RESISTANCE , CAPACITANCE & INDUCTANCE (R,L & C SERIES & PARALLEL CIRCUIT

Subject Name: Electrical Fundamentals

Prepared By:
Nikesh I Patel

Approved By:

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INDUCTIVE LOADS

Inductance

The inductance of an inductor (L) is measured in henries (H). It depends upon the length (*l*), cross-sectional area (*A*), number of turns of wire (*N*), and the permeability of the material contained in the core (μ).

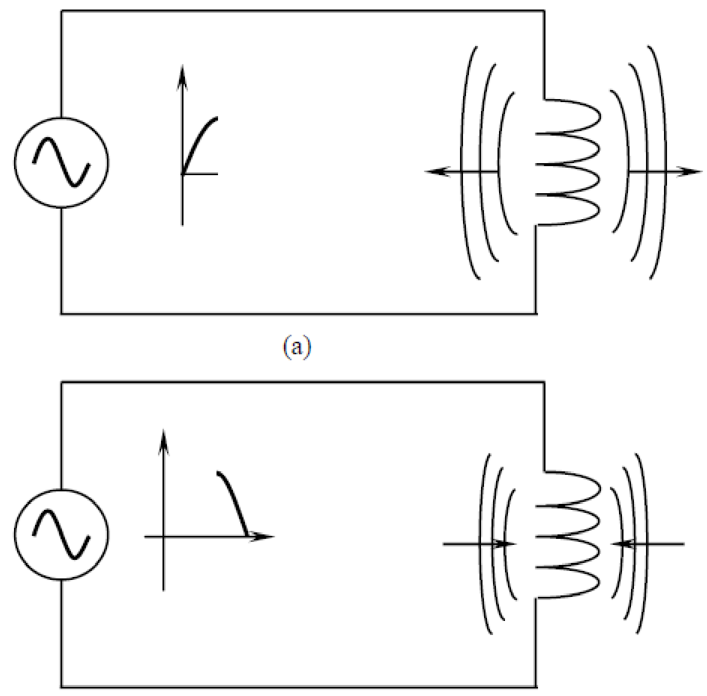
This is expressed mathematically as

$$L = \frac{N^2 \mu A}{l}$$

Circuits contain inductance when load contains a coil. Loads such as motors, transformers, and chocks all contain coils of wire.

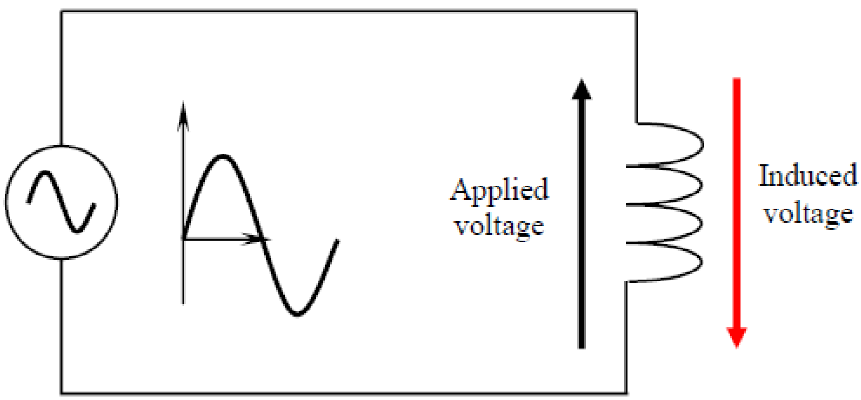
Whenever current flows through a coil, a magnetic field is created around the wire.
In the case of a circuit having inductance, the opposing force of the counter EMF would be enough to keep the current from remaining in phase with the applied voltage.

The inductor is connected to an AC voltage source.
The magnetic field changes magnitude and
Direction as a result a voltage is induced in the
Coil as shown in Figure.
This induced voltage is 180^{o} out of phase
with the applied voltage. The induced
Voltage limits the flow of current
through the circuit in a manner similar to
resistance. This current-limiting property of the
inductor is called reactance (X).
Since this reactance is caused by
inductance, it is called inductive
reactance (XL). It is measured in ohms.

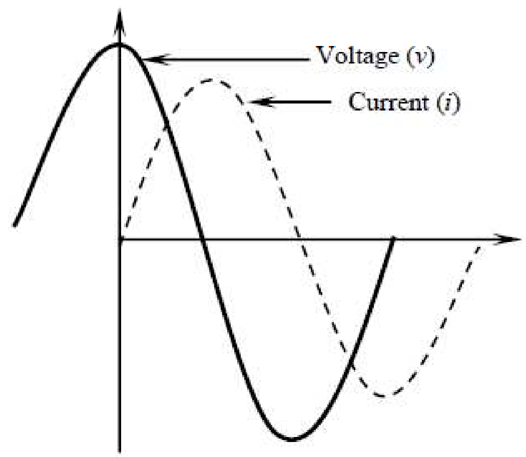


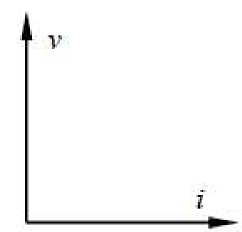
(a)

$$v_L = (\omega L) I_P \sin(\omega t + 90^\circ)$$



The equation shows that there is a phase shift of 90°. Figure below shows the voltage is leading the current when AC current passes through an inductor. The current is at 0° and the voltage drop across the inductor is at +





$$X_L = \omega L = 2\pi f L$$

$$v_L = L\frac{di}{dt}$$

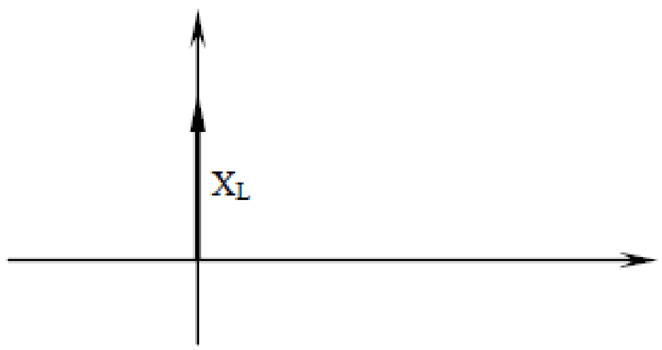
$$i = I_P \sin(\omega t) = I_{rms}\angle 0^\circ$$

$$\frac{di}{dt} = \omega I_P \sin(\omega t + 90^\circ)$$

$$Z_L = \frac{(X_L I_{rms} \angle 90^o)}{(I_{rms} \angle 0^o)}$$

$$Z_L = (X_L \angle 90^o)$$

The impedance diagram of an inductor is shown in Figure. The length of the phasor XL lies entirely along the imaginary (+*y*) axis.



In a pure inductive circuit, however, no true power is produced.

CAPACTIVE LOADS

It opposes a change in voltage. Capacitors block DC in electronic circuits. The current through a capacitance depends on how rapidly the voltage across it changes.

$$i_C = C\frac{dv}{dt}$$

where

$$v = V_p \sin(\omega t) = V_{rms}\angle 0^\circ$$

Taking the derivative of a sinusoidal voltage

$$\frac{dv}{dt} = \omega V_p \sin(\omega t + 90^\circ)$$

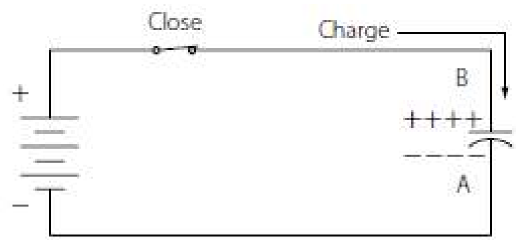
$$I_C = (\omega C)\times(V_{rms}\angle 90^\circ)$$

$$X_C = \frac{1}{\omega C}$$

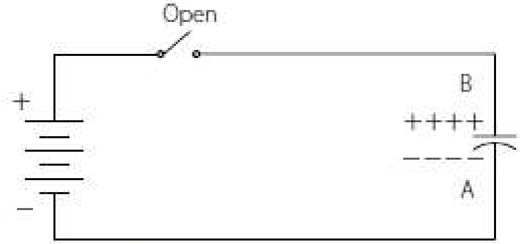
The impedance of a capacitor is a frequency-dependent complex quantity varying as an inverse function of frequency.

Acts as a short circuit at high frequencies and as an open circuit at low frequencies.

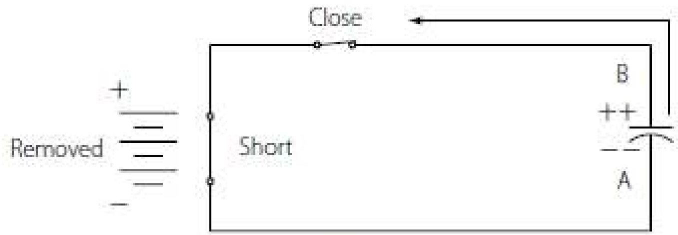
The current is shifted 90° ahead of the voltage.



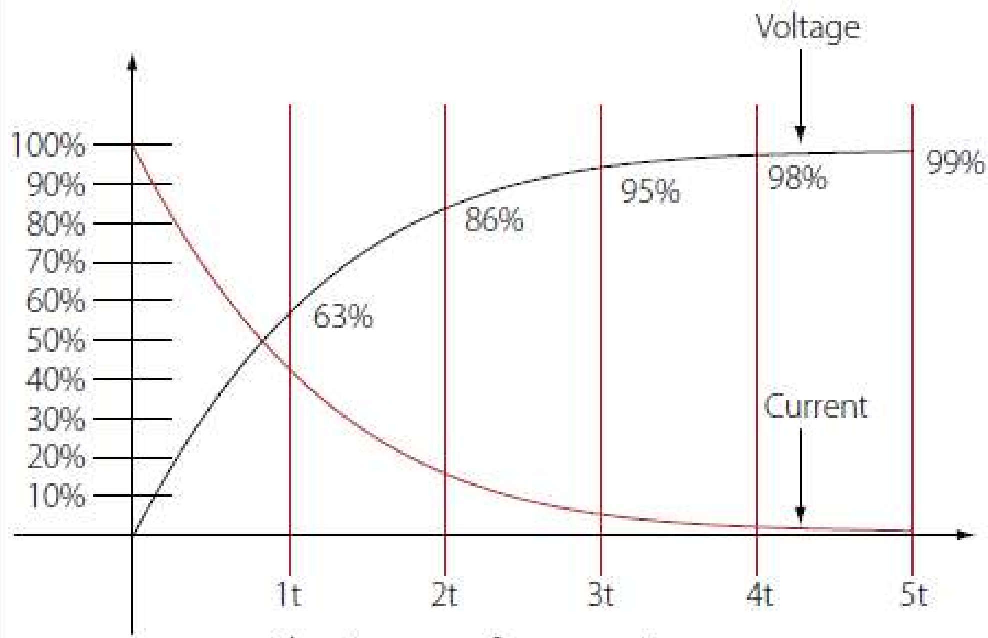
(A) Capacitor being charged



(B) Capacitor retains charge



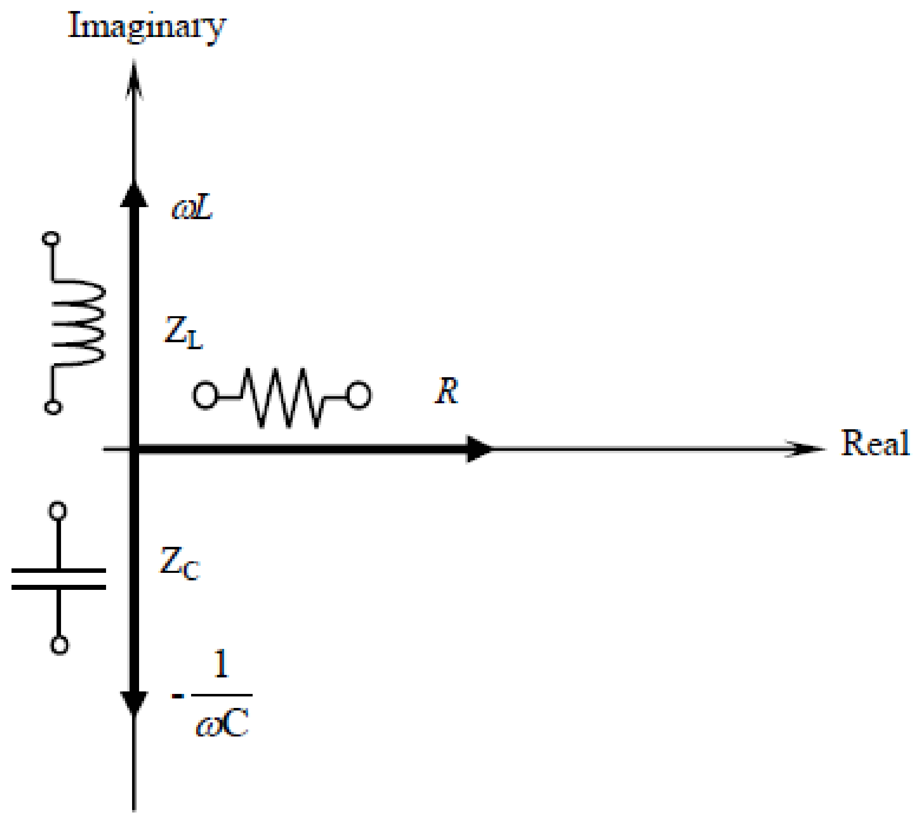
(C) Capacitor discharges



Charging curve for a capacitor

The RC Time Constant

•When a capacitior charges or discharges through a resistance, a certain amount of time is required for a full charge or discharge.
•The voltage across the capacitor will not change instantaneously.
•The rate of charging or discharging is determined by the time constant of the circuit.
•The time constant of a series RC (resistor/capacitor) circuit is a time interval that equals the product of the resistance in ohms and the capacitance in farad and is symbolized by the greek letter tau (τ). $\tau = RC$
•The time constant is the time required to charge it to 63% of the voltage of the source.
•The time required to bring the charge to about 99% of the source voltage is approximately 5 τ.



Impedances of *R*, *L*, and *C* in the complex plane.

- All the rules and laws learned in the study of DC circuits apply to AC circuits including Ohm's law, Kirchhoff's laws, and network analysis methods.
- All variables must be expressed in complex form, taking into account phase as well as magnitude, and all voltages and currents must be of the same frequency (in order that their phase relationships remain constant).
- The impedance of circuit elements is either purely real (for resistors) or purely imaginary (for inductors and capacitors), the impedance for an arbitrary circuit have both a real and imaginary part.

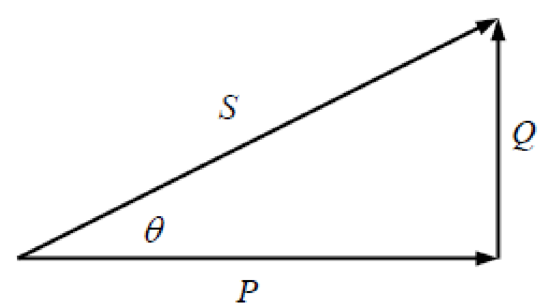
POWER AND POWER FACTOR

The power consumed by a load will be comprised of several individual power components, such as apparent power, reactive power, and active or real power.

The active or real power component of the load is that portion of the load that performs real work.
The reactive power component of the load is used to supply energy that is stored in either a magnetic or electrical field.
An example of reactive power being used to supply a magnetic field is the magnetizing current consumed by a transformer or an electric motor.

An example of a device that supplies reactive power is the capacitor.

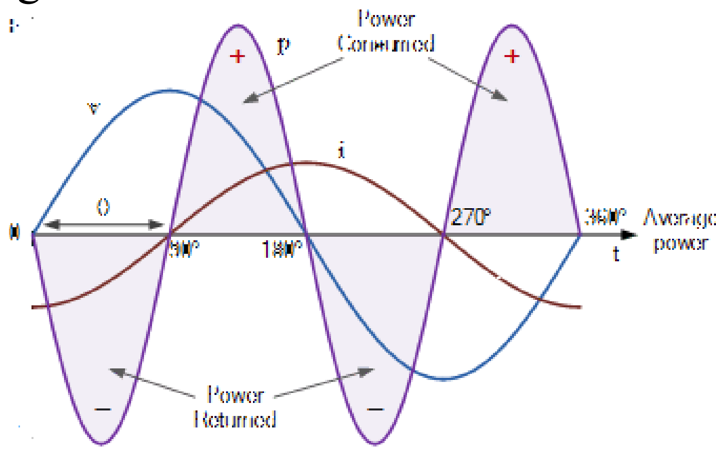
The relationship between these electrical power quantities is best realized by using the power triangle shown in fig.
Where:
S = magnitude of power apparent in VA
P = magnitude of real (active) power in W
Q = magnitude of reactive power in VAR



When an AC power is applied to a reactive load, the voltage is 90° out of phase with the current.
When the instantaneous amplitudes of the voltage and current are multiplied, the resultant wave represents the instantaneous power of the reactor.

$$p = V \times I \cos\theta$$

Where **V** and **I** are the sinusoids rms values, and **θ** (Theta) is the phase angle between the voltage and the current. The units of power are in watts (W).
The average power is zero, which means that reactive loads do not dissipate power as shown in figure.



The base of the power triangle represents the real power component.
The vertical component represents the reactive power component.
The hypotenuse of the triangle represents the apparent power component

$$S = \sqrt{P^2 + Q^2}$$

Power Factor

The ratio of real power to apparent power is called *power factor. It is expressed as*

$$\mathrm{PF} = \frac{P}{S} = \cos(\theta)$$

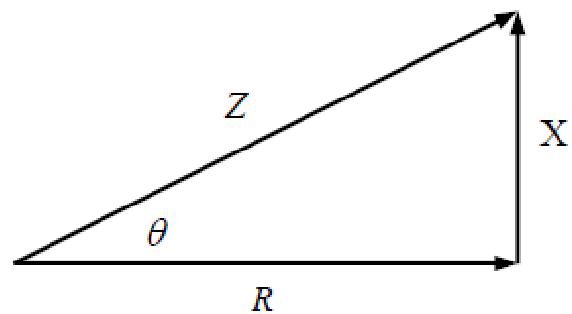
The power factor is a measure of how well the load is converting the total power consumed into real work.
A power factor equal to 1.0 indicates that the load is converting all the power consumed into real work.
Power factor of 0.0 indicates that the load is not producing any real work.
The power factor of a load will be between 0.0 and 1.0.

The ratio of reactive power to apparent power is referred to as the *reactive factor of the load. It is expressed as*

$$\mathrm{RF} = \frac{Q}{S} = \sin(\theta)$$

Only the resistive portion of an AC circuit dissipates power.
The ratio of the circuit resistance to the amplitude of the circuit impedance is called power factor. This is expressed mathematically as

$$\text{Power factor} = \frac{R}{|Z|}$$



Impedance triangle

The impedance of an AC circuit is resistive (Z= R). Therefore, the power factor is 1.

When the impedance is reactive (Z = *jX),* the power factor is zero.

When an AC power is applied to a reactive load, the voltage is 90o out of phase with the current.

The power factor is related to the phase angle through the impedance diagram. This is expressed as

$$\text{Power factor} = \cos(\theta) = R/|Z|$$

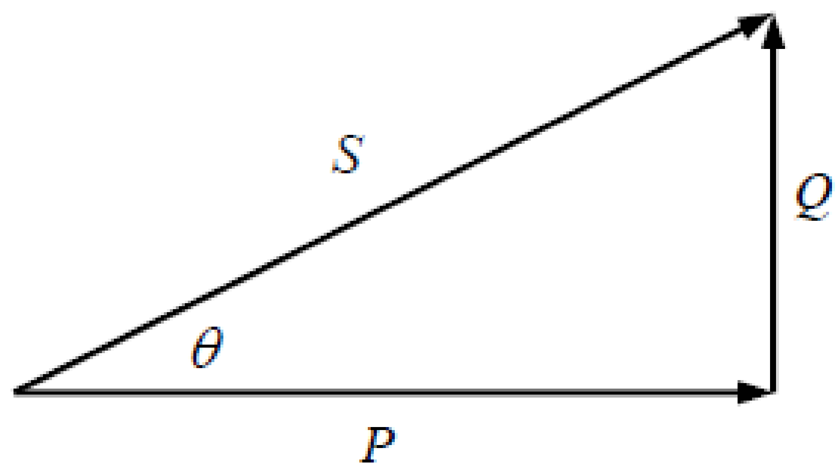
Leading and Lagging Power Factor

A load in which the current lags the applied voltage is said to have a lagging power factor.

A load in which the current leads the applied voltage is said to have a leading power factor.

The current in an inductive load will lag the applied voltage by certain angle as shown in Figure. Therefore, an inductive load will have a lagging power factor.

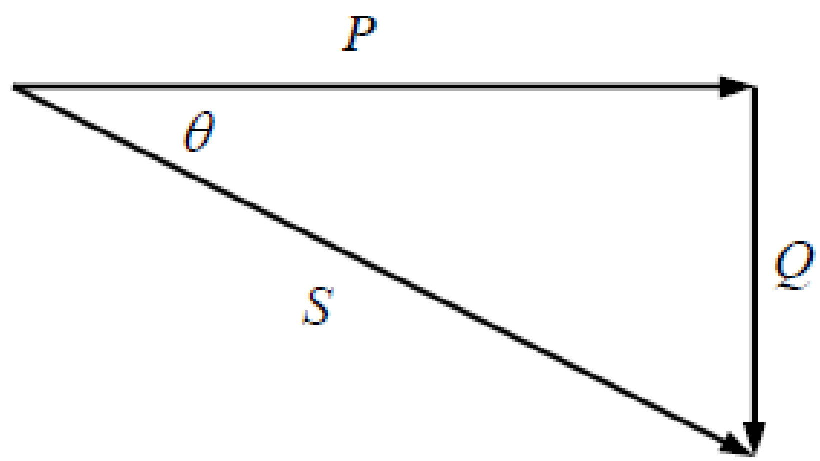
Good examples of inductive loads are transformers, motors, generators, and typical residential loads.



Power triangle for lagging power factor.

A leading power factor is one in which the current leads the applied voltage by certain angle as shown in Figure.

A power factor correction capacitor is an example for a load having a leading power factor.



Power triangle for leading power factor.

Example

A three-phase load consumes 100 kW, and 50 kVAR.
Determine the apparent
power, reactive factor, and the power factor angle.

Solution: The apparent power

$$S = \sqrt{(100{,}000)^2 + (50{,}000)^2} = 111.8\,\mathrm{kVA}$$

the power factor

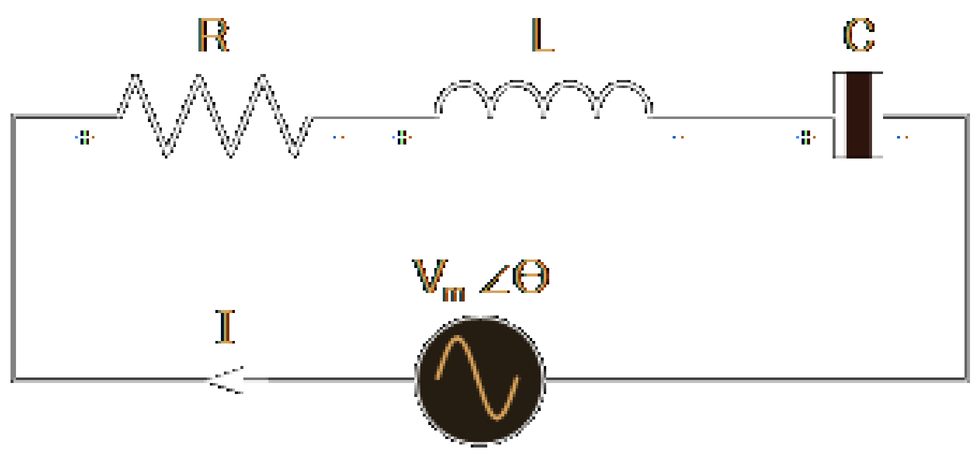
$$\mathrm{PF} = \frac{100{,}000}{111{,}803} = 0.8944$$

the reactive power

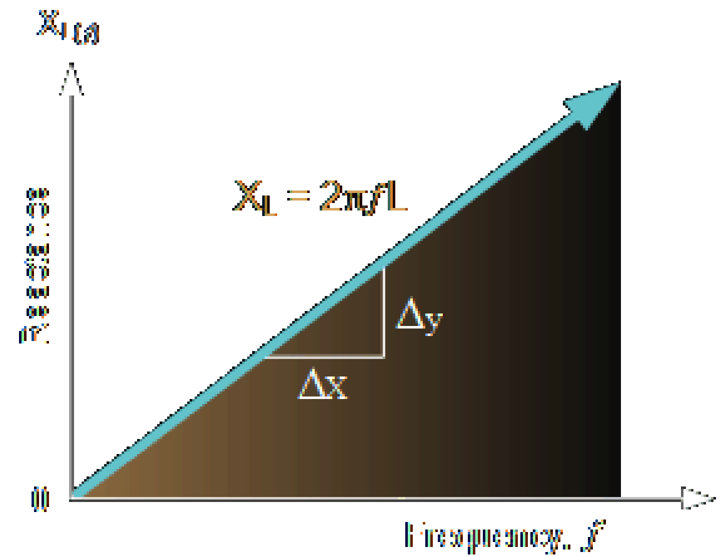
$$\mathrm{RF} = \frac{50{,}000}{111{,}803} = 0.4472$$

Series Resonance Circuit

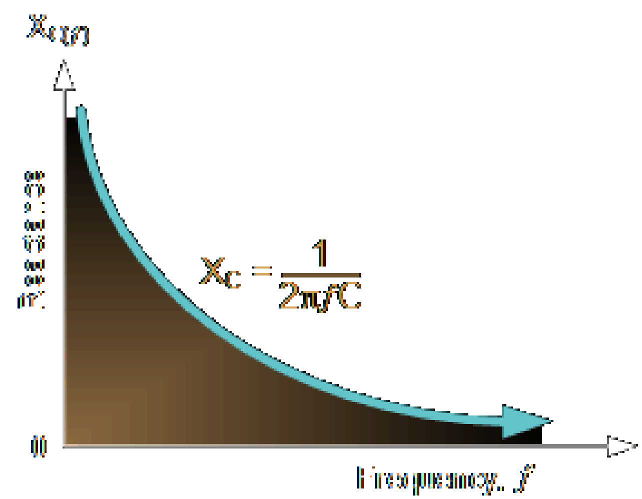
In a circuit where the inductor and capacitor are in series, and the frequency is the resonant frequency, or frequency of resonance, the circuit is said to be "in resonance" and is referred to as a series resonant circuit. The symbol for resonant frequency is Fn.



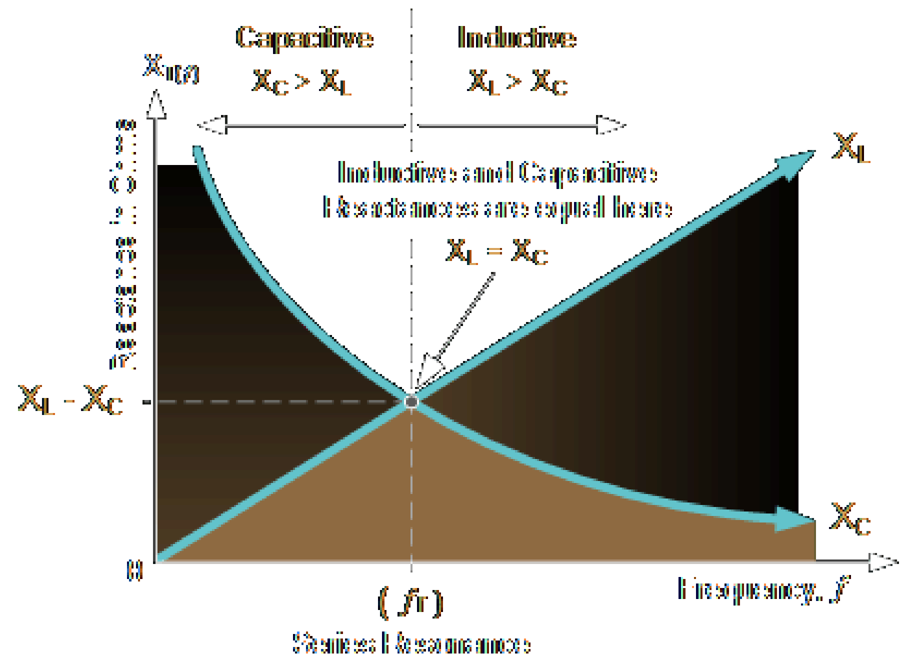
The inductive reactance value of an inductor increases linearly as the frequency across it increases ($X_L \propto f$) as shown in graph



The capacitive reactance would decrease If the **Frequency** is increased. Capacitive reactance is "**Inversely proportional**" to frequency for any given value of capacitance as shown in the graph. Capacitive reactance is negative and is inversely proportional to frequency ($X_C \propto f^{-1}$)



The values of these resistances X_L and X_C depends upon the frequency of the supply. At a higher frequency X_L is high and at a low frequency X_C is high. There js a frequency point were the value of X_L is the same as the value of X_C. The point of intersection of X_L and X_C is the series resonance frequency point, (f_r or ω_r) as shown.



Electrical resonance occurs in an AC circuit when the two reactances which are opposite and equal cancel each other out as $X_L = X_C$ and the point on the graph at which this happens is were the two reactance curves cross each other. In a series resonant circuit, the resonant frequency, f_r point can be calculated as follows.

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}}\,(\mathrm{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}\,(\mathrm{rads})$$

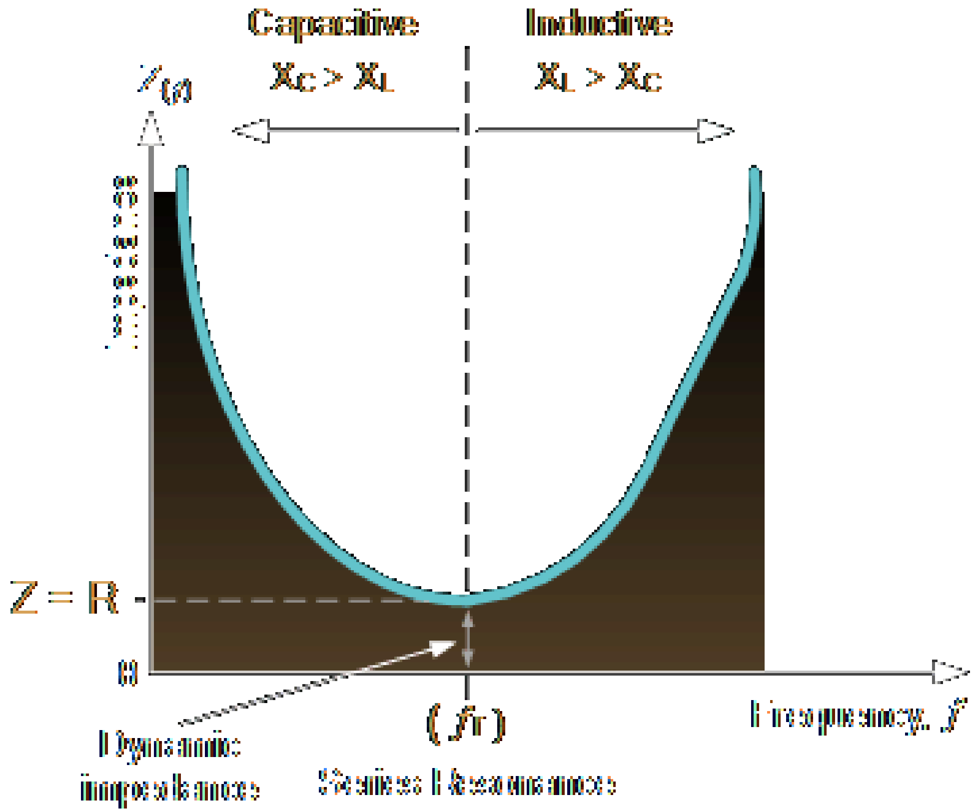
The only opposition to current flow in a series resonance circuit being the resistance, R.

In complex form, the resonant frequency is the frequency at which the total impedance of a series RLC circuit becomes purely *"real"*.

The total impedance of the series circuit becomes just the value of the resistance and therefore: $Z = R$.

The circuit impedance at resonance is called the "dynamic impedance" of the circuit and depending upon the frequency, X_C or X_L will dominate either side of resonance as shown.

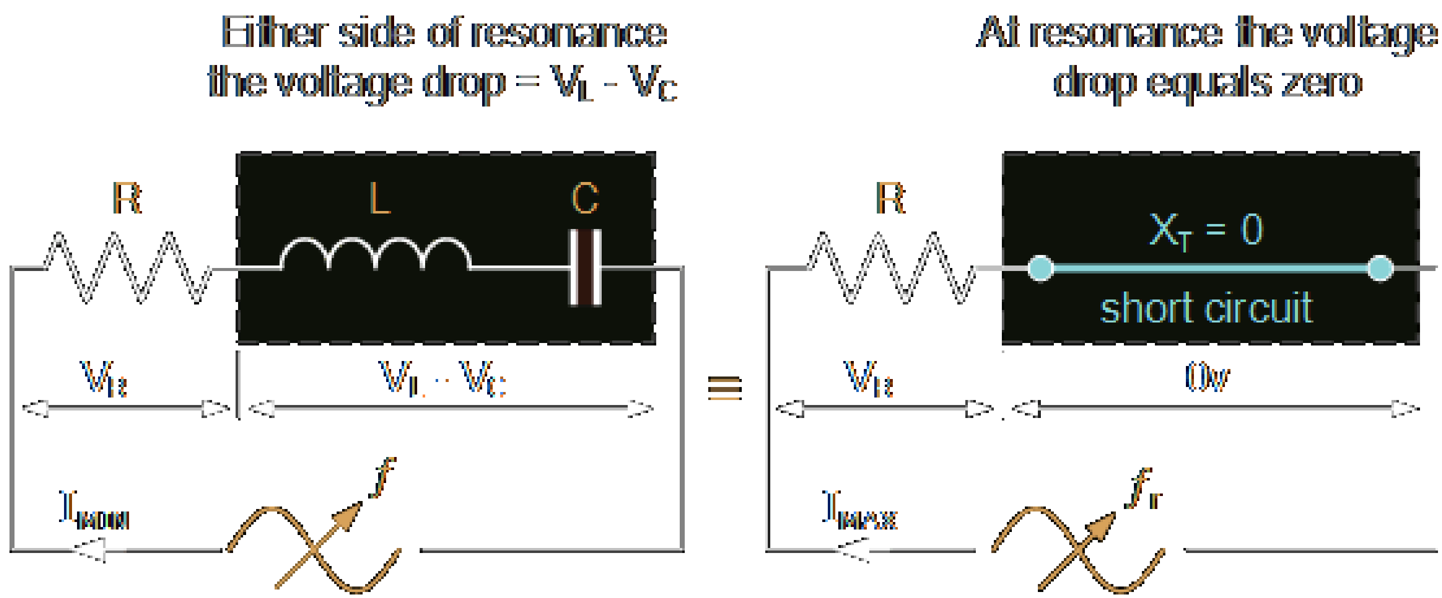
Impedance in a Series Resonance Circuit



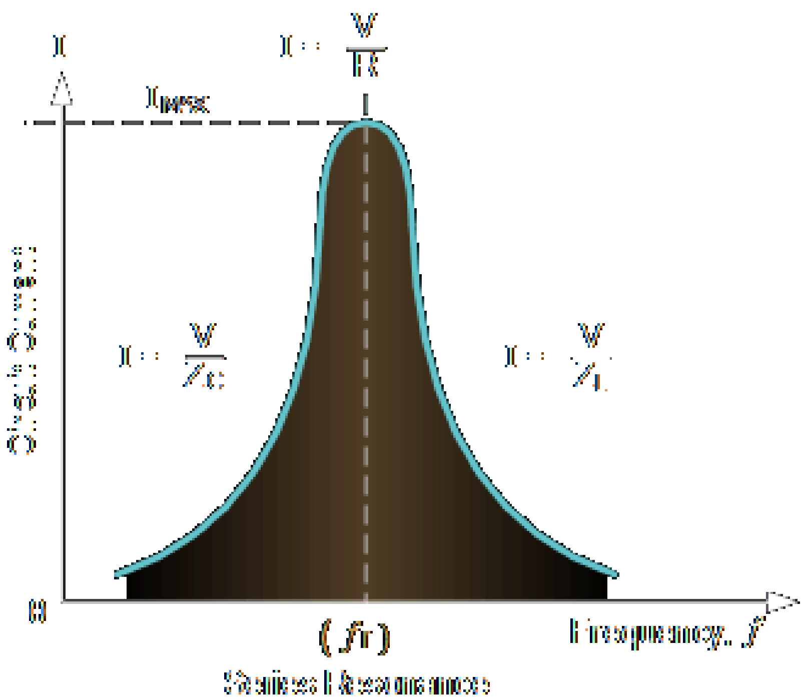
A very low value of resistance at resonance means that the resulting current flowing through the circuit may be dangerously high.

As $V_L = -V_C$ the resulting reactive voltages are zero and all the supply voltage is dropped across the resistor. Therefore, $V_R = V_{supply}$ and it is for this reason that series resonance circuits are known as voltage resonance circuits

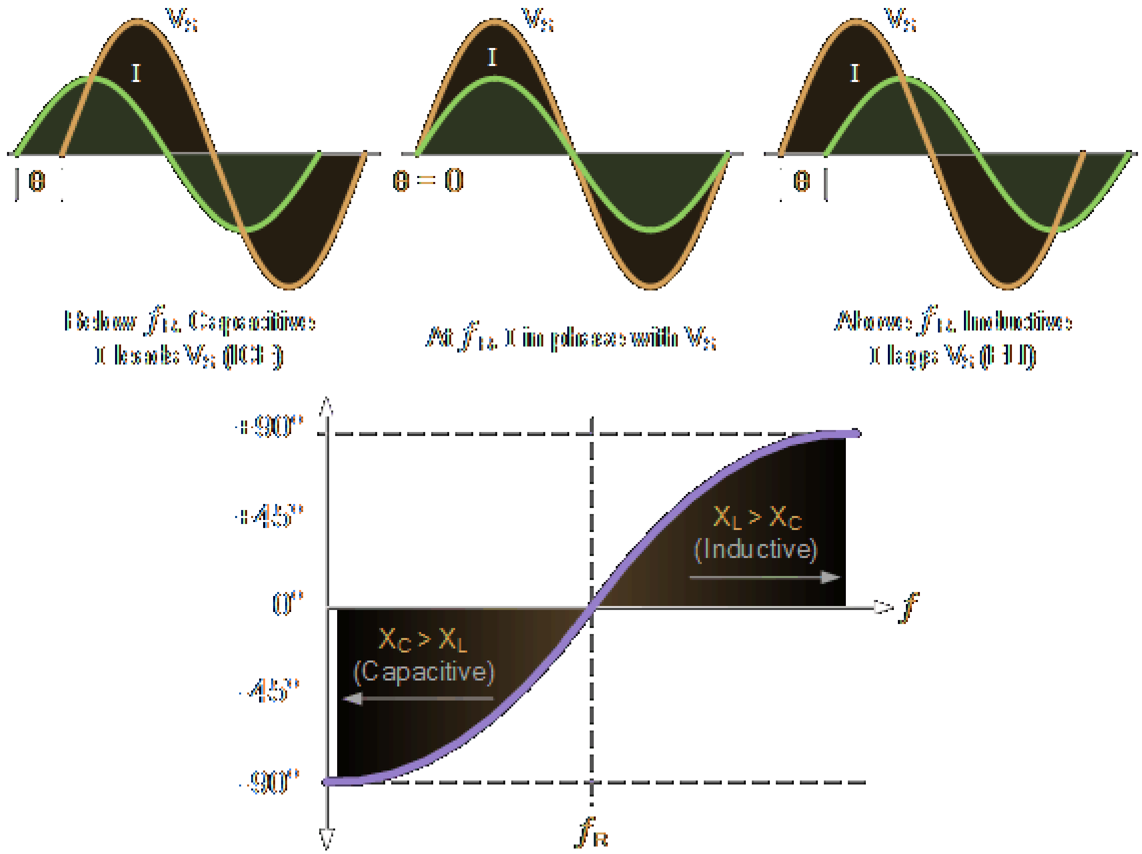
Series RLC Circuit at Resonance



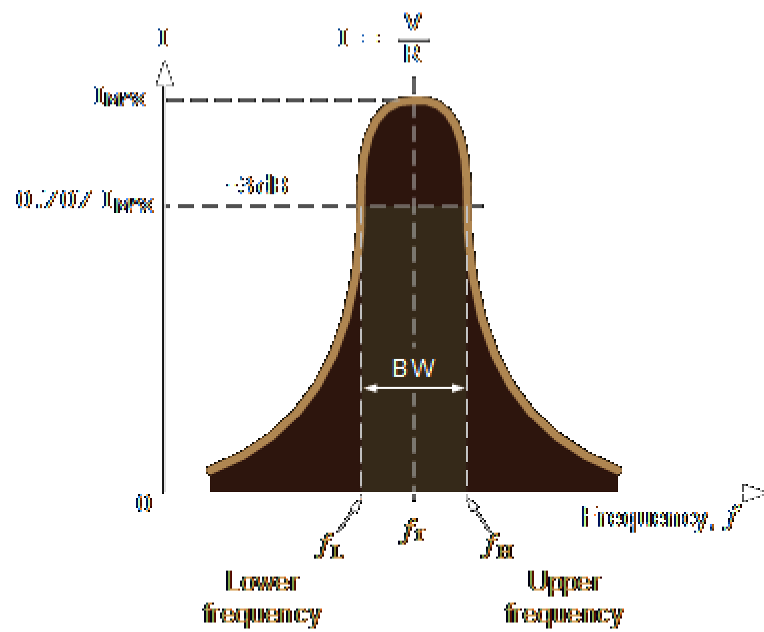
Since the current flowing through a series resonance circuit is the product of voltage divided by impedance, at resonance the impedance, Z is at its minimum value, (=R). Therefore, the circuit current at this frequency will be at its maximum value of V/R



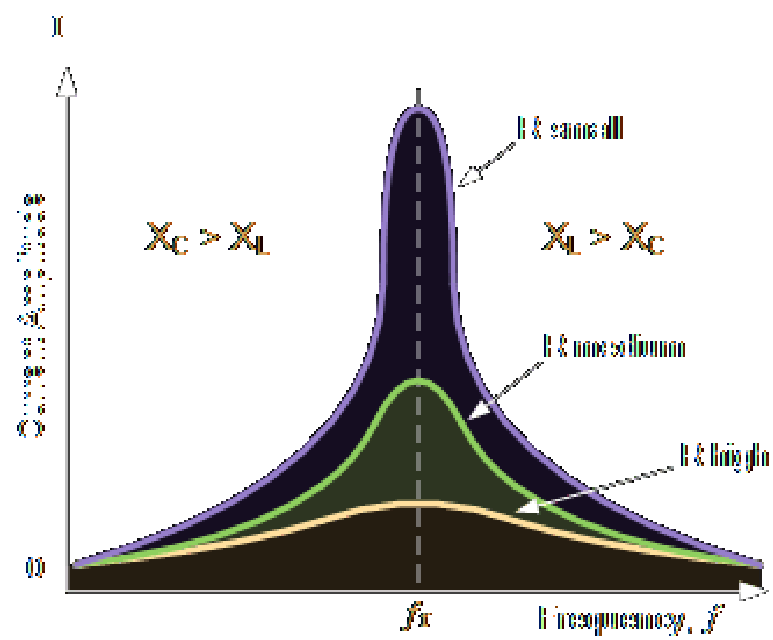
Phase Angle of a Series Resonance Circuit



The frequency response of the circuits current magnitude relates to the “sharpness” of the resonance in a series resonance circuit. The sharpness of the peak is measured quantitatively and is called the **Quality factor, Q** of the circuit. The quality factor relates the maximum or peak energy stored in the circuit (the reactance) to the energy dissipated (the resistance) during each cycle of oscillation meaning that it is a ratio of resonant frequency to bandwidth and the higher the circuit Q, the smaller the bandwidth, Q = f_r /BW.

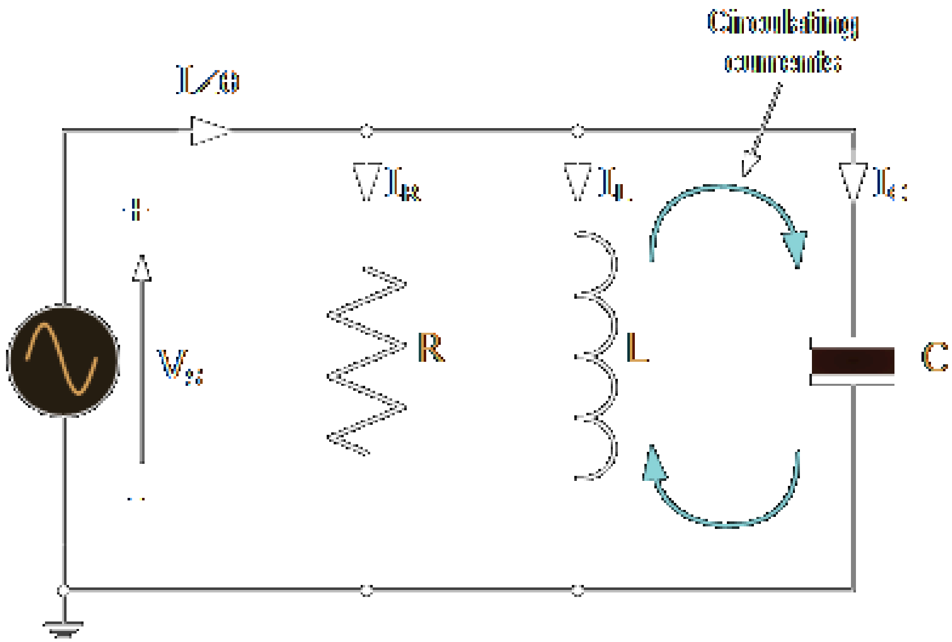


As the bandwidth is taken between the two -3dB points, the **selectivity** of the circuit is a measure of its ability to reject any frequencies either side of these points. The selectivity can be controlled by adjusting the value of the resistance only, keeping all the other components the same, since Q = (X_L or X_C)/R.



Quality Factor, (Q) $$Q = \frac{\omega_r L}{R} = \frac{X_L}{R} = \frac{1}{\omega_r C R} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Parallel Resonance Circuit



A parallel circuit containing a resistance, R, an inductance, L and a capacitance, C will produce a **parallel resonance** (also called anti-resonance) circuit when the resultant current through the parallel combination is in phase with the supply voltage. At resonance there is a large circulating current between the inductor and the capacitor due to the energy of the oscillations, producing current resonance.

The resonance takes place when $V_L = -V_C$ and this situation occurs when the two reactances are equal, $X_L = X_C$. The admittance of a parallel circuit is given as:

$$Y = G + B_L + B_C$$

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

or

$$Y = \frac{1}{R} + \frac{1}{2\pi f L} + 2\pi f C$$

Resonance occurs when $X_L = X_C$ and the imaginary parts of Y become zero. Then:

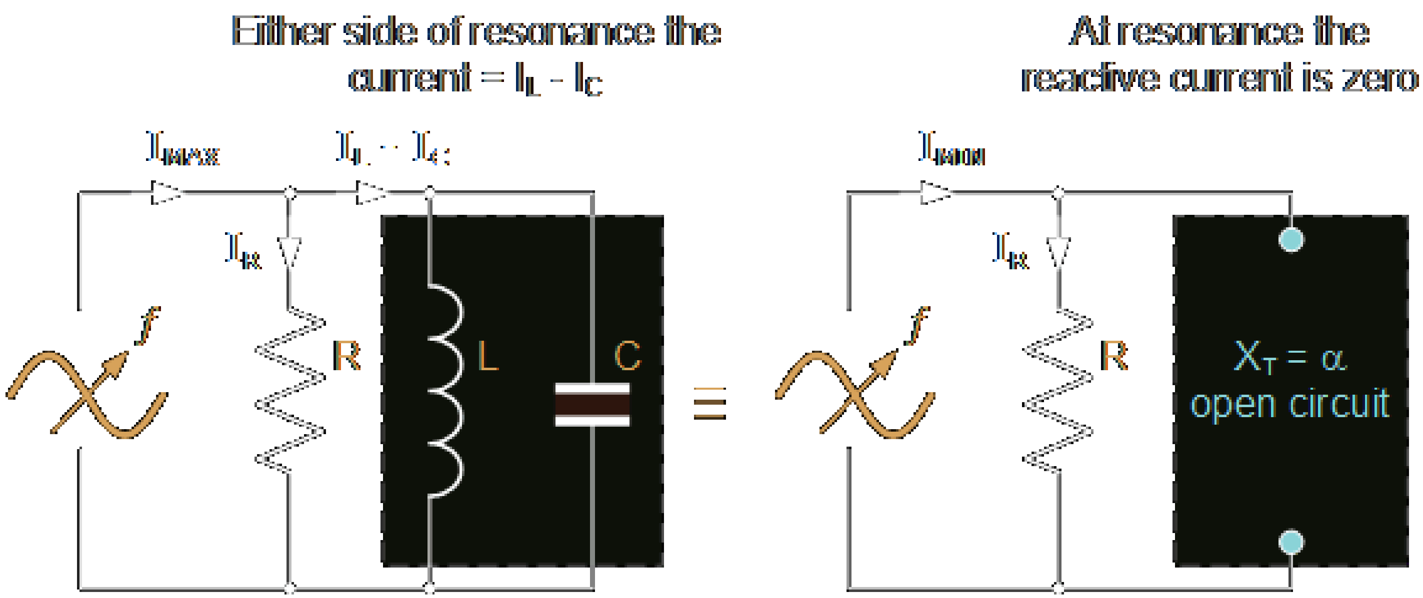
$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

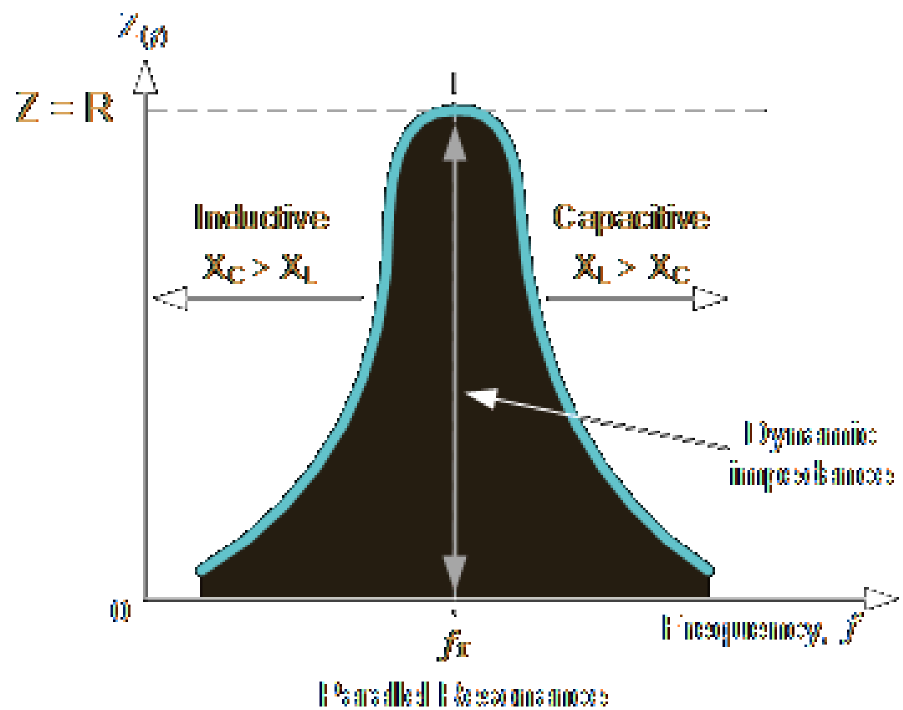
$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}} \,(\mathrm{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \,(\mathrm{rads})$$

At resonance the parallel LC tank circuit acts like an open circuit with the circuit current being determined by the resistor, R only. The total impedance of a parallel resonance circuit at resonance is the value of the resistance in the circuit and Z = R as shown.

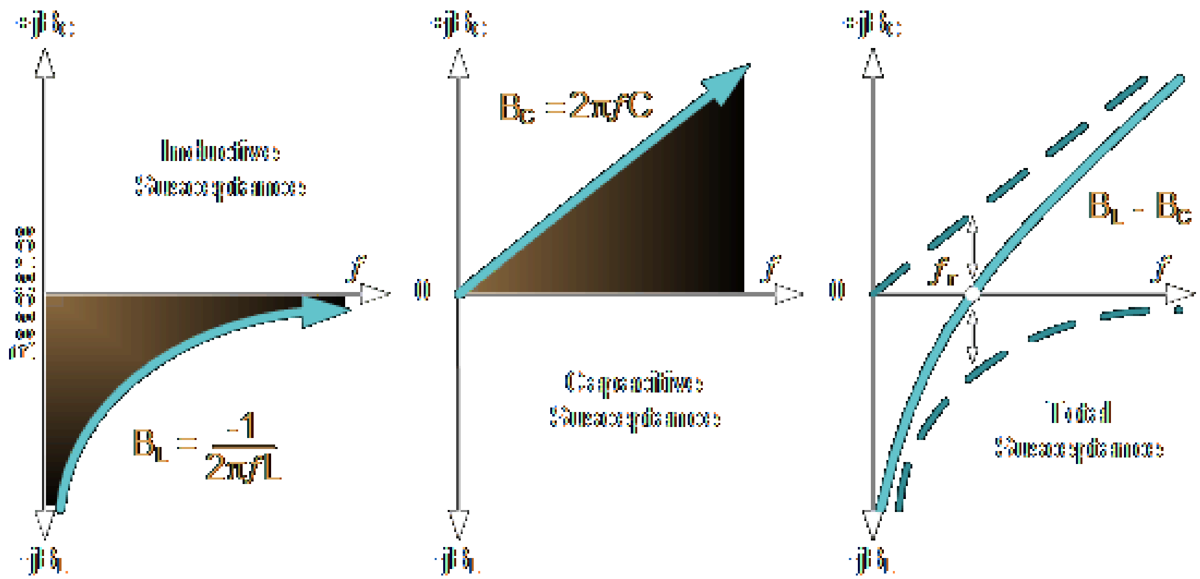


At resonance, the impedance of the parallel circuit is at its maximum value and equal to the resistance of the circuit creating a circuit condition of high resistance and low current.

At resonance, as the impedance of the circuit is that of resistance only, the total circuit current, I is “in-phase” with the supply voltage, V_S.



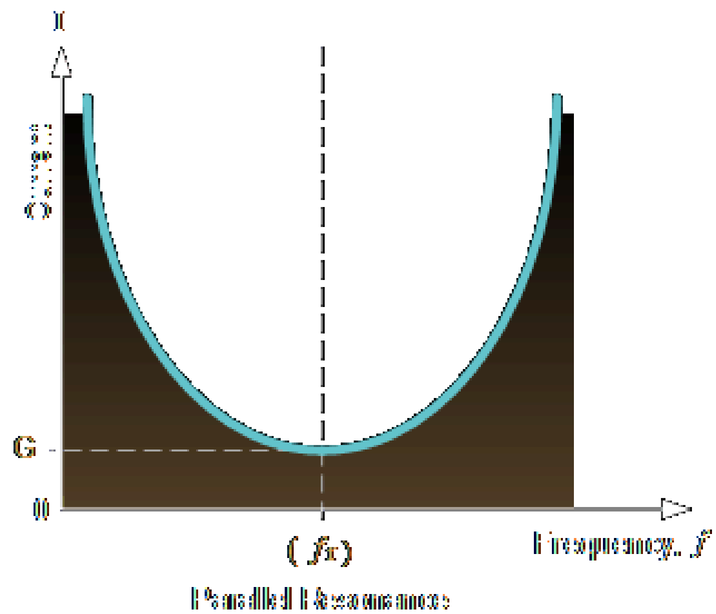
At the resonant frequency, f_r the admittance of the circuit is at its minimum and is equal to the conductance, G given by 1/R because in a parallel resonance circuit the imaginary part of admittance, i.e. the susceptance, B is zero because $B_L = B_C$ as shown.



The maximum dynamic impedance (Z_d) of the circuit as shown.

$$Z_d = \frac{L}{RC}$$

At resonance the impedance, *Z* is at its maximum value, (=R). The circuit current at this frequency will be at its minimum value of V/R and the graph of current against frequency for a parallel resonance circuit is as shown

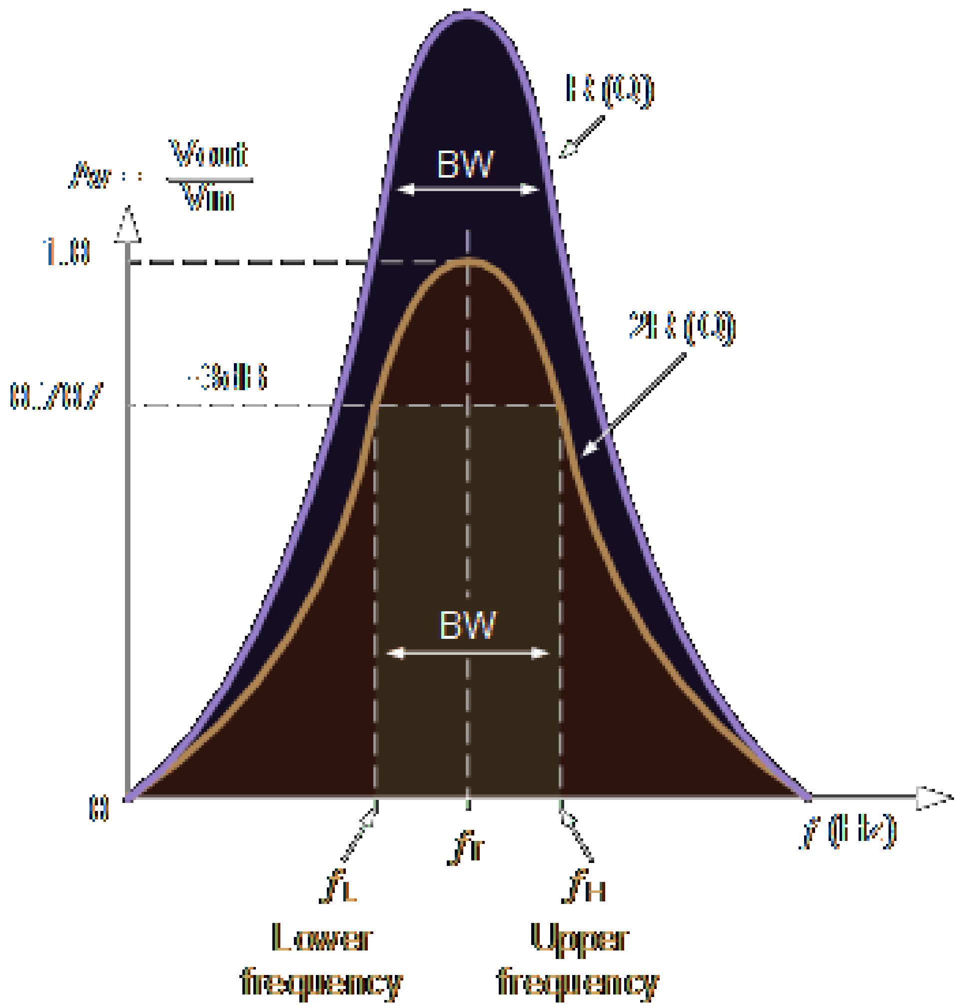


Bandwidth & Selectivity of a Parallel Resonance Circuit

The bandwidth of a parallel resonance circuit is defined as the difference of the upper and lower cut-off frequencies given as: f_{upper} and f_{lower} of the half-power frequencies or where the power dissipated in the circuit is half of the full power dissipated at the resonant frequency i.e 0.5(I^2 R) or -3dB points or 70.7% of its maximum resonant value, $(0.707 \times I)^2 R$

If the resonant frequency remains constant, an increase in the quality factor, **Q** will cause a decrease in the bandwidth and likewise, a decrease in the quality factor will cause an increase in the bandwidth as defined by:

$$BW = f_r / Q \quad \text{or} \quad BW = f_{upper} - f_{lower}$$



Av = Vout/Vin
1.0
0.707
-3dB
0
R (Ω)
2R (Ω)
BW
BW
fr
f (Hz)
fL
Lower frequency
fH
Upper frequency

THE SELECTIVITY OR Q-FACTOR

The selectivity or **Q-factor** for a parallel resonance circuit is generally defined as the ratio of the circulating branch currents to the supply current and is given as

$$\text{Quality Factor, } Q = \frac{R}{2\pi f L} = 2\pi f C R = R\sqrt{\frac{C}{L}}$$