7

PERMUTATIONS
<u>AND COMBINATIONS</u>

The other day, I wanted to travel from Bangalore to Allahabad by train. There is no direct train from Bangalore to Allahabad, but there are trains from Bangalore to Itarsi and from Itarsi to Allahabad. From the railway timetable I found that there are two trains from Bangalore to Itarsi and three trains from Itarsi to Allahabad. Now, in how many ways can I travel from Bangalore to Allahabad?

There are **counting problems** which come under the branch of Mathematics called **combinatorics.**

Suppose you have five jars of spices that you want to arrange on a shelf in your kitchen. You would like to arrange the jars, say three of them, that you will be using often in a more accessible position and the remaining two jars in a less accessible position. In how many ways can you do it?

In another situation suppose you are painting your house. If a particular shade or colour is not available, you may be able to create it by mixing different colours and shades. While creating new colours this way, the order of mixing is not important. It is the combination or choice of colours that determine the new colours; but not the order of mixing.

To give another similar example, when you go for a journey, you may not take all your dresses with you. You may have 4 sets of shirts and trousers, but you may take only 2 sets. In such a case you are choosing 2 out of 4 sets and the order of choosing the sets doesn't matter. In these examples, we need to find out the number of choices in which it can be done.

In this lesson we shall consider simple counting methods and use them in solving such simple counting problems.

MODULE - I
Algebra

OBJECTIVES

After studying this lesson, you will be able to :

- find out the number of ways in which a given number of objects can be arranged;

- state the Fundamental Principle of Counting;

- define n! and evaluate it for defferent values of n;

- state that permutation is an arrangement and write the meaning of $^{n}P_{r}$;

- state that $^{n}P_{r} = \dfrac{n!}{(n-r)!}$ and apply this to solve problems;

- show that (i) $(n+1)$ $^{n}P_{n} = {}^{n+1}P_{n}$ (ii) $^{n}P_{r+1} = (n-r)$ $^{n}P_{r}$;

- state that a combination is a selection and write the meaning of $^{n}C_{r}$;

- distinguish between permutations and combinations;

- derive $^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$ and apply the result to solve problems;

- derive the relation $^{n}P_{r} = r!\,^{n}C_{r}$;

- verify that $^{n}C_{r} = {}^{n}C_{n-r}$ and give its interpretation; and

- derive $^{n}C_{r} + {}^{n}C_{n-r} = {}^{n+1}C_{r}$ and apply the result to solve problems.

EXPECTED BACKGROUND KNOWLEDGE

- Number Systems

- Four Fundamental Operations

7.1 COUNTING PRINCIPLE

Let us now solve the problem mentioned in the introduction. We will write t_1, t_2 to denote trains from Bangalore to Itarsi and T_1, T_2, T_3, for the trains from Itarsi to Allahabad. Suppose I take t_1 to travel from Bangalore to Itarsi. Then from Itarsi I can take T_1 or T_2 or T_3. So the possibilities are t_1T_1, t_2T_2 and t_3T_3 where t_1T_1 denotes travel from Bangalore to Itarsi by t_1 and travel from Itarsi to Allahabad by T_1. Similarly, if I take t_2 to travel from Bangalore to Itarsi, then the possibilities are t_2T_1, t_2T_2 and t_2T_3. Thus, in all there are $6 (2 \times 3)$ possible ways of travelling from Bangalore to Allahabad.

Here we had a small number of trains and thus could list all possibilities. Had there been 10 trains from Bangalore to Itarsi and 15 trains from Itarsi to Allahabad, the task would have been

very tedious. Here the **Fundamental Principle of Counting** or simply the **Counting Principle** comes in use :

If any event can occur in *m* ways and after it happens in any one of these ways, a second event can occur in *n* ways, then both the events together can occur in $m \times n$ ways.

Example 7.1 How many multiples of 5 are there from 10 to 95 ?

Solution : As you know, multiples of 5 are integers having 0 or 5 in the digit to the extreme right (i.e. the unit's place).

The first digit from the right can be chosen in 2 ways.

The second digit can be any one of 1,2,3,4,5,6,7,8,9.

i.e. There are 9 choices for the second digit.

Thus, there are $2 \times 9 = 18$ multiples of 5 from 10 to 95.

Example 7.2 In a city, the bus route numbers consist of a natural number less than 100, followed by one of the letters *A,B,C,D,E* and *F*. How many different bus routes are possible?

Solution : The number can be any one of the natural numbers from 1 to 99.
There are 99 choices for the number.

The letter can be chosen in 6 ways.

∴ Number of possible bus routes are $99 \times 6 = 594$.

CHECK YOUR PROGRESS 7.1

1.   (a) How many 3 digit numbers are multiples of 5?

   (b) A coin is tossed thrice. How many possible outcomes are there?

   (c) If you have 3 shirts and 4 pairs of trousers and any shirt can be worn with any pair of trousers, in how many ways can you wear your shirts and pairs of trousers?

   (d) A tourist wants to go to another country by ship and return by air. She has a choice of 5 different ships to go by and 4 airlines to return by. In how many ways can she perform the journey?

2.   (a) In how many ways can two vacancies be filled from among 4 men and 12 women if one vacancy is filled by a man and the other by a woman?

   (b) Flooring and painting of the walls of a room needs to be done. The flooring can be done in 3 colours and painting of walls can be done in 12 colours. If any colour combination is allowed, find the number of ways of flooring and painting the walls of the room.

   So far, we have applied the counting principle for two events. But it can be extended to three or more, as you can see from the following examples :

Example 7.3 There are 3 questions in a question paper. If the questions have 4,3 and 2 solutionsvely, find the total number of solutions.

Solution : Here question 1 has 4 solutions,

question 2 has 3 solutions

and question 3 has 2 solutions.

∴   By the multiplication (counting) rule,

total number of solutions $= 4 \times 3 \times 2$

$= 24$

Example 7.4 Consider the word ROTOR. Whichever way you read it, from left to right or from right to left, you get the same word. Such a word is known as *palindrome*. Find the maximum possible number of 5-letter palindromes.

Solution : The first letter from the right can be chosen in 26 ways because there are 26 alphabets.

Having chosen this, the second letter can be chosen in 26 ways

∴ The first two letters can chosen in $26 \times 26 = 676$ ways

Having chosen the first two letters, the third letter can be chosen in 26 ways.

∴ All the three letters can be chosen in $676 \times 26 = 17576$ ways.

It implies that the maximum possible number of five letter palindromes is 17576 because the fourth letter is the same as the second letter and the fifth letter is the same as the first letter.

Note : In Example 7.4 we found the maximum possible number of five letter palindromes. There cannot be more than 17576. But this does not mean that there are 17576 palindromes. Because some of the choices like CCCCC may not be meaningful words in the English language.

Example 7.5 How many 3-digit numbers can be formed with the digits 1,4,7,8 and 9 if the digits are not repeated.

Solution : Three digit number will have unit's, ten's and hundred's place.

Out of 5 given digits any one can take the unit's place.

This can be done in 5 ways.   ... (*i*)

After filling the unit's place, any of the four remaining digits can take the ten's place.

This can be done in 4 ways.   ... (*ii*)

After filling in ten's place, hundred's place can be filled from any of the three remaining digits.

This can be done in 3 ways. ... (iii)

∴ By counting principle, the number of 3 digit numbers = $5 \times 4 \times 3 = 60$

Let us now state the **General Counting Principle**

If there are *n* events and if the first event can occur in m_1 ways, the second event can occur in m_2 ways after the first event has occured, the third event can occur in m_3 ways after the second event has ocurred, and so on, then all the *n* events can occur in $m_1 \times m_2 \times \ldots \times m_{n-1} \times m_n$ ways.

Example 7.6 Suppose you can travel from a place *A* to a place *B* by 3 buses, from place *B* to place *C* by 4 buses, from place *C* to place *D* by 2 buses and from place *D* to place *E* by 3 buses. In how many ways can you travel from *A* to *E*?

Solution : The bus from *A* to *B* can be selected in 3 ways.

The bus from *B* to *C* can be selected in 4 ways.

The bus from *C* to *D* can be selected in 2 ways.

The bus from *D* to *E* can be selected in 3 ways.

So, by the General Counting Principle, one can travel from *A* to *E* in $3 \times 4 \times 2 \times 3$ ways = 72 ways.

CHECK YOUR PROGRESS 7.2

1. (a) What is the maximum number of 6-letter palindromes?

    (b) What is the number of 6-digit palindromic numbers which do not have 0 in the first digit?

2. (a) In a school there are 5 English teachers, 7 Hindi teachers and 3 French teachers. A three member committee is to be formed with one teacher representing each language. In how many ways can this be done?

    (b) In a college students union election, 4 students are contesting for the post of President. 5 students are contesting for the post of Vice-president and 3 students are contesting for the post of Secretary. Find the number of possible results.

3. (a) How many three digit numbers greater than 600 can be formed using the digits 1,2,5,6,8 without repeating the digits?

    (b) A person wants to make a time table for 4 periods. He has to fix one period each for English, Mathematics, Economics and Commerce. How many different time tables can he make?

7.2 PERMUTATIONS

Suppose you want to arrange your books on a shelf. If you have only one book, there is only

Notes

one way of arranging it. Suppose you have two books, one of History and one of Geography.

You can arrange the Geography and History books in two ways. Geography book first and the History book next, *GH* or History book first and Geography book next; *HG*. In other words, there are two arrangements of the two books.

Now, suppose you want to add a Mathematics book also to the shelf. After arranging History and Geography books in one of the two ways, say *GH*, you can put Mathematics book in one of the following ways: *MGH*, *GMH* or *GHM*. Similarly, corresponding to *HG*, you have three other ways of arranging the books. So, by the Counting Principle, you can arrange Mathematics, Geography and History books in 3×2 ways $= 6$ ways.

By permutation we mean an arrangement of objects in a particular order. In the above example, we were discussing the number of permutations of one book or two books.

In general, if you want to find the number of permutations of *n* objects $n \geq 1$, how can you do it? Let us see if we can find an answer to this.

Similar to what we saw in the case of books, there is one permutation of 1 object, 2×1 permutations of two objects and $3 \times 2 \times 1$ permutations of 3 objects. It may be that, there are $n \times (n-1) \times (n-2) \times ... \times 2 \times 1$ permutations of *n* objects. In fact, it is so, as you will see when we prove the following result.

Theorem 7.1 The total number of permutations of *n* objects is *n* (*n* – 1) ....2.1.

Proof : We have to find the number of possible arrangements of *n* different objects.

The first place in an arrangement can be filled in *n* different ways. Once it has been done, the second place can be filled by any of the remaining (*n*–1) objects and so this can be done in (*n*–1) ways. Similarly, once the first two places have been filled, the third can be filled in (*n*–2) ways and so on. The last place in the arrangement can be filled only in one way, because in this case we are left with only one object.

Using the counting principle, the total number of arrangements of *n* different objects is

$n(n-1)(n-2) .... 2.1.$ **.....(7.1)**

The product *n* (*n* – 1) ... 2.1 occurs so often in Mathematics that it deserves a name and notation. It is usually denoted by ***n*!** (or by $\lfloor n$ read as ***n* factorial**).

     ***n*! = *n* (*n* – 1) ... 3.2.1**

Here is an example to help you familiarise yourself with this notation.

Example 7.7 Evaluate (a) 3!   (b) 2! + 4!   (c) 2!×3!

Solution : (a) $3! = 3 \times 2 \times 1 = 6$

         (b) $2! = 2 \times 1 = 2$

              $4! = 4 \times 3 \times 2 \times 1 = 24$

Therefore, $2! + 4! = 2 + 24 = 26$

(c) $2! \times 3! = 2 \times 6 = 12$

Notice that $n!$ satisfies the relation

$$n! = n \times (n-1)! \qquad \textbf{... (7.2)}$$

This is because, $n\,(n-1)! = n\,[(n-1).(n-2)...\,2.1]$

$$= n.(n-1).(n-2)...2.1$$

$$= n!$$

Of course, the above relation is valid only for $n \geq 2$ because 0! has not been defined so far. Let us see if we can define 0! to be consistent with the relation. In fact, if we define

$$0! = 1 \qquad \textbf{... (7.3)}$$

then the relation 7.2 holds for $n = 1$ also.

Example 7.8 Suppose you want to arrange your English, Hindi, Mathematics, History, Geography and Science books on a shelf. In how many ways can you do it?

Solution : We have to arrange 6 books.

The number of permutations of n objects is $n! = n.(n-1).(n-2)...2.1$

Here $n = 6$ and therefore, number of permutations is $6.5.4.3.2.1 = 720$

CHECK YOUR PROGRESS 7.3

1.   (a) Evaluate : (i) 6!   (ii) 7!   (iii) 7! + 3!   (iv) 6! $\times$ 4!   (v) $\dfrac{5!}{3!.2!}$

(b)   Which of the following statements are true?

(i) $2! \times 3! = 6!$ (ii) $2! + 4! = 6!$

(iii) 3! divides 4!   (iv) $4! - 2! = 2!$

2.   (a) 5 students are staying in a dormitory. In how many ways can you allot 5 beds to them?

(b) In how many ways can the letters of the word 'TRIANGLE' be arranged?

(c) How many four digit numbers can be formed with digits 1, 2, 3 and 4 and with distinct digits?

7.3 PERMUTATION OF r OBJECTS OUT OF n OBJECTS

Suppose you have five story books and you want to distribute one each to Asha, Akhtar and Jasvinder. In how many ways can you do it? You can give any one of the five books to Asha

and after that you can give any one of the remaining four books to Akhtar. After that, you can give one of the remaining three books to Jasvinder. So, by the Counting Principle, you can distribute the books in $5 \times 4 \times 3$ *ie.*60 ways.

More generally, suppose you have to arrange r objects out of n objects. In how many ways can you do it? Let us view this in the following way. Suppose you have n objects and you have to arrange r of these in r boxes, one object in each box.

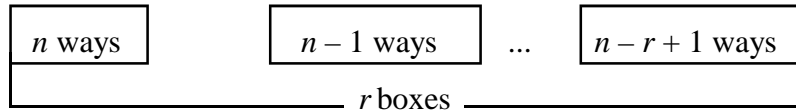| n ways		$n-1$ ways	...	$n-r+1$ ways

r boxes

Fig. 7.1

Suppose there is one box. $r = 1$. You can put any of the n objects in it and this can be done in n ways. Suppose there are two boxes. $r = 2$. You can put any of the objects in the first box and after that the second box can be filled with any of the remaining $n - 1$ objects. So, by the counting principle, the two boxes can be filled in $n(n-1)$ ways. Similarly, 3 boxes can be filled in $n(n-1)(n-2)$ ways.

In general, we have the following theorem.

Theorem 7.2 The number of permutations of r objects out of n objects is

$$n(n-1)\cdots(n-r+1).$$

The number of permutations of r objects out of n objects is usually denoted by ${}^nP_r$.

Thus,

$${}^nP_r = n(n-1)(n-2)..(n-r+1) \quad \textbf{... (7.4)}$$

Proof : Suppose we have to arrange r objects out of n different objects. In fact it is equivalent to filling r places, each with one of the objects out of the given n objects.

The first place can be filled in n different ways. Once this has been done, the second place can be filled by any one of the remaining $(n-1)$ objects, in $(n-1)$ ways. Similarly, the third place can be filled in $(n-2)$ ways and so on. The last place, the rth place can be filled in $[n-(r-1)]$ i.e. $(n-r+1)$ different ways. You may easily see, as to why this is so.

Using the Counting Principle, we get the required number of arrangements of r out of n objects is $n(n-1)(n-2)............(n-r+1)$

Example 7.9 Evaluate : (a) ${}^4P_2$ (b) ${}^6P_3$ (c) $\dfrac{{}^4P_3}{{}^3P_2}$ (d) ${}^6P_3 \times {}^5P_2$

Solution : (a) ${}^4P_2 = 4(4-1) = 4\times 3 = 12.$

(b) ${}^6P_3 = 6(6-1)(6-2) = 6\times 5\times 4 = 120.$

(c) $\quad \dfrac{^4P_3}{^3P_2} = \dfrac{4(4-1)(4-2)}{3(3-1)} = \dfrac{4\times3\times2}{3\times2} = 4$

(d) $\quad ^6P_3 \times {}^5P_2 = 6\,(6-1)\,(6-2)\times5\,(5-1)$

$\qquad\qquad = 6\times5\times4\times5\times4 = 2400$

Example 7.10 If you have 6 New Year greeting cards and you want to send them to 4 of your friends, in how many ways can this be done?

Solution : We have to find number of permutations of 4 objects out of 6 objects.

This number is $^6P_4 = 6(6-1)(6-2)(6-3) = 6.5.4.3 = 360$

Therefore, cards can be sent in 360 ways.

Consider the formula for ^nP_r, namely, $^nP_r = n\,(n-1) \ldots (n-r+1)$. This can be obtained by removing the terms $n-r, n-r-1,\ldots,2, 1$ from the product for n!. The product of these terms is $(n-r)(n-r-1)\ldots2.1$, i.e., $(n-r)!$.

Now, $\quad \dfrac{n!}{(n-r)!} = \dfrac{n\,(n-1)\,(n-2)\ldots(n-r+1)\,(n-r)\ldots2.1}{(n-r)\,(n-r-1)\ldots2.1}$

$\qquad\qquad = n\,(n-1)\,(n-2)\ldots(n-r+1)$

$\qquad\qquad = {}^nP_r$

So, using the factorial notation, this formula can be written as follows :

$$^nP_r = \dfrac{n!}{(n-r)!} \qquad\qquad\qquad\text{... (7.5)}$$

Example 7.11 Find the value of $^nP_0$.

Solution : Here $r = 0$. Using relation 7.5, we get

$$^nP_0 = \dfrac{n!}{n!} = 1$$

Example 7.12 Show that $(n + 1)\,^nP_r = {}^{n+1}P_{r+1}$

Solution : $(n + 1)\,^nP_r = (n+1)\dfrac{n!}{(n-r)!} = \dfrac{(n+1)\,n!}{(n-r)!}$

$\qquad\qquad = \dfrac{(n+1)!}{[(n+1)-(r+1)]!}$ [writing $n - r$ as $[(n+1)-(r+1)]$]

$\qquad\qquad = {}^{n+1}P_{r+1}$ $\qquad\qquad\qquad\qquad$ (By definition)

Notes

CHECK YOUR PROGRESS 7.4

1. (a) Evaluate : (i) $^4P_2$ (ii) $^6P_3$ (iii) $\dfrac{^4P_3}{^3P_2}$ (iv) $^6P_3 \times ^5P_2$ (v) ^nP_n

(b) Verify each of the following statements :

(i) $6 \times ^5P_2 = ^6P_2$ (ii) $4 \times ^7P_3 = ^7P_4$

(iii) $^3P_2 \times ^4P_2 = ^{12}P_4$ (iii) $^3P_2 + ^4P_2 = ^7P_4$

2. (a) (i) What is the maximum possible number of 3- letter words in English that do not contain any vowel?

(ii) What is the maximum possible number of 3- letter words in English which do not have any vowel other than 'a'?

(b) Suppose you have 2 cots and 5 bedspreads in your house. In how many ways can you put the bedspreads on your cots?

(c) You want to send Diwali Greetings to 4 friends and you have 7 greeting cards with you. In how many ways can you do it?

3. Show that $^nP_{n-1} = ^nP_n$.

4. Show that $(n-r) \, ^nP_r = ^nP_{r+1}$.

7.4 PERMUTATIONS UNDER SOME CONDITIONS

We will now see examples involving permutations with some extra conditions.

Example 7.13 Suppose 7 students are staying in a hall in a hostel and they are allotted 7 beds. Among them, Parvin does not want a bed next to Anju because Anju snores. Then, in how many ways can you allot the beds?

Solution : Let the beds be numbered 1 to 7.

Case 1 : Suppose Anju is allotted bed number 1.

Then, Parvin cannot be allotted bed number 2.

So Parvin can be allotted a bed in 5 ways.

After alloting a bed to Parvin, the remaining 5 students can be allotted beds in 5! ways.
So, in this case the beds can be allotted in $5 \times 5! \, ways = 600$ ways.
Case 2 : Anju is allotted bed number 7.
Then, Parvin cannot be allotted bed number 6
As in Case 1, the beds can be allotted in 600 ways.

Case 3 : Anju is allotted one of the beds numbered 2,3,4,5 or 6.

Parvin cannot be allotted the beds on the right hand side and left hand side of Anju's bed. For example, if Anju is allotted bed number 2, beds numbered 1 or 3 cannot be allotted to Parvin.

Therefore, Parvin can be allotted a bed in 4 ways in all these cases.

After allotting a bed to Parvin, the other 5 can be allotted a bed in 5! ways.

Therefore, in each of these cases, the beds can be allotted in $4 \times 5! = 480$ ways.

∴   The beds can be allotted in

$(2 \times 600 + 5 \times 480) \, ways = (1200 + 2400) \, ways = 3600$ ways.

Example 7.14 In how many ways can an animal trainer arrange 5 lions and 4 tigers in a row so that no two lions are together?

Solution : They have to be arranged in the following way :

L	T	L	T	L	T	L	T	L

The 5 lions should be arranged in the 5 places marked 'L'.

This can be done in 5! ways.

The 4 tigers should be in the 4 places marked 'T'.

This can be done in 4! ways.

Therefore, the lions and the tigers can be arranged in $5! \times 4! \, ways = 2880$ ways.

Example 7.15 There are 4 books on fairy tales, 5 novels and 3 plays. In how many ways can you arrange these so that books on fairy tales are together, novels are together and plays are together and in the order, books on fairytales, novels and plays.

Solution : There are 4 books on fairy tales and they have to be put together.

They can be arranged in 4! ways.
Similarly, there are 5 novels.
They can be arranged in 5! ways.
And there are 3 plays.

They can be arranged in 3! ways.

So, by the counting principle all of them together can be arranged in $4! \times 5! \times 3!$ ways $= 17280$ ways.

Example 7.16 Suppose there are 4 books on fairy tales, 5 novels and 3 plays as in Example 7.15. They have to be arranged so that the books on fairy tales are together, novels are together and plays are together, but we no longer require that they should be in a specific order. In how many ways can this be done?

Solution : First, we consider the books on fairy tales, novels and plays as single objects.

These three objects can be arranged in 3!ways = 6 ways.

Let us fix one of these 6 arrangements.

This may give us a specific order, say, novels $\rightarrow$ fairy tales $\rightarrow$ plays.

Given this order, the books on the same subject can be arranged as follows.

The 4 books on fairy tales can be arranged among themselves in 4! = 24 ways.

The 5 novels can be arranged in 5! = 120 ways.

The 3 plays can be arranged in 3! = 6 ways.

For a given order, the books can be arranged in $24\times120\times6 = 17280$ ways.

Therefore, for all the 6 possible orders the books can be arranged in $6\times17280 = 103680$ ways.

Example 7.17 In how many ways can 4 girls and 5 boys be arranged in a row so that all the four girls are together?

Solution : Let 4 girls be one unit and now there are 6 units in all.

They can be arranged in 6! ways.

In each of these arrangements 4 girls can be arranged in 4! ways.

$\therefore$ Total number of arrangements in which girls are always together

$= 6! \times 4!$

$= 720 \times 24$

$= 17280$

Example 7.18 How many arrangements of the letters of the word 'BENGALI' can be made

(i) if the vowels are never together.

(ii) if the vowels are to occupy only odd places.

Solution : There are 7 letters in the word 'Bengali; of these 3 are vowels and 4 consonants.

(i) Considering vowels a, e, i as one letter, we can arrange 4+1 letters in 5! ways in each of which vowels are together. These 3 vowels can be arranged among themselves in 3! ways.

$\therefore$ Total number of words   = $5! \times 3!$

= $120 \times 6 = 720$

(ii) There are 4 odd places and 3 even places. 3 vowels can occupy 4 odd places in $^4P_3$ ways and 4 constants can be arranged in $^4P_4$ ways.

$\therefore$ Number of words $= {}^4P_3 \times {}^4P_4 = 24 \times 24$

$= 576.$

CHECK YOUR PROGRESS 7.5

1.   Mr. Gupta with Ms. Gupta and their four children is travelling by train. Two lower berths, two middle berths and 2 upper berths have been allotted to them. Mr. Gupta has undergone a knee surgery and needs a lower berth while Ms. Gupta wants to rest during the journey and needs an upper berth. In how many ways can the berths be shared by the family?

2.   Consider the word UNBIASED. How many words can be formed with the letters of the word in which no two vowels are together?

3.   There are 4 books on Mathematics, 5 books on English and 6 books on Science. In how many ways can you arrange them so that books on the same subject are together and they are arranged in the order Mathematics $\rightarrow$ English $\rightarrow$ Science.

4.   There are 3 Physics books, 4 Chemistry books, 5 Botany books and 3 Zoology books. In how many ways can you arrange them so that the books on the same subject are together?

5.   4 boys and 3 girls are to be seated in 7 chairs such that no two boys are together. In how many ways can this be done?

6.   Find the number of permutations of the letters of the word 'TENDULKAR', in each of the following cases :

(i) beginning with T and ending with R.

(ii) vowels are always together.

(iii) vowels are never together.

7.5 COMBINATIONS

Let us consider the example of shirts and trousers as stated in the introduction. There you have 4 sets of shirts and trousers and you want to take 2 sets with you while going on a trip. In how many ways can you do it?

Let us denote the sets by S_1, S_2, S_3, S_4. Then you can choose two pairs in the following ways :

1. $\{S_1, S_2\}$ 2. $\{S_1, S_3\}$ 3. $\{S_1, S_4\}$

4. $\{S_2, S_3\}$ 5. $\{S_2, S_4\}$ 6. $\{S_3, S_4\}$

[Observe that $\{S_1, S_2\}$ is the same as $\{S_2, S_1\}$]. So, there are 6 ways of choosing the two sets that you want to take with you. Of course, if you had 10 pairs and you wanted to take 7 pairs, it will be much more difficult to work out the number of pairs in this way.

Now as you may want to know the number of ways of wearing 2 out of 4 sets for two days, say Monday and Tuesday, and the order of wearing is also important to you. We know from section 7.3, that it can be done in $^4P_2 = 12$ ways. But note that each choice of 2 sets gives us two ways of wearing 2 sets out of 4 sets as shown below :

1. $\{S_1, S_2\} \to S_1$ on Monday and S_2 on Tuesday or S_2 on Monday and S_1 on Tuesday

2. $\{S_1, S_3\} \to S_1$ on Monday and S_3 on Tuesday or S_3 on Monday and S_1 on Tuesday

3. $\{S_1, S_4\} \to S_1$ on Monday and S_4 on Tuesday or S_4 on Monday and S_1 on Tuesday

4. $\{S_2, S_3\} \to S_2$ on Monday and S_3 on Tuesday or S_3 on Monday and S_2 on Tuesday

5. $\{S_2, S_4\} \to S_2$ on Monday and S_4 on Tuesday or S_4 on Monday and S_2 on Tuesday

6. $\{S_3, S_4\} \to S_3$ on Monday and S_4 on Tuesday or S_4 on Monday and S_3 on Tuesday

Thus, there are 12 ways of wearing 2 out of 4 pairs.

This argument holds good in general as we can see from the following theorem.

Theorem 7.3 Let $n \geq 1$ be an integer and $r \leq n$. Let us denote the number of ways of choosing r objects out of n objects by ^nC_r. Then

$$^nC_r = \frac{^nP_r}{r!} \qquad\qquad \textbf{... (7.6)}$$

Proof : We can choose r objects out of n objects in ^nC_r ways. Each of the r objects chosen can be arranged in $r!$ ways. The number of ways of arranging r objects is $r!$. Thus, by the counting principle, the number of ways of choosing r objects and arranging the r objects chosen can be done in $^nC_r r!$ ways. But, this is precisely ^nP_r. In other words, we have

$$^nP_r = r!\,.^nC_r \qquad\qquad \textbf{... (7.7)}$$

Dividing both sides by $r!$, we get the result in the theorem.

Here is an example to help you to familiarise yourself with ^nC_r.

Example 7.19 Evaluate each of the following :

(a) $^5C_2$ (b) $^5C_3$

(c) $^4C_3 + {^4C_2}$ (d) $\dfrac{^6C_3}{^4C_2}$

Solution : (a) $^5C_2 = \dfrac{^5P_2}{2!} = \dfrac{5.4}{1.2} = 10$.   (b) $^5C_3 = \dfrac{^5P_3}{3!} = \dfrac{5.4.3}{1.2.3} = 10$.

(c) $\quad {}^4C_3 + {}^4C_2 = \dfrac{{}^4P_3}{3!} + \dfrac{{}^4P_2}{2!} = \dfrac{4.3.2}{1.2.3} + \dfrac{4.3}{1.2} = 4 + 6 = 10$

(d) $\quad {}^6C_3 = \dfrac{{}^6P_3}{3!} = \dfrac{6.5.4}{1.2.3} = 20$ and ${}^4C_2 = \dfrac{4.3}{1.2} = 6$

$\therefore \quad \dfrac{{}^6C_3}{{}^4C_2} = \dfrac{20}{6} = \dfrac{10}{3}.$

Example 7.20 Find the number of subsets of the set {1,2,3,4,5,6,7,8,9,10,11} having 4 elements.

Solution : Here the order of choosing the elements doesn't matter and this is a problem in combinations.

We have to find the number of ways of choosing 4 elements of this set which has 11 elements.

By relation (7.6), this can be done in

$$ {}^{11}C_4 = \dfrac{11.10.9.8}{1.2.3.4} = 330 \text{ ways.} $$

Example 7.21 12 points lie on a circle. How many cyclic quadrilaterals can be drawn by using these points?

Solution : For any set of 4 points we get a cyclic quadrilateral. Number of ways of choosing 4 points out of 12 points is ${}^{12}C_4 = 495$. Therefore, we can draw 495 quadrilaterals.

Example 7.22 In a box, there are 5 black pens, 3 white pens and 4 red pens. In how many ways can 2 black pens, 2 white pens and 2 red pens can be chosen?

Solution : Number of ways of choosing 2 black pens from 5 black pens

$$ = {}^5C_2 = \dfrac{{}^5P_2}{2!} = \dfrac{5.4}{1.2} = 10. $$

Number of ways of choosing 2 white pens from 3 white pens

$$ = {}^3C_2 = \dfrac{{}^3P_2}{2!} = \dfrac{3.2}{1.2} = 3. $$

Number of ways of choosing 2 red pens from 4 red pens

$$ = {}^4C_2 = \dfrac{{}^4P_2}{2!} = \dfrac{4.3}{1.2} = 6. $$

$\therefore$ By the Counting Principle, 2 black pens, 2 white pens, and 2 red pens can be chosen in $10\times3\times6=180$ ways.

Example 7.23 A question paper consists of 10 questions divided into two parts A and B. Each part contains five questions. A candidate is required to attempt six questions in all of which at least 2 should be from part A and at least 2 from part B. In how many ways can the candidate select the questions if he can answer all questions equally well?

Solution : The candidate has to select six questions in all of which at least two should be from Part A and two should be from Part B. He can select questions in any of the following ways :

	Part A	Part B
(i)	2	4
(ii)	3	3
(iii)	4	2

If the candidate follows choice (i), the number of ways in which he can do so is

$$^5C_2\times{}^5C_4 = 10\times5 = 50$$

If the candidate follows choice (ii), the number of ways in which he can do so is

$$^5C_3\times{}^5C_3 = 10\times10 = 100 .$$

Similarly, if the candidate follows choice (iii), then the number of ways in which he can do so is

$$^5C_4\times{}^5C_2 = 50 .$$

Therefore, the candidate can select the question in $50+100+50=200$ ways.

Example 7.24 A committee of 5 persons is to be formed from 6 men and 4 women. In how many ways can this be done when

(i) at least 2 women are included?
(ii) atmost 2 women are included?

Solution : (i) When at least 2 women are included.
The committee may consist of

3 women, 2 men : It can be done in $^4C_3\times{}^6C_2$ ways.

or,   4 women, 1 man : It can be done in $^4C_4\times{}^6C_1$ ways.

or,   2 women, 3 men : It can be done in $^4C_2\times{}^6C_3$ ways.

$\therefore$ Total number of ways of forming the committee

$$= {}^4C_2.{}^6C_3+{}^4C_3.{}^6C_2+{}^4C_4.{}^6C_1$$

$$= 6\times20 + 4\times15 + 1\times6$$

$$= 120 + 60 + 6 = 186$$

(ii) When atmost 2 women are included

The committee may consist of

2 women, 3 men : It can be done in $^4C_2.^6C_3$ ways

or, 1 woman, 4 men : It can be done in $^4C_1.^6C_4$ ways

or, 5 men : It can be done in $^6C_5$ ways

∴ Total number of ways of forming the committee

$= {}^4C_2.{}^6C_3 + {}^4C_1.{}^6C_4 + {}^6C_5$

$= 6 \times 20 + 4 \times 15 + 6$

$= 120 + 60 + 6 = 186$

Example 7.25 The Indian Cricket team consists of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can a cricket eleven be selected if we have to select 1 wicket keeper and atleast 4 bowlers?

Solution : We are to choose 11 players including 1 wicket keeper and 4 bowlers

or, 1 wicket keeper and 5 bowlers.

Number of ways of selecting 1 wicket keeper, 4 bowlers and 6 other players

$= {}^2C_1.{}^5C_4.{}^9C_6$

$= 2 \times \dfrac{5 \times 4 \times 3 \times 2.1}{4.3.2.1.} \times \dfrac{9 \times 8 \times 7 \times 6 \times 5 \times 4}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= 2 \times 5 \times \dfrac{9 \times 8 \times 7}{3 \times 2 \times 1} = 840$

Number of ways of selecting 1 wicket keeper, 5 bowlers and 5 other players

$= {}^2C_1.{}^5C_5.{}^9C_5$

$= 2 \times 1 \times \dfrac{9 \times 8 \times 7 \times 6 \times 5}{5 \times 4 \times 3 \times 2 \times 1} = 2 \times 1 \times \dfrac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 252$

∴ Total number of ways of selecting the team

$= 840 + 252 = 1092$

Q CHECK YOUR PROGRESS 7.6

1. (a) Evaluate :

(i) $^{13}C_3$ (ii) $^9C_5$ (iii) $^8C_2 + {}^8C_3$ (iv) $\dfrac{^9C_3}{^6C_3}$

(b) Verify each of the following statement :

(i) $^5C_2 = {}^5C_3$ (ii) $^4C_3 \times {}^3C_2 = {}^{12}C_6$

(iii) $^4C_2 + {}^4C_3 = {}^8C_5$ (iv) $^{10}C_2 + {}^{10}C_3 = {}^{11}C_3$

2. Find the number of subsets of the set $\{1, 3, 5, 7, 9, 11, 13, ..., 23\}$ each having 3 elements.

3. There are 14 points lying on a circle. How many pentagons can be drawn using these points?

4. In a fruit basket there are 5 apples, 7 plums and 11 oranges. You have to pick 3 fruits of each type. In how many ways can you make your choice?

5. A question paper consists of 12 questions divided into two parts A and B, containing 5 and 7 questions repectively. A student is required to attempt 6 questions in all, selecting at least 2 from each part. In how many ways can a student select a question?

6. Out of 5 men and 3 women, a committee of 3 persons is to be formed. In how many ways can it be formed selecting (i) exactly 1 woman. (ii) atleast 1 woman.

7. A cricket team consists of 17 players. It includes 2 wicket keepers and 4 bowlers. In how many ways can a playing eleven be selected if we have to select 1 wicket keeper and atleast 3 bowlers?

8. To fill up 5 vacancies, 25 applications were recieved. There were 7 S.C. and 8 O.B.C. candidates among the applicants. If 2 posts were reserved for S.C. and 1 for O.B.C. candidates, find the number of ways in which selection could be made?

7.6 SOME SIMPLE PROPERTIES OF ^nC_r

In this section we will prove some simple properties of ^nC_r which will make the computations of these coefficients simpler. Let us go back again to Theorem 7.3. Using relation 7.7 we can rewrite the formula for ^nC_r as follows :

$$^nC_r = \frac{n!}{r!(n-r)!} \qquad ....(7.8)$$

Example 7.26 Find the value of $^nC_0$.

Solution : Here $r = 0$. Therefore, $^nC_0 = \dfrac{n!}{0!\,n!} = \dfrac{1}{0!} = 1$,

since we have defined $0! = 1$.

The formula given in Theorem 7.3 was used in the previous section. As we will see shortly, the formula given in Equation 7.8 will be useful for proving certain properties of ^nC_r.

$$^nC_r = {}^nC_{n-r} \qquad\qquad ...(7.9)$$

This means just that the number of ways of choosing r objects out of n objects is the same as the number of ways of not choosing $(n - r)$ objects out of n objects. In the example described in the introduction, it just means that the number of ways of selecting 2 sets of dresses is the same as the number of ways of rejecting $4 - 2 = 2$ dresses. In Example 7.20, this means that the number of ways of choosing subsets with 4 elements is the same as the number of ways of rejecting 8 elements since choosing a particular subset of 4 elements is equivalent to rejecting its complement, which has 8 elements.

Let us now prove this relation using Equation 7.8. The denominator of the right hand side of this equation is $r!\,(n - r)!$. This does not change when we replace r by $n - r$.

$$(n - r)!.[n - (n - r)]! = (n - r)!.r!$$

The numerator is independent of r. Therefore, replacing r by $n - r$ in Equation 7.8 we get result.

How is the relation 7.9 useful? Using this formula, we get, for example, $^{100}C_{98}$ is the same as $^{100}C_2$. The second value is much more easier to calculate than the first one.

Example 7.27 Evaluate :

(a) $^7C_5$ (c) $^{10}C_9$

(b) $^{11}C_9$ (d) $^{12}C_9$

Solution : (a) From relation 7.9, we have

$$^7C_5 = {}^7C_{7-5} = {}^7C_2 = \frac{7.6}{1.2} = 21$$

(b) Similarly $^{10}C_9 = {}^{10}C_{10-9} = {}^{10}C_1 = 10$

(c) $^{11}C_9 = {}^{11}C_{11-9} = {}^{11}C_2 = \dfrac{11.10}{1.2} = 55$

(d) $^{12}C_{10} = {}^{12}C_{12-10} = {}^{12}C_2 = \dfrac{12.11}{1.2} = 66$

There is another relation satisfied by ^nC_r which is also useful. We have the following relation:

$$^{n-1}C_{r-1} + {}^{n-1}C_r = {}^nC_r \qquad\qquad ....(7.10)$$

$$^{n-1}C_{r-1} + {}^{n-1}C_r = \frac{(n-1)!}{(n-r)!(r-1)!} + \frac{(n-1)!}{(n-r-1)!r!}$$

$$= \frac{(n-1)!}{(n-r)(n-r-1)!(r-1)!} + \frac{(n-1)!}{r(n-r-1)!(r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!(r-1)!}\left[\frac{1}{n-r} + \frac{1}{r}\right]$$

$$= \frac{(n-1)!}{(n-r-1)!(r-1)!}\left[\frac{n}{(n-r)r}\right]$$

$$= \frac{n(n-1)!}{(n-r)(n-r-1)!r(r-1)!}$$

$$= \frac{n!}{(n-r)!r!} = {}^nC_r$$

Example 7.28 Evaluate :

(a) $^6C_2 + {}^6C_1$
(b) $^8C_2 + {}^8C_1$

(c) $^5C_3 + {}^5C_2$
(d) $^{10}C_2 + {}^{10}C_3$

Solution : (a) Let us use relation (7.10) with $n = 7, r = 2$. So, $^6C_2 + {}^6C_1 = {}^7C_2 = 21$

(b) Here $n = 9,\ r = 2$. Therefore, $^8C_2 + {}^8C_1 = {}^9C_2 = 36$

(c) Here $n = 6,\ r = 3$. Therefore, $^5C_3 + {}^5C_2 = {}^6C_3 = 20$

(d) Here $n = 11, r = 3$. Therefore, $^{10}C_2 + {}^{10}C_3 = {}^{11}C_3 = 165$

To understand the relation 7.10 better, let us go back to Example 7.20 In this example let us calculate the number of subsets of the set, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}. We can subdivide them into two kinds, those that contain a particular element, say 1, and those that do not contain 1. The number of subsets of the set having 4 elements and containing 1 is the same as the number of subsets of {2, 3, 4, 5, 6, 7, 8, 9, 10, 11} having 3 elements. There are $^{10}C_3$ such subsets.

The number of subsets of the set having 4 elements and not containing 1 is the same as the number of subsets of the set $\{2,3,4,5,6,7,8,9,10,11,\}$ having 4 elements. This is $^{10}C_4$. So, the number of subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ having four elements is $^{10}C_3 + {}^{10}C_4$. But, this is $^{11}C_4$ as we have seen in the example. So, $^{10}C_3 + {}^{10}C_4 = {}^{11}C_4$. The same argument works for the number of r–element subsets of a set with n elements.

This reletion was noticed by the French Mathematician **Blaise Pascal** and was used in the so called **Pascal Triangle,** given below.

```
n = 0                          1

n = 1                       1      1

n = 2                    1    2    1

n = 3                 1    3    3    1

n = 4              1    4    6    4    1

n = 5           1    5   10   10    5    1
```

The first row consists of single element $^0C_0 = 1$. The second row consists of $^1C_0$ and $^1C_1$. From the third row onwards, the rule for writing the entries is as follows. Add consecutive elements in the previous row and write the answer between the two entries. After exhausting all possible pairs, put the number 1 at the begining and the end of the row. For example, the third row is got as follows. Second row contains only two elements and they add up to 2. Now, put 1 before and after 2. For the fourth row, we have $1 + 2 = 3$, $2 + 1 = 3$. Then, we put $1 + 2 = 3$, $2 + 1 = 3$. Then we put 1 at the beginning and the end. Here, we are able to calculate, for example, $^3C_1$, $^3C_2$, from $^2C_0$, $^2C_1$, $^2C_2$ by using the relation 7.10. The important thing is we are able to do it using addition alone.

The numbers ^nC_r occur as coefficents in the binomial expansions, and therefore, they are also called **binomial coefficents** as we will see in lesson 8. In particular, the property 7.10 will be used in the proof of the binomial theorem.

Example 7.29 If $^nC_{10} = {}^nC_{12}$ find n,

Solution : Using $^nC_r = {}^nC_{n-r}$ we get

$\qquad n - 10 = 12$

or, $\quad n = 12 + 10 = 22$

Notes

CHECK YOUR PROGRESS 7.7

1. (a) Find the value of $^nC_{n-1}$. Is $^nC_{n-1}={^nC_n}$? (b) Show that $^nC_n = {^nC_0}$

2. Evaluate :

   (a) $^9C_5$ (b) $^{14}C_{10}$

   (c) $^{13}C_9$ (d) $^{15}C_{12}$

3. Evaluate :

   (a) $^7C_3 + {^7C_2}$ (b) $^8C_4 + {^8C_5}$

   (c) $^9C_3 + {^9C_2}$ (d) $^{12}C_3 + {^{12}C_2}$

4. If $^{10}C_r = {^{10}C_{2r+1}}$, find the value of r. 5. If $^{18}C_r = {^{18}C_{r+2}}$ find $^rC_5$

PROBLEMS INVOLVING BOTH PERMUTATIONS AND COMBINATIONS

So far, we have studied problems that involve either permutation alone or combination alone. In this section, we will consider some examples that need both of these concepts.

Example 7.30 There are 5 novels and 4 biographies. In how many ways can 4 novels and 2 biographies can be arranged on a shelf ?

Soluton : 4 novels can be selected out of 5 in $^5C_4$ ways. 2 biographies can be selected out of 4 in $^4C_2$ ways.

Number of ways of arranging novels and biographies

$$= {^5C_4} \times {^4C_2} = 5 \times 6 = 30$$

After selecting any 6 books (4 novels and 2 biographies) in one of the 30 ways, they can be arranged on the shelf in $6! = 720$ ways.

By the Counting Principle, the total number of arrangements $= 30 \times 720 = 21600$

Example 7.31 From 5 consonants and 4 vowels, how many words can be formed using 3 consonants and 2 vowels ?

Solution : From 5 consonants, 3 consonants can be selected in $^5C_3$ ways.

From 4 vowels, 2 vowels can be selected in $^4C_2$ ways.

Now with every selection, number of ways of arranging 5 letters is $^5P_5$

Notes

$\therefore$ Total number of words $= {}^5C_3 \times {}^4C_2 \times {}^5P_5$

$$= \frac{5 \times 4}{2 \times 1} \times \frac{4 \times 3}{2 \times 1} \times 5!$$

$$= 10 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7200$$

CHECK YOUR PROGRESS 7.8

1.   There are 5 Mathematics, 4 Physics and 5 Chemistry books. In how many ways can you arrange 4 Mathematics, 3 Physics and 4 Chemistry books.

(a) if the books on the same subjects are arranged together, but the order in which the books are arranged within a subject doesn't matter ?

(b) if books on the same subjects are arranged together and the order in which books are arranged within subject matters ?

2.   There are 9 consonants and 5 vowels. How many words of 7 letters can be formed using 4 consonents and 3 vowels ?

3.   In how many ways can you invite at least one of your six friends to a dinner?

4.   In an examination, an examinee is required to pass in four different subjects. In how many ways can he fail?

LET US SUM UP

•   Fundamental principle of counting states.

If there are n events and if the first event can occur in m_1 ways, the second event can occur in m_2 ways after the first event has occurred, the third event can occur in m_3 ways after the second event has occurred and so on, then all the n events can occur in

$m_1 \times m_2 \times m_3 \times \ldots\ldots \times m_{n-1} \times m_n$ ways.

•   The number of permutations of n objects taken all at a time is $n!$

• $\quad {}^nP_r = \dfrac{n!}{(n-r)!}$

• $\quad {}^nP_n = n!$

•   The number of ways of selecting r objects out of n objects $\quad {}^nC_r = \dfrac{n!}{r!(n-r)!}$

• $\quad {}^nC_r = {}^nC_{n-r}$

• $\quad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

SUPPORTIVE WEB SITES

http://www.wikipedia.org

http://mathworld.wolfram.com

TERMINAL EXERCISE

1. There are 8 true - false questions in an examination. How many responses are possible ?

2. The six faces of a die are numbered 1,2,3,4,5 and 6. Two such dice are thrown simultaneously. In how many ways can they turn up ?

3. A restaurant has 3 vegetables, 2 salads and 2 types of bread. If a customer wants 1 vegetable, 1 salad and 1 bread, how many choices does he have ?

4. Suppose you want to paper your walls. Wall papers are available in 4 diffrent backgrounds colours with 7 different designs of 5 different colours on them. In how many ways can you select your wall paper ?

5. In how many ways can 7 students be seated in a row on 7 seats ?

6. Determine the number of 8 letter words that can be formed from the letters of the word *ALTRUISM*.

7. If you have 5 windows and 8 curtains in your house, in how many ways can you put the curtains on the windows ?

8. Determine the maximum number of 3- letter words that can be formed from the letters of the word *POLICY*.

9. There are 10 athletes participating in a race and there are three prizes, 1st, 2nd and 3rd to be awarded. In how many ways can these be awarded ?

10. In how many ways can you arrange the letters of the word *ATTAIN* so that the *T*s are together?

11. A group of 12 friends meet at a party. Each person shake hands once with all others. How many hand shakes will be there. ?

12. Suppose that you own a shop which sells televisions. You are selling 5 different kinds of television sets, but your show case has enough space for display of 3 televison sets only. In how many ways can you select the television sets for the display ?

13. A contractor needs 4 carpenters. Five equally qualified carpenters apply for the job. In how many ways can the contractor make the selection ?

14. In how many ways can a committe of 9 can be selected from a group of 13?

15. In how many ways can a committee of 3 men and 2 women be selected from a group of 15 men and 12 women ?

16. In how ways can 6 persons be selected from 4 grade 1 and 7 grade II officers, so as to include at least two officers from each category ?

17. Out of 6 boys and 4 girls, a committee of 5 has to be formed. In how many ways can this be done if we take :

     (a) 2 girls.

     (b) at least 2 girls.

18. The English alphabet has 5 vowels and 21 consonants. What is the maximum number of words, that can be formed from the alphabet with 2 different vowels and 2 different consonants?

19. From 5 consonants and 5 vowels, how many words can be formed using 3 consonants and 2 vowels?

20. In a school annual day function a variety programme was organised. It was planned that there would be 3 short plays, 6 recitals and 4 dance programmes. However, the chief guest invited for the function took much longer time than expected to finish his speech. To finish in time, it was decided that only 2 short plays, 4 recitals and 3 dance programmes would be perfomed, How many choices were available to them ?

     (a) if the programmes can be perfomed in any order ?

     (b) if the programmes of the same kind were perfomed at a stretch?

     (c) if the programmes of the same kind were perfomed at a strech and considering the order of performance of the programmes of the same kind ?

Notes

ANSWERS

CHECK YOUR PROGRESS 7.1

1.   (a) 180   (b) 8   (c) 12   (d) 20

2.   (a) 48   (b) 36

CHECK YOUR PROGRESS 7.2

1.   (a)   17576   (b)   900

2.   (a)   105   (b)   60

3.   (a)   24   (b)   24

CHECK YOUR PROGRESS 7.3

1.   (a) (i) 720   (ii) 5040   (iii) 5046   (iv) 17280   (v) 10

     (b) (i) False   (ii) False   (iii) True   (iv) False

2.   (a) 120   (b) 40320   (c) 24

CHECK YOUR PROGRESS 7.4

1.   (a) (i) 12   (ii) 120   (iii) 4   (iv) 7200   (v) $n!$

     (b) (i) False   (ii) True   (iii) False   (iv) False

2.   (a) (i) 7980   (ii) 9240   (b) 20   (c) 840

CHECK YOUR PROGRESS 7.5

1.   96   2. 1152   3. 2073600   4. 2488320

5.   144   6. (i) 5040   (ii) 30240   (iii) 332640

CHECK YOUR PROGRESS 7.6

1.   (a)   (i) 286

                  (ii) 126

                  (iii) 84

                  (iv) $\dfrac{21}{5}$

     (b)   (i) True

                  (ii) False

(iii) False

(iv) True

2.   1771

3.   2002

4.   57750

5.   805

6.   (i)   30

      (ii)   46

7.   3564

8.   7560

CHECK YOUR PROGRESS 7.7

1.   (a) n , No

2.   (a) 126

      (b) 1001

      (c) 715

      (d) 455

3.   (a) 56

      (b) 126

      (c) 120

      (d) 286

4.   3

5.   56

CHECK YOUR PROGRESS 7.8

1   (a) 600

      (b) 207 3600

2.   6350400

3.   63

4.   15

TERMINAL EXERCISE

1. 256

2. 36

3. 12

4. 140

5. 5040

6. 40320

7. 6720

8. 120

9. 720

10. 120

11. 66

12. 10

13. 5

14. 715

15. 30030

16. 371

17. (a) 120

     (b) 186

18. 50400

19. 12000

20. (a) 65318400

     (b) 1080

     (c) 311040