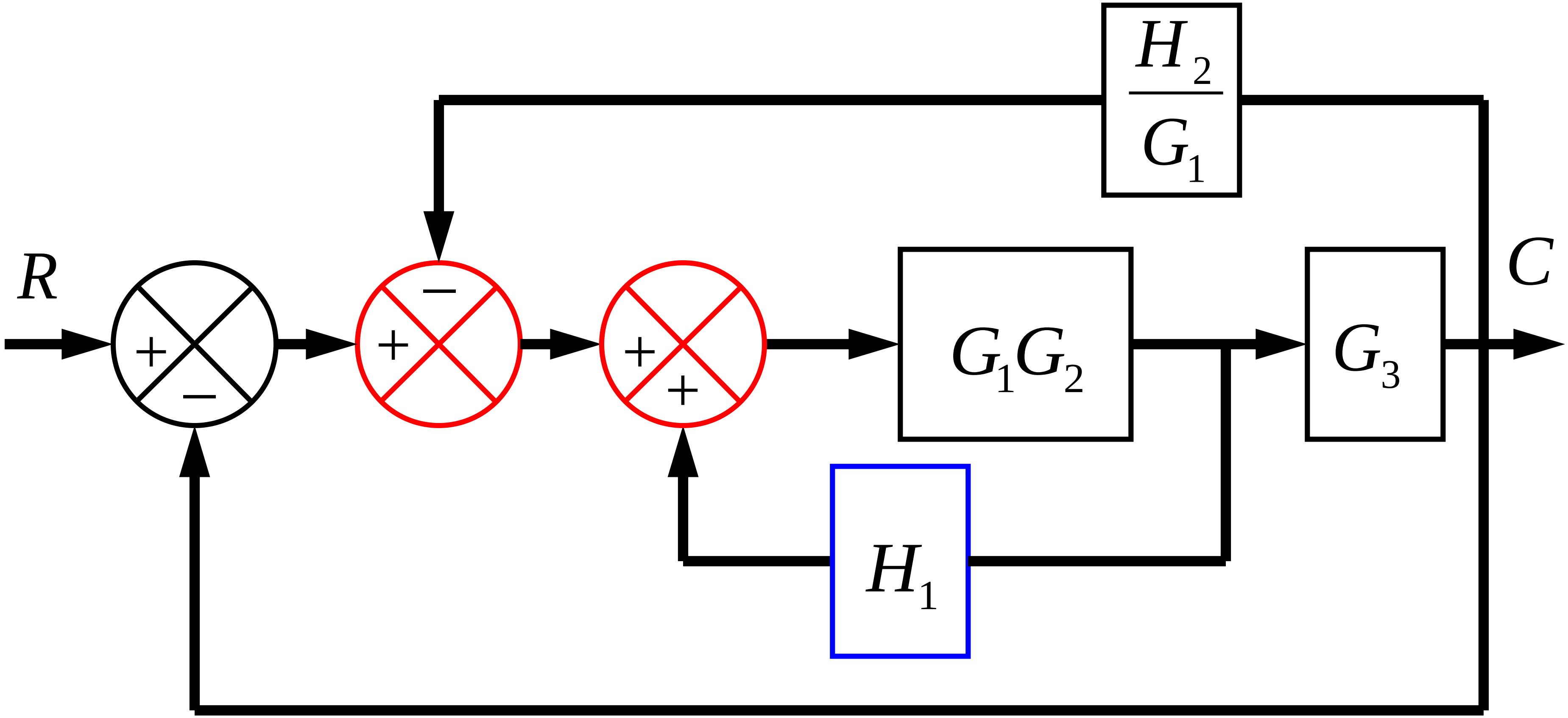
block diagram: reduction example



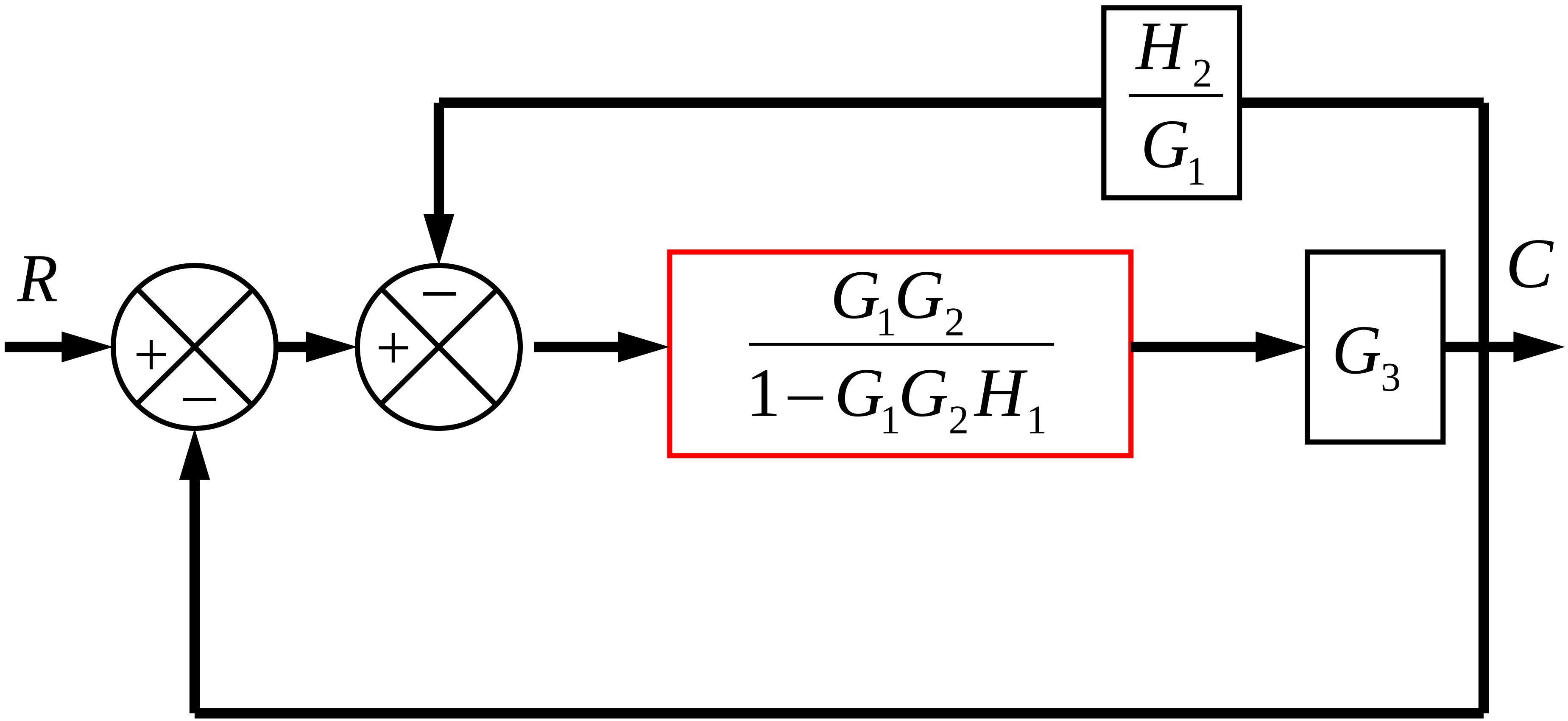
block diagram: reduction example



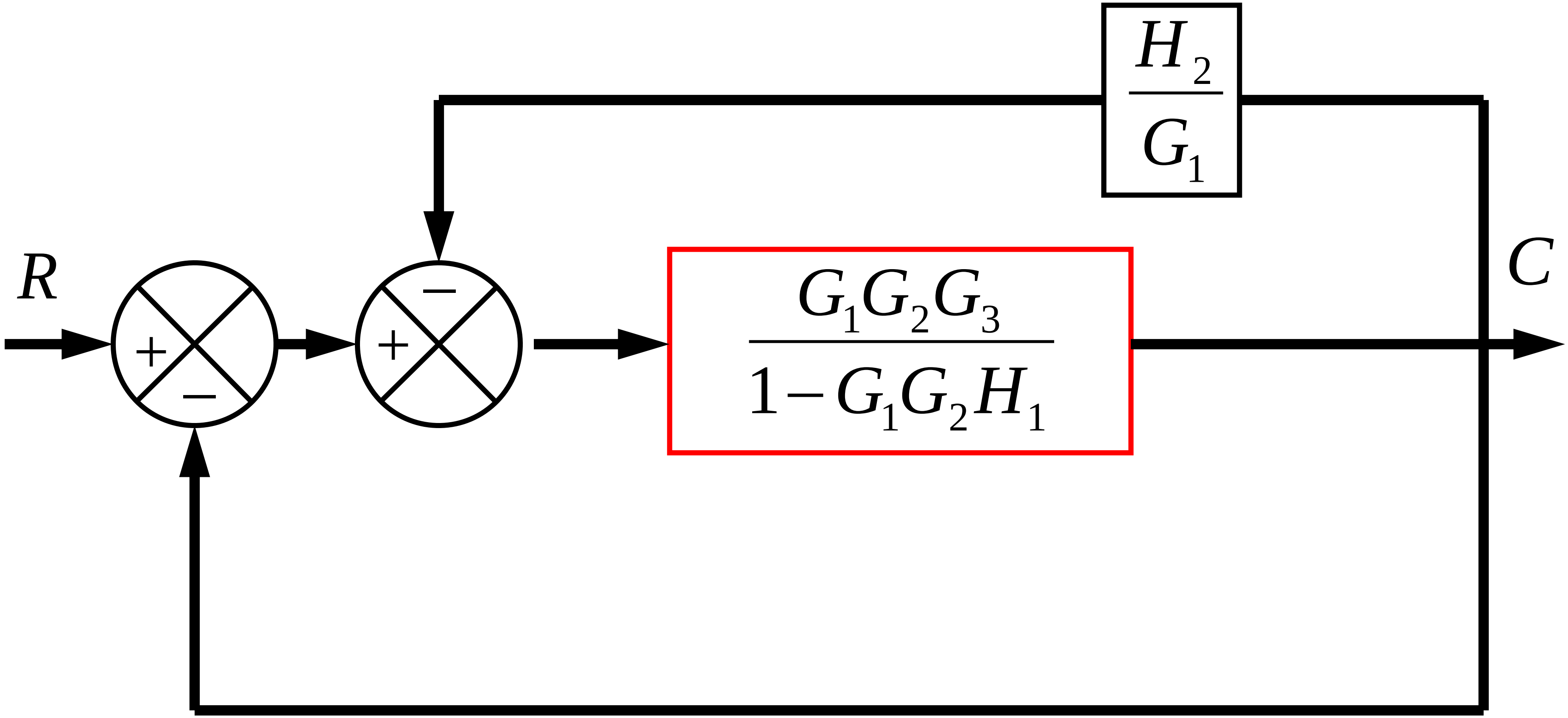
block diagram: reduction example



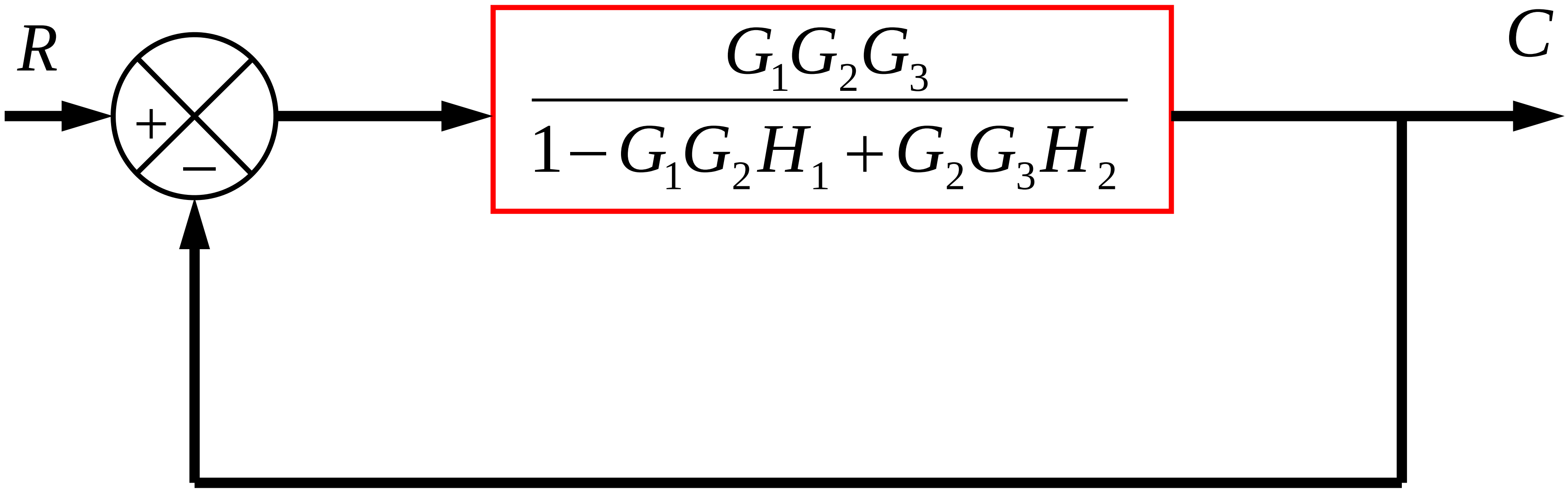
block diagram: reduction example



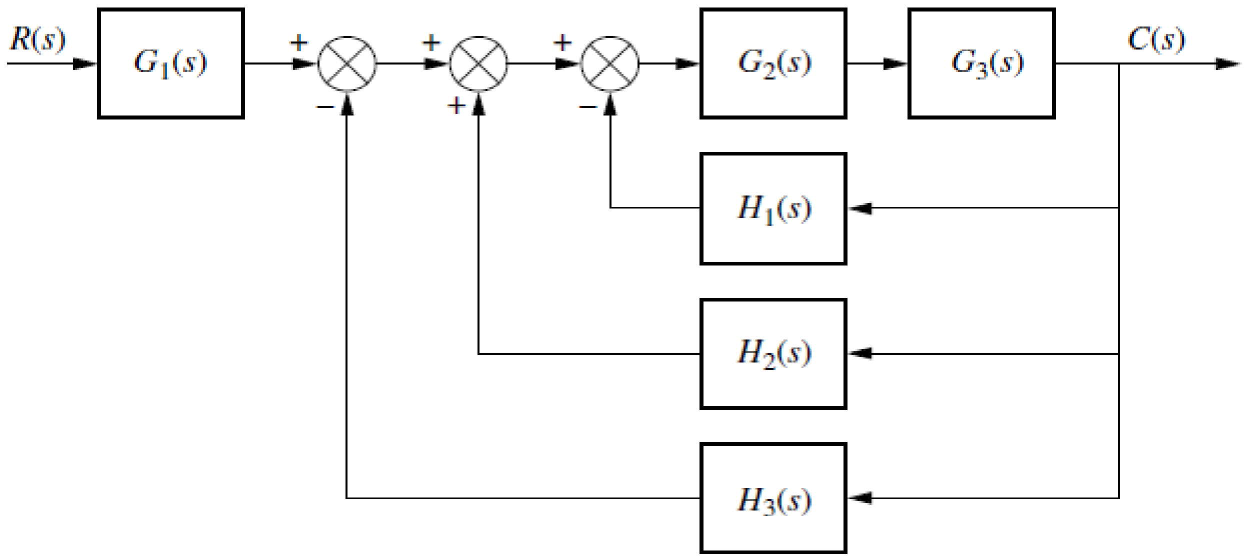
block diagram: reduction example



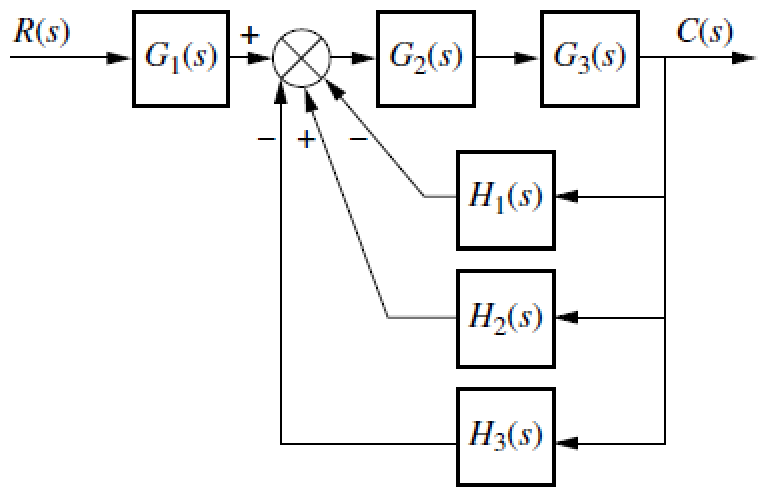
block diagram: reduction example



Reduce the Block Diagram. (from Nise: page-242)

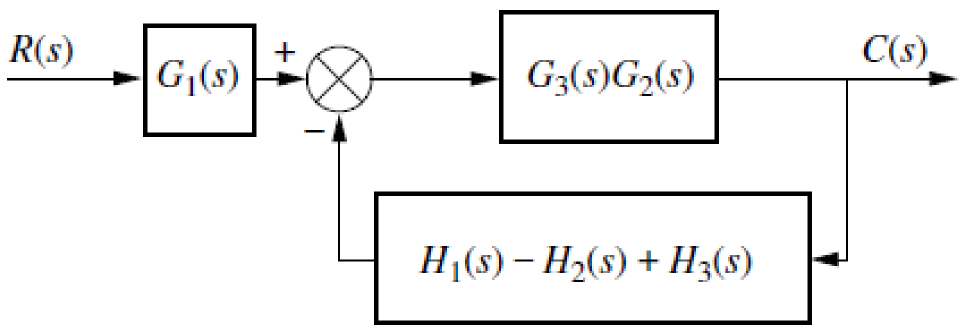


First, the three summing junctions can be collapsed into a single summing junction,

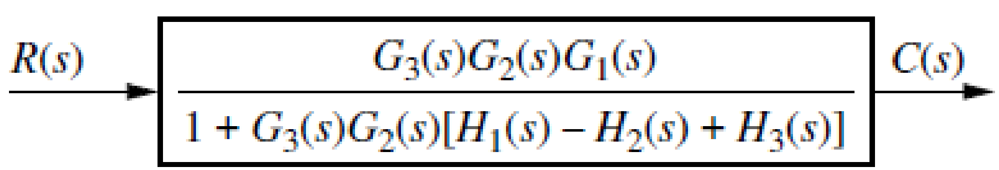


Continue.

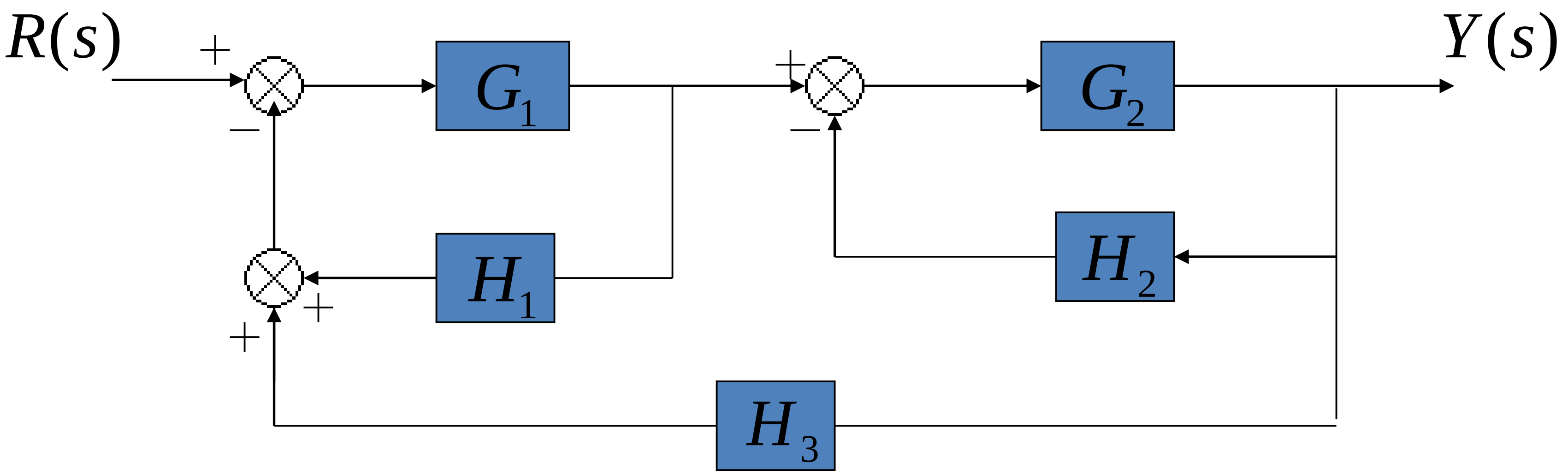
Second, recognize that the three feedback functions, $H_1(s)$, $H_2(s)$, and $H_3(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. Also recognize that $G_2(s)$ and $G_3(s)$ are connected in cascade.



Finally, the feedback system is reduced and multiplied by $G_1(s)$ to yield the equivalent transfer function shown in Figure

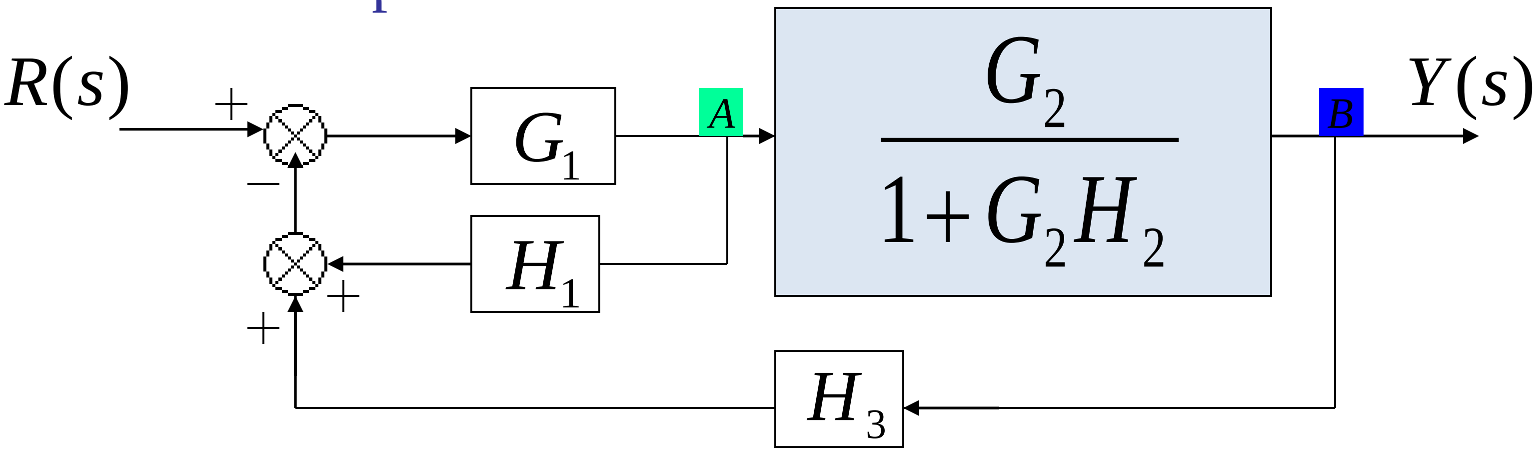


Find the transfer function of the following block diagrams.

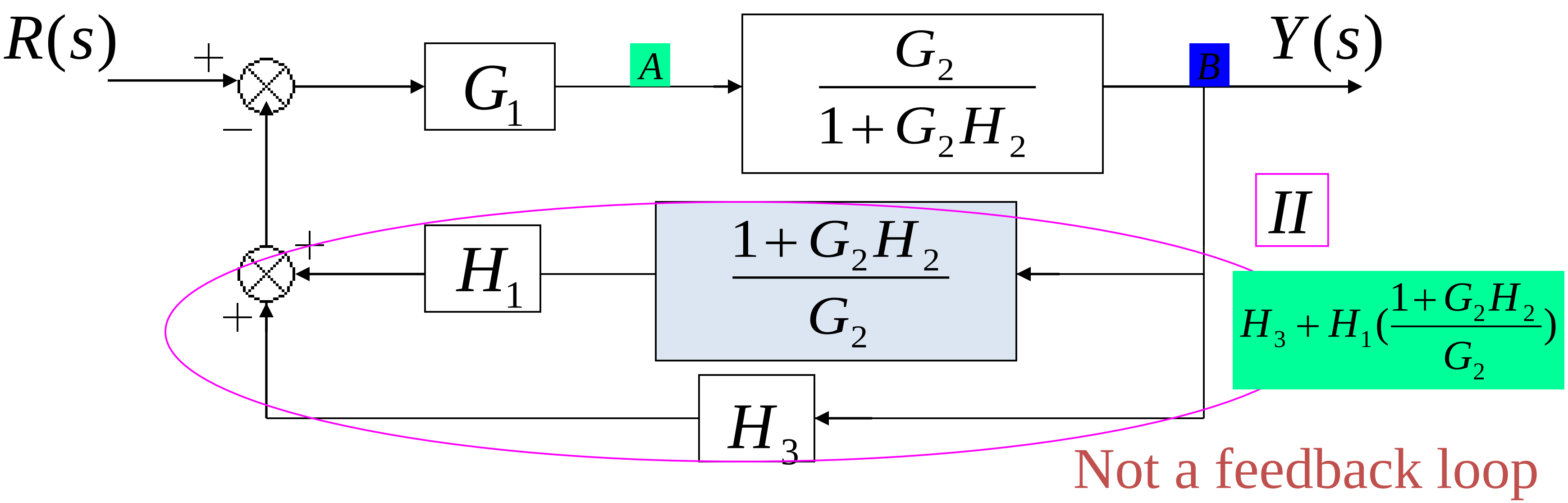


Solution:

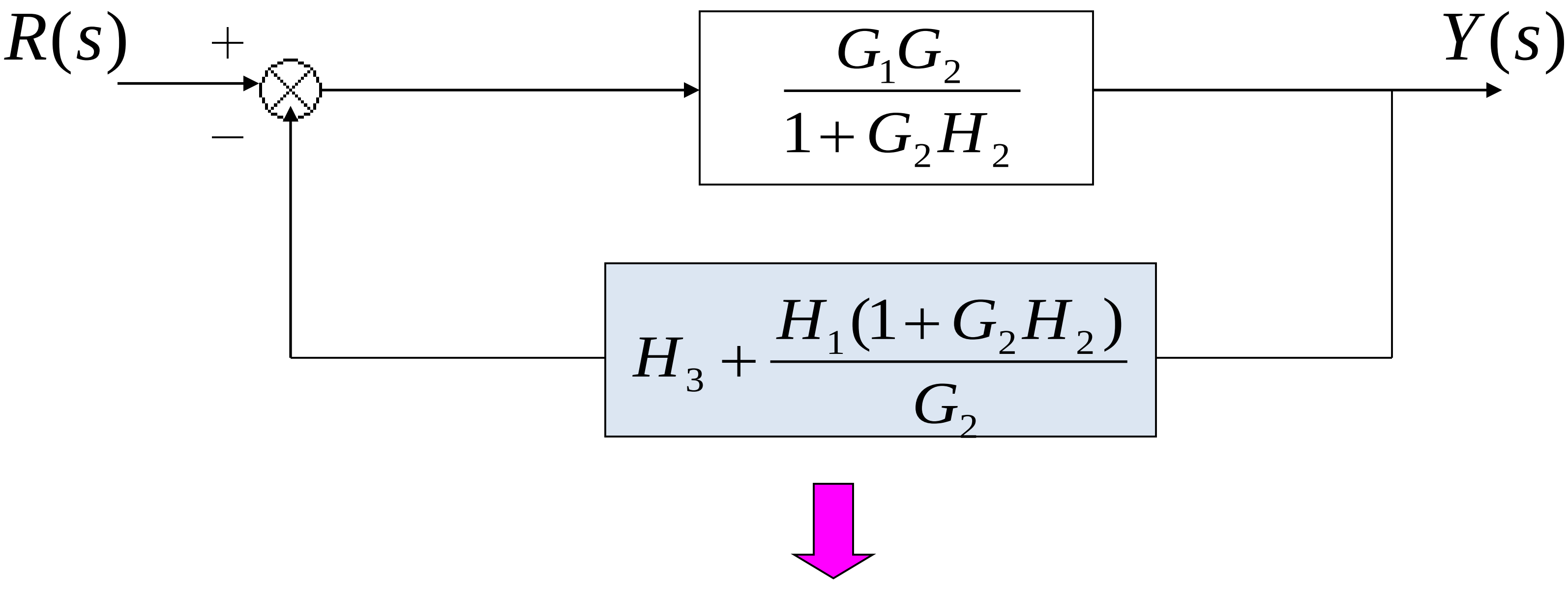
1. Eliminate loop I



2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$



3. Eliminate loop II



$$\frac{Y(s)}{R(s)} = \frac{G_1G_2}{1 + G_2H_2 + G_1G_2H_3 + G_1H_1 + G_1G_2H_1H_2}$$

End