Hilbert space →

Every finite - dimensional vector space is trivially a Hilbert space. It can be also defined as a complete set of inner product space.

Inner product :- $\langle \alpha | \beta \rangle$

$$\langle \alpha | \beta \rangle = a_1^* b_1 + a_2^* b_2 + a_3^* b_3 + \cdots a_n^* b_n$$

Inner product of two functions $f_1(x)$ and $f_2(x)$ is given by,

$$\langle f_1 | f_2 \rangle = \int_a^b f_1^* f_2 \, dx$$

Orthonormal set of functions is defined as,

$$\langle f_m | f_n \rangle = \delta_{mn}$$

Any other function in Hilbert ~~space~~ space can be written as a linear combination of f_n's,

$$g(x) = \sum_{n=1}^{\infty} a_n f_n(x)$$

where $a_n = \langle f_n | g \rangle$

$\Rightarrow$ Wave function live in Hilbert Space.

"In Q.M. any observables like (position, momentum, energy) are always represented in terms of Hermitian operators."

Hermitian operator :-

$$A^{\dagger} = A$$

→ eigenvalue of Hermitian operators are real.

In other words,

$$\langle f_1 | \hat{A} f_2 \rangle = \langle \hat{A} f_1 | f_2 \rangle$$

Orthogonal operator :-

$$O^T O = O^{-1} O = \mathbb{1}$$

Unitary operator :-

$$U^{\dagger} U = \mathbb{1} = U^{-1} U$$

$$\Rightarrow \boxed{U^{\dagger} = U^{-1}}$$

Eigenvectors and Eigenvalues :-

An eigenvalue eqn. is written as,

$$\hat{A}\psi = a\psi$$

where a is the eigenvalue of $\hat{A}$ op. and ψ are the Corres. eigen-vectors.

$$\therefore \left|\hat{A} - a\hat{\mathbb{1}}\right|\psi = 0$$

Hence, $|\hat{A} - a\hat{\mathbb{1}}| = 0 \longrightarrow$ is the characteristics eqn. By solving this we can find the eigenvalues and eigenvectors.

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Basis vectors $\rightarrow$

A set of nonzero N vectors $\varphi_1, \varphi_2, \dots \varphi_N$ is said to be linearly independent if,

$$\sum_{i=1}^{N} a_i \varphi_i = 0$$

$\Rightarrow \quad a_1 = a_2 = \dots = a_N = 0$

Any vector ψ can be expressed as a linear combination,

$$\left\{ \psi = \sum_{i=1}^{N} a_i \varphi_i \right\}$$