FLEXURAL STRENGTH

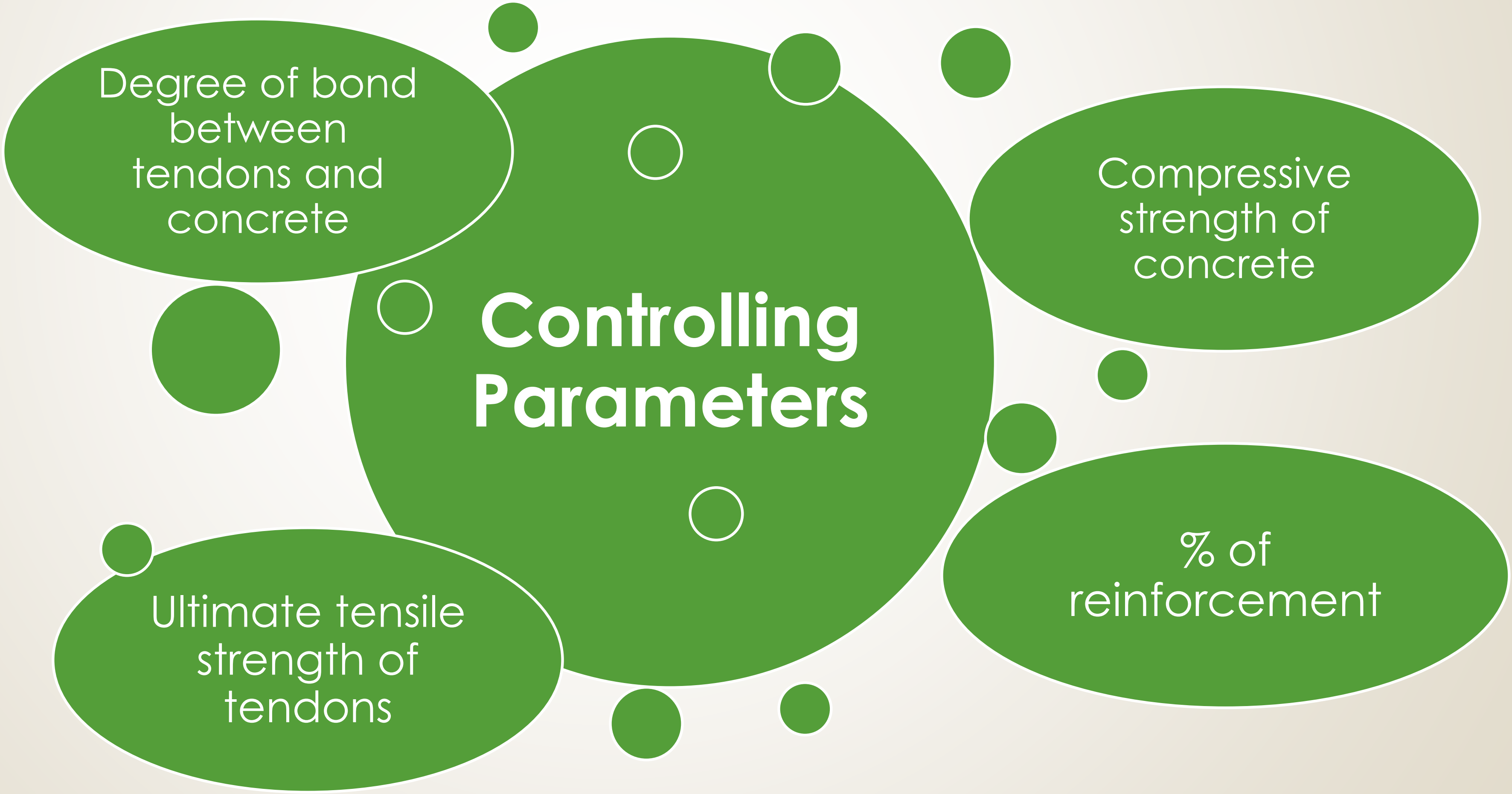
OF

PRESTRESSED CONCRETE

SECTIONS

Types of Flexural Failures

- **When prestressed concrete members are subjected to bending loads, different types of flexural failures are possible at critical sections depending upon principal controlling parameters.**



Types of Flexural Failures

- In the post-cracking stage, the behavior of a prestressed concrete member is more akin that of a RC member.
- The theories used for estimating the flexural strength of RC section may as well be used for prestressed concrete sections.

Various Types of Flexural Failure

1. Fracture of steel in tension

- Sudden failure of prestressed member without any warning is generally due to fracture of steel in tension zone.
- Imminent when % of steel provided is low.
- Can be prevented by providing a certain minimum % of steel in the c/s.

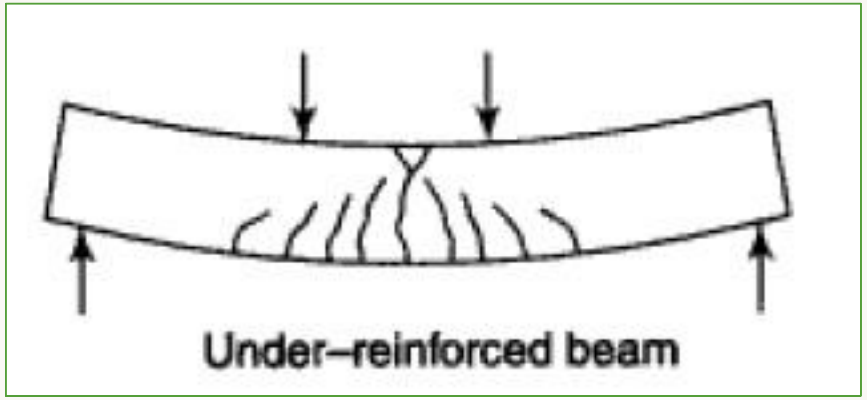
IS : 1343

- ➔ Minimum longitudinal reinforcement 0.2% of c/s area except in case of pretensioned units of small sections.
- ➔ For HYSD reinforcement 0.15%.
- ➔ % of steel provided, both tensioned and untensioned taken together, should be sufficient to avoid sudden failure.

2. Failure of Under Reinforced Sections

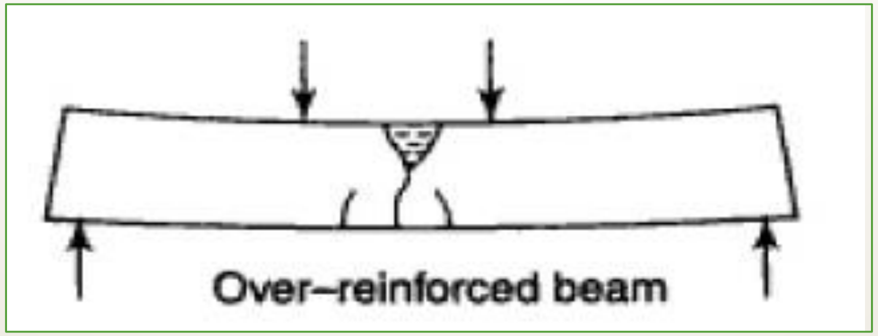
- If the c/s is provided with an amount of steel greater than the minimum prescribed in case 1, the failure is characterized by an excessive elongation of steel followed by crushing of concrete.
- As bending loads are increased, excessive elongation of steel raises the NA closer to the compression face at the critical section.
- Member approaches failure due to the gradual reduction of compression zone, which exhibits large deflections and cracks at the soffit of the beam.
- When the area of concrete in compression zone is insufficient to resist the resultant internal compressive force, ultimate flexural failure of member takes place through the crushing of concrete.
- Large deflection and wide cracks are characteristic features of under reinforced sections at failure.

- **It is common practice to design under reinforced sections which is more important in statically indeterminate structures.**
- **Upper limit of maximum area of steel is generally prescribed in various codes for under-reinforced sections.**



3. Failure of Over Reinforced Sections

- Generally this type of section fails by sudden crushing of concrete.
- Failure is characterized by small deflections and narrow cracks.
- Area of steel is comparatively large, the stresses developed in steel at failure of the member may not reach the tensile strength and in many cases it may be well within the proof stress of tendons.
- Amount of reinforcement should preferably not exceed required for balanced section.
- In this connection, most of the codes follow a conservative approach in formulating the evaluation procedures for flexural strength calculations of over-reinforced sections.

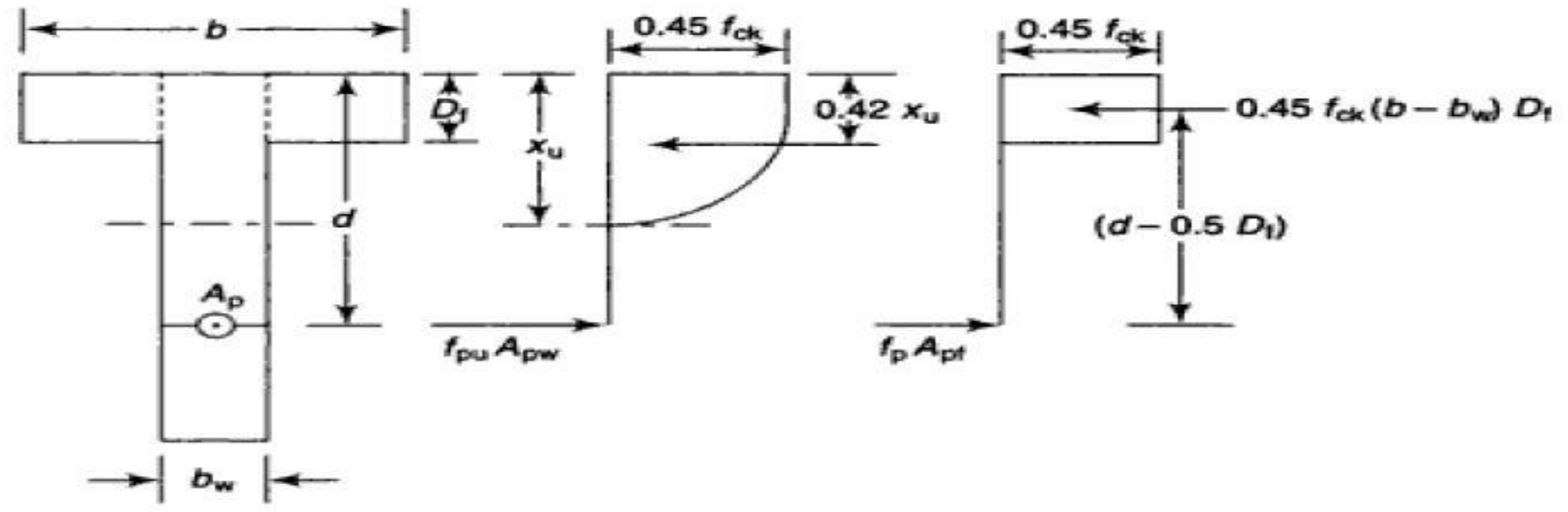


4. Other modes of Failure

- Member subjected to transverse loads may fail in shear before their full flexural strength is attained, if it is not adequately designed for shear.
- Web shear cracks may develop if principal stresses are excessive and if thin webs are used, failure may occur due to web crushing.
- In pretensioned members, failure of bond between steel and surrounding concrete may be due to inadequate transmission lengths at the ends of the member.
- In post-tensioned members, anchorage failures may take place if the end block is not properly designed to resist transverse tensile forces.

SIMPLIFIED CODE PROCEDURE

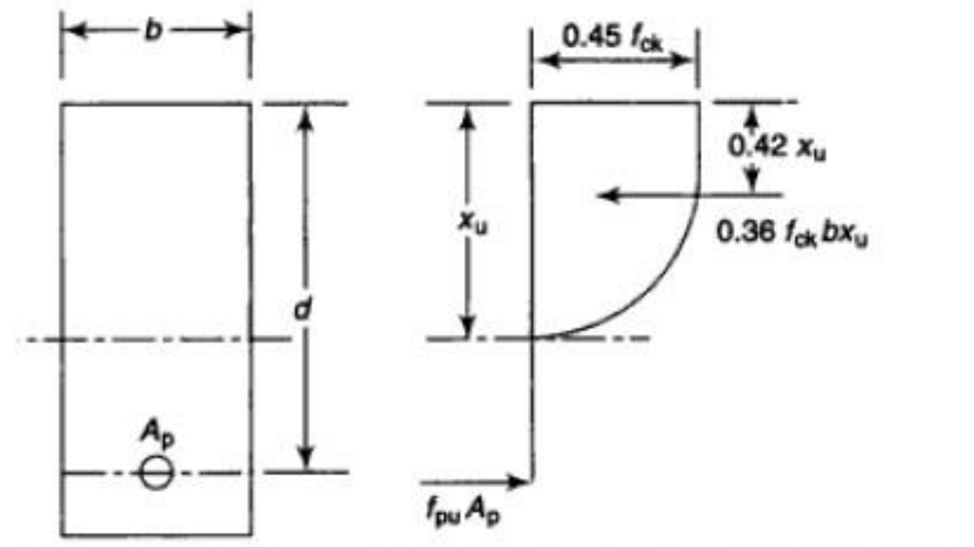
Ultimate moment of resistance of flanged sections in which neutral axis lies outside the flange is computed by combining moment of resistance of the web and flange portions and considering stress blocks shown in fig.



Moment of Resistance of Flanged Sections ($x_u > D_f$) (IS: 1343–1980)

SIMPLIFIED CODE PROCEDURE - INDIAN CODE PROVISIONS

- IS Code Method (IS: 1343) for computing the flexural strength of rectangular sections or T-sections in which neutral axis lies within the flange is based on rectangular and parabolic stress block as shown in figure below.



Moment of Resistance of Rectangular Sections (IS:1343–1980)

- Moment of resistance is obtained from following equation:

$M_u = f_{pu}\,A_p\,(d - 0.42x_u)$

INDIAN CODE PROVISIONS

- Moment of resistance is obtained from following equation:

$M_u = f_{pu}\ A_p\ (d - 0.42x_u)$

Where, M_u = Ultimate moment of resistance of section

f_{pu} = Tensile stress developed in tendons at the failure stage of the beam

A_p = Area of prestressing tendons

x_u = Neutral axis depth

- The value of f_{pu} depends on effective reinforcement ratio

$$\left(\frac{A_p f_p}{bdf_{ck}}\right)$$

f_p = Characteristic tensile strength of the prestressing steel

f_{pe} = Effective prestress in tendons after losses

d = Effective depth

- For pretensioned and post-tensioned members with an effective bond between concrete and tendons, the value of f_{pu} and x_u are given in Table below:

Table 11 Conditions at the ultimate limit state for Rectangular beams with Pretensioned Tendons or with post-tensioned Tendons having Effective Bond

(Clause D-1)

Sr No	$\frac{A_{ps} \cdot f_{pu}}{bdf_{ck}}$	Stress in tendons as propagation of the design strength $\frac{f_{Pu}}{0.87f_p}$		Ratio of depth of neutral axis to that of centroid of the tendon in tension zone x_u/d	
		Pretensioning	**Post-tensioning with effective bond**	**Pretensioning**	**Post-tensioning with effective bond**
(1)	(2)	(3)	(4)	(5)	(6)
I	**0.025**	1.0	1.0	0.054	0.054
II	**0.05**	1.0	1.0	0.109	0.109
III	**0.10**	1.0	1.0	0.217	0.217
IV	**0.15**	1.0	1.0	0.326	0.316
V	**0.20**	1.0	0.95	0.435	0.414[1)]
VI	**0.25**	1.0	0.90	0.542	0.488[1)]
VII	**0.30**	1.0	0.85	0.655	0.558[1)]
VIII	**0.40**	0.9	0.75	0.783	0.653[1)]

[1)] The neutral axis depth in these cases is too low to provide the necessary elongation for developing $0.87f_{pu}$ stress level. Hence, it is essential that the strength provided exceeds the required strength by 15 percent for these cases.

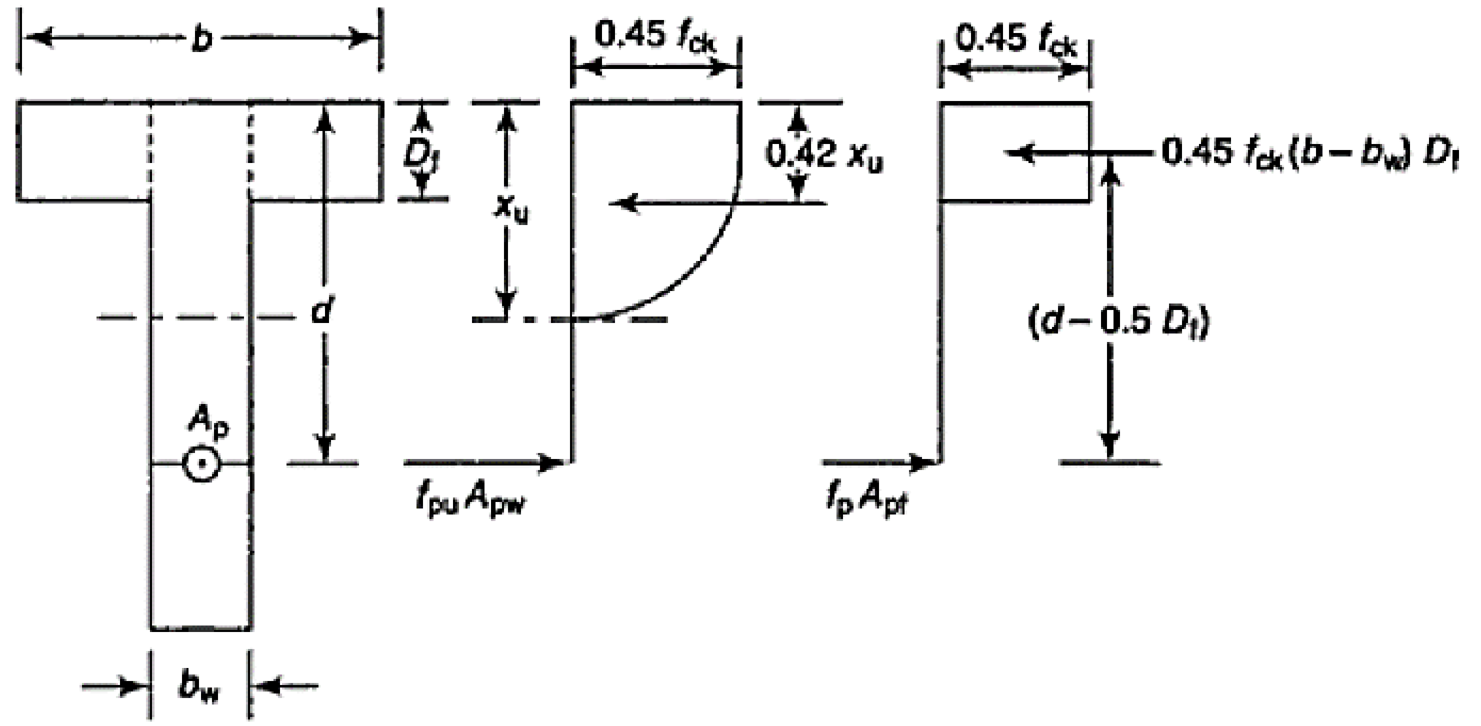
The effective prestress f_{pe} after all losses should not be less than $0.451f_p$.

- **For post-tensioned rectangular beams with unbonded tendons, the values of fpu and xu are influenced by the effective span to depth rations and their values for different span/depth ratios are shown in Table below:**

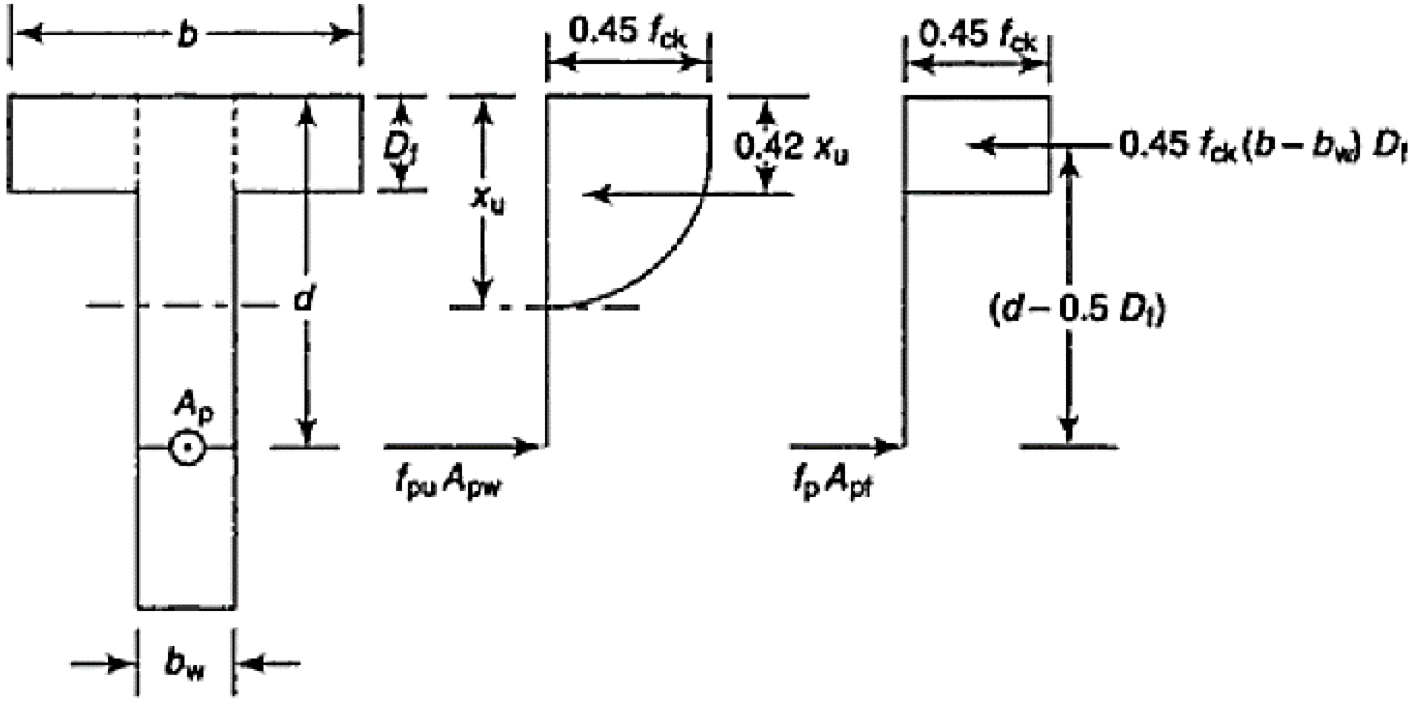
Table 12 Conditions at the ultimate limit state for Post-tensioned Rectangular beam having un-bonded tendons (IS: 1343)

$\left(\frac{A_p . f_{pe}}{bdf_{ck}}\right)$	Stress in tendons as a proportion of the effective pre-stress (f_{pu}/f_{pe}) for values of (L/D)			Ratio of depth of neutral axis to that of tendons on tension zone (x_u/d) for values of (L/D)		
	$\frac{\text{Effective span}}{\text{Effective depth}}$			$\frac{\text{Effective span}}{\text{Effective depth}}$		
	30	**20**	**10**	**30**	**20**	**10**
0.025	1.23	1.34	1.45	0.10	0.10	0.10
0.05	1.21	1.32	1.45	0.16	0.16	0.18
0.10	1.18	1.26	1.45	0.30	0.32	0.36
0.15	1.14	1.20	1.36	0.44	0.46	0.52
0.20	1.11	1.16	1.27	0.56	0.58	0.64

- The ultimate moment of resistance of flanged sections in which the neutral axis falls outside the flange is computed by combining the moment of resistance of the web and flange portions considering the stress blocks shown in figure below.



Moment of Resistance of Flanged Sections ($x_u > D_f$) (IS: 1343–1980)



Moment of Resistance of Flanged Sections ($x_u > D_f$) (IS: 1343–1980)

- If, A_{pw} = Area of prestressing steel in web

  A_{pf} = Area of prestressing steel in flange

  D_f = Thickness of flange

  Then, $A_p = A_{pw} + A_{pf}$

  but, $A_{pf} = 0.45\ f_{ck}\ (b - b_w)\ (D_f/f_p)$

  After that, $A_{pw} = A_p - A_{pf}$

- For the effective reinforcement ratio of $\left(\frac{A_{pw} f_p}{b_w d f_{ck}}\right)$, the corresponding values of $\left(\frac{f_p}{0.87 f_{pu}}\right)$ and $\left(\frac{x_u}{d}\right)$ are obtained from Table 11.

- The ultimate moment of resistance of the flanged section is obtained from the expression, $M_u = f_{pu}\ A_{pw}\ (d - 0.42x_u) + 0.45\ f_{ck}(b - b_w)\ D_f\ (d - 0.5D_f)$

Ex: A pretensioned concrete beam having a rectangular section, 150 mm wide and 350 mm deep has an effective cover of 50 mm. If, f_{ck} = 40 N/mm², f_p = 1600 N/mm² and the area of prestressing steel A_p = 461 mm², calculate the ultimate flexural strength of the section using IS:1343 provisions.

Solution:

f_{ck} = 40 N/mm², f_p = 1600 N/mm², A_p = 461 mm², b = 150 mm and d = 300 mm

Effective reinforcement ratio is given by $\left(\dfrac{A_p f_p}{f_{ck} bd}\right) = \left(\dfrac{1600 \text{ x } 461}{40 \text{ x } 150 \text{ x } 300}\right) = 0.40$

From table 11 of IS:1343, corresponding values of $\left(\dfrac{f_{pu}}{0.87 f_p}\right) = 0.9$ and $\left(\dfrac{x_u}{d}\right) = 0.783$

f_{pu} = 0.87 x 0.9 x 1600 = 1253 N/mm²

X_u = 0.783 x 300 = 234.9 mm

$M_u = f_{pu} A_p (d - 0.42x_u)$ = 116 x 10^6 Nmm

= 116 kNm

Ex: A pretensioned, T section has a flange which is 300 mm wide and 200 mm thick. The rib is 150 mm wide by 350 mm deep. The effective depth of cross section is 500 mm. If, f_{ck} = 50 N/mm², f_p = 1600 N/mm² and the area of prestressing steel A_p = 200 mm², estimate the ultimate moment capacity of T-section using the Indian Standard Code regulations.

Solution:

f_{ck} = 50 N/mm², f_p = 1600 N/mm², A_p = 200 mm², b = 300 mm and d = 500 mm

Effective reinforcement ratio is given by $\left(\frac{A_p f_p}{f_{ck} bd}\right) = \left(\frac{1600 \text{ x } 200}{50 \text{ x } 300 \text{ x } 500}\right) = 0.04$

From table 11 of IS:1343, corresponding values of $\left(\frac{f_{pu}}{0.87 f_p}\right) = 1.0$ and $\left(\frac{x_u}{d}\right) = 0.09$

f_{pu} = 0.87 x 1.0 x 1600 = 1392 N/mm²

X_u = 0.09 x 500 = 45 mm (Assumption is correct that the NA lies in flange)

Hence, the ultimate moment of resistance of section is,

$M_u = f_{pu} A_p (d - 0.42x_u)$ = 134 x 10^6 Nmm

= 134 kNm

Ex: A pretensioned, T section has a flange 1200 mm wide and 150 mm thick. The rib is 300 mm wide by 1500 mm deep. The high tensile steel has an area of 4700 mm^2, which is located at an effective depth of 1600 mm. If, f_{ck} = 40 N/mm^2, f_p = 1600 N/mm^2. Calculate the flexural strength of T-section.

Solution:

f_{ck} = 40 N/mm^2, f_p = 1600 N/mm^2, A_p = 4700 mm^2, b = 1200 mm, bw = 300 mm,
d = 500 mm and Df = 150 mm

Here, we can use $A_p = A_{pw} + A_{pf}$

Where, $A_{pf} = 0.45\, f_{ck}\, (b - b_w)\, (D_f/f_p) = 0.45 \times (40)\, (1200 – 300)\, (150/1600) = 1518$ mm^2

$A_{pw} = A_p – A_{pf} = 4700 – 1518 = 3182$ mm^2

Now, $\left(\frac{A_{pw} f_p}{b_w d f_{ck}}\right)$ = 0.265 and from table 11 $\left(\frac{f_{pu}}{0.87 f_p}\right) = 1.0$ and $\left(\frac{x_u}{d}\right) = 0.56$

So, $f_{pu} = 0.87 \times 1600 = 1392$ N/mm^2

$X_u = 0.56 \times 1600 = 896$ mm

The ultimate moment of resistance of the flanged section is obtained from the expression,

$$M_u = f_{pu} A_{pw} (d - 0.42x_u) + 0.45 f_{ck}(b - b_w) D_f (d - 0.5D_f)$$

= (1392 x 3182) (1600 – 0.42 x 896) + 0.45 x 40 x 900 x 150 (1600 – 75)

= [(5420 x 10^6) + (3705 x 10^6)]

= 9125 x 10^6 Nmm = 9125 kNm