DEFLECTION OF PRESTRESSED CONCRETE MEMBERS

TOPICS TO BE COVERED

- FACTORS INFLUENCING DEFLECTIONS
- SHORT TERM DEFLECTIONS OF UNCRACKED MEMBERS
- EFFECT OF TENDON PROFILE ON DEFLECTIONS
- DEFLECTIONS DUE TO SELF-WEIGHT AND IMPOSED LOADS
- PREDICTION OF LONG TIME DEFLECTIONS
- DEFLECTION OF CRACKED MEMBERS
- REQUIREMENTS OF VARIOUS CODES OF PRACTICE

Why Control On Deflections Is Required ????

- Philosophy of design – "Limit State Approach" adopted by the Russian code in 1954 and American and British codes in 1971, requires a proper knowledge of the behaviour of structural concrete members at the multiple limit states, of which deflection forms an important criterion for the safety of the structures.
- It is the general practice, according to various national codes, that structural concrete members should be designed to have adequate stiffness to limit deflections, which may adversely affect the strength or serviceability of the structure at working loads.

Why Control On Deflections Is Required????

- **Suitable control on deflections is very essential for the following reasons:**

1. **Excessive sagging of principal structural members is not only unsightly, but at times, also renders the floor unsuitable for the intended use.**
2. **Large deflections under dynamic effects and under the influence of variable loads may cause discomfort to the users.**
3. **Excessive deflections are likely to cause damage to finishes, partitions and associated structures.**

Why Control On Deflections Is Required????

- In recent years, damage to partitions and finishes has been the most important consequence of excessive deflections.
- A field survey conducted by Mayer in Germany revealed over 80 examples of damage to partition walls, of which twenty-one had estimated deflections within the prescribed code-limits.
- The survey also indicated that a maximum limit on deflection should be specified in addition to a limiting deflection - span ratio, since it was recognized that as the span increases, the former limitation is likely to control.
- For a reasonably accurate assessment of deflections, it is very essential to consider the various factors which influence them.

FACTORS INFUENCING DEFLECTIONS

- Imposed load and self-weight
- Magnitude of the prestressing force
- Cable profile
- Second moment of area of cross-section
- Modulus of elasticity of concrete
- Shrinkage, creep and relaxation of steel stress
- Span of the member
- Fixity conditions

Computation of Deflections

Precracking stage

Gross MI

Called as short term or Instantaneous Deflection

Mohr's Theorem

Post-cracking stage

Effective MI of cracked Section

Moment – curvature relationships (section, properties of cracked beam)

- **In both cases, the effect of creep and shrinkage of concrete is to increase.**
- **The long term deflections under sustained loads, which IS estimated by using empirical methods that involve the use of effective (long term) modulus of elasticity or by multiplying short-term deflections by suitable factors.**

SHORT TERM DEFLECTIONS OF UNCRACKED MEMBERS

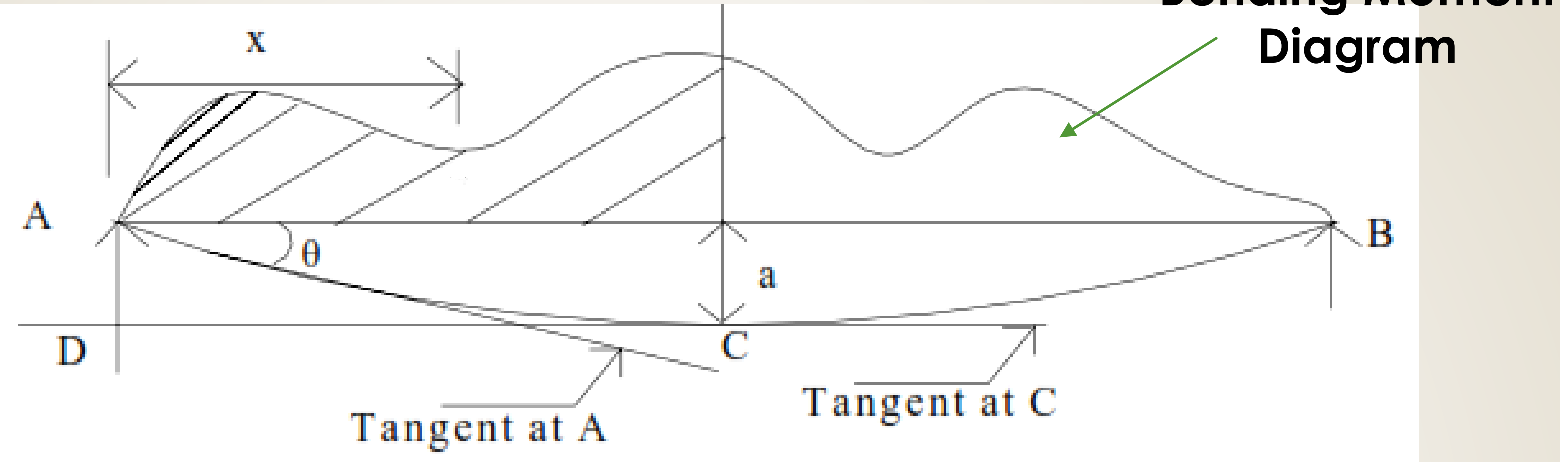
- Short term deflections are governed by the BM distribution along the span and flexural rigidity of the members.
- Mohr's moment area theorems are readily applicable for the estimation of deflections due to prestressing force, self weight and imposed loads.

Moment Area Theorem

- The vertical deviation of a point A on an elastic curve with respect to the tangent which is extended from another point B equals the moment of area under (M/EI) diagram between those two points (A and B). This moment is computed about point A where the deviation from B to A is to be determined.

$$y_b = \int_A^B \frac{M}{EI} x \, dx$$

Mohr's Theorems



Slope and deflection of beam

- If, θ = Slope of the elastic curve at A
- AD = Intercept between the tangent at C & vertical at A
- a = Deflection at the centre for symmetrically loaded simply supported beam
- A = Area of the beam between A and C
- x = Distance of the centroid of the BMD between A and C from the left support
- EI = Flexural rigidity of the beam

$$\theta = \left(\frac{\text{Area of BMD}}{\text{Flexural Rigidity}} \right) = \left(\frac{A}{EI} \right)$$

Theorem 1

$$\text{a} = \left(\frac{\text{Moment of the area of BMD}}{\text{Flexural Rigidity}} \right) = \left(\frac{A\,\text{x}}{EI} \right)$$

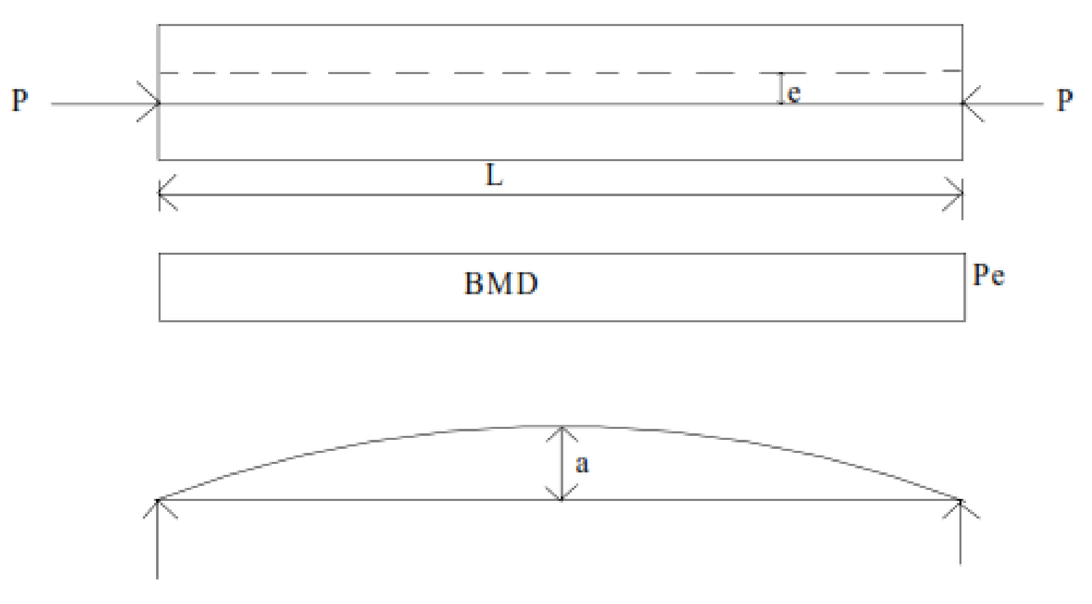
Theorem 2

EFFECT OF TENDON PROFILE ON DEFLECTIONS

- In most of the cases of prestressed beams, tendons are located With eccentricities towards the soffit of the beams to counteract the sagging bending moments due to transverse loads.
- Consequently, the concrete beams deflect upwards (camber) on the application or transfer of prestress.
- Since the bending moment at every section is the product of the prestressing force and eccentricity, the tendon profile itself will represent the shape of the B.M.D.
- The method of computing deflections of beams with different cable profiles is outlined in the following sections.

EFFECT OF TENDON PROFILE ON DEFLECTIONS

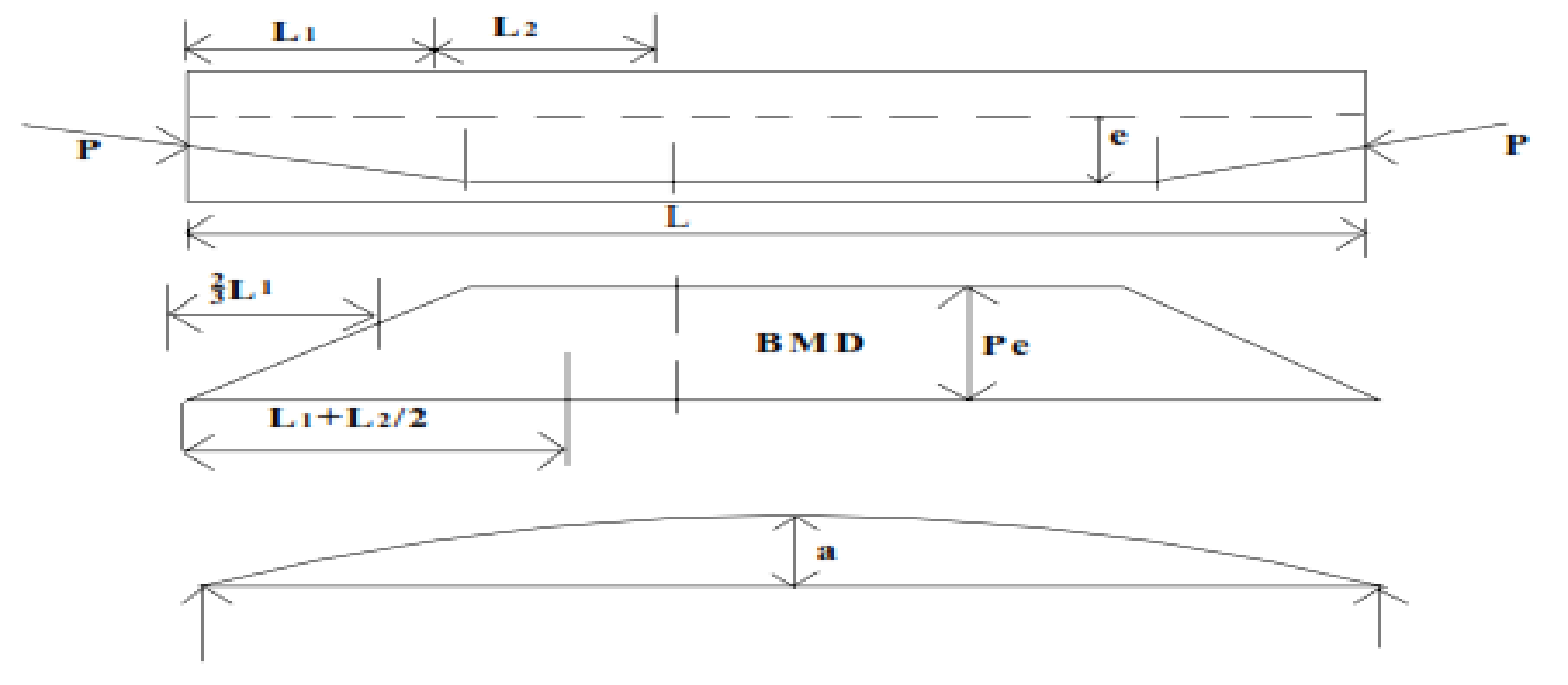
1. **Straight Tendons**



$$a = -\frac{\left(Pe\frac{L}{2}\right)\left(\frac{L}{4}\right)}{EI} = -\frac{PeL^2}{8EI}$$

EFFECT OF TENDON PROFILE ON DEFLECTIONS

2. Trapezoidal Tendons

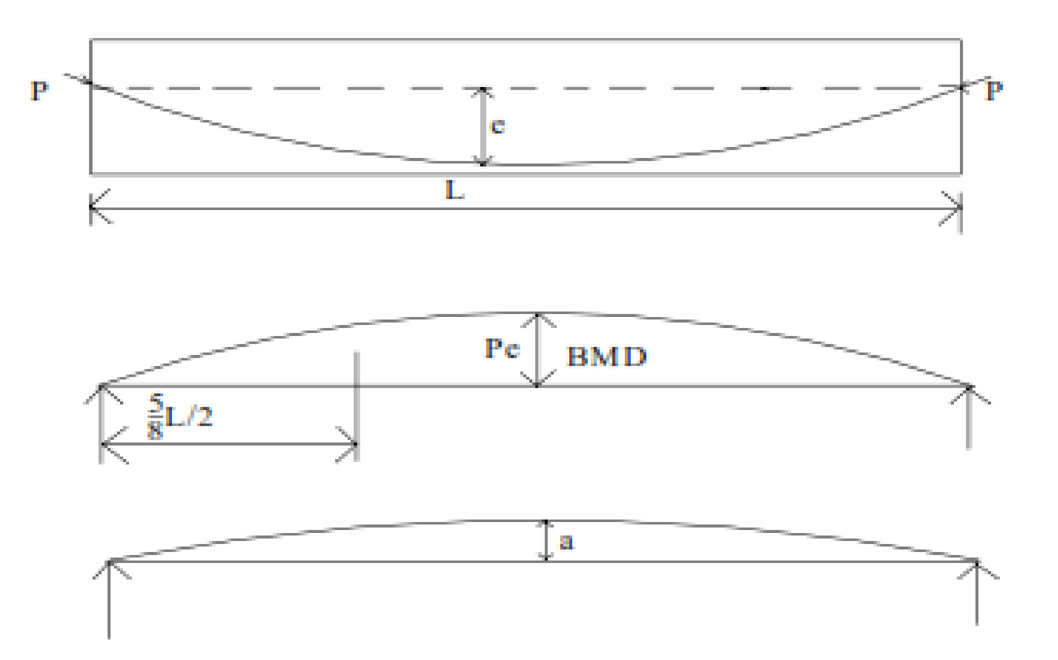


$$a = -\frac{Pe}{EI}[l_2(l_1 + l_2/2) + (l_1/2)(2/3l_1)]$$

$$= -\frac{Pe}{6EI}[2l_1^2 + 6l_1l_2 + 3l_2^2]$$

EFFECT OF TENDON PROFILE ON DEFLECTIONS

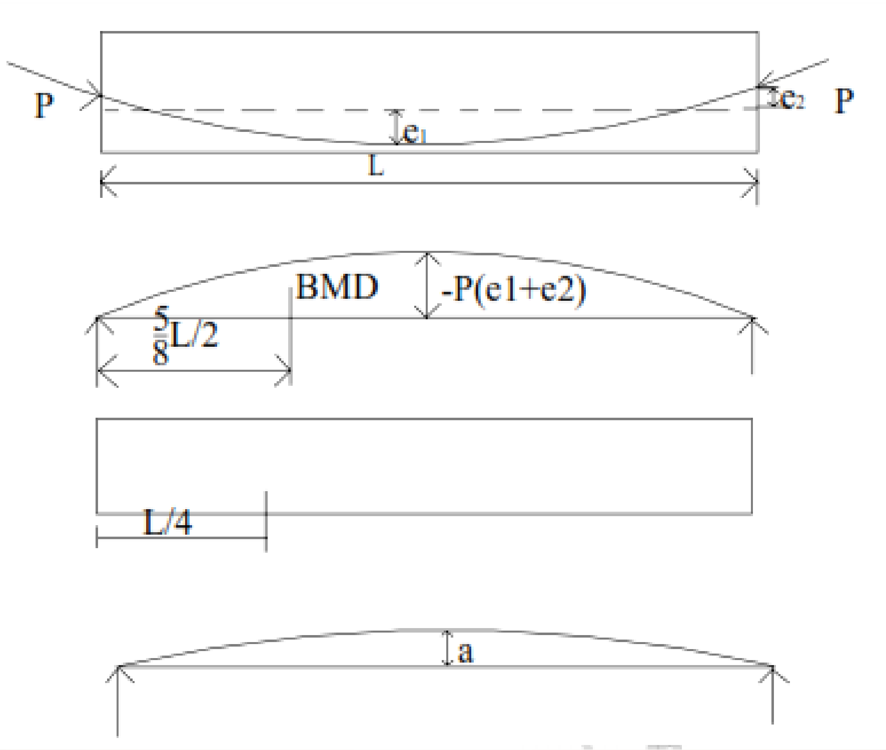
3. Parabolic Tendons (Concentric Anchors)



$$a = -\left(\frac{5PeL^2}{48EI}\right)$$

EFFECT OF TENDON PROFILE ON DEFLECTIONS

4. Parabolic Tendons (Eccentric Anchors)

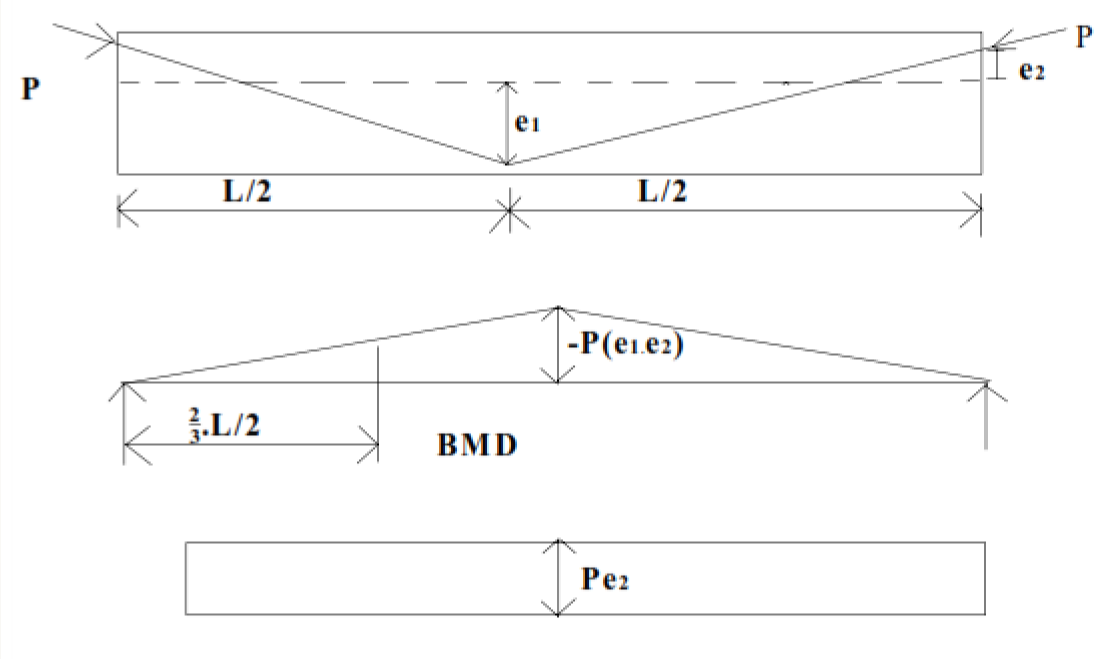


$$a = -\left(\frac{5PL^2}{48EI}(e_1 + e_2)\right) + \left(\frac{Pe_2L^2}{8EI}\right)$$

$$= \frac{PL^2}{48EI}(-5e_1 + e_2)$$

EFFECT OF TENDON PROFILE ON DEFLECTIONS

5. Sloping Tendons (Eccentric Anchors)

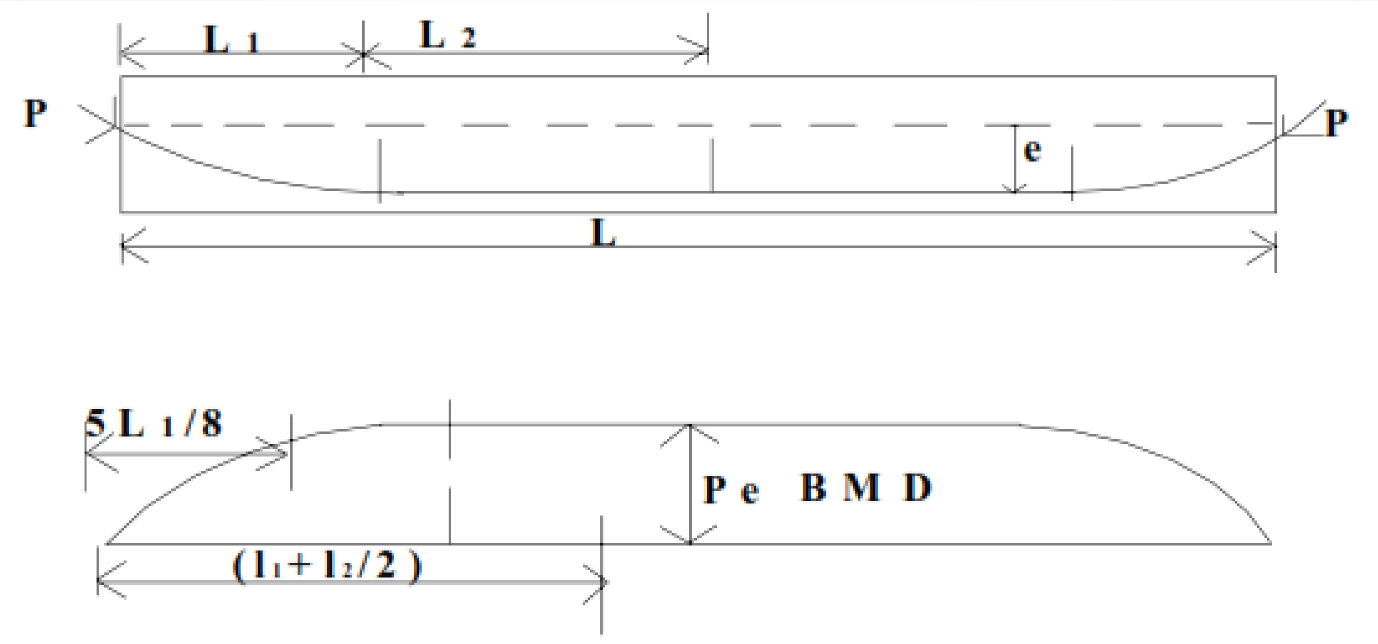


$$a = -\left(\frac{PL^2}{12EI}(e_1 + e_2)\right) + \left(\frac{Pe_2L^2}{8EI}\right)$$

$$= \frac{PL^2}{24EI}(-2e_1 + e_2)$$

EFFECT OF TENDON PROFILE ON DEFLECTIONS

6. **Parabolic and Straight Tendons**

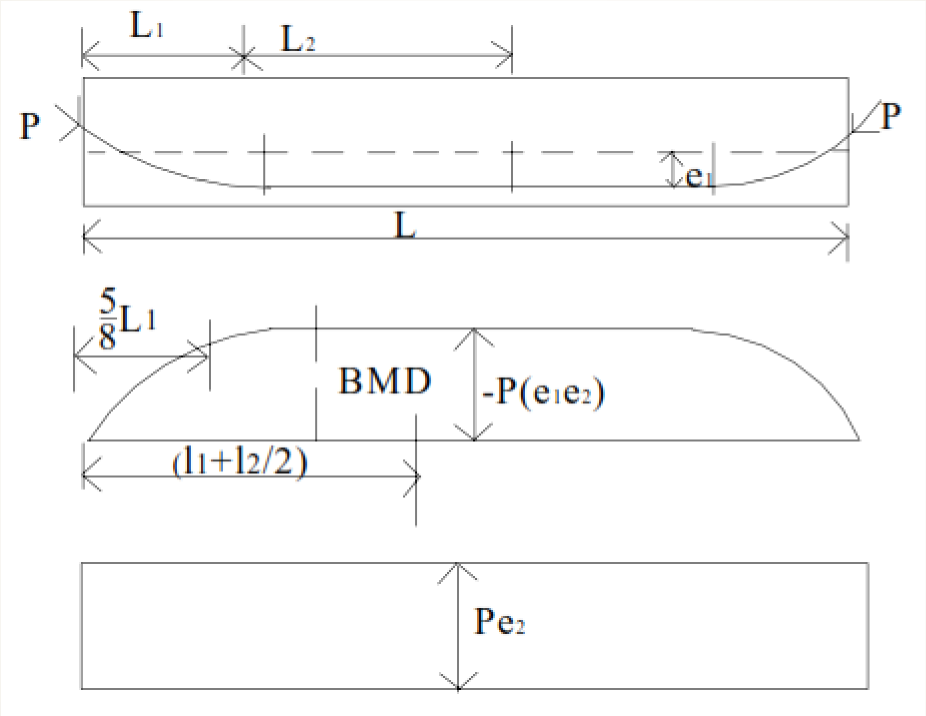


$$a = -\frac{Pe}{EI}\left[\frac{2}{3}l_1\left(\frac{5}{8}l_1\right) + l_2\left(l_1 + \frac{l_2}{2}\right)\right]$$

$$= -\frac{Pe}{EI}\left[(5l_1^2 + 12l_1l_2 + 6l_2^2)\right]$$

EFFECT OF TENDON PROFILE ON DEFLECTIONS

7. Parabolic and Straight Tendons (Eccentric Anchors)



$$a = -\frac{P(e_1+e_2)}{EI}[5l_1^{\,2}+12l_1l_2+6l_2^{\,2}]+\left[\frac{Pe_2L^2}{8EI}\right]$$

DEFLECTIONS DUE TO SELF-WEIGHT AND IMPOSED LOADS

- At the time of transfer of prestress, the beam hogs up due to effect of prestressing.
- At this stage, the self-weight of the beam induces downward deflections, which further increase due to effect of imposed load on the beam.
- If, g =Self-weight of the beam / m
- q = Imposed load / m (uniformly distributed)
- The downward deflection is computed as,

$$a = \frac{5(g+q)wl^4}{384EI}$$

Ex: A rectangular concrete beam of cross-section 150 mm wide and 300 mm deep is simply supported over a span of 8 m and is prestressed by means of a symmetric parabolic cable at a distance of 75 mm from the bottom of the beam at mid span and 125 mm from the top of the beam at support sections. If the force in the cable is 350 kN and the modulus of elasticity of concrete is 38 kN/mm^2, calculate:

(a) The deflection at mid-span when the beam is supporting its own weight and

(b) The concentrated load which must be applied at mid-span to restore it to the level of supports.

Solution :

P = 350 kN, Ec = 38 kN/mm^2, I = 3375 x 10^6 mm^4, e_1 = 75 mm and e_2 = 25 mm

Deflection due to prestressing force

$$= \frac{PL^2}{48EI}(-5e_1 + e_2)$$

$$= \left(\frac{350 \text{ x } 8000^4}{48 \text{ x } 38 \text{ x } 337.5 \text{ x } 10^6}\right)(-5 \text{ x } 75 + 25) = 12.74\text{mm (upwards)}$$

Self-weight of the beam, g = (0.15 x 0.30 x 24) = 1.08 kN/m

Downward deflection due to self-weight $= \left(\dfrac{5 \text{ x } 0.00108 \text{ x } 8000^4}{384 \text{ x } 38 \text{ x } 337.5 \text{ x } 10^6} \right) = 4.5\text{mm (downwards)}$

(a) Deflection due to (prestress + self-weight) = -12.7 + 4.5 = -8.2 mm

(b) If, Q = concentrated loads required at the center of span, Then

$$\left(\frac{QL^3}{48EI} \right) = 8.2 \text{ mm}$$

$$Q = \left(\frac{8.2 \text{ x } 48 \text{ x } 38 \text{ x } 3375 \text{ x } 10^5}{8000^3} \right) = 9.9 \text{ kN}$$

ESTIMATION OF LONG-TIME DEFLECTIONS

- Deformations of prestressed members change with time as a result of creep and shrinkage of concrete and relaxation of stress in steel.
- Deflection of prestressed members can be computed relative to a given datumn, if the magnitude and longitudinal distribution of curvatures for the beam span are known for that instant based on load history, which includes the prestressing forces and the live loads.
- The prestressed concrete member develops deformations under the influence of two usually opposing effects, which are the prestress and transverse loads.
- The net curvature (φ_t)at a section at any given stage is obtained.

$$\Phi_t = \Phi_{mt} + \Phi_{pt}$$

- Where, Φ_{mt} = change of curvature caused by transverse loads
- Φ_{pt} = change of curvature caused by prestress
- Under the section of sustained transverse loads , the compressive stress distribution in the concrete changes with time.

- However, in practical cases, the change of stress being small, it may assumed that the concrete creeps under constant stress.
- The creep strain due to transverse loads is directly computed as a function of the creep coefficient so that the change of curvature can be estimated by the expression,

$$\Phi_{mt} = (1 + \Phi)\,\varphi_i$$

- Where, Φ = Creep coefficient and
- φ_i = Initial curvature immediately after the application of transverse loads
- The change of curvature due to the sustained prestress (Φ_{pt}) upon the cumulative effects of creep and shrinkage of concrete and relaxation of stress in steel.
- Several methods have been proposed to evaluate the curvature under simplified assumptions.
- The important ones are attributed to Busemann, McHenry, Douglass and Corley, Sozen and Siess.

- According to Neville and the ACI committee report, the creep curvature due to prestress is obtained on the simplified assumption that creep is induced by the average prestress acting over the given time. Using this approach,
- P_i = Initial prestress and P_t = Prestress at a time, t
- Loss of prestressing force due to relaxation,
- $L_p = (P_i – P_t)$
- If, e = eccentricity of the prestressing force at a section and
- EI = flexural rigidity
- The curvature due to prestressing force after time t can be expressed as

$$\varphi_{pt} = -\frac{P_i e}{EI}\left(1 - \frac{L_p}{P_i} + \left(1 - \frac{L_p}{2P_i}\right)\varphi\right)$$

- If, a_{i1} = Initial deflection due to transverse loads
- a_{ip} = Initial deflection due to prestress
- Then, the total long time deflection after time t is obtained from the expression,

$$a_f = a_{i1}(1+\phi) - a_{ip}\left(\left(1-\frac{L_p}{P_i}\right)+\left(1-\frac{L_p}{2P_i}\right)\varphi\right)$$

- In this expression, the negative sign refers to deflections in upward direction (camber).

- A much simplified but an approximate is suggested by **Lin** for computing long time deflections.
- In this method, the initial deflection due to prestress and transverse loads is modified to account for the loss of prestress which tends to decrease the deflection, and the creep effect which tends to increase the deflection.
- The principle of reduced modulus involving the creep coefficient is used to amplify the initial deflections.
- According to this method. the final long time deflection is expressed as,

$$a_f = \left[a_{i1} - a_{ip} \times \frac{P_t}{P_i} \right] (1+\varphi)$$

Ex: A simply supported beam with a uniform section spanning over 6 m is post-tensioned by two cables, both of which have an eccentricity of 100 mm below the centroid of the section at mid-span. The first cable is parabolic and is anchored at an eccentricity of 100 mm above the centroid at each end, the second cable is straight and parallel to the line joining the supports. The cross-sectional area of each cable is 100 mm^2 and they carry an initial stress of 1200 N/mm^2. The concrete has a cross-section of 20000 mm^2 and a radius of gyration of 120 mm. The beam supports two concentrated loads of 20 kN each at the third points of the span, E_c = 38 kN/mm^2. Calculate using Lin's simplified method. Take creep coefficient as 2.

(a) The instantaneous deflection at the center of span and

(b) The deflection at the center of span after 2 years, assuming 20 per cent loss in prestress and the effective modulus of elasticity to be one-third of the short-term modulus of elasticity.

Solution :

- $A = 20000\ mm^2$, $i = 120\ mm$ ➔ $I = 288 \times 10^6\ mm^4$
- $P = 120\ kN$, $e_1 = e_2 = 100\ mm$, $L = 6000\ mm$
- Self-weight of the beam = 0.00048 kN/mm
- Concentrated loads at the third points of span, Q = 20 kN
- Downward deflection due to self-weight

$$= \left(\frac{5 \text{ x } 0.00048 \text{ x } 6000^4}{384 \text{ x } 38 \text{ x } 288 \text{ x } 10^6}\right) = 0.74\text{mm (downwards)}$$

- Downward deflection due to concentrated loads

$$= \frac{23QL^3}{648EI} = \left(\frac{23 \text{ x } 20 \text{ x } 6000^3}{648 \text{ x } 38 \text{ x } 288 \text{ x } 10^6}\right) = 14.10\text{mm (upwards)}$$

- Deflection due to parabolic cable

$$= \frac{PL^2}{48EI}(-5e_1 + e_2) = 3.29\text{mm (upwards)}$$

- Deflection due to prestressing of straight cable

$$= -\frac{PeL^2}{8EI} = 4.92\text{mm (upwards)}$$

- Instantaneous deflection due to prestress + dead load + live load

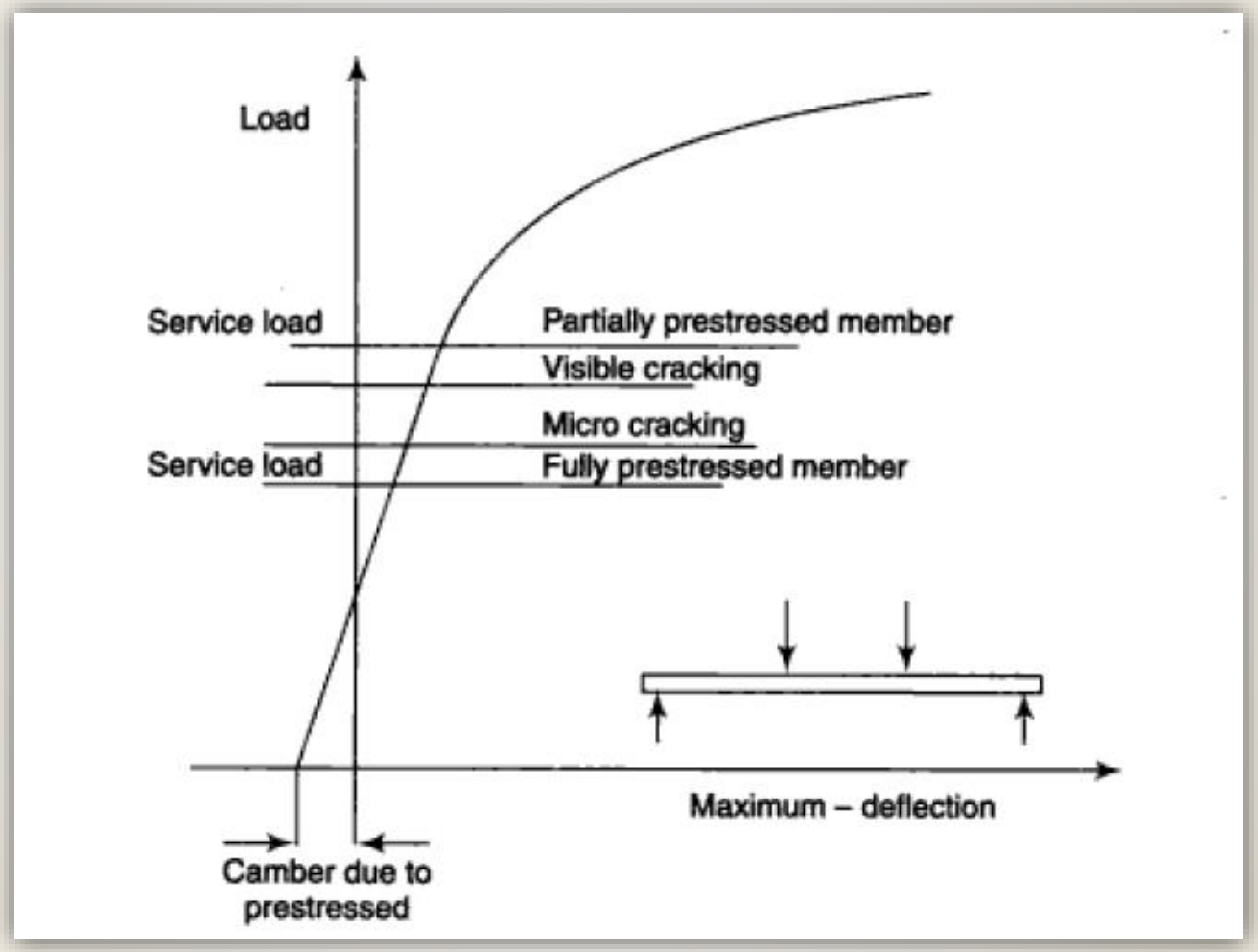
= + 0.74 + 14.10 – 3.29 – 4.92 = 6.65 mm (downwards)

b) At the end of two years, $E_{ce} = E_c / 3$ and loss of prestress = 20%

- Upward deflection = 3 [0.8 (3.27 + 4.29)] = 19.65 mm
- Downward deflection = 3 (0.74 + 14.10) = 44.52 mm
- Net deflection = -19.65 + 44.52 = 24.87 mm

DEFLECTION OF CRACKED MEMEBRS

Short term Deflections of Cracked Members



Load-Deflection Characteristics of Typical Prestressed Member under Flexural Loading

DEFLECTIONS OF CRACKED MEMBERS

- When the tensile stresses exceeds the tensile strength of the concrete, cracks develop in the member.
- Experimental Investigations
  - Micro-cracks - 3 N/mm^2
  - Visible cracks - 3.5 to 7 N/mm^2
- Curve is approximately linear up to stage of visible cracking.
- Deflection increases at a faster rate beyond this stage due to reduced stiffness of the beam.
- In post cracking stage behavior is same as that of RC members.

- Deflection of cracked structural concrete members may be estimated by the unilinear or bilinear method recommended by European concrete committee.

Unilinear Method

$$a = \frac{\beta L^2 M}{E_c I_r}$$

Where, a = maximum deflection

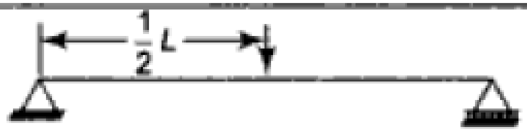
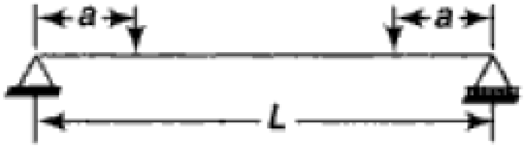
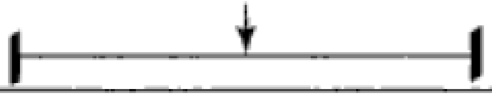
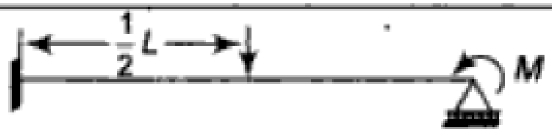
L = effective span

M = Maximum bending moment in the beam

E_c = Modulus of elasticity of concrete

I_r = Second moment of area of equivalent or transformed cracked section

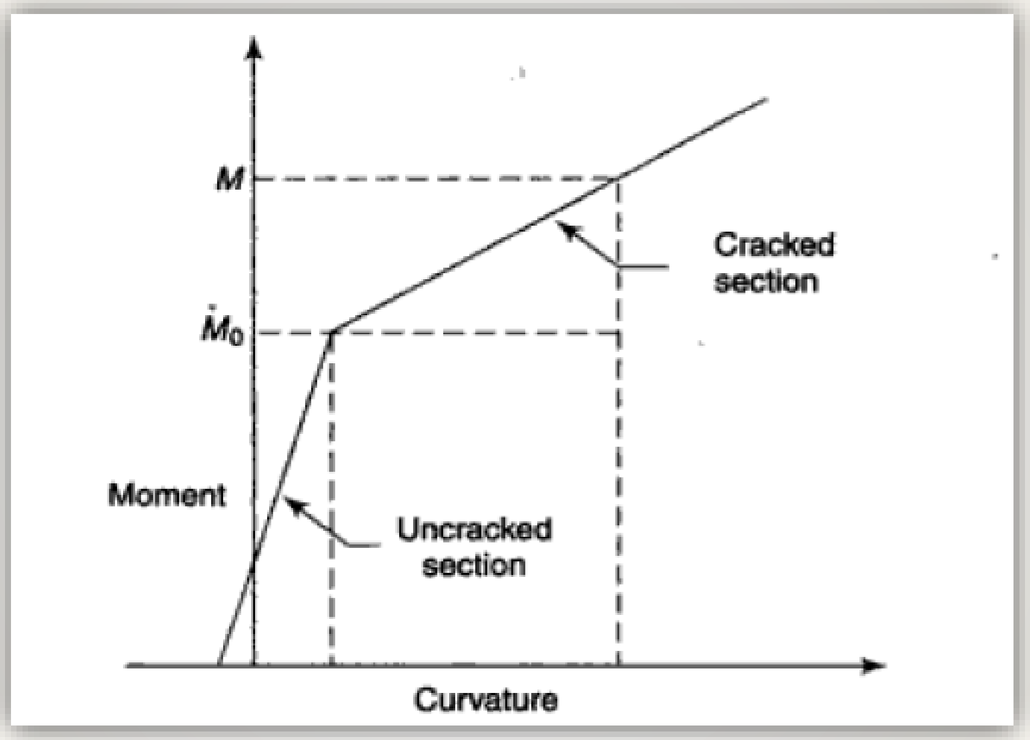
β = Constant depending upon the end conditions, position of the given section and load distribution

Support conditions and load	Values of β
	$\frac{5}{48}$
	$\frac{1}{12}$
	$\frac{1}{24}\left[3-4\left(\frac{a}{L}\right)^2\right]$
	$\frac{1}{8}$
	$\frac{1}{24}$
	$\frac{1}{16}$
	$\frac{1}{3}$
	$\frac{1}{4}$
	$\frac{1}{20}$
	$\frac{1}{23}$

Values of constant β for different types of loading and support

Bilinear Method

- The moment curvature is approximated by two straight lines.



Bilinear Moment Curvature Relationship

- Slope of the first line - stiffness of the un-cracked section
- Slope of the second line - stiffness of the cracked section

- Instantaneous deflection in the post cracking stage = deflection upto the cracking load based on gross section + deflection beyond the cracking load for cracked section.
- Deflection is estimated by the following expression:

$$a = \beta L^2 \left[\frac{M_{cr}}{E_c I_c} + \left(\frac{M - Mcr}{0.85\, EcIr}\right)\right]$$

Where, M_{cr} = Cracking moment

M = Moment at which the deflection is required

I_c = Second moment of area of the un-cracked equivalent concrete section

I_r = Second moment of area of the cracked equivalent concrete section

β = Constant depending upon the end conditions, position of the given section and load distribution

- The revised American code (ACI 318-1989) considers the bilinear character of the load-deflection characteristic by incorporating a suitable effective value of flexural rigidity in the unilinear formula.
- The modulus of elasticity is expressed as a function of the cylinder compressive strength of the form,

$$E_c = W_c^{1.5}\, 0.043 \sqrt{f_c'} \ (\text{N/mm}^2)$$

- Above equation is used when density of concrete (W_c) is between 1500 to 2500 kg/m^3.
- For normal density concrete, modulus of elasticity is expressed as

$$E_c = 4700 \sqrt{f_c'}$$

- Where, f_c' = cylindrical compressive strength in N/mm^2
- Conversion from Cylindrical to Cubical Strength

$$f_{bk} = \frac{(fc_k - 1.77)}{0.83}$$

- Effective Moment of Inertia is expressed as

$$I_{eff} = \left(\frac{M_{cr}}{M}\right)^{a} I_{gr} + \left[1 - \left(\frac{M_{cr}}{M}\right)^{a}\right] I_{cr} \le I_{gr}$$

Where, M = Moment at which deflection is required

M_{cr}= Cracking moment = $\frac{f_{cr} Ig_r}{y_t}$

I_{gr} = MI of the gross section about the centroidal axis neglecting the reinforcement

y_t = Distance from the centroidal axis of the gross section neglecting the reinforcement to the extreme fibre in tension

I_{cr} = MI of the cracked transformed section

I_{eff} = Effective MI for computation of deflection

f_{cr} = Modulus of rupture of concrete = 0.7 * $\sqrt{f_{ck}}$

Ex: A prestressed concrete beam having a c/s area (A) of 5×10^4 mm^2 is simply supported over a span of 10 m. It supports a UDL (imposed) of 3kN/m, half of which is non-permenanant. The tendon follows a trapezoidal profile with an eccentricity of 100mm within the middle-third of the span and varies linearly from the third-span points to zero at the supports. The area of tendons, A_p = 350 mm^2 have effective prestress of 1290 N/mm^2 immediately after transfer. Using the following data, calculate

1. **Short term deflections and**
2. **Long term deflections**

Assume, I_g = 4.5E8 mm^2, D_c = 23.6 kN/m^3, E_c = 34 kN/mm^2, Creep co-efficient = 2, Concrete shrinkage = 450E-6, E_s = 200 kN/mm^2, Relaxation of steel stress = 10%

1. Short term deflection

Prestressing force, P= 350 x 1290 = 451500 N

Self weight of the beam = Dc x A = 1.18 kN/m

Non-permanent load = 1.5 kN/m

Permanent load = DL + Sustained LL = 1.18 +1.5 = 2.68 kN/m

(i) Deflection due to prestressing force

$$a_p = \frac{-Pe}{6EI}[2L_1^2 + 6L_1L_2 + 3L_2^2]$$

L_1 = 3.33 m , L_2 = 1.66 m, e = 100 mm

= -31 mm (upwards)...........................(1)

(ii) Deflection due to non-permanent load (LL)

$$a_q = \left(\frac{5qL^4}{384EI}\right)$$

= 12.8 mm (downwards)...............(2)

(iii) Deflection due to permanent load (sustained load)

$$a_g = \left(\frac{5gL^4}{384EI}\right)$$

= 22.8 mm (downwards)...............(3)

Short term deflections:

a) When non-permanent load is acting a_s = (1) + (2) + (3) = -31 + 12.8 + 22.8 = 4.6 (downwards)

b) When non-permanent load is not acting a_s = -31 + 22.8 = -8.2 (upwards)

2. Long-term deflection

Stress in concrete at level of steel,

$$f_c = \left(\frac{451.5\ X\ 1000}{5E4}\right) + \left(\frac{451.5\ X\ 1000\ X\ 100\ X\ 100}{4.5E8}\right) = 19\ N/mm^2$$

a_e = 5.88

a) Loss due to relaxation = 10% = 129 N/mm^2

b) Loss due to shrinkage = $(450 \times 10^{-6} \times 200 \times 10^3)$ = 90 N/mm^2

c) Loss due to creep = (2 x 5.88 x 19) = 223 N/mm^2

Total loss = 442 N/mm^2

Loss of prestressing force = (442 x 350) = 154700 N

Final prestressing force = 451.5 – 154.7 = 296.8 kN

Average prestressing force = $\frac{451.5 + 296.8}{2}$ = 374.15 kN

(i) Long term deflection due to prestress

a_{lp} = (1) - Deflection due to loss of prestress + (Deflection due to avg prestressing force due to creep with φ = 2)

$$= 31 - \left(\frac{154.7\ X\ 31}{451.5}\right) + \left(\frac{374.15\ X\ 31}{451.5}\right)2 = 31 - 10.6 + 51.4 = 72 \text{ mm (upwards)}......................(4)$$

(ii) Long-term deflection due to permanent load

a_{lg}= (1 + φ) (short term deflection) = (1 + 2) 22.8 = 68.4 mm (downwards)….(5)

(iii) Long-term deflection due to non-permanent load

a_{lq} = 12.8 mm (downwards)……………………………………………….(6)

Total long term deflection

a) When non-permanent load is acting a_l = -(4) + (5) + (6)=9.2mm(downwards)

b) When non-permanent load is not acting a_l = -(4) + (5) = -3.6mm (upwards)

Requirement of various codes of practice

IS : 1343 – 1980

1) Final deflection including effects of temp., creep and shrinkage should not exceed span /250.
2) Deflection including effects of temp., creep and shrinkage occuring after the erection of partition and application of finishes should not exceed span/350 or 20 mm whichever is less.
3) If finishes are to be applied to be prestressed concrete members, the total upward deflection should not exceed span/300, unless uniformity of camber between two adjacent units can be assured.

Contd...

British code (BS : 8110 – 1985)

A maximum deflection limit of span/250, beyond which the sag in member will usually become noticable.

To prevent damage in non-structural elements, deflection after installation of finishes and finishes should not exceed following values:

1) Span/250 or 20 mm, whichever is less for brittle materials.
2) Span/2350 or 20 mm, whichever is less for non-brittle partitions or finishes.

Contd…

FIP - 1984

Suitable deflections for floors, roofs and other horizontal members in buildings are as folllows:

1) **Total deflection below level of supports: span/200 to span/300**
2) **Deflection after addition of partitions: span/500 to span/1000**