

# LOSSES OF PRESTRESS

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# Nature of Losses of Prestress

- Initial prestress in concrete undergoes a gradual reduction with time from the stage of transfer due to various causes and is referred as “**Loss of Prestress**”.
- From design point of view, a reasonable good estimate of such is required.

No	Pretensioning
1.	Elastic deformation of concrete
2.	Relaxation of stress in steel
3.	Shrinkage of concrete
4.	Creep of concrete

No	Post-tensioning
1.	<ul style="list-style-type: none"> <li>•No loss due to elastic deformation if all wires are tensioned simultaneously</li> <li>•If wires are tensioned successively then there will be loss due to elastic deformation of concrete</li> </ul>
2.	Relaxation of stress in steel
3.	Shrinkage of concrete
4.	Creep of concrete
5.	Friction
6.	Anchorage sleep

- In addition to above, there may be **losses of prestress due to sudden changes in temperature, especially in steam curing of pretensioned units.**
- The rise in temperature causes a partial transfer of prestress **(due to elongation of the tendons between adjacent units in long-line process)** which may cause a large amount of creep if the concrete is not properly cured.
- If there is possibility of a change of temperature between the times of tensioning and transfer, the corresponding loss should be allowed for in the design.

# Loss due to Elastic Deformation of Concrete

- ▶ The loss of pre-stress due to elastic deformation of concrete depends on the modular ratio and the average stress in concrete at the level of steel.

- ▶ If  $f_c$  = Prestress in concrete at the level of steel

$E_s$  = Modulus of elasticity of steel

$E_c$  = Modulus of elasticity of concrete

Modular ratio,  $\alpha_e = E_s / E_c$

Strain in concrete at the level of steel =  $f_c / E_c$

Stress in steel corresponding to this strain =  $(f_c / E_c) \times E_s$

Loss of stress in steel =  $\alpha_e f_c$

If the initial stress in steel is known, % loss of stress in steel due to the elastic deformation of concrete can be computed.

**EX:** A pretensioned concrete beam, 100 mm wide by 300 mm deep, is prestressed by straight wires carrying an initial force of 150 kN at an eccentricity of 50 mm.  $E_s = 210 \text{ kN/mm}^2$ ,  $E_c = 35 \text{ kN/mm}^2$ . Estimate the percentage loss of stress in steel due to elastic deformation of concrete if the area of steel wires is  $188 \text{ mm}^2$ .

➤  $P = 150 \text{ kN}$

➤  $e = 50 \text{ mm}$

➤  $A = 3E4 \text{ mm}^2$

➤  $I = 225E6 \text{ mm}^4$

➤  $\alpha_e = E_s / E_c = 6$

➤ Initial stress in steel =  $(150E3 / 188) = 800 \text{ N/mm}^2$

➤ Stress in concrete,  $f_c = (150E3 / 3E4) + (150E3 \times 50 \times 150 / 225E6) = 6.66 \text{ N/mm}^2$

➤ Loss of stress due to elastic deformation of concrete =  $\alpha_e \times f_c = (6 \times 6.66) = 40 \text{ N/mm}^2$

➤ Percentage loss of stress in steel =  $40 \times 100 / 800 = 5\%$

**EX:** A post-tensioned concrete beam, 100 mm wide by 300 mm deep, is prestressed by three cables, each with a cross-sectional area of 50 mm<sup>2</sup> and with an initial stress of 1200 N/mm<sup>2</sup>. All 3 cables are straight and located 100 mm from the soffit of the beam. If  $\alpha_e = 6$ , calculate loss of stress in 3 cables due to elastic deformation of concrete for following cases:

a) Simultaneous tensioning and anchoring of all the cables  
 b) Successive tensioning of 3 cables, one at a time.

► Force in each cable,  $P = (50 \times 1200) = 60E3 \text{ N}$

►  $A = 3E4 \text{ mm}^2$

$I = 225E6 \text{ mm}^4$

►  $e = 50 \text{ mm}$

$y = 50 \text{ mm}$

► Stress in concrete at level of steel,  $f_c = (P/A) + (Pe/Z) = 2.7 \text{ N/mm}^2$

a) When the cables are tensioned simultaneously

Loss of stress = 0

b) When the cables are tensioned successively

**Cable 1 is tensioned and anchored** – no loss due to elastic deformation

**Cable 2 is tensioned and anchored** – loss of stress in cable 1 is given by

$$\alpha_e f_c = (6 \times 2.7) = 16.2 \text{ N/mm}^2$$

**Cable 3 is tensioned and anchored** – loss of stress in cable 1 & 2 is given by,

$$\text{Loss of stress in cable 1} = (6 \times 2.7) = 16.2 \text{ N/mm}^2$$

$$\text{Loss of stress in cable 2} = (6 \times 2.7) = 16.2 \text{ N/mm}^2$$

► Total loss in stress due to elastic deformation of concrete is

$$\text{Cable 1} = (16.2 + 16.2) = 32.4 \text{ N/mm}^2$$

$$\text{Cable 2} = 16.2 \text{ N/mm}^2$$

$$\text{Cable 3} = 0$$

**Average loss of stress considering all cables = 16.2 N/mm<sup>2</sup>**

Note: It can be shown that if the number of wires, bars or strands are large, the loss due to elastic shortening approaches (but does not exceed) one-half of the corresponding load with pretensioning, i.e. loss of stress =  $(1/2) \times \alpha_e \times f_c$

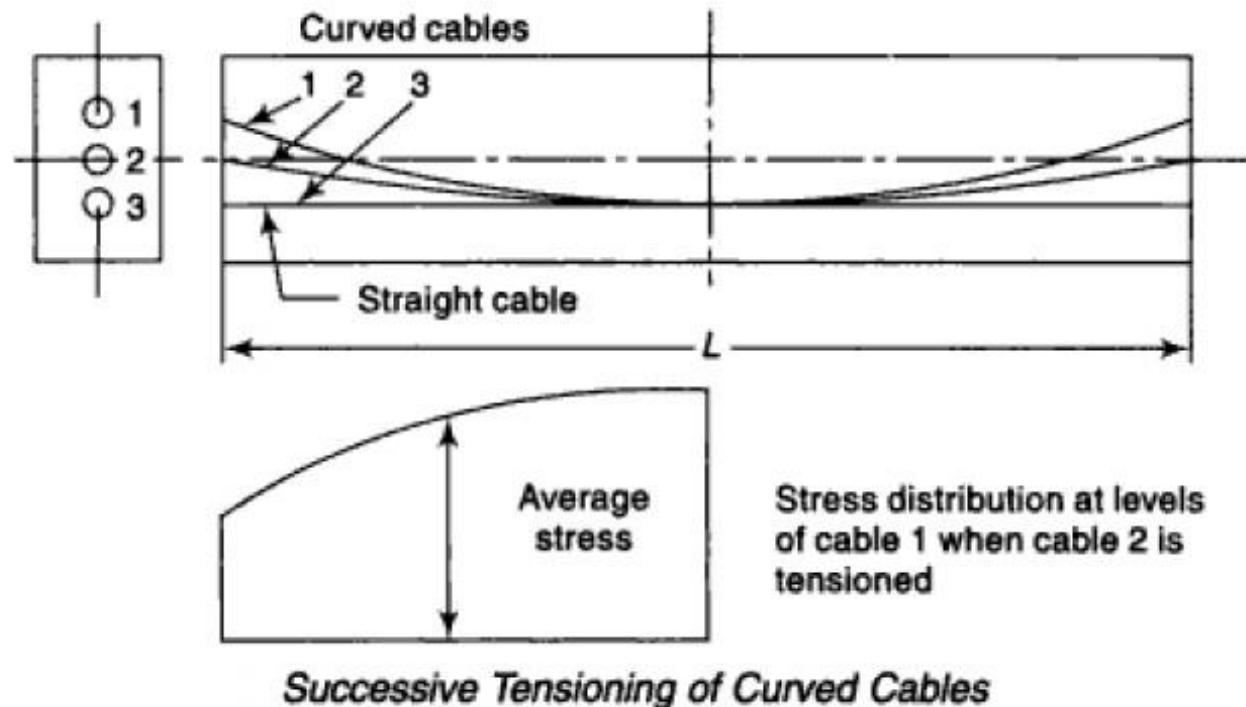
Where,  $f_c$  = stress in concrete at the level of steel due to the effect of all the cables simultaneously tensioned.

**Applying this to above problem,**

$$\text{Loss of stress} = (1/2) \times 6 \times (3 \times 2.7) = 24.3 \text{ N/mm}^2$$

# Loss of Stress due to Successive Tensioning of Curved Cables

- In most bridge girders cables are curved with maximum eccentricity at the centre of the span.
- In such cases the loss of stress due to the elastic deformation of concrete is estimated by considering the average stress in concrete at the level of steel.
- Consider a beam shown in figure, which is post-tensioned by 3 parabolic cables.
- The stress distribution in concrete at the level of cable 1 is also shown in figure, when cable 2 is tensioned.
- For computing the loss of stress, the average stress (shown in figure) is considered. When cable 3 is tensioned, there will be losses of stress in both cables 1 & 2.





**EX:** A post-tensioned concrete beam, 100 mm wide by 300 mm deep, spanning over 10 m is stressed by successive tensioning and anchoring of 3 cables 1, 2 and 3 respectively. The c/s area of each cable is 200 mm<sup>2</sup> and the initial stress in the cable is 1200 N/mm<sup>2</sup>.  $\alpha_e = 6$ . The first cable is parabolic with an eccentricity of 50 mm below the centroidal axis at the centre span and 50 mm above the centroidal axis at the support sections. The second cable is parabolic with zero eccentricity at the supports and an eccentricity of 50 mm at the centre span. The 3<sup>rd</sup> cable is straight with a uniform eccentricity of 50 mm below the centroidal axis. Estimate the % loss of stress in each of the cables, if they are successively tensioned and anchored.

➤ Force in each cable,  $P = 240 \text{ kN}$

➤  $A = 3E4 \text{ mm}^2$

➤  $I = 225E6 \text{ mm}^4$

➤  $\alpha_e = E_s / E_c = 6$

➤ When cable 1 is tensioned and anchored – no loss of stress due to elastic deformation of concrete.

➤ When cable 2 is tensioned and anchored – stress at the level of cable 1 is given by,

$$\text{Stress at support section} = (240E3 / 3E4) = 8 \text{ N/mm}^2$$

➤ Stress at the centre of span =  $(240E3 / 3E4) + (240E3 \times 50 \times 50 / 225E6) = 10.7 \text{ N/mm}^2$

➤ Average stress in concrete =  $8 + (2 \times 2.7 / 3) = 9.8 \text{ N/mm}^2$

➤ Loss of stress in cable 1 =  $(6 \times 9.8) = 58.8 \text{ N/mm}^2$

- When cable 3 is tensioned and anchored, stress distribution at the levels of cable 1, cable 2 and the average stress and loss of stress is obtained as follows:

	Cable 1	Cable 2
<b><u>Stress at support</u></b>	$(240E3 / 3E4) - (240E3 \times 50 \times 50 / 225E6)$ <b>= 5.3 N/mm<sup>2</sup></b>	$(240E3 / 3E4)$ <b>= 8 N/mm<sup>2</sup></b>
<b><u>Stress at the center of span</u></b>	$(240E3 / 3E4) + (240E3 \times 50 \times 50 / 225E6)$ <b>= 10.7 N/mm<sup>2</sup></b>	$(240E3 / 3E4) + (240E3 \times 50 \times 50 / 225E6)$ <b>= 10.7 N/mm<sup>2</sup></b>
<b><u>Average stress in concrete</u></b>	$[5.3 + (2 \times 5.4 / 3)]$ <b>= 8.9 N/mm<sup>2</sup></b>	$[8 + (2 \times 2.7 / 3)]$ <b>= 9.8 N/mm<sup>2</sup></b>
<b><u>Loss of stress in cable</u></b>	$(6 \times 8.9) =$ <b>53.4 N/mm<sup>2</sup></b>	$(6 \times 9.8) =$ <b>58.8 N/mm<sup>2</sup></b>

<b>Total losses</b>		
<b>Cable No.</b>	<b>Loss of stress</b>	<b>Percentage Loss</b>
Cable 1	$58.8 + 53.4$ <b>= 112.2 N/mm<sup>2</sup></b>	<b>9.8%</b>
Cable 2	<b>58.8 N/mm<sup>2</sup></b>	<b>4.9%</b>
Cable 3	<b>No loss of stress</b>	<b>0%</b>

**EX:** A simply supported concrete beam of uniform section is post-tensioned by means of two cables, both of which have an eccentricity of 100 mm below centroid of the section at mid-span. The first cable is parabolic and is anchored at an eccentricity of 100 mm above the centroid at each end. The second cable is straight and parallel to the line joining the supports. If the c/s area of each cable is  $100 \text{ mm}^2$ , the concrete beam has a sectional area of  $2E4 \text{ mm}^2$  and  $i = 120 \text{ mm}$ , calculate the loss of stress in the first cable when the second is tensioned to a stress of  $1200 \text{ N/mm}^2$ . take the  $\alpha_e = 6$  and neglect friction.

➤  $A = 2E4 \text{ mm}^2$

➤  $i = 120 \text{ mm}$

➤  $I = (2E4 \times 120^2) = 288E6 \text{ mm}^4$

➤ Prestressing force,  $P = (1200 \times 100) = 12E4 \text{ N}$

➤ When cable 2 is tensioned and anchored, stress at the level of cable 1 is given by,

➤ Stress in concrete =  $(12E4 / 2E4) \pm (12E4 \times 100 \times 100 / 288E6) = (6 \pm 4.2)$

=  $10.2 \text{ N/mm}^2$  at central section and  $1.8 \text{ N/mm}^2$  at end section

Average stress in concrete =  $[1.8 + (2/3) 8.4] = 7.4 \text{ N/mm}^2$

Loss of stress in cable 1 =  $(6 \times 7.4) = 44.4 \text{ N/mm}^2$

# Loss due to Shrinkage of Concrete

- Shrinkage of concrete in pre-stressed members results in a shortening of tensioned wires and hence contributes to the loss of stress.
- Shrinkage of concrete is affected by the type of cement, aggregates and method of curing used.
- Use of high strength concrete with low water cement ratios results in a reduction in shrinkage and consequent loss of pre-stress.
- Primary cause of drying shrinkage → loss of water
- For pre-tensioned members, generally moist curing is done until time of transfer.
- So, total residual shrinkage strain will be larger.

*IS: 1343*

$$\text{Loss of Stress} = \epsilon_{cs} \times E_s$$

$\epsilon_{cs}$  = total residual shrinkage strain = 300E-6 for prestressing

$\left[ \frac{200 \times 10^{-6}}{\log_{10}(t+2)} \right]$  for post-tensioning

Where, t = age of concrete at transfer in days

**EX:** A concrete beam is pre-stressed by a cable carrying an initial pre-stressing force of 300 kN. The c/s area of wires in cable is 300 mm<sup>2</sup>. Calculate % loss of stress in the cable only due to shrinkage of concrete using IS:1343 assuming beam to be a) Pre-tensioned & b) post-tensioned.  $E_s = 210 \text{ kN/mm}^2$ , age of concrete at transfer = 8 days.

➤  $P_i = \left( \frac{300E3}{300} \right) = 1000 \text{ N/mm}^2$

a) If the beam is pre-tensioned and residual shrinkage is strain = 300E-6

Loss of pre-stress =  $(300E-6 \times 210E3) = 63 \text{ N/mm}^2$

% loss of stress =  $\left( \frac{63 \times 100}{1000} \right) = 6.3\%$

b) If the beam is post-tensioned and residual shrinkage is strain =  $\left[ \frac{200 \times 10^{-6}}{\log_{10}(t+2)} \right] = 200E-6$

Loss of prestress =  $(200E-6 \times 210E3) = 42 \text{ N/mm}^2$

% loss of stress =  $\left( \frac{42 \times 100}{1000} \right) = 4.2\%$

# Loss due to Creep of Concrete

- Sustained pre-stress in the concrete of a prestressed member results in creep of concrete which effectively reduces the stress in high-strength tensile steel.
- Factors influencing creep
- Loss of stress in steel due to creep of concrete can be estimated if the magnitude of **ultimate creep strain** or **creep co-efficient** is known.

## Methods:

### 1. Ultimate creep strain method

If  $\epsilon_{cc}$  = ultimate creep strain for a sustained unit stress

$f_c$  = pre-stress in concrete at the level of steel

$E_s$  = Modulus of elasticity of steel

The loss of stress in steel due to creep of concrete =  $\epsilon_{cc} f_c E_s$

## 2. Creep Co-efficient Method

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If  $\varepsilon_c$  = Creep strain

$\varepsilon_e$  = Elastic strain

$\Phi$  = Creep co-efficient

$f_c$  = Stress in concrete

$E_s$  = Modulus of elasticity of steel

$E_c$  = Modulus of elasticity of concrete

$\alpha_e$  = Modular ratio

The loss of stress in steel due to creep of concrete =  $\varepsilon_{cc} f_c E_s$

➔ Creep co-efficient =  $\left( \frac{\text{Creep Strain}}{\text{Elastic Strain}} \right) = \left( \frac{\varepsilon_c}{\varepsilon_e} \right)$

➔  $\varepsilon_c = \Phi \times \varepsilon_e = \Phi \times (f_c / E_c)$

➔ Loss of stress in steel =  $\varepsilon_c E_c \Phi E_s = \Phi \times (f_c / E_c) E_s = \Phi \times f_c \times \alpha_e$

➔ The magnitude of  $\Phi$  varies depending on humidity, concrete quality, duration of applied loading and the age of concrete when loaded.

**EX:** A concrete beam of rectangular section 100 mm x 300 mm is prestressed by 5 wires of 7 mm diameter located at an eccentricity of 50 mm, the initial stress in wire is 1200 N/mm<sup>2</sup>. Estimate the loss of stress in steel due to creep of concrete using ultimate creep strain method and creep co-efficient method (IS : 1343) for following data:  $E_s = 210 \text{ kN/mm}^2$ ,  $E_c = 35 \text{ kN/mm}^2$ ,  $A = 3\text{E}4 \text{ mm}^2$ ,  $I = 225\text{E}6 \text{ mm}^4$ , Ultimate creep strain,  $\epsilon_{cc} = 41\text{E}-6 \text{ mm/mm per N/mm}^2$ , Creep co-efficient =  $\Phi = 1.6$

➤  $P = (5 \times 38.5 \times 1200) = 23\text{E}4 \text{ N}$

➤  $\alpha_e = \left(\frac{E_s}{E_c}\right) = 6$

➤ Stress in concrete at level of steel is given by,  $f_c = \left[\frac{23\text{E}4}{3\text{E}4} + \frac{23\text{E}4 \times 50 \times 50}{225\text{E}6}\right] = 10.2 \text{ N/mm}^2$

**a) Ultimate Creep Strain Method**

$$\begin{aligned} \text{Loss of stress in steel} &= \epsilon_{cc} f_c E_s \\ &= 41\text{E}-6 \times 10.2 \times 210\text{E}6 \\ &= 88 \text{ N/mm}^2 \end{aligned}$$

**b) Creep co-efficient Method**

$$\begin{aligned} \text{Loss of stress in steel} &= \Phi \times f_c \times \alpha_e \\ &= 1.6 \times 10.2 \times 6 \\ &= 97.92 \text{ N/mm}^2 \end{aligned}$$



# Loss due to Relaxation of Stress in Steel

- IS:1343 provides guidelines for calculating this loss as a % of initial stress in steel.
- IS code recommends a value varying from 0 to 90 N/mm<sup>2</sup> for stress in wires varying from 0.5f<sub>pu</sub> to 0.8f<sub>pu</sub>.
- Temporary overstressing by 5-10% for period of 2 min is sometimes used to reduce this loss for drawn wires.
- Over-stressing does not appear to be beneficial for stabilized wires which has 0.1% proof stress in excess of 85% of the tensile strength.

Relaxation losses for pre-stressing steel at 1000 h at 27 ± 2°C (IS:1343)

Sr. No.	Initial Stress	Relaxation Loss (%)	
		Normal	Low
1	0.5f <sub>pu</sub>	0	0
2	0.6f <sub>pu</sub>	0.3	1.0
3	0.7f <sub>pu</sub>	5.0	2.5
4	0.8f <sub>pu</sub>	8.0	4.5

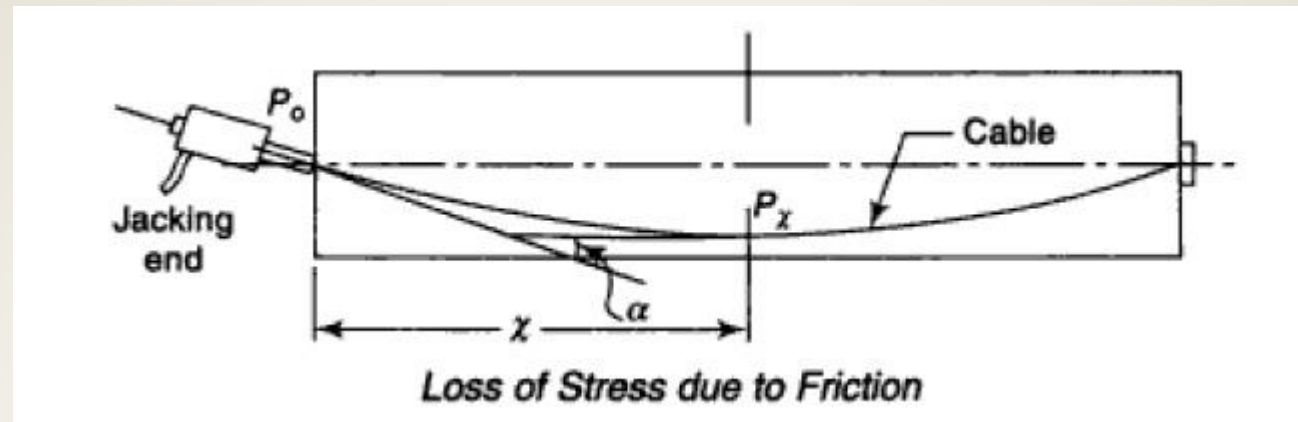
# Loss due to Friction

- For post-tensioned members, the tendons are housed in ducts preformed in concrete.
- Ducts may be either straight or follow curved profile.
- Consequently, on tensioning curved tendons, loss of stress occurs in post-tensioned members due to friction.

The magnitude of loss may be of two types:

1. Loss of pre-stress due to curvature effect
2. Loss of stress due to the wobble / wave effect

Takes place due to accidental or unavoidable misalignment.



For the beam shown in figure,

If magnitude of pre-stressing force is  $P_x$  at a distance  $x$  from the tensioning end then it follows an exponential function of the type,

$$P_x = P_0 e^{-(\mu\alpha + Kx)}$$

- $P_0$  = Pre-stressing force at jacking end
- $\mu$  = Co-efficient of friction between duct and cable
- $\alpha$  = Cumulative angle in radians through which the tangent to the cable profile has turned between any two points under consideration
- $K$  = Friction co-efficient for wave effect
- $e = 2.7183$

### Values of Co-efficient of friction $\mu$

Condition	Value
Steel moving on smooth concrete	0.55
Steel moving on steel fixed to duct	0.35
Steel moving on steel fixed to concrete	0.25
Steel moving on lead	0.25
Multi layer wire rope cables in rigid rectangular steel sheaths	0.18-0.30
Multi layer wire rope cables with spacer plates providing lateral separation	0.15-0.25

### Values of friction co-efficient for wave effect k

Condition	Value
Normal Conditions	0.15 per 100 m
For Thin walled ducts and where heavy vibrations are encountered, other adverse conditions	1.5 per 100 m

k may be reduced to 0 where

- The clearance between the duct and cable is sufficiently large to eliminate the 'wave effect'.
- Sheath is made of heavy gauge steel tube with water tight joints. (Deformation of the duct profile is prevented during vibration of concrete)

## How to Reduce Frictional Losses?

- By using various lubricants
- Over tensioning the tendons by an amount equal to maximum frictional loss
- Jacking the tendons from both the ends of the beam (generally adopted when the tendons are too long or angles of bending are large).

**EX:** A concrete beam of 10 m span, 100mm wide and 300 mm deep is prestressed by 3 cables. Area of each cable is 200 mm<sup>2</sup> and initial stress in the cable is 1200 N/mm<sup>2</sup>. Cable 1 is parabolic with an eccentricity of 50 mm above the centroid at supports and 50 mm below at the centre of span. Cable 2 is also parabolic with 0 eccentricity at supports and 50 mm below centroid at the centre span. Cable 3 is straight with uniform eccentricity of 50 mm below the centroid. If the cables are tensioned from one end only, estimate the % loss of stress in each cable due to friction. Assume  $\mu = 0.35$  and  $K = 0.0015$  per m.

➤ Slope = ???

➤ Equation of parabola,  $y = (4e/L^2) * x * (L - x)$

➤ Slope at the ends ( $X = 0$ )  $\frac{dy}{dx} = \left(\frac{4e}{L^2}\right) (L - 2x) = \left(\frac{4e}{L}\right)$

➤ For cable 1 Slope at End =  $\frac{4 \times 100}{10 \times 1000} = 0.04$

➤ Cumulative angle between tangents,  $\alpha = (2 \times 0.04) = 0.08$  radians

➤ For cable 2 Slope at End =  $\frac{4 \times 50}{10 \times 1000} = 0.02$

➤ Cumulative angle between tangents,  $\alpha = (2 \times 0.02) = 0.04$  radians

➤ Initial prestressing force in each cable,  $P_o = (200 \times 1200) = 24000$  N

➔ If  $P_x$  = Prestressing force (stress) in cable at the farther end,

$$P_x = P_o e^{-(\mu\alpha + Kx)}$$

For, small values of  $(\mu\alpha + Kx)$ , we can take

$$P_x = P_o [1 - (\mu\alpha + Kx)]$$

$$\text{Loss of stress} = P_o(\mu\alpha + Kx)$$

$$\text{Cable 1} = P_o(0.35 \times 0.08 + 0.0015 \times 10) = 0.043 P_o$$

$$\text{Cable 2} = P_o(0.35 \times 0.04 + 0.0015 \times 10) = 0.029 P_o$$

$$\text{Cable 3} = P_o(0 + 0.0015 \times 10) = 0.015 P_o$$

Cable No	Loss of Stress, N/mm <sup>2</sup>	Percentage loss
1	51.6	4.3
2	34.8	2.9
3	18.0	1.5

**EX:** A post-tensioned concrete beam, 200 mm wide and 450 mm deep is prestressed by a circular cable (total area = 800 mm<sup>2</sup>) with zero eccentricity at the ends and 150 mm at the centre. The span of the beam is 10 m. The cable is to be stressed from one end such that an initial stress of 840 N/mm<sup>2</sup> is available in theunjacked and immediately after anchoring. Determine the stress in the wires at the jacking end and % loss of stress due to friction. Take co-efficient of friction for curvature effect = 0.6, Friction co-efficient for wave effect = 0.003/m.

➤ Slope = ???

➤ If R is the radius of the circular cable, then  $(R - 0.15)^2 + 5^2 = R^2$

$$\rightarrow R = 84 \text{ m}$$

➤ If  $\alpha$  is the angle between the horizontal and the tangent drawn to the cable at support, then

$$\sin \alpha = 5 / 84 = 0.06 \text{ radians}$$

$$P_x = P_o [1 - (\mu\alpha + Kx)]$$

$$P_x = P_o [1 - (0.6 \times 0.12 + 0.03 \times 10)] = 0.898 P_o$$

➤  $P_o = \frac{P_x}{0.898} = \frac{840}{0.898} = 940 \text{ N/mm}^2$

➤ Loss of stress in cable = 940 – 840 = 100 N/mm<sup>2</sup>

➤ % Loss of stress = 10.6%



# Loss due to Anchorage Slip

- In most post-tensioning systems, when the cable is tensioned and jack is released to transfer prestress to concrete, the friction wedges, employed to grip the wires, slip over a small distance before the wires are firmly housed between the wedges.
- The magnitude of sleep depends upon the type of wedge and stress in the wires.
- When anchor plates are employed, it may be necessary to allow for the small settlement of the plate into the end of the concrete member.
- Loss during anchoring, which occurs with wedge type grips, is normally allowed for on the site by over-extending the tendon in the prestressing operation by the amount of the draw in before anchoring.
- This method is satisfactory provided the momentary over-stress does not exceed the prescribed limits of 80-85 % of ultimate strength of the wire.

- Magnitude of the loss of stress due to slip in anchorage is computed as follows:

$\Delta$  = slip of anchorage, mm

L = length of the cable, mm

A = Cross-sectional area of the cable, mm<sup>2</sup>

E<sub>s</sub> = Modulus of elasticity of steel, N/ mm<sup>2</sup>

P = pre-stressing force in the cable, N

$$\Delta = PL / A E_s$$

$$\text{Loss of stress due to anchorage slip} = \left( \frac{P}{A} \right) = \frac{E_s \Delta}{L}$$

- Loss of stress is caused by definite total amount of shortening.
- For shorter members the loss is higher compared to long ones.
- For long line pre-tensioning system, the slip at the anchorage is normally very small in comparison with the length of the tensioned wire and hence is generally ignored.
- For shorter member due care should be taken to allow for the loss of stress due to anchorage slip.

**EX:** A concrete beam is post-tensioned by a cable carrying an initial stress of 1000 N/mm<sup>2</sup>. the slip at the jacking end was observed to be 5 mm.  $E_s = 210 \text{ kN/mm}^2$ . Estimate the % loss of stress due to anchorage slip if the length of the beam is (a) 30 m and (b) 3 m.

**(a)** For a 30 m long beam,

$$\text{Loss of stress} = \left[ \frac{210E3 \times 5}{30 \times 1000} \right] = 35 \text{ N/mm}^2$$

$$\% \text{ loss of stress} = \frac{35}{1000} \times 100 = 3.5\%$$

**(b)** For a 3 m long beam,

$$\text{Loss of stress} = \left[ \frac{210E3 \times 5}{3 \times 1000} \right] = 350 \text{ N/mm}^2$$

$$\% \text{ loss of stress} = \frac{35}{100} \times 100 = 35\%$$

# TOTAL LOSSES ALLOWED IN DESIGN

- ▶ It is normal practice in design of pre-stressed concrete members to assume the total loss of stress as a % of the initial stress and provide for this in the design computations.
- ▶ Since the loss of pre-stress depends on several factors, such as properties of concrete and steel, method of curing, degrees of pre-stress and method of pre-stressing, so it is difficult to generalize exact amount of total loss of pre-stress.

TYPE OF LOSS	% LOSS OF STRESS	
	PRE-TENSIONING	POST-TENSIONING
Elastic shortening & Bending of concrete	4	1
Creep of concrete	6	5
Shrinkage of concrete	7	6
Creep in steel	8	8
<b>TOTAL</b>	<b>25</b>	<b>20</b>

- $f_{pe}$  = Effective stress in tendons after losses
- $f_{pi}$  = Stress in tendons at transfer
- $\eta$  = Reduction factor for loss of prestress
- $\eta = \left( \frac{f_{pe}}{f_{pi}} \right)$
- Value of  $\eta$  is generally taken as 0.75 for pretensioned and 0.80 for post-tensioned members.

**EX:** A concrete 200 mm wide and 300 mm deep, is prestressed with wires (area 320 mm<sup>2</sup>) located at a constant eccentricity of 50 mm and carrying an initial stress of 1000 N/mm<sup>2</sup>. The span of the beam is 10 m. Calculate the percentage loss of stress in wires if (a) the beam is pre-tensioned, and (b) the beam is post-tensioned, using the following data:  
 $E_s = 210 \text{ kN/mm}^2$  and  $E_c = 35 \text{ kN/mm}^2$ .  
 Relaxation of steel stress = 5 percent of the initial stress.  
 Shrinkage of concrete for pretensioning =  $300 \times 10^{-6}$  and  $200 \times 10^{-6}$  for post-tensioning. Creep co-efficient = 1.6, Slip of anchorage 1 mm  
 Frictional coefficient for wave effect = 0.0015/m

**Solution:**

- Prestressing force =  $\frac{(320 \times 1000)}{1000} = 320 \text{ kN}$
- Cross-sectional area,  $A = 60000 \text{ mm}^2$
- $\alpha_e = \left(\frac{E_s}{E_c}\right) = 6$
- Moment of Inertia,  $I = \frac{(200 \times 300^3)}{12} = 45 \times 10^7 \text{ mm}^4$

➤ Stress in concrete at the level of steel,  $f_c = \frac{(320 \times 10^3)}{6 \times 10^4} + \frac{(320 \times 10^3 \times 50 \times 50)}{45 \times 10^7} = 7 \text{ N/mm}^2$

### Losses of Stress

TYPE OF LOSS	PRETENSIONED BEAM (N/mm <sup>2</sup> )	POST-TENSIONED BEAM (N/mm <sup>2</sup> )
Elastic deformation of concrete	$6 \times 7 = 42$	--
Relaxation of stress in steel	5 % of 1000 = 50	5 % of 1000 = 50
Creep of concrete	$(1.6 \times 7 \times 6) = 67.20$	$(1.6 \times 7 \times 6) = 67.20$
Shrinkage of concrete	$(300 \times 10^{-6} \times 210 \times 10^3) = 63$	$(200 \times 10^{-6} \times 210 \times 10^3) = 42$
<i>Friction effect</i>	--	$(10 \times 0.0015 \times 1000) = 15$
<i>Slip of anchorage</i>	--	$(1 \times 210 \times 10^3) (10 \times 1000) = 21$
<b>TOTAL loss of stress</b>	<b>222.20</b>	<b>195.20</b>
<b>% loss of stress</b>	<b>22.2%</b>	<b>19.52%</b>

**EX:** A prestressed concrete pile, 250 mm square, contains 60 pre-tensioned wires, each of 2 mm diameter, uniformly distributed over the section. The wires are initially tensioned on the prestressing bed with a total force of 300 kN. Calculate the final stress in concrete and the percentage loss of stress in steel after all losses, given the following data:  
 $E_s = 210 \text{ kN/mm}^2$ ,  $E_c = 32 \text{ kN/mm}^2$ ,  
 Shortening due to creep =  $30 \times 10^{-6} \text{ mm/mm}$  per  $\text{N/mm}^2$  of stress. Total shrinkage strain =  $200 \times 10^{-6}$  per unit length. Relaxation of steel stress = 5 percent of the initial stress.

**Solution:**

- Prestressing force,  $P = 300 \text{ kN}$
- Average initial stress in concrete  $= \frac{(300 \times 1000)}{250 \times 250} = 4.8 \text{ N/mm}^2$
- $\alpha_e = \left(\frac{E_s}{E_c}\right) = 6.56$
- Initial stress in wires  $= \frac{(300 \times 10^3)}{188.4} = 1590 \text{ N/mm}^2$



### Losses of stress:

1. Elastic deformation of concrete =  $(6.58 \times 4.8) = 31.5 \text{ N/mm}^2$
2. Creep of concrete =  $(30 \times 10^{-6}) \times 4.8 \times 210 \times 10^3 = 30 \text{ N/mm}^2$
3. Shrinkage of concrete =  $(200 \times 10^{-6}) \times 210 \times 10^3 = 42 \text{ N/mm}^2$
4. Relaxation of steel stress =  $(0.05 \times 1590) = 79.5 \text{ N/mm}^2$

$$\text{Total loss} = 183 \text{ N/mm}^2$$

$$\text{Effective prestress} = (1590 - 183) = 1407 \text{ N/mm}^2$$

$$\text{Final stress in concrete} = \frac{(1407 \times 188.4)}{250 \times 250} = 4.26 \text{ N/mm}^2$$

$$\text{Percentage loss of stress in steel} = \frac{183}{1590} \times 100 = 11.6\%$$