SUBJECT: FLUID MECHANICS

BASIC EQUATIONS OF COMPRESSIBLE FLOW

The basic equations of the compressible flows are

1. Continuity Equation,
2. Bernoulli's Equation or Energy Equation,
3. Momentum Equation,
4. Equation of state.

Continuity Equation. This is based on law of conservation of mass which states tha matter cannot be created nor destroyed. Or in other words, the matter or mass is constant. For one-dimensional steady flow, the mass per second = ρAV

where ρ = Mass density, A = Area of cross-section, V = Velocity

As mass or mass per second is constant according to law of conservation of mass. Hence

$$\rho AV = \text{Constant.}$$

Differentiating equation (15.6), $d(\rho AV) = 0$ or $\rho d(AV) + AVd\rho = 0$

or $\rho[AdV + VdA] + AVd\rho = 0$ or $\rho AdV + \rho VdA + AVd\rho = 0$

Dividing by ρAV, we get $\dfrac{dV}{V} + \dfrac{dA}{A} + \dfrac{d\rho}{\rho} = 0.$

Equation (15.7) is also known as continuity equation in differential form.

Bernoulli's Equation. Bernoulli's equation has been derived for incompressible fluids in Chapter 6. The same procedure is followed. The flow of a fluid particle along a stream-line in the direction of S is considered. The resultant force on the fluid particle in the direction of S is equated to the mass of the fluid particle and its acceleration. As the flow of compressible fluid is steady, the same Euler's equation as given by equation is obtained as

$$\frac{dp}{\rho} + VdV + gdZ = 0$$

Integrating the above equation, we get

$$\int \frac{dp}{\rho} + \int VdV + \int gdZ = \text{Constant}$$

or

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gZ = \text{Constant}$$

Problem *A gas is flowing through a horizontal pipe at a temperature of 4°C. The diameter of the pipe is 8 cm and at a section 1-1 in this pipe, the pressure is 30.3 N/cm² (gauge). The diameter of the pipe changes from 8 cm to 4 cm at the section 2-2, where pressure is 20.3 N/cm² (gauge). Find the velocities of the gas at these sections assuming an isothermal process. Take R = 287.14 Nm/kg K, and atmospheric pressure = 10 N/cm².*

Solution. Given :

For the section 1-1,

Temperature, $t_1 = 4°C$

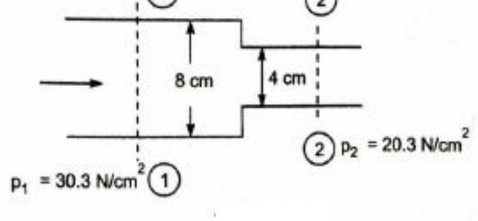
∴ Absolute temperature, $T_1 = 4 + 273 = 277°K$

Diameter pipe, $D_1 = 8 \text{ cm} = 0.08 \text{ m}$

∴ Area of pipe, $A_1 = \dfrac{\pi}{4} D_1^2 = \dfrac{\pi}{4}(.08)^2 = .005026 \text{ m}^2$

Pressure, $p_1 = 30.3 \text{ N/cm}^2$ (gauge)

$= 30.3 + 10 = 40.3 \text{ N/cm}^2 \text{ (absolute)} = 40.3 \times 10^4 \text{ N/m}^2$ (abs.)



For the section 2-2,

Diameter of pipe, $D_2 = 4 \text{ cm} = .04 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} (.04)^2 = .0012565 \text{ m}^2$

Pressure, $p_2 = 20.3 + 10 = 30.3 \text{ N/cm}^2 \text{ (abs.)} = 30.3 \times 10^4 \text{ N/m}^2 \text{ (abs.)}$

Gas constant, $R = 287.14 \text{ N-m/kg°K}$

Ratio of specific heat, $k = 1.4$.

Applying continuity equation at sections (1) and (2), we get

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

or

$$\frac{V_2}{V_1} = \frac{\rho_1 A_1}{\rho_2 A_2} = \frac{\rho_1 \times .005026}{\rho_2 \times .0012565} = 4 \times \frac{\rho_1}{\rho_2} \quad \text{...}(i)$$

For isothermal process using equation (15.3),

$$\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2} \text{ or } \frac{\rho_1}{\rho_2} = \frac{p_1}{p_2} = \frac{40.3 \times 10^4}{30.3 \times 10^4} = 1.33$$

Substituting the value of $\frac{\rho_1}{\rho_2} = 1.33$ in equation (i), we get

$$\frac{V_2}{V_1} = 4 \times 1.33 = 5.32$$

$$\therefore \quad V_2 = 5.32\, V_1 \quad \text{...}(ii)$$

Applying Bernoulli's equation at sections 1-1 and 2-2 for isothermal process which is given by

$$\frac{p_1}{\rho_1 g} \log_e p_1 + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho_2 g} \log_e p_2 + \frac{V_2^2}{2g} + Z_2$$

For horizontal pipe, $Z_1 = Z_2$

$$\therefore \quad \frac{p_1}{\rho_1 g} \log_e p_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho_2 g} \log_e p_2 + \frac{V_2^2}{2g}$$

or

$$\frac{p_1}{\rho_1 g} \log_e p_1 - \frac{p_2}{\rho_2 g} \log_e p_2 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

But for isothermal process, $\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2}$

$$\therefore \quad \frac{p_1}{\rho_1 g} \log_e p_1 - \frac{p_1}{\rho_1 g} \log_e p_2 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

or

$$\frac{p_1}{\rho_1 g}\left[\log_e \frac{p_1}{p_2}\right] = \frac{(5.32\, V_1)^2}{2g} - \frac{V_1^2}{2g} \quad (\because \text{ From } (ii),\ V_2 = 5.32\, V_1)$$

$$\frac{p_1}{\rho_1 g} \log_e \left(\frac{40.3 \times 10^4}{30.3 \times 10^4}\right) = \frac{V_1^2}{2g}(5.32^2 - 1) = 27.30 \frac{V_1^2}{2g}$$

$$\frac{p_1}{\rho_1 g} \log_e 1.33 = 27.30 \frac{V_1^2}{2g}$$

$$\frac{p_1}{\rho_1 g} \times 0.285 = 27.30 \frac{V_1^2}{2g}$$

$$\frac{p_1}{\rho_1} = \frac{27.30}{2 \times .285} V_1^2 = 47.894 V_1^2$$

from equation of state, *i.e.*, from equation (15.2), we have

$$\frac{p}{\rho} = RT \text{ or at section 1, } \frac{p_1}{\rho_1} = RT_1$$

$$\frac{p_1}{\rho_1} = RT_1 = 287.14 \times 277 = 79537.4$$

Momentum Equations. The momentum per second of a flowing fluid (or momentum flux) is equal to the product of mass per second and the velocity of the flow. Mathematically, the momentum per second of a flowing fluid (compressible or incompressible) is

$$= \rho AV \times V, \quad \text{where } \rho AV = \text{Mass per second.}$$

The term ρAV is constant at every section of flow due to continuity equation. This means the momentum per second at any section is equal to the product of a constant quantity and the velocity. This also implies that momentum per second is independent of compressible effect. Hence the momentum equation for incompressible and compressible fluid is the same. The momentum equation for compressible fluid for any direction may be expressed as,

Net force in the direction of S = Rate of change of momentum in the direction of S

= Mass per second [change of velocity]

$= \rho AV[V_2 - V_1]$

where V_2 = Final velocity in the direction of S,

V_1 = Initial velocity in the direction of S.

VELOCITY OF SOUND OR PRESSURE WAVE IN A FLUID

The disturbance in a solid, liquid or gas is transmitted from one point to the other. The velocity with which the disturbance is transmitted depends upon the distance between the molecules of the medium. In case of solids, molecules are closely packed and hence the disturbance is transmitted instantaneously. In case of liquids and gases (or fluids) the molecules are relatively apart. The disturbance will be transmitted from one molecule to the next molecule. But in case of fluids, there is some distance between two adjacent molecules. Hence each molecule will have to travel a certain distance before it can transmit the disturbance. Thus the velocity of disturbance in case of fluids will be less than the velocity of the disturbance in solids.

The distance between the molecules is related with the density, which in turn depends upon pressure in case of fluids. Hence the velocity of disturbance depends upon the changes of pressure and density of the fluid.

S.No.	Physical Quantity	Symbol	Dimensions
	(*b*) **Geometric**		
4.	Area	A	L^2
5.	Volume	$\forall$	L^3
	(*c*) **Kinematic Quantities**		
6.	Velocity	v	LT^{-1}
7.	Angular Velocity	ω	T^{-1}
8.	Acceleration	a	LT^{-2}
9.	Angular Acceleration	α	T^{-2}
10.	Discharge	Q	L^3T^{-1}
11.	Acceleration due to Gravity	g	LT^{-2}
12.	Kinematic Viscosity	ν	L^2T^{-1}
	(*d*) **Dynamic Quantities**		
13.	Force	F	MLT^{-2}
14.	Weight	W	MLT^{-2}
15.	Density	ρ	ML^{-3}
16.	Specific Weight	w	$ML^{-2}T^{-2}$
17.	Dynamic Viscosity	μ	$ML^{-1}T^{-1}$
18.	Pressure Intensity	p	$ML^{-1}T^{-2}$
19.	Modulus of Elasticity	$\begin{cases} K \\ E \end{cases}$	$ML^{-1}T^{-2}$
20.	Surface Tension	σ	MT^{-2}
21.	Shear Stress	τ	$ML^{-1}T^{-2}$
22.	Work, Energy	W or E	ML^2T^{-2}
23.	Power	P	ML^2T^{-3}
24.	Torque	T	ML^2T^{-2}
25.	Momentum	M	MLT^{-1}

DIMENSIONAL HOMOGENEITY

Dimensional homogeneity means the dimensions of each terms in an equation on both sides are equal. Thus if the dimensions of each term on both sides of an equation are the same the equation is known as dimensionally homogeneous equation. The powers of fundamental dimensions (*i.e.*, *L*, *M*, *T*) on both sides of the equation will be identical for a dimensionally homogeneous equation. Such equations are independent of the system of units.

Let us consider the equation, $V = \sqrt{2gH}$

Dimension of L.H.S. $= V = \dfrac{L}{T} = LT^{-1}$

Dimension of R.H.S. $= \sqrt{2gH} = \sqrt{\dfrac{L}{T^2} \times L} = \sqrt{\dfrac{L^2}{T^2}} = \dfrac{L}{T} = LT^{-1}$

Dimension of L.H.S. = Dimension of R.H.S. $= LT^{-1}$

$\therefore$ Equation $V = \sqrt{2gH}$ is dimensionally homogeneous. So it can be used in any system of units.

METHODS OF DIMENSIONAL ANALYSIS

If the number of variable involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods :

1. Rayleigh's method, and
2. Buckingham's π-theorem.

Rayleigh's Method. This method is used for determining the expression for a variable which depends upon maximum three or four variables only. If the number of independent variables becomes more than four, then it is very difficult to find the expression for the dependent variable.

Problem *The resisting force R of a supersonic plane during flight can be considered as dependent upon the length of the aircraft l, velocity V, air viscosity μ, air density ρ and bulk modulus of air K. Express the functional relationship between these variables and the resisting force.*

Solution. The resisting force R depends upon

(i) density, l, (ii) velocity, V,

(iii) viscosity, μ, (iv) density, ρ,

(v) Bulk modulus, K.

$$\therefore \quad R = Al^a \cdot V^b \cdot \mu^c \cdot \rho^d \cdot K^e \qquad ...(i)$$

where A is the non-dimensional constant.

Substituting the dimensions on both sides of the equation (i),

$$MLT^{-2} = AL^a \cdot (LT^{-1})^b \cdot (ML^{-1}T^{-1})^c \cdot (ML^{-3})^d \cdot (ML^{-1}T^{-2})^e$$

Equating the powers of M, L, T on both sides,

Power of M, $\quad 1 = c + d + e$

Power of L, $\quad 1 = a + b - c - 3d - e$

Power of T, $\quad -2 = -b - c - 2e.$

There are five unknowns but equations are only three. Expressing the three unknowns in terms of two unknowns (μ and K).

∴ Express the values of a, b and d in terms of c and e.

Solving,

$$d = 1 - c - e$$

$$b = 2 - c - 2e$$

$$a = 1 - b + c + 3d + e = 1 - (2 - c - 2e) + c + 3(1 - c - e) + e$$

$$= 1 - 2 + c + 2e + c + 3 - 3c - 3e + e = 2 - c.$$

Substituting these values in (i), we get

$$R = A\, l^{2-c} \cdot V^{2-c-2e} \cdot \mu^c \cdot \rho^{1-c-e} \cdot K^e$$

$$= A\, l^2 \cdot V^2 \cdot \rho(l^{-c}\, V^{-c}\, \mu^c \rho^{-c}) \cdot (V^{-2e} \cdot \rho^{-e} \cdot K^e)$$

$$= A\, l^2V^2\rho \left(\frac{\mu}{\rho VL}\right)^c \cdot \left(\frac{K}{\rho V^2}\right)^e$$

$$= \mathbf{A\rho l^2V^2}\, \phi \left[\left(\frac{\mu}{\rho \mathbf{VL}}\right) \cdot \left(\frac{\mathbf{K}}{\rho \mathbf{V}^2}\right)\right]. \textbf{ Ans.}$$

Ref: R.K. Bansal

Buckingham's π-Theorem. The Rayleigh's method of dimensional analysis becomes more laborious if the variables are more than the number of fundamental dimensions (M, L, T). This difficulty is overcame by using Buckingham's π-theorem, which states, "If there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into $(n - m)$ dimensionless terms. Each term is called π-term".

Let $X_1, X_2, X_3, ..., X_n$ are the variables involved in a physical problem. Let X_1 be the dependent variable and $X_2, X_3, ..., X_n$ are the independent variables on which X_1 depends. Then X_1 is a function of $X_2, X_3, ..., X_n$ and mathematically it is expressed as

$$X_1 = f(X_2, X_3, ..., X_n) \quad ...(\ .1)$$

Equation (.1) can also be written as

$$f_1(X_1, X_2, X_3, ..., X_n) = 0. \quad ...(\ 2)$$

Equation (2) is a dimensionally homogeneous equation. It contains n variables. If there are m fundamental dimensions then according to Buckingham's π-theorem, equation (12.2) can be written in terms of number of dimensionless groups or π-terms in which number of π-terms is equal to $(n - m)$. Hence equation (2) becomes as

$$f(\pi_1, \pi_2, ..., \pi_{n-m}) = 0. \quad ...(\ .3)$$

Each of π-terms is dimensionless and is independent of the system. Division or multiplication by a constant does not change the character of the π-term. Each π-term contains $m + 1$ variables, where m is the number of fundamental dimensions and is also called repeating variables. Let in the above case X_2, X_3 and X_4 are repeating variables if the fundamental dimension m (M, L, T) = 3. Then each π-term is written as

$$\left.\begin{aligned} \pi_1 &= X_2^{a_1} . X_3^{b_1} . X_4^{c_1} . X_1 \\ \pi_2 &= X_2^{a_2} . X_3^{b_2} . X_4^{c_2} . X_5 \\ &\vdots \\ \pi_{n-m} &= X_2^{a_{n-m}} . X_3^{b_{n-m}} . X_4^{c_{n-m}} . X_n \end{aligned}\right\} \quad 4)$$

Each equation is solved by the principle of dimensional homogeneity and values of a_1, b_1, c_1 etc., are obtained. These values are substituted in equation (4) and values of $\pi_1, \pi_2, ..., \pi_{n-m}$ are obtained. These values of π's are substituted in equation (3). The final equation for the phenomenon is obtained by expressing any one of the π-terms as a function of others as

$$\pi_1 = \phi\,[\pi_2, \pi_3, ..., \pi_{n-m}]$$

or

$$\pi_2 = \phi_1\,[\pi_1, \pi_3, ..., \pi_{n-m}] \quad ...(\ .5)$$

Method of Selecting Repeating Variables. The number of repeating variables are equal to the number of fundamental dimensions of the problem. The choice of repeating variables is governed by the following considerations :

1. As far as possible, the dependent variable should not be selected as repeating variable.
2. The repeating variables should be choosen in such a way that one variable contains geometric property, other variable contains flow property and third variable contains fluid property.

Variables with Geometric Property are

(*i*) Length, l (*ii*) d (*iii*) Height, H etc.

Variables with flow property are

(*i*) Velocity, V (*ii*) Acceleration etc.

Variables with fluid property : (*i*) μ, (*ii*) ρ, (*iii*) ω etc.

3. The repeating variables selected should not form a dimensionless group.
4. The repeating variables together must have the same number of fundamental dimensions.
5. No two repeating variables should have the same dimensions.

In most of fluid mechanics problems, the choice of repeating variables may be (*i*) d, v, ρ (*ii*) l, v, ρ or (*iii*) l, v, μ or (*iv*) d, v, μ.

Procedure for Solving Problems by Buckingham's π-theorem. The procedure for solving problems by Buckingham's π-theorem is explained by considering the which is also solved by the Rayleigh's method. The problem is :

The resisting force R of a supersonic plane during flight can be considered as dependent upon the length of the aircraft l, velocity V, air viscosity μ, air density ρ and bulk modulus of air K. Express the functional relationship between these variables and the resisting force.

Solution. Step 1. The resisting force R depends upon (*i*) l, (*ii*) V, (*iii*) μ, (*iv*) ρ and (*v*) K. Hence R is a function of l, V, μ, ρ and K. Mathematically,

$$R = f(l, V, \mu, \rho, K) \qquad ...(i)$$

or it can be written as $f_1(R, l, V, \mu, \rho, K) = 0 \qquad ...(ii)$

∴ Total number of variables, $n = 6$.

Number of fundamental dimensions, $m = 3$.

[m is obtained by writing dimensions of each variables as $R = MLT^{-2}$, $V = LT^{-1}$, $\mu = ML^{-1}T^{-1}$, $\rho = ML^{-3}$, $K = ML^{-1}T^{-2}$. Thus as fundamental dimensions in the problem are M, L, T and hence $m = 3$.]

Number of dimensionless π-terms $= n - m = 6 - 3 = 3$.

Thus three π-terms say π_1, π_2 and π_3 are formed. Hence equation (*ii*) is written as

$$f_1(\pi_1, \pi_2, \pi_3) = 0. \qquad ...(iii)$$

Step 2. Each π term $= m + 1$ variables, where m is equal to 3 and also called repeating variables. Out of six variables R, l, V, μ, ρ and K, three variables are to be selected as repeating variable. R is a dependent variable and should not be selected as a repeating variable. Out of the five remaining

Step 3. Each π-term is written as according to equation (

$$\left.\begin{aligned} \pi_1 &= l^{a_1} . V^{b_1} . \rho^{c_1} . R \\ \pi_2 &= l^{a_2} . V^{b_2} . \rho^{c_2} . \mu \\ \pi_3 &= l^{a_3} . V^{b_3} . \rho^{c_3} . K \end{aligned}\right\} \qquad ...(iv)$$

Step 4. Each π-term is solved by the principle of dimensional homogeneity. For the first π-term, we have

$$\pi_1 = M^0L^0T^0 = L^{a_1} . (LT^{-1})^{b_1} . (ML^{-3})^{c_1} . MLT^{-2}.$$

Equating the powers of M, L, T on both sides, we get

Power of M, $\quad 0 = c_1 + 1 \qquad \therefore \; c_1 = -1$

Power of L, $\quad 0 = a_1 + b_1 - 3c_1 + 1,$

$\therefore \quad a_1 = -b_1 + 3c_1 - 1 = 2 - 3 - 1 = -2$

Power of T, $\quad 0 = -b_1 - 2 \qquad \therefore \; b_1 = -2$

Substituting the values of a_1, b_1 and c_1 in equation (*iv*),

$$\pi_1 = l^{-2} . V^{-2} . \rho^{-1} . R$$

or

$$\pi_1 = \frac{R}{l^2V^2\rho} = \frac{R}{\rho l^2 V^2} \qquad ...(v)$$

Similarly for the 2nd π-term, we get $\pi_2 = M^0L^0T^0 = L^{a_2} \cdot (LT^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot ML^{-1}T^{-1}$.

Equating the powers of M, L, T on both sides

Power of M, $\quad 0 = c_2 + 1, \quad \therefore \quad c_2 = -1$

Power of L, $\quad 0 = a_2 + b_2 - 3c_2 - 1,$

$$a_2 = -b_2 + 3c_2 + 1 = 1 - 3 + 1 = -1$$

Power of T, $\quad 0 = -b_2 - 1, \quad \therefore \quad b_2 = -1$

Substituting the values of a_2, b_2 and c_2 in π_2 of (iv)

$$\pi_2 = l^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu = \frac{\mu}{lV\rho}.$$

3rd π-term

$$\pi_3 = l^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot K$$

or

$$M^0L^0T^0 = L^{a_3} \cdot (LT^{-3})^{b_3} \cdot (ML^{-3})^{a_3} \cdot ML^{-1}T^{-2}$$

Equating the powers of M, L, T on both sides, we have

Power of M, $\quad 0 = c_3 + 1, \quad \therefore \quad c_3 = -1$

Power of L, $\quad 0 = a_3 + b_3 - 3c_3 - 1, \quad \therefore \quad a_3 = -b_3 + 3c_3 + 1 = 2 - 3 + 1 = 0$

Power of T, $\quad 0 = -b_3 - 2, \quad \therefore \quad b_3 = -2$

Substituting the values of a_3, b_3 and c_3 in π_3 term

$$\pi_3 = l^0 \cdot V^{-2} \cdot \rho^{-1} \cdot K = \frac{K}{V^2\rho}.$$

Step 5. Substituting the values of π_1, π_2 and π_3 in equation (iii), we get

$$f_1\left(\frac{R}{\rho l^2 V^2}, \frac{\mu}{lV\rho}, \frac{K}{V^2\rho}\right) = 0 \quad \text{or} \quad \frac{R}{\rho l^2 V^2} = \phi\left[\frac{\mu}{lV\rho}, \frac{K}{V^2\rho}\right]$$

or

$$R = \rho l^2 V^2 \phi\left[\frac{\mu}{lV\rho}, \frac{K}{V^2\rho}\right]. \textbf{ Ans.}$$

DIMENSIONLESS NUMBERS

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force. As this is a ratio of one force to the other force, it will be a dimensionless number. These dimensionless numbers are also called non-dimensional parameters. The followings are the important dimensionless numbers :

1. Reynold's number,
2. Froude's number,
3. Euler's number,
4. Weber's number,
5. Mach's number.

Reynold's Number (R_e). It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained as

Inertia force (F_i) = Mass × Acceleration of flowing fluid

$$= \rho \times \text{Volume} \times \frac{\text{Velocity}}{\text{Time}} = \rho \times \frac{\text{Volume}}{\text{Time}} \times \text{Velocity}$$

$$= \rho \times AV \times V \quad \{\because \text{ Volume per sec} = \text{Area} \times \text{Velocity} = A \times V\}$$

$$= \rho AV^2 \quad \text{...(12.11)}$$

Viscous force (F_v) = Shear stress × Area $\left\{\because \tau = \mu \frac{du}{dy} \therefore \text{ Force} = \tau \times \text{Area}\right\}$

$$= \tau \times A$$

$$= \left(\mu \frac{du}{dy}\right) \times A = \mu \cdot \frac{V}{L} \times A \quad \left\{\because \frac{du}{dy} = \frac{V}{L}\right\}$$

By definition, Reynold's number,

$$R_e = \frac{F_i}{F_v} = \frac{\rho AV^2}{\mu \cdot \frac{V}{L} \times A} = \frac{\rho VL}{\mu}$$

$$= \frac{V \times L}{(\mu / \rho)} = \frac{V \times L}{\nu} \quad \left\{\because \frac{\mu}{\rho} = \nu = \text{Kinematic viscosity}\right\}$$

In case of pipe flow, the linear dimension L is taken as diameter, d. Hence Reynold's number for pipe flow,

$$R_e = \frac{V \times d}{\nu} \quad \text{or} \quad \frac{\rho Vd}{\mu}.$$

Froude's Number (F_e). The Froude's number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force. Mathematically, it is expressed as

$$F_e = \sqrt{\frac{F_i}{F_g}}$$

where F_i from equation (12.11) $= \rho AV^2$

and F_g = Force due to gravity

= Mass × Acceleration due to gravity

$= \rho \times \text{Volume} \times g = \rho \times L^3 \times g$ $\{\because \text{ Volume} = L^3\}$

$= \rho \times L^2 \times L \times g = \rho \times A \times L \times g$ $\{\because L^2 = A = \text{Area}\}$

$$\therefore \quad F_e = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho AV^2}{\rho ALg}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}}$$

Euler's Number (E_u). It is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force. Mathematically, it is expressed as

$$E_u = \sqrt{\frac{F_i}{F_p}}$$

where F_P = Intensity of pressure × Area = $p \times A$
and $F_i = \rho A V^2$

$$\therefore \quad E_u = \sqrt{\frac{\rho A V^2}{p \times A}} = \sqrt{\frac{V^2}{p/\rho}} = \frac{V}{\sqrt{p/\rho}}$$

Weber's Number (W_e). It is defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

Weber's Number, $$W_e = \sqrt{\frac{F_i}{F_s}}$$

where F_i = Inertia force = $\rho A V^2$
and F_s = Surface tension force
= Surface tension per unit length × Length = $\sigma \times L$

$$\therefore \quad W_e = \sqrt{\frac{\rho A V^2}{\sigma \times L}} = \sqrt{\frac{\rho \times L^2 \times V^2}{\sigma \times L}} \qquad \{\because A = L^2\}$$

$$= \sqrt{\frac{\rho L \times V^2}{\sigma}} = \sqrt{\frac{V^2}{\sigma/\rho L}} = \frac{V}{\sqrt{\sigma/\rho L}}.$$

Mach's Number (M). Mach's number is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force. Mathematically, it is defined as

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{F_i}{F_e}}$$

where $F_i = \rho A V^2$
and F_e = Elastic force = Elastic stress × Area
$= K \times A = K \times L^2$ $\{\because K = \text{Elastic stress}\}$

$$\therefore \quad M = \sqrt{\frac{\rho A V^2}{K \times L^2}} = \sqrt{\frac{\rho \times L^2 \times V^2}{K \times L^2}} = \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}}$$

But $$\sqrt{\frac{K}{\rho}} = C = \text{Velocity of sound in the fluid}$$

$$\therefore \quad M = \frac{V}{C}.$$