Unit 1

Content

- Introduction to simple numerical methods for solving coupled differential equations and for studying correlations
  - Gaussian Method
  - LD decomposition Method
  - Euler's Method
  - Modified Euler's method
- Applications

Introduction to Numerical Methods

- Numerical methods gives approximate solutions.
- Accuracy of the solutions increased by reducing Δx.
- Numerical methods are easy to implement in computer system.
- Numerical methods are taking very less time to solve the problems.
- Algebra, Differentiation and integrations problems are solved by simple math like addition, subtraction, multiplication and division.
- Application: In computational materials science, the unknown and known parameters are expressed by set of linear or non-linear equations. Numerical methods are helps to find the unknown parameters.

Recap of Matrix

- Matrix: an array of numbers

$$A = \overbrace{\begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 8 \\ -6 & 3 & 9 \end{bmatrix}}^{\text{Columns}} \left.\vphantom{\begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}}\right\} \text{Rows}$$

- Matrix dimensions: Rows X columns

$$B = \begin{bmatrix} -8 & 2 \\ 23 & 10 \\ 18 & -5 \end{bmatrix}$$

Dimension of B is = 3 X 2

Dimension of A is = 3 X 3

- Matrix elements: A matrix entry. Each element in a matrix represented by naming the row and column in which it appears.

$$B = \begin{bmatrix} -8 & 2 \\ 23 & 10 \\ 18 & -5 \end{bmatrix}$$

The element $B_{2,1}$ is 23

- Matrix addition: add the numbers in the matching positions.

3+4=7

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

- Matrix subtraction: Subtract the numbers in the matching positions

3-4=-1

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

- Multiply by a constant:

2x4=8

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

- Multiply a matrix by another matrix: dot product of rows and columns

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} X \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ & \end{bmatrix}$$

(1x7)+(2x9)+(3x11)=58

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} X \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ & \end{bmatrix}$$

(1x8)+(2x10)+(3x12)=64

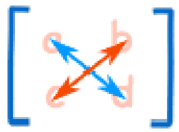
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} X \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

Condition: the number of columns of the 1^{st} matrix must equal to the number of rows of 2^{nd} matrix.
In this case: 2x<u>3</u> and <u>3</u>x2

- Matrix multiplication is not commutative. AB≠BA
- Determinant of a matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

|A| = ad-bc

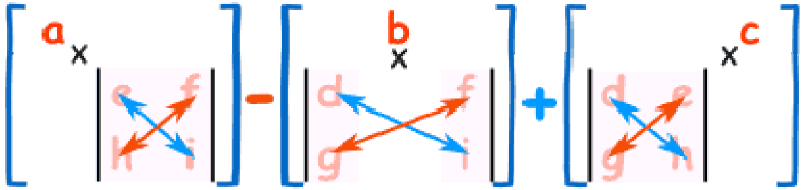


Blue is +ve

Red is –ve

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

|A| = a(ei − fh) − b(di − fg) + c(dh − eg)



- Identity matrix:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- It is square matrix
- It has 1 on the diagonal and 0 everywhere else
- Its symbol is the capital letter I.

3 x3 identity matrix

- Matrix inverse:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\underbrace{ad-bc}_{\text{determinant}}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example:

$$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \frac{1}{4\text{x}6-7\text{x}2} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

- Transposing: Swap the rows and columns

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^{T} = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

- Representing system of equations in matrix:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n
\end{aligned}
$$

- The equation can be written as

$$Ax = b$$

$$
A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ . & . & \ldots & . \\ a_{n1} & a_{n21} & \ldots & a_{nn} \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ . \\ x_n \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ . \\ b_n \end{bmatrix}
$$

Gaussian Elimination Method

The procedure for Gaussian elimination are

- to make the upper-left corner element a 1, use elementary row operations to get 0s in all positions underneath that first 1,
- get 1s for leading coefficients in every row diagonally from the upper-left to lower-right corner, & get 0s beneath all leading coefficients.
- Basically, eliminate all variables in the last row except for one, all variables except for two in the equation above that one, and so on and so forth to the top equation, which has all the variables. Then you can use back substitution to solve for one variable at a time by plugging the values you know into the equations from the bottom up.

Step 1

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Step 2

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Step 3

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Following operation are allowed.

- Multiply an row by a constant (other than zero) $-2r_3 \rightarrow r_3$
- Switch any two rows. $r_1 \rightarrow r_2$
- Add any rows together. $r_1 + r_2 \rightarrow r_2$

- Solve the set of linear equations by Gaussian elimination method.

$$3x_1 + 2x_2 - x_3 = -1$$
$$6x_1 - 6x_2 + 7x_3 = -7$$
$$3x_1 - 4x_2 + 4x_3 = -6$$

- It can be written as

$$\begin{bmatrix} -3 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -7 \\ -6 \end{bmatrix}$$

- Form an augmented matrix by combining matrix A with the column vector b

$$\begin{pmatrix} -3 & 2 & -1 & -1 \\ 6 & -6 & 7 & -7 \\ 3 & -4 & 4 & -6 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 2 & -1 & -1 \\ 6 & -6 & 7 & -7 \\ 3 & -4 & 4 & -6 \end{pmatrix}$$

Step 1: $r_1+r_3 \rightarrow r_3$

$$\begin{pmatrix} -3 & 2 & -1 & -1 \\ 6 & -6 & 7 & -7 \\ 0 & -2 & 3 & -7 \end{pmatrix}$$

Step 2:2 $r_1+r_2 \rightarrow r_2$

$$\begin{pmatrix} -3 & 2 & -1 & -1 \\ 0 & -2 & 5 & -9 \\ 0 & -2 & 3 & -7 \end{pmatrix}$$

Step 4:-2 $r_2+r_3 \rightarrow r_3$

$$\begin{pmatrix} -3 & +2 & -1 & -1 \\ +0 & -2 & +5 & -9 \\ +0 & +0 & -2 & +2 \end{pmatrix}$$

Step 5: relating the matrix and equations,

$$-3x_1 + 2x_2 - x_3 = -1$$
$$-2x_2 + 5x_3 = -9$$
$$-2x_3 = +2$$

Step 6: Solve the equation

$$-2x_3 = 2 \Rightarrow \boxed{x_3 = -1}$$
$$-2x_2 + 5x_3 = -9 \Rightarrow -2x_2 + 5(-1) = -9 \Rightarrow \boxed{x_2 = 2}$$
$$-3x_1 + 2x_2 - x_3 = -1$$
$$-3x_1 + 2(2) - (-1) = -1 \Rightarrow \boxed{x_1 = 2}$$

Therefore,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$

LU Decomposition Method

- Another method to solve the equations using triangular matrix.

In this method, matrix A is decomposed into lower and upper triangular parts to find the solution for *AX=B*.

$$A=LU$$

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}; L=\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}; U=\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

There fore, LUX=B

Let Y=UX, so that LY=B. Solve this triangular system for Y.

Finally solve the triangular system UX=Y for X.

Find the solution of $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in the system $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$

Step 1: *A = LU*

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}; L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}; U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{21}U_{11} + L_{32}U_{23} + U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$$

Therefore, $U_{11}=1$; $U_{12}=2$; $U_{13}=4$

$L_{21}U_{11}=3$

$$L_{21}U_{11} = 3 \Rightarrow L_{12}X(3) = 3 \Rightarrow L_{21} = 3$$

$$L_{12}U_{12} + U_{22} = 8 \Rightarrow (3X2) + U_{22} = 8 \Rightarrow U_{22} = 2$$

$$L_{21}U_{13} + U_{23} = 14 \Rightarrow (3X4) + U_{23} = 14 \Rightarrow U_{23} = 2$$

$$L_{31}U_{11}=2 \Rightarrow L_{31}X1=2 \Rightarrow L_{31}=2$$

$$L_{31}U_{12}+L_{32}U_{22}=6 \Rightarrow (2X2)+L_{32}X2=6 \Rightarrow L_{32}=1$$

$$L_{31}U_{13}+L_{32}U_{23}+U_{33}=13 \Rightarrow (2X4)+(1X2)+U_{33}=13 \Rightarrow U_{33}=3$$

$$A=\begin{bmatrix}1 & 2 & 4\\ 3 & 8 & 14\\ 2 & 6 & 13\end{bmatrix};L=\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 2 & 1 & 1\end{bmatrix};U=\begin{bmatrix}1 & 2 & 4\\ 0 & 2 & 2\\ 0 & 0 & 3\end{bmatrix}$$

Step 2: LY=B

$$LY=\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 2 & 1 & 1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}=\underbrace{\begin{bmatrix}3\\ 13\\ 4\end{bmatrix}}_{B}$$

Forward substitution:

$$y_1=3$$

$$3y_1+y_2=13 \Rightarrow 3(3)+y_2=13 \Rightarrow y_2=4$$

$$2y_1+y_2+y_3=4 \Rightarrow 2(3)+4+y_3=4 \Rightarrow y_3=-6$$

UX=Y

$$U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}; X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; Y = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix}$$

Backward substitution:

$$3x_3 = -6 \Rightarrow x_3 = -2$$

$$2x_2 + 2x_3 = 4 \Rightarrow 2(x_2) + 2(-2) = 4 \Rightarrow 2x_2 = 4 + 4 \Rightarrow x_2 = 4$$

$$x_1 + 2x_2 + 4x_3 = 3 \Rightarrow x_1 + 2(4) + 4(-2) = 3 \Rightarrow x_1 = 3$$

Limitation of LU decomposition method

- Matrix should be square
- Leading sub-matrices should be non-zero determinants.

Which of these matrices have an LU decomposition

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$$

The leading sub-matrices are,

$$A_1 = 1; A_2 = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}; A_3 = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$$

Determinant $|A_1|=1$; $|A_2|=2$; $|A_3|=6$

$$A = \begin{bmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{bmatrix}$$

Determinant $|A_1|=1$;
$|A_2|=0$

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Determinant $|A_1|=3$:|A2|=3

$$A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$$

Determinant |A1|=0;|A2|=-3

Euler's Method

The Euler method is the most straightforward method to integrate a differential equation.

Consider the equation, $\dfrac{dy}{dx} = f(x, y)$ ....*(1)*

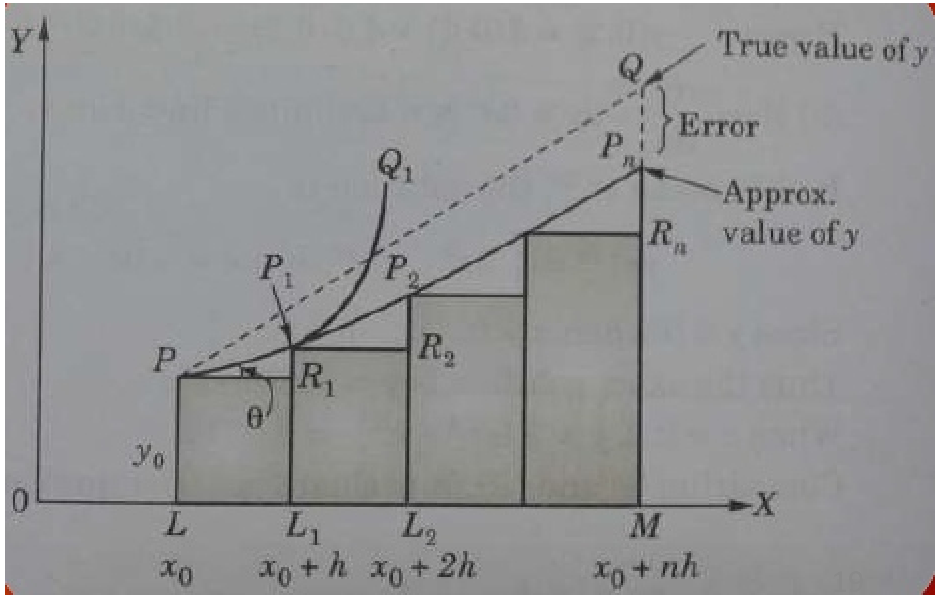
Given that $y(x_0)=y_0$ its curve of solution through $P(_{x0,y0})$ is shown dotted in figure. Now we have to find ordinate of any other point at Q on this curve.

Dividing the curve into 'n' equal sub-interval each of width 'h'.

So we approximate the tangent for LL_1

So $Y_1=L_1P_1=LP+R_1P_1$

$=y_0+PR_1 \tan\theta = y_0+hf(x_0,y_0)$

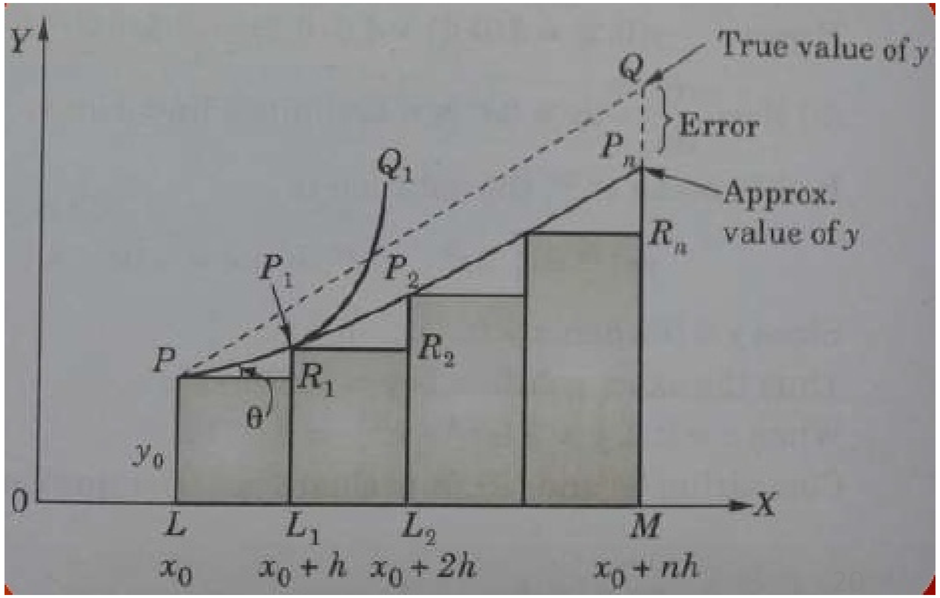


Repeating this process *n times*

$$y_{n+1} = y_n + hf(x_n, y_n) \quad \text{..........(2)}$$

Where $x_n = x_0 + nh$

Equation 2 is called as Euler's Method for finding an approximate solution.



Working of Euler Method

- Given function is taken for the first approximation. Using initial boundary condition and value of ‘h’.

→If the value of ‘*h*’ is not given than the initial and final value(required value) is divided into ‘*n*’ sub-intervals for finding value of ‘*h*’.

- After getting the first approximation the second approximation is taken for the function of x_0+h and y_1 .

Than the Euler’s Method become $y_2=y_1+hf(x_0+h,y_1)$

where y_1 is the value obtain from first approximation .

- Similerly approximations are taken out until $x_0+h=x_n$ (Required value of y at any point x_n)

Problem: Apply Euler's Method to solve y'=x+y .Given y(0)=0. Find y at x=0.8 using step length 0.2

Given: $$\frac{dy}{dx} = x + y$$

y(0)=0 f(x,y)=x+y

h=0.2 Find y(0.8)=?

Remember:

$f(x,y) = x+y$

$f(x_0,y_0)= x_0+y_0$

$f(x_1,y_1)= x_1+y_1$

$f(x_2,y_2)= x_2+y_2$

....

n	x	Y(x) $[y_{n+1}=y_n+hf(x_n,y_n)]$
0	0	0
1	0.2	$y_1=y_0+hf(x_0,y_0)$ $= y_0+h[x_0 +y_0]$ =0+0.2[0+0]=0
2	0.4	$y_2=y_1+hf(x_1,y_1)$ $=y_1+h[x_1 +y_1]$ =0+0.2[0.2+0]=0.4
3	0.6	$y_3=y_2+hf(x_2,y_2)$ $=y_2+h[x_2 +y_2]$ =0.4+0.2[0.4+0.4]=0.432
4	0.8	$y_4=y_3+hf(x_3,y_3)$ $=y_3+h[x_3 +y_3]$ =0.432+0.2[0.6+0.432] =0.6384

Solve the initial value problem: $y' = x + 2y$, $y(0) = 0$ numerically, finding a value for the solution at $x = 1$, and using steps of size $h = 0.25$

Given: $\frac{dy}{dx} = x + 2y$

Y(0) =0 and h = 0.25

Find: y(1)=?

Remember:

$f(x,y) = x+2y$

$f(x_0,y_0) = x_0+2y_0$

$f(x_1,y_1)= x_1+2y_1$

$f(x_2,y_2)= x_2+2y_2$

....

n	x	Y(x) $[y_{n+1}=y_n+hf(x_n,y_n)]$
0	$X_0=0$	$Y_0=0$
1	$X_1= X_0+h$ $= 0+0.25$ $= 0.25$	$y_1=y_0+hf(x_0,y_0)$ $= y_0+h[x_0 +2y_0]$ $=0+0.25[0+0]=0$
2	$X_2= X_1+h$ $= 0.25+0.25$ $= 0.5$	$y_2=y_1+hf(x_1,y_1)$ $=y_1+h[x_1 +2y_1]$ $=0+0.25[0.25+0]=0.0625$
3	$X_3= X_2+h$ $= 0.5+0.25$ $= 0.75$	$y_3=y_2+hf(x_2,y_2)$ $=y_2+h[x_2 +2y_2]$ $=0.0625+0.25[0.5+2(0.0625)]$ $=0.21875$
4	$X_4= X_3+h$ $= 0.75+0.25$ $= 1$	$y_4=y_3+hf(x_3,y_3)$ $=y_3+h[x_3 +2y_3]$ $=0.21875+0.25[0.75+2(0.21875)]$ $=0.515625$

Modified Euler's Methods

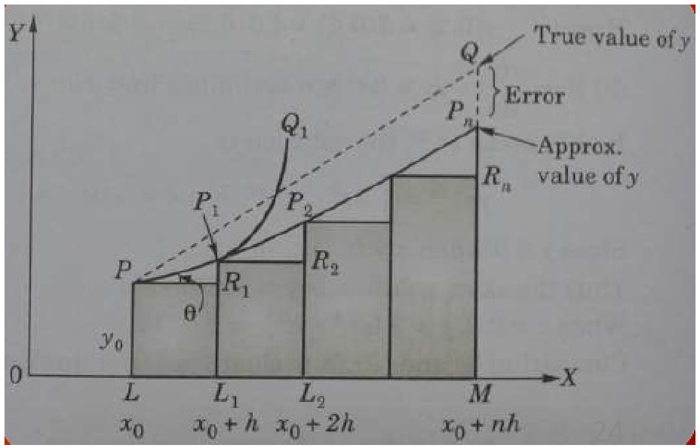
In the Euler's Modified method , The curve of the solution in the interval "LL_1 " is approximates by the tangent at P such as at P_1 we have, $y_1 = y_0 + hf(x_0, y_0)$.......(1)

Than the slope of the curve of the solution through P_1 is computed at the tangent at P_1 to P_1Q_1 is drawn meeting the ordinate through L_2 in $P_2(x_0+2h,y_2)$

Now we find better approximation y_1' of $y(x_0+h)$ by taking the slope of the curve as the mean of the slope of the tangent at P and P_1 i.e.

$$y_1 = y_0 + \frac{h}{2}\{f(x_0, y_0) + f(x_0 + h, y_1)\}.....(2)$$

As the slope of the tangent at P1 is not known, We take Y1 is found in equation (1)



- Euler method,

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Where $x_n = x_0 + nh$

- Euler's Modified method is

$$\mathrm{y}_{\mathrm{n+1}} = \mathrm{y}_{\mathrm{n}} + \frac{h}{2}\{\mathrm{f}(\mathrm{x}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}}) + \mathrm{f}(\mathrm{x}_{\mathrm{n+1}}, \mathrm{y}_{\mathrm{n+1}})\}$$

Working of Modified Euler's Method

1. First we find the first approximation using Euler's Method.
2. The approximated value of y_1 is than modified using Euler modified method.
3. The approximated value of y_1 from Euler modified method is again approximated until the equal value of y_1 is found.
4. The value of y_1 is taken for the approximation of y_2 using Euler method.
5. And the process continues.

Apply Euler's Modified Method to solve y'=x+y .Given y(0)=1.Find y at x=0.2 using step length 0.1

Given, $\frac{dy}{dx} = x + y$

Y(0)=1 h=0.1

Y(0.2)=?

Step 1:

$X_0=0$ $y_0=1$ $x_1=x_0+h=0.1$

By Euler method , calculate y_1

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$y_1=y_0+hf(x_0,y_0)$

$= y_0+h[x_0 +y_0]$

$=1+0.1[0+1]=1.1$

Step 2: $X_0=0$ $y_0=1$ $x_1=0.1$ $y_1=1.1$

$f(x_0,y_0))=x_0+y_0=0+1=1$

$f(x_1,y_1))=x_1+y_1=0.1+1.1=1.2$

$y_1=y_0+(h/2)\{f(x_0,y_0)+f(x_1,y_1)\}$

$= 1+(0.1/2)[\{1+1.2\}$

$=1+(0.05)\{2.2\}=1.11$

Step 3: $X_0=0$ $y_0=1$ $x_1=0.1$ $y_1=1.11$

$f(x_1,y_1))=x_1+y_1=0.1+1.11=1.21$

$y_1=y_0+(h/2)\{f(x_0,y_0)+f(x_1,y_1)\}$

$= 1+(0.1/2)[\{1+1.21\}$

$=1+(0.05)\{2.21\}=1.1105$

Repeat the process to find y(0.2)=?

Step 1: $X_1=0.1$ $y_1=1.1105$ $x_2=x_1+h=0.2$

By Euler method , calculate y_2

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$y_2=y_1+hf(x_1,y_1)$

$= y_1+h[x_1 +y_1]$

$=1.1105+0.1[0.1+1.1105]$

$=1.23155$

Step 2: $X_1=0.1$ $y_1=1.1105$

$X_2=0.2$ $y_2=1.23155$

$f(x_1,y_1))=x_1+y_1=0.1+1.1105$

$=1.2105$

$f(x_2,y_2))=x_2+y_2=0.2+1.23155$

$=1.43155$

$y_2=y_1+(h/2)\{f(x_1,y_1)+f(x_2,y_2)\}$

$= 1.1105+(0.1/2)\{1.2105+1.43155\}$

$=1.1105+(0.05)\{2.64205\}$

$=1.1105+0.1321025$

$=1.2426025$

~ 1.2426

Step 3:

$X_1=0.1$ $y_1=1.1105$

$X_2=0.2$ $y_2=1.2426$

$f(x_2,y_2))=x_2+y_2=0.2+1.2426=1.4426$

$y_2=y_1+(h/2)\{f(x_1,y_1)+f(x_2,y_2)\}$

$= 1.1105+(0.1/2)\{1.2105+1.4426\}$

$= 1.1105+(0.05)\{2.6531\}$

$=1.243155\sim 1.2432$

$\frac{dy}{dx} = x - y - 2$ and y(-1)=3 . Use Euler's modified method with three steps of equal size to approximate y(2).

$x_{-1}=-1$, $y_{-1}=3$

$f(x_{-1}, y_{-1})=-1-3-2=-6$

By Euler method , calculate y_0

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$Y_0=y_{-1}+hf(x_{-1}, y_{-1})$

$=3+1(-6)=-3$

Step 2: $x_{-1}=-1$, $y_{-1}=3$

$x_0=0$, $y_0=-3$

In modified Euler's method

$y_0=y_{-1}+(h/2)\{f(x_{-1},y_{-1})+f(x_0,y_0)\}$

$y_0=3+(1/2)(1-6) =3+(0.5)(-5)$

$=3-2.5=0.5$

Step 3: $x_{-1}=-1$, $y_{-1}=3$ $x_0=0$, $y_0=0.5$

$y_0=y_{-1}+(h/2)\{f(x_{-1},y_{-1})+f(x_0,y_0)\}$

$y_0=3+(1/2)(1-2.5) =3+(0.5)(-1.5)$

$=3-0.75=2.25$

Step 1: $x_0=0$, $y_0=2.25$ $x_1=1$, $y_1=?$

By Euler method , calculate y_1

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$Y_1=y_0+hf(x_0, y_0)=2.25+1(4.25)=6.50$

In modified Euler's method

$y_1=y_0+(h/2)\{f(x_1,y_1)+f(x_0,y_0)\}$

$y_0=2.5+(1/2)(-7.5-2.5) =2.5+(0.5)(-10)$

$=7.5$