SUBJECT: FLUID MECHANICS

INTRODUCTION

Fluid mechanics is that branch of science which deals with the behaviour of the fluids (liquids or gases) at rest as well as in motion. Thus this branch of science deals with the static, kinematics and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics.

PROPERTIES OF FLUIDS

Density or Mass Density. Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic metre, *i.e.*, kg/m^3. The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}.$$

The value of density of water is 1 gm/cm^3 or 1000 kg/m^3.

Specific Weight or Weight Density. Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.

Thus mathematically,
$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$
$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$
$$= \rho \times g \qquad \left\{\because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho\right\}$$

Specific Volume. Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume. Mathematically, it is expressed as

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume of fluid}}} = \frac{1}{\rho}$$

Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg. It is commonly applied to gases.

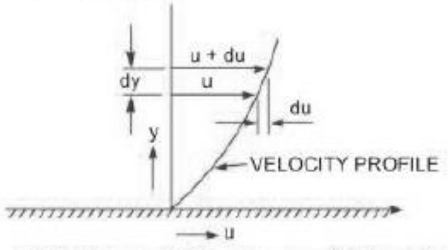
Specific Gravity. Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol S.

$$\text{Mathematically, } S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

VISCOSITY

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say u and $u + du$ as shown in Fig. 1.1, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by symbol τ (Tau).



Velocity variation near a solid boundary.

Mathematically, $$\tau \propto \frac{du}{dy}$$

or $$\tau = \mu \frac{du}{dy}$$

where μ (called *mu*) is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

Kinematic Viscosity. It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the Greek symbol (ν) called 'nu'. Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{\because \text{Force} = \text{Mass} \times \text{Acc.} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2}\right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

In MKS and SI, the unit of kinematic viscosity is metre^2/sec or m^2/sec while in CGS units it is written as cm^2/s. In CGS units, kinematic viscosity is also known as stoke.

Thus, one stoke $= \text{cm}^2/\text{s} = \left(\frac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4}\ \text{m}^2/\text{s}$

Centistoke means $= \frac{1}{100}$ stoke.

Newton's Law of Viscosity. It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity. Mathematically, it is expressed as given by equation

$$\tau = \mu \frac{du}{dy}.$$

Types of Fluids. The fluids may be classified into the following five types :

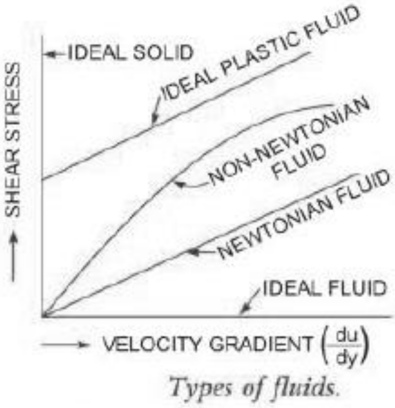
1. Ideal fluid,
2. Real fluid,
3. Newtonian fluid,
4. Non-Newtonian fluid, and
5. Ideal plastic fluid.

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. Real Fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

3. Newtonian Fluid. A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. Non-Newtonian Fluid. A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.



Types of fluids.

5. Ideal Plastic Fluid. A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

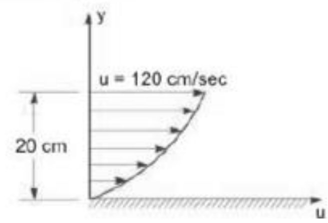
Problem *If the velocity profile of a fluid over a plate is parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stresses at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.*

Solution. Given :

Distance of vertex from plate = 20 cm

Velocity at vertex, $u = 120$ cm/sec

Viscosity, $\mu = 8.5 \text{ poise} = \frac{8.5}{10} \frac{\text{N s}}{\text{m}^2} = 0.85.$



The velocity profile is given parabolic and equation of velocity profile is

$$u = ay^2 + by + c \qquad ...(i)$$

where a, b and c are constants. Their values are determined from boundary conditions as :

(*a*) at $y = 0$, $u = 0$

(*b*) at $y = 20$ cm, $u = 120$ cm/sec

(*c*) at $y = 20$ cm, $\frac{du}{dy} = 0$.

Substituting boundary condition (*a*) in equation (*i*), we get

$$c = 0.$$

Boundary condition (*b*) on substitution in (*i*) gives

$$120 = a(20)^2 + b(20) = 400a + 20b \qquad ...(ii)$$

Boundary condition (*c*) on substitution in equation (*i*) gives

$$\frac{du}{dy} = 2ay + b \qquad ...(iii)$$

or
$$0 = 2 \times a \times 20 + b = 40a + b$$

Solving equations (*ii*) and (*iii*) for a and b

From equation (*iii*), $b = -40a$

Substituting this value in equation (*ii*), we get

$$120 = 400a + 20 \times (-40a) = 400a - 800a = -400a$$

$$\therefore \quad a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$\therefore \quad b = -40 \times (-0.3) = 12.0$$

Substituting the values of a, b and c in equation (*i*),

$$u = -0.3y^2 + 12y.$$

Velocity Gradient

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

at $y = 0$, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 =$ **12/s. Ans.**

at $y = 10$ cm, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 =$ **6/s. Ans.**

at $y = 20$ cm, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 =$ **0. Ans.**

Shear Stresses

Shear stress is given by, $\tau = \mu \frac{du}{dy}$

(*i*) Shear stress at $y = 0$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2 \text{ N/m}^2$.

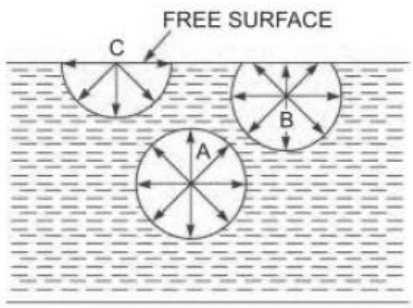
(*ii*) Shear stress at $y = 10$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1 \text{ N/m}^2$.

(*iii*) Shear stress at $y = 20$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 =$ **0. Ans.**

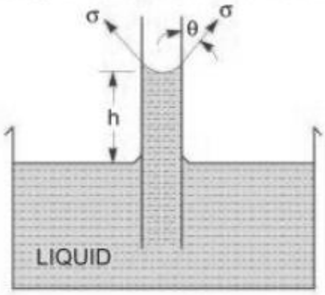
SURFACE TENSION AND CAPILLARITY

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ (called sigma). In MKS units, it is expressed as kgf/m while in SI units as N/m.

The phenomenon of surface tension is explained by Fig. 1.10. Consider three molecules *A*, *B*, *C* of a liquid in a mass of liquid. The molecule *A* is attracted in all directions equally by the surrounding molecules of the liquid. Thus the resultant force acting on the molecule *A* is zero. But the molecule *B*, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced. Thus a net resultant force on molecule *B* is acting in the downward direction. The molecule *C*, situated on the free surface of liquid, does experience a resultant downward force. All the molecules on the free surface experience a downward force. Thus the free surface of the liquid acts like a very thin film under tension of the surface of the liquid act as though it is an elastic membrane under tension.



Capillarity. Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.



Capillary rise.

Expression for Capillary Rise. Consider a glass tube of small diameter 'd' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

Let σ = Surface tension of liquid

θ = Angle of contact between liquid and glass tube.

The weight of liquid of height h in the tube = (Area of tube × h) × ρ × g

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$
$$= \sigma \times \pi d \times \cos\theta$$

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or
$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d}$$

The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos\theta$ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d}$$

Expression for Capillary Fall. If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in Fig.

Let h = Height of depression in tube.

Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos\theta$.

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' × Area

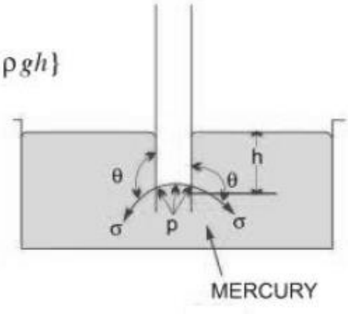
$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \ \{\because p = \rho g h\}$$

Equating the two, we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore \quad h = \frac{4\sigma\cos\theta}{\rho g d} \quad \ldots$$

Value of θ for mercury and glass tube is 128°.

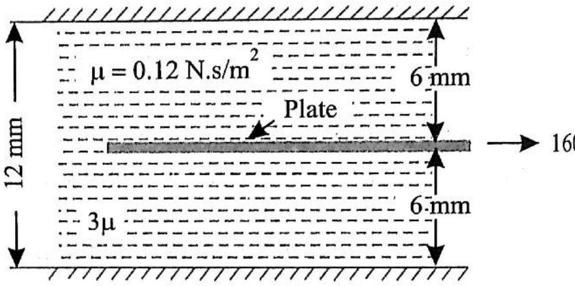


Example *In the Fig. is shown a central plate of area 6 m² being pulled with a forc of 160 N. If the dynamic viscosities of the two oils are in the ratio of 1:3 and the viscosity of top o is 0.12 N.s/m² determine the velocity at which the central plate will move.*

Solution: Area of the plate, $A = 6 \text{ m}^2$

Force applied to the plate, $F = 160$ N

Viscosity of top oil, $\mu = 0.12 \text{ N.s/m}^2$



Velocity of the plate, u:

Let F_1 = Shear force in the upper side of *thin* (assumed) plate,

F_2 = Shear force on the lower side of the thin plate, and

F = Total force required to drag the plate $(= F_1 + F_2)$

Then, $F = F_1 + F_2 = \tau_1 \times A + \tau_2 \times A$

$$= \mu\left(\frac{\partial u}{\partial y}\right)_1 \times A + 3\mu\left(\frac{du}{dy}\right)_2 \times A$$

(where τ_1 and τ_2 are the shear stresses on the two sides of the plate)

$$160 = 0.12 \times \frac{u}{6 \times 10^{-3}} \times 6 + 3 \times 0.12 \times \frac{u}{6 \times 10^{-3}} \times 6$$

or $$160 = 120u + 360u = 480u \quad \text{or} \quad u = \frac{160}{480} = \mathbf{0.333 \text{ m/s (Ans.)}}$$

Problem *Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.*

Solution. Given :

Capillary rise, $h = 2.0 \text{ mm} = 2.0 \times 10^{-3}$ m

Surface tension, $\sigma = 0.073575$ N/m

Let dia. of tube $= d$

The angle θ for water $= 0^\circ$

The density for water, $\rho = 1000 \text{ kg/m}^3$

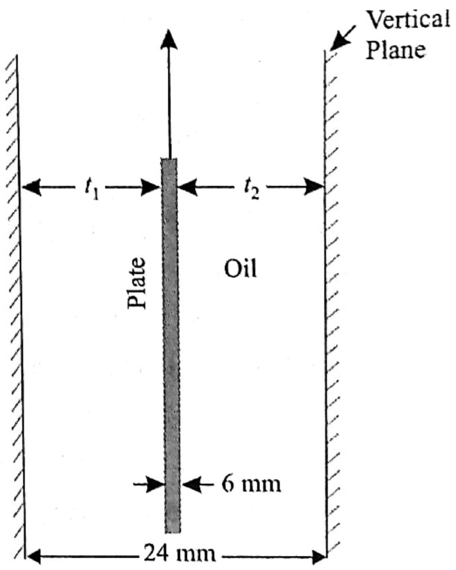
$$h = \frac{4\sigma}{\rho \times g \times d} \quad \text{or} \quad 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

$$\therefore \quad d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m} = \mathbf{1.5 \text{ cm. Ans.}}$$

Example *A metal plate 1.25 m × 1.25 m × 6 mm thick and weighing 90 N is place midway in the 24 mm gap between the two vertical plane surfaces as shown in the Fig. Th gap is filled with an oil of specific gravity 0.85 and dynamic viscosity 3.0 N.s/m². Determine th force required to lift the plate with a constant velocity of 0.15 m/s.*

Solution. *Given*: Dimensions of the plate $= 1.25 \text{ m} \times 1.25 \text{ m} \times 6 \text{ mm}$

$\therefore$ Area of the plate, $A = 1.25 \times 1.25 = 1.5625 \text{ m}^2$



Thickness of the plate = 6 mm

$$\therefore \quad t_1 = t_2 = \frac{24 - 6}{2} = 9 \text{ mm}$$

(Since the plate is situated midway in the gap)

Specific gravity of oil = 0.85

Dynamic viscosity of oil = 3 N.s/m²

Velocity of the plate = 0.15 m/s

Weight of the plate = 90 N

Force required to lift the plate:

Drag force (or viscous resistance) against the motion of the plate,

$$F = \tau_1 . A + \tau_2 . A$$

(where τ_1 and τ_2 are the shear stresses on two sides of the plate)

$$= \mu . \left(\frac{du}{dy}\right)_1 \times A + \mu \left(\frac{du}{dy}\right)_2 \times A$$

$$= \mu . \frac{u}{t_1} \times A + \mu . \frac{u}{t_2} \times A$$

$$= \mu A u . \left(\frac{1}{t_1} + \frac{1}{t_2}\right)$$

or $$F = 3 \times 1.5625 \times 0.15 \left(\frac{1}{9 \times 10^{-3}} + \frac{1}{9 \times 10^{-3}}\right)$$

$$= 3 \times 1.5625 \times 0.15 \times \frac{2}{9 \times 10^{-3}} = 156.25 \text{ N}$$

Upward thrust or buoyant force on the plate = specific weight × volume of oil displaced

$$= 0.85 \times 9810 \times (1.25 \times 1.25 \times 0.006) = 78.17 \text{ N}$$

Effective weight of the plate = 90 – 78.17 = 11.83 N

$\therefore$ Total force required to lift the plate at velocity of 0.15 m/s = F + effective weight of the plate

$$= 156.25 + 11.83 = \mathbf{168.08 \text{ N}} \textbf{ (Ans.)}$$

Example *In order to form a stream of bubbles, air is introduced through a nozzle into a tank of water at 20°C. If the process requires 3.0 mm diameter bubbles to be formed, by how much the air pressure at the nozzle must exceed that of the surrounding water?*

What would be the absolute pressure inside the bubble if the surrounding water is at 100.3 kN/m²?

Take surface tension of water at 20°C = 0.0735 N/m.

Solution. Diameter of a bubble, $d = 3.0 \text{ mm} = 3 \times 10^{-3}$ m

Surface tension of water at 20°C, $\sigma = 0.0735$ N/m

The excess pressure intensity of air over that of surrounding water, $\Delta p = p$.

We know, $$p = \frac{4\sigma}{d} = \frac{4 \times 0.0735}{3 \times 10^{-3}} = \mathbf{98\ N/m^2\ (Ans.)}$$

Absolute pressure inside the bubble, p_{abs}:

$$p_{abs} = p + p_{atm}$$
$$= 98 \times 10^{-3} + 100.3$$
$$= 0.098 + 100.3 = \mathbf{100.398\ kN/m^2\ (Ans.)}$$

Example *A soap bubble 62.5 mm diameter has an internal pressure in excess of the outside pressure of 20 N/m². What is tension in the soap film?*

Solution. *Given*: Diameter of the bubble, $d = 62.5 \text{ mm} = 62.5 \times 10^{-3}$ m;

Internal pressure in excess of the outside pressure, $p = 20$ N/m².

Surface tension, σ:

Using the relation, $$p = \frac{8\sigma}{d}$$

i.e., $$20 = \frac{8\sigma}{62.5 \times 10^{-3}} \quad \therefore \quad \sigma = 20 \times \frac{62.5 \times 10^{-3}}{8} = \mathbf{0.156\ N/m\ (Ans.)}$$

Example *What do you mean by surface tension? If the pressure difference between the inside and outside of the air bubble of diameter 0.01 mm is 29.2 kPa, what will be the surface tension at air-water interface?*

Solution. Surface tension is defined *as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.* The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by the letter σ and is expressed as N/m.

$$p \times \frac{\pi}{4} d^2 = \sigma (\pi d)$$

or $$\sigma = p \times \frac{d}{4}$$

Substituting the values; $d = 0.01 \times 10^{-3}$ m; $p = 29.2 \times 10^3$ Pa (or N/m²), we get

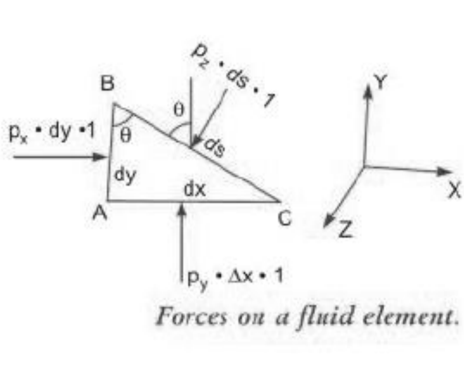
$$\sigma = 29.2 \times 10^3 \times \frac{0.01 \times 10^{-3}}{4} = \mathbf{0.073\ N/m\ (Ans.)}$$

PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as :

The fluid element is of very small dimensions *i.e.*, *dx*, *dy* and *ds*.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in Fig. . Let the width of the element perpendicular to the plane of paper is unity and p_x,



Forces on a fluid element.

p_y and p_z are the pressures or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

1. Pressure forces normal to the surfaces, and
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face AB $= p_x \times$ Area of face AB

$= p_x \times dy \times 1$

Similarly force on the face $AC = p_y \times dx \times 1$

Force on the face BC $= p_z \times ds \times 1$

Weight of element $=$ (Mass of element) $\times g$

$$= (\text{Volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g,$$

where ρ = density of fluid.

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p\,(ds \times 1) \sin(90° - \theta) = 0$$

or $$p_x \times dy \times 1 - p_z\, ds \times 1 \cos\theta = 0.$$

But from Fig $$ds \cos\theta = AB = dy$$

$\therefore$ $$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or $$p_x = p_z$$

Similarly, resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos(90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

or $$p_y \times dx - p_z\, ds \sin\theta - \frac{dxdy}{2} \times \rho \times g = 0.$$

But $ds \sin\theta = dx$ and also the element is very small and hence weight is negligible.

$\therefore$ $$p_y dx - p_z \times dx = 0$$

$$p_y = p_z$$

we have

$$p_x = p_y = p_z$$

The above equation shows that the pressure at any point in x, y and z directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

Problem *A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 30 kN. Find the force required at the plunger.*

Solution. Given :

Dia. of ram, $D = 20 \text{ cm} = 0.2 \text{ m}$

$\therefore$ Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(.2)^2 = 0.0314 \text{ m}^2$

Dia. of plunger $d = 3 \text{ cm} = 0.03 \text{ m}$

$\therefore$ Area of plunger, $a = \frac{\pi}{4}(.03)^2 = 7.068 \times 10^{-4} \text{ m}^2$

Weight lifted, $W = 30 \text{ kN} = 30 \times 1000 \text{ N} = 30000 \text{ N}.$

Pressure intensity developed due to plunger $= \frac{\text{Force}}{\text{Area}} = \frac{F}{a}.$

By Pascal's Law, this pressure is transmitted equally in all directions

Hence pressure transmitted at the ram $= \frac{F}{a}$

$\therefore$ Force acting on ram = Pressure intensity $\times$ Area of ram

$$= \frac{F}{a} \times A = \frac{F \times .0314}{7.068 \times 10^{-4}} \text{ N}$$

But force acting on ram = Weight lifted = 30000 N

$\therefore$ $$30000 = \frac{F \times .0314}{7.068 \times 10^{-4}}$$

$\therefore$ $$F = \frac{30000 \times 7.068 \times 10^{-4}}{.0314} = \mathbf{675.2\ N.\ Ans.}$$

MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices :

1. Manometers 2. Mechanical Gauges.

Manometers. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

Mechanical Gauges. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge, (*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and (*d*) Bellows pressure gauge.

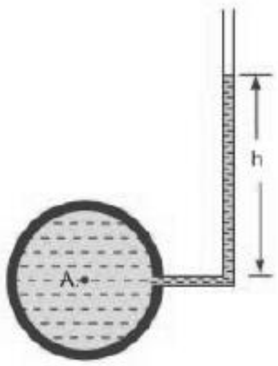
SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer,`
2. U-tube Manometer, and
3. Single Column Manometer.

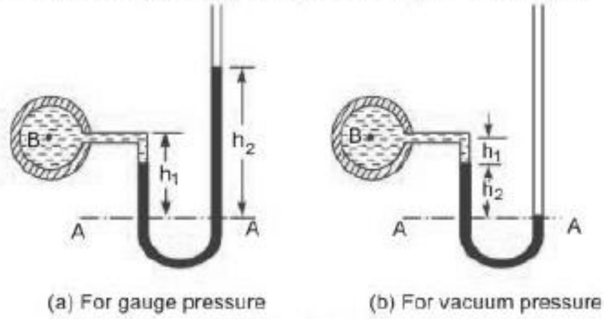
Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. : . The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$= \rho \times g \times h \ \frac{\text{N}}{\text{m}^2}.$$



Piezometer.

U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



U-tube Manometer.

(*a*) **For Gauge Pressure.** Let B is the point at which pressure is to be measured, whose value is p. The datum line is A-A.

Let h_1 = Height of light liquid above the datum line

h_2 = Height of heavy liquid above the datum line

S_1 = Sp. gr. of light liquid

ρ_1 = Density of light liquid = $1000 \times S_1$

S_2 = Sp. gr. of heavy liquid

ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$\therefore$ $p = (\rho_2 g h_2 - \rho_1 \times g \times h_1)$.

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig.

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A $= 0$

$\therefore$ $\rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore$ $p = -(\rho_2 g h_2 + \rho_1 g h_1)$.

Example *A U-tube manometer is used to measure the pressure of oil of specific gravity 0.85 flowing in a pipe line. Its left end is connected to the pipe and the right-limb is open to the atmosphere. The centre of the pipe is 100 mm below the level of mercury (specific gravity = 13.6) in the right limb. If the difference of mercury level in the two limbs is 160 mm, determine the absolute pressure of the oil in the pipe.*

Solution. Specific gravity of oil, $S_1 = 0.85$

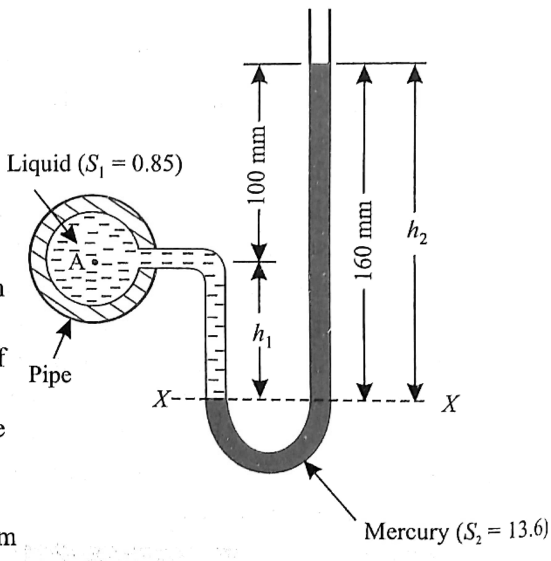
Specific gravity of mercury, $S_2 = 13.6$

Height of the oil in the left limb,

$h_1 = 160 - 100 = 60$ mm $= 0.06$ m

Difference of mercury level,

$h_2 = 160$ mm $= 0.16$ m.



Absolute pressure of oil:

Let, h_1 = Gauge pressure in the pipe in terms of head of water, and

p = Gauge pressure in terms of kN/m^2.

Equating the pressure heads above the datum line X–X, we get:

$$h + h_1 S_1 = h_2 S_2$$

or, $h + 0.06 \times 0.85 = 0.16 \times 13.6 = 2.125$ m

The pressure p is given by:

$$p = wh$$
$$= 9.81 \times 2.125 \text{ kN/m}^2$$
$$= 20.84 \text{ kPa} \quad (\because w = 9.81 \text{ kN/m}^3 \text{ in S.I. units})$$

Absolute pressure of oil in the tube,

$$p_{abs.} = p_{atm.} + p_{gauge}$$
$$= 100 + 20.84 = \mathbf{120.84\ kPa\ (Ans.)}$$

Example *An inverted differential manometer is connected to two pipes A and B carrying water under pressure as shown in Fig. 2.26. The fluid in the manometer is oil of specific gravity 0.75. Determine the pressure difference between A and B.*

Solution. Specific gravity of oil, $S = 0.75$

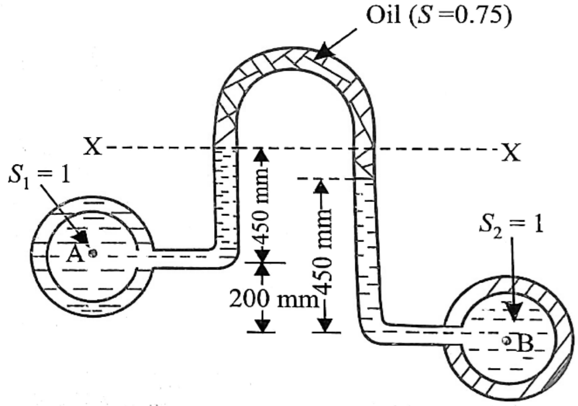
Specific gravity of water, $S_1, S_2 = 1$

Difference of oil in the two limbs $= (450 + 200) - 450 = 200$ mm

We know that pressure heads on the left and right limbs below the datum line X–X are *equal.*

Pressure head in the left limb below X–X

$$= h_A - \frac{450}{1000} \times 1 = h_A - 0.45$$



Pressure head in the right limb below X–X

$$= h_B - \frac{450}{1000} \times 1 - \frac{200}{1000} \times 0.75$$

$$= h_B - 0.45 - 0.15 = h_B - 0.6$$

Equating the two pressure heads, we get:

$$h_A - 0.45 = h_B - 0.6$$

$$h_B - h_A = \mathbf{0.15\ m\ (Ans.)}$$

or, $\frac{p_B}{w} - \frac{p_A}{w} = 0.15$ or $p_B - p_A$

$= w \times 0.15 = 9.81 \times 0.15 = 1.47$ kN/m^2 = **1.47 kPa (Ans.)**

INTRODUCTION

In this chapter, the equilibrium of the floating and sub-merged bodies will be considered. Thus the chapter will include : 1. Buoyancy, 2. Centre of buoyancy, 3. Metacentre, 4. Metacentric height, 5. Analytical method for determining metacentric height, 6. Conditions of equilibrium of a floating and sub-merged body, and 7. Experimental method for metacentric height.

BUOYANCY

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

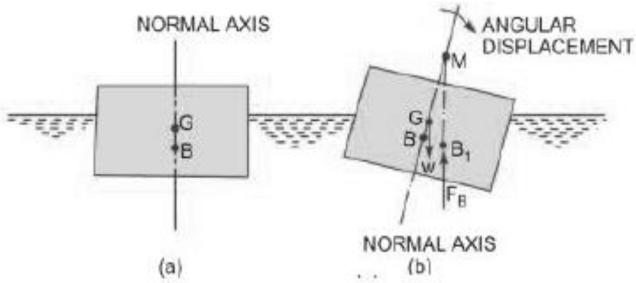
CENTRE OF BUOYANCY

It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

META-CENTRE

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

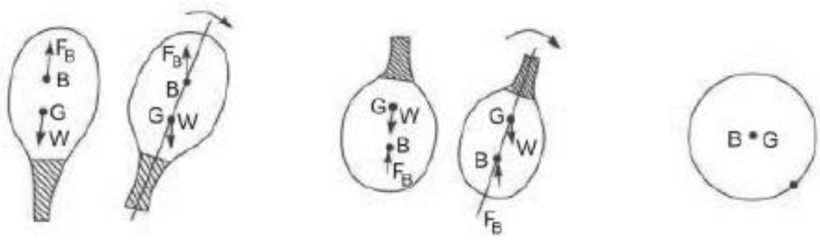
Consider a body floating in a liquid as shown in Fig. (*a*). Let the body is in equilibrium and *G* is the centre of gravity and *B* the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.



Meta-centre

Let the body is given a small angular displacement in the clockwise direction as shown in Fig. (*b*). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at B_1 as shown in Fig. (*b*). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say *M*. This point *M* is called **Meta-centre.**

Stability of a Sub-merged Body. The position of centre of gravity and centre of buoyancy in case of a completely sub-merged body are fixed. Consider a balloon, which is completely sub-merged in air. Let the lower portion of the balloon contains heavier material, so that its centre of gravity is lower than its centre of buoyancy as shown in Fig. (*a*). Let the weight of the balloon is *W*. The weight *W* is acting through *G*, vertically in the downward direction, while the buoyant force F_B is acting vertically up, through *B*. For the equilibrium of the balloon $W = F_B$. If the balloon is given an angular displacement in the clockwise direction as shown in Fig. (*a*), then *W* and F_B constitute a couple acting in the anti-clockwise direction and brings the balloon in the original position. Thus the balloon in the position, shown by Fig (*a*) is in stable equilibrium.



(a) STABLE EQUILIBRIUM (b) UNSTABLE EQUILIBRIUM (c) NEUTRAL EQUILIBRIUM

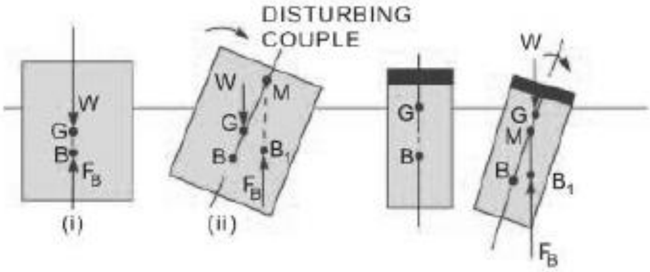
Stabilities of sub-merged bodies.

(*a*) **Stable Equilibrium.** When $W = F_B$ and point *B* is above *G*, the body is said to be in stable equilibrium.

(*b*) **Unstable Equilibrium.** If $W = F_B$, but the centre of buoyancy (*B*) is below centre of gravity (*G*), the body is in unstable equilibrium as shown in Fig. (*b*). A slight displacement to the body, in the clockwise direction, gives the couple due to *W* and F_B also in the clockwise direction. Thus the body does not return to its original position and hence the body is in unstable equilibrium.

(*c*) **Neutral Equilibrium.** If $F_B = W$ and *B* and *G* are at the same point, as shown in Fig. (*c*), the body is said to be in neutral equilibrium.

(*a*) **Stable Equilibrium**. If the point *M* is above *G*, the floating body will be in stable equilibrium as shown in Fig. (*a*). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from *B* to B_1 such that the vertical line through B_1 cuts at *M*. Then the buoyant force F_B through B_1 and weight *W* through *G* constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.



(a) Stable equilibrium *M* is above *G* (b) Unstable equilibrium *M* is below *G*.

Stability of floating bodies.

(*b*) **Unstable Equilibrium.** If the point *M* is below *G*, the floating body will be in unstable equilibrium as shown in Fig. (*b*). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and *W* is also acting in the clockwise direction and thus overturning the floating body.

(*c*) **Neutral Equilibrium.** If the point *M* is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig

Let A = Total area of the surface

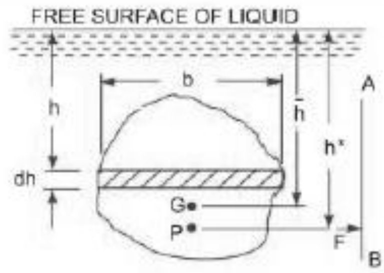
$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

(*a*) **Total Pressure (F).** The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.



Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig.

Pressure intensity on the strip, $p = \rho g h$

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho g h \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $$\int b \times h \times dh = \int h \times dA$$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$\therefore$ $$F = \rho g A \bar{h}$$

For water the value of $\rho = 1000$ kg/m^3 and $g = 9.81$ m/s^2. The force will be in Newton.

(*b*) **Centre of Pressure (h*).** Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. Hence moment of the force F about free surface of the liquid $= F \times h^*$

Moment of force dF, acting on a strip about free surface of liquid

$= dF \times h$ $\quad [\because \; dF = \rho g h \times b \times dh]$

$= \rho g h \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 \, dh = \rho g \int h^2 \, dA \qquad (\because \; b dh = dA)$$

But $$\int h^2 dA = \int b h^2 dh$$

= Moment of Inertia of the surface about free surface of liquid

$= I_0$

$\therefore$ Sum of moments about free surface

$= \rho g I_0$

Equating

$$F \times h^* = \rho g I_0$$

But $$F = \rho g A \bar{h}$$

$\therefore$ $$\rho g A \bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}}$$

By the theorem of parallel axis, we have

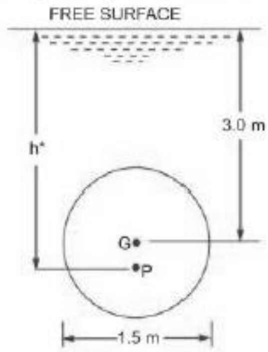
$$I_0 = I_G + A \times \bar{h}^2$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

Problem *Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.*



Solution. Given : Dia. of plate, $d = 1.5$ m

$\therefore$ Area, $$A = \frac{\pi}{4}(1.5)^2 = 1.767 \text{ m}^2$$

$$\bar{h} = 3.0 \text{ m}$$

Total pressure is given by equation (3.1),

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 1.767 \times 3.0 \text{ N}$$
$$= \mathbf{52002.81 \text{ N. Ans.}}$$

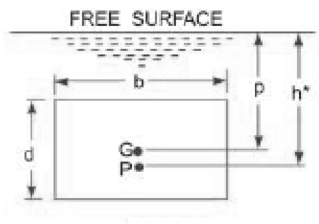
Position of centre of pressure (h^*) is given by equation (3.5),

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi d^4}{64} = \frac{\pi \times 1.5^4}{64} = 0.2485 \text{ m}^4$

$\therefore$ $$h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0 = 0.0468 + 3.0$$
$$= \mathbf{3.0468 \text{ m. Ans.}}$$

Problem *A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 'd' metres in length and depth of centroid of the area is 'p' m below the water surface. Prove that the depth of pressure is equal to* $\left(p + \frac{d^2}{12p}\right)$.



Solution. Given :

Depth of vertical gate $= d$ m

Let the width of gate $= b$ m

$\therefore$ Area, $A = b \times d \text{ m}^2$

Depth of C.G. from free surface

$$\bar{h} = p \text{ m.}$$

Let h^* is the depth of centre of pressure from free surface, which is given by equation

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}, \text{ where } I_G = \frac{bd^3}{12}$$

$\therefore$ $$h^* = \left(\frac{bd^3}{12} \Big/ b \times d \times p\right) + p = \frac{\mathbf{d^2}}{\mathbf{12p}} + \mathbf{p} \quad \text{or} \quad \mathbf{p} + \frac{\mathbf{d^2}}{\mathbf{12p}} \text{. Ans.}$$