ANALYSIS

OF

PRESTRESS

AND

BENDING STRESSES

BASIC ASSUMPTIONS

1. Concrete is a homogeneous elastic material.
2. Within range of working stresses, both concrete and steel behave elastically. (Does not withstand the small amount of creep which occurs in both the materials under sustained loading)
3. A plane section before bending remains plane even after bending, which implies a linear strain distribution across the depth of the member.

- As long as tensile stresses do not exceed the limit of modulus of rupture of concrete (corresponding to the stage of visible cracking of concrete), any change in the loading of the member results in a change of stress in concrete only, the sole function of the prestressing tendon being to impart and maintain prestress in concrete.
- Up to the stage of visible cracking on concrete, the changes in the stress of steel, the loading being negligibly small, are generally not considered in the computations.

ANALYSIS OF PRESTRESS

- The stresses due to prestressing lone are generally combined stresses due to the action of direct load and bending resulting from and eccentrically applied load.
- The stresses are evaluated by using the well known relationship for combined stresses used in the case of columns.
- Following notations and sign conventions are used for the analysis of prestress:

P = Prestressing force (positive when producing direct compression)

e = Eccentricity of prestressing force (M = P.e = Moment)

A = Cross sectional area of the concrete member

I = Second moment of area of section about its centroid

Z_t and Z_b = Section modulus of the top and bottom fibres

f_{sup} and f_{inf} = Prestress in concrete developed at the top and bottom fibres (positive when compressive and negative when tensile in nature)

y_t an y_b = Distance of the top and bottom fibres from the centroid of the section

i = Radius of gyration

ANALYSIS OF PRESTRESS

- Stresses due to pre-stressing alone are generally combined stresses due to action of direct load and bending resulting from an eccentrically applied load.
- Stresses are evaluated by using relationship for combined stresses in columns.
- <u>Concentric Tendon</u>

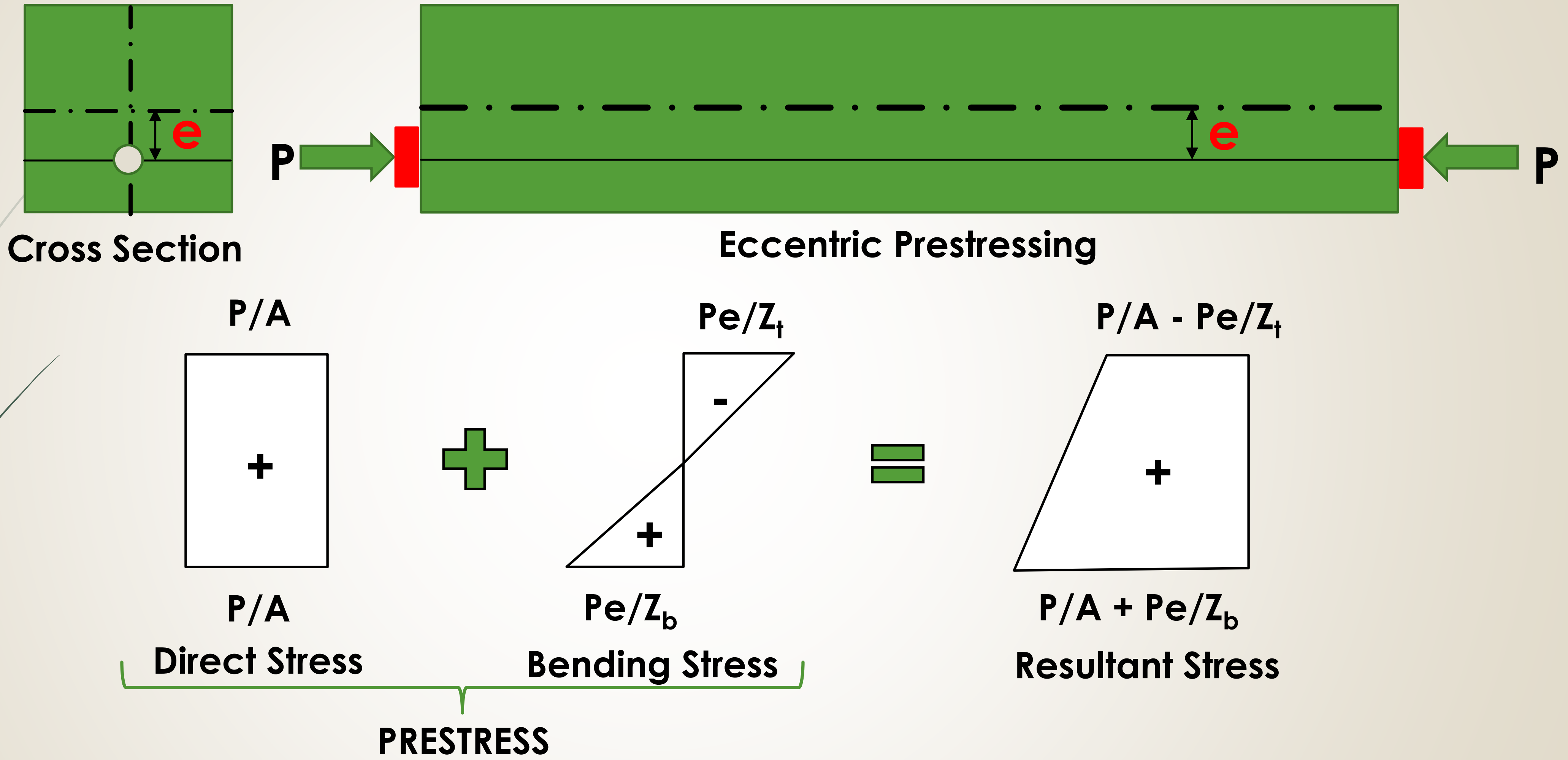
Cross Section

Concentric Prestressing

Stress = P/A

- Which is compressive across the depth of the beam.
- Generally applied loads and dead loads induce tensile stresses towards soffit of the beam and are counterbalanced more effectively by eccentric tendons.

Eccentric Tendon

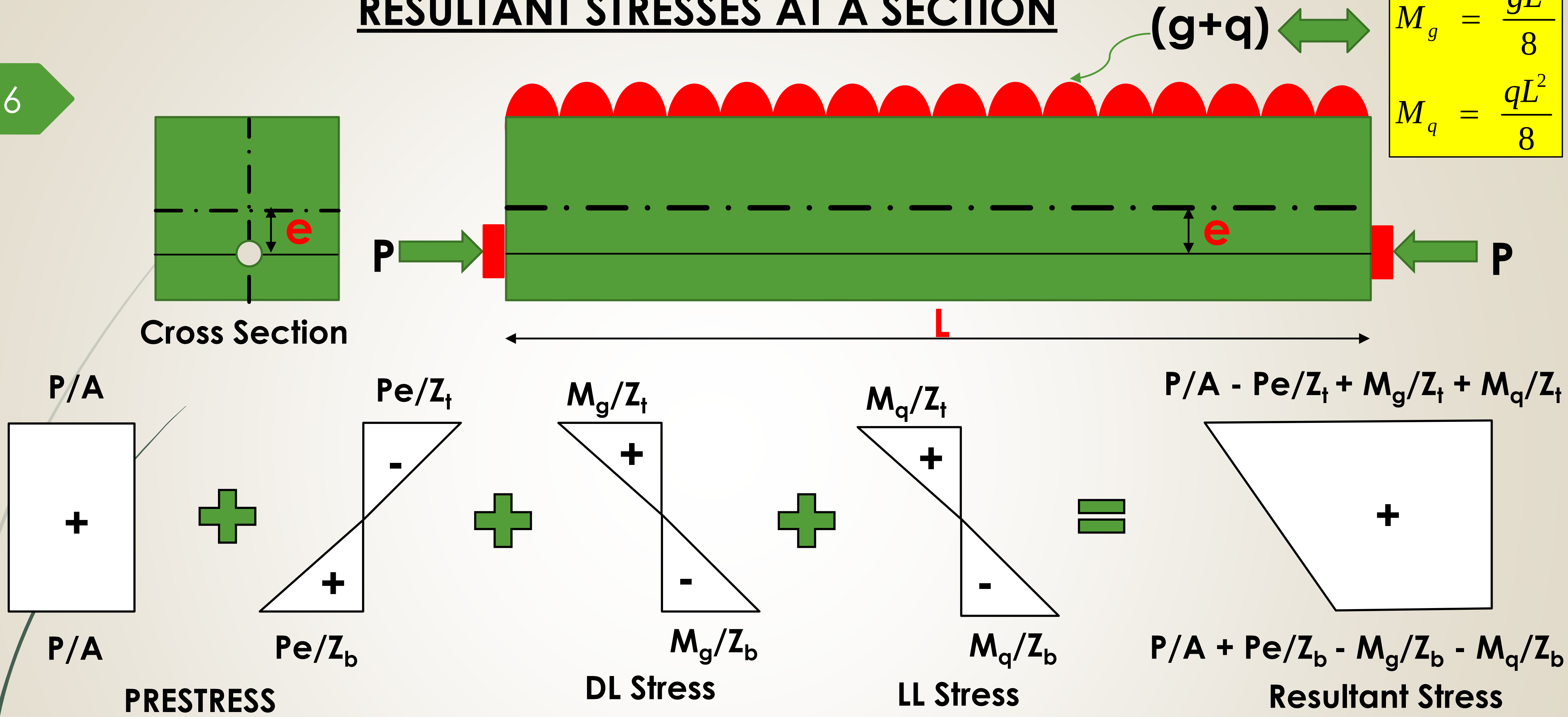


$$f_{inf} = \left(\frac{P}{A} + \frac{P\,e}{Z_b} \right) = \frac{P}{A}\left(1 + \frac{e\,y_b}{i^2} \right) \quad \& \quad f_{sup} = \left(\frac{P}{A} - \frac{P\,e}{Z_t} \right) = \frac{P}{A}\left(1 - \frac{e\,y_t}{i^2} \right)$$

RESULTANT STRESSES AT A SECTION

$$M_g = \frac{gL^2}{8}$$

$$M_q = \frac{qL^2}{8}$$



$$f_{inf} = \left(\frac{P}{A} + \frac{Pe}{Z_b}\right) - \frac{M_g}{Z_b} - \frac{M_q}{Z_b} \quad \& \quad f_{sup} = \left(\frac{P}{A} - \frac{Pe}{Z_t}\right) + \frac{M_g}{Z_t} + \frac{M_q}{Z_t}$$

Ex:1 A concrete beam of rectangular section, 150 mm wide by 300 mm deep, prestressed by 4 high-tensile wires of 5 mm diameter stressed to 1200 N/mm². The wires are located at an eccentricity of 50 mm. Examine the stresses developed at the soffit of the beam by considering the 'nominal concrete' and 'equivalent concrete' section.

Prestressing force = 1200 x 80 = 96000 N

For, Nominal Concrete Section

A = 150 x 300 = 45000 mm²

I = 150 x 300³ /12 = 3375 x 10⁵ mm⁴

Stress at the soffit of the section

$$= \frac{P}{A} + \frac{Pe}{Z} = \frac{96000}{45000} + \frac{96000(50)(6)}{150 \text{ x } (300)^2}$$

= 2.13 + 2.13

= 4.27 N/mm²

For, Equivalent Concrete Section

Assuming modular ratio, m = 6

Equivalent area of steel = 400 mm²

A_e = 45000 + (m - 1) 80 = 45400 mm²

Position of centroid of the section from the soffit = 149 mm

Y = (44600 x 150) + (400 x 50) / (44600 + 400) = 149.11 mm

I_e = (3375 x 10⁵) + ((150 x 300) x 1²) + (400 x 49²)

= 3385 x 10⁵ mm⁴

$$\text{Stresses at the soffit} = \frac{96000}{45000} + \frac{(96000 \text{ x } 49)\ (150)}{3385 \text{ x } 10^6}$$

= 4.20 N/mm²

Ex:2 A concrete beam of rectangular section 100 mm wide by 250 mm deep spanning over 8 m is pre-stressed by a straight cable carrying an effective prestressing force of 250 kN located at an eccentricity of 40 mm. The beam supports LL of 1.2 kN/m.

A) Calculate resultant stress distribution for the center of span c/s of the beam assuming the density of concrete as 24 kN/m^3.

B) Find the magnitude of the prestressing force with an eccentricity of 40 mm which can balance the stress due to Dead & LL at the soffit of the center span.

P = 250 kN = 250 x 1000 N

A = 100 x 250 = 25000 mm^2

e = 40 mm

Self weight of the beam = 0.1 x 0.25 x 24 = 0.6 kN/m

LL on the beam = 1.2 kN/m

Total load on the beam = (1.2 + 0.6) = 1.8 kN/m

Section modulus, Z = bd^2 / 6 = 1.04 x 10^6 mm^3

M = wL^2 / 8 = 14.4 kNm

- Stress due to loads = $M / Z = \pm 13.8\ N/mm^2$
- Pre-stress at top and bottom fibres = $[P/A \pm Pe/Z] = [10 \pm 9.6]\ N/mm^2$

A) Resultant stress at

Top fibre = $10 - 9.6 + 13.8 = 14.2\ N/mm^2$ (comp)

Bottom fibre = $10 + 9.6 - 13.8 = 5.8\ N/mm^2$ (comp)

B) If P is the pre-stressing force required to balance the stresses at soffit, then

$$[P/A + Pe/Z] = [M/Z]$$

$$P\,[1/A + e/Z] = [M/Z]$$

P = 176.39 kN

Ex:3 **A rectangular concrete of beam with c/s 30 cm deep and 20 cm wide is prestressed by means of 15 wires of 5 mm diameter located 6.5 cm from the bottom of the beam and 3 wires of diameter 5 mm, 2.5 cm from the top. Assuming the pre-stress in the steel as 840 N/mm^2, calculate the stresses at the extreme fibres of the mid-span section when the beam is supporting its own weight over a span of 6 m. If a UDL of 6 kN/m is imposed, evaluate the maximum working stress in the concrete. Take, Dc = 24 kN/m^3.**

Centroid of the prestressing force from the base

$$y = \frac{[(15 \times 65) + (3 \times 275)]}{18} = 100 \text{ mm}$$

e = 50 mm

$P = (18 \times 840 \times 19.7) = 3 \times 10^5$ N

$A = 300 \times 200 = 6 \times 10^4 \text{ mm}^2$

Self wt. of the beam = 0.3 x 0.2 x 24 = 1.44 kN/m

$M_g = 1.44 \times 6^2 / 8 = 6.48$ kNm

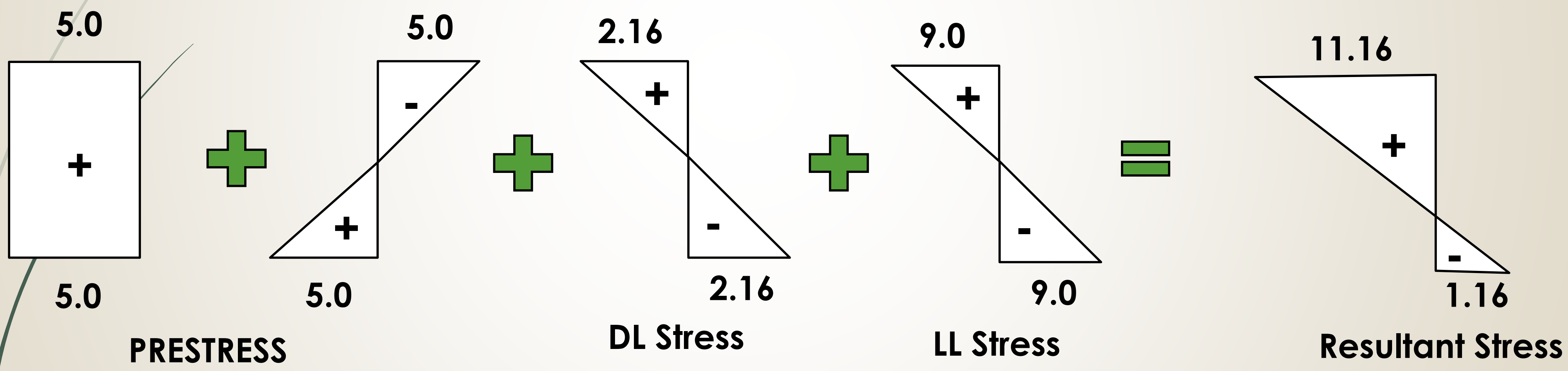
$I = bD^3 / 12 = 45 \times 10^7 \text{ mm}^4$

Z_t and $Z_b = 45 \times 10^7 / 150 = 3 \times 10^6 \text{ mm}^3$

LL on the beam = 6 kN/m

$M_q = 6 \times 6^2 / 8 = 27$ kNm

- Direct stress due to pre-stress (P/A) = 5 N/mm²
- Bending stress due to prestress (Pe/Z) = 5 N/mm²
- Self weight stress (M_g/Z) = 2.16 N/mm²
- LL Stress (M_q/Z)= 9 N/mm²

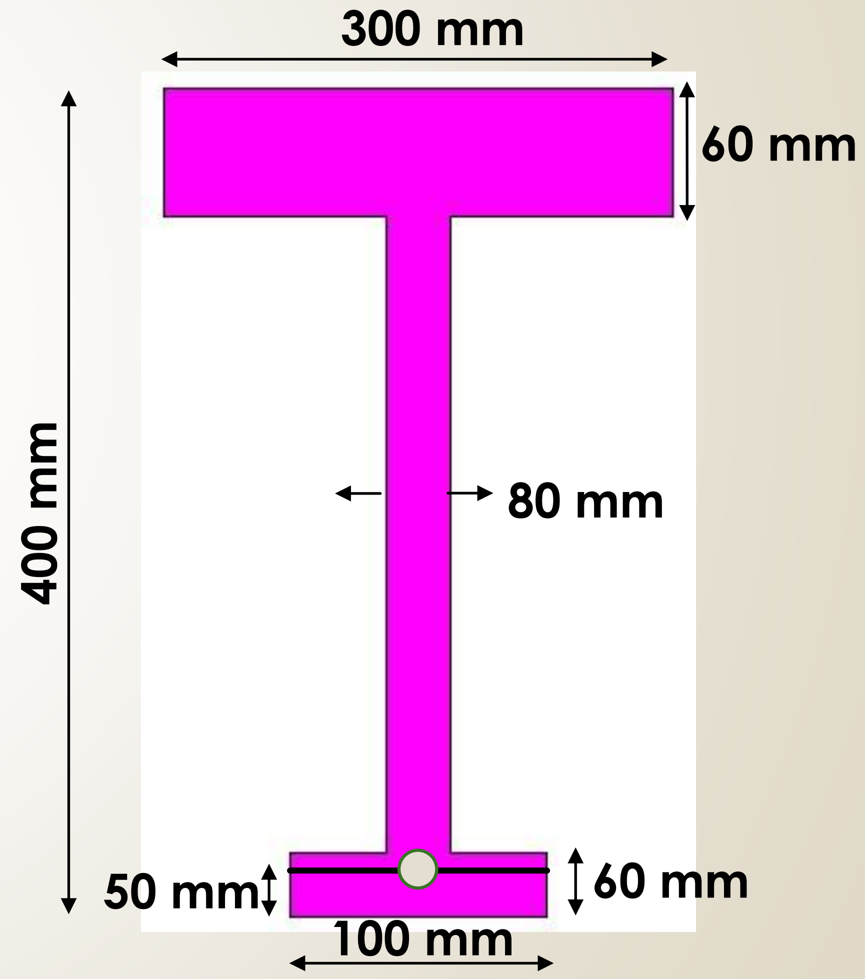


Stress distribution at mid span

Ex:3 An unsymmetrical I-section beam is used to support an imposed load of 2 kN/m over a span of 8 m. At the centre of the span, the effective prestressing force of 100 kN is located at 50 mm from the soffit of the beam. Estimate the stresses at the centre of span section of the beam for following Load calculations:

A) Prestress + self-weight

B) Prestress + self-weight + live load



$P = 100$ kN

$A = 46400$ mm^2

$y = 156$ mm (from top)

$e = 400 - 156 - 50 = 194$ mm

$I = 75.8 \times 10^7$ mm^4

$Z_t = (75.8 \times 10^7 / 156) = 485 \times 10^4$ mm^3

$Z_b = (75.8 \times 10^7 / 244) = 310 \times 10^4$ mm^3

$g = (0.0464 \times 1 \times 24) = 1.12$ kN/m

$M_g = 1.12 \times 8^2 / 8 = 8.96$ kNm

$M_q = 2 \times 8^2 / 8 = 16$ kNm

Stresses at the centre of span			
Sr No.	Type of stress	At top fibre (N/mm²)	At bottom fibre (N/mm²)
1.	Prestress	$P/A = +2.15$ $P_e/Z_t = -4.0$	$P/A = +2.15$ $P_e/Z_b = +6.25$
2.	Self weight stress	$M_g/Z_t = +1.85$	$M_g/Z_b = -2.9$
3.	Live load stress	$M_q/Z_t = +3.3$	$M_q/Z_b = -5.15$
Resultant Stresses (N/mm²)			
(1+2)		0	+5.5
(1+2+3)		+3.3	+0.35

PRESSURE LINE or THRUST LINE

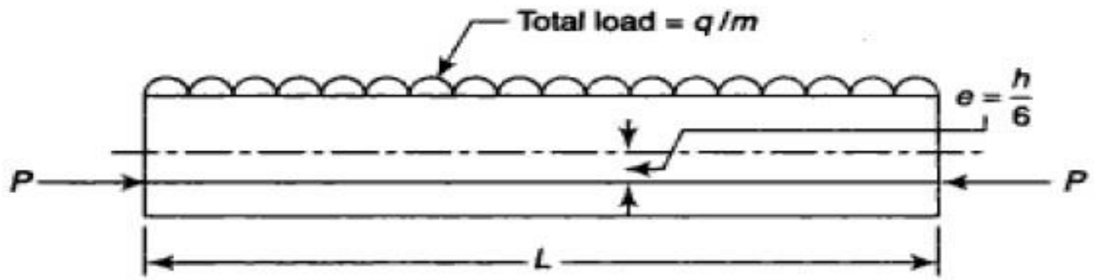
- At any given section of a prestressed concrete beam, the combined effect of the prestressing force and the externally applied load will result in a distribution of concrete stresses that can be resolved into a single force.

- The locus of points of the points of application of this resultant force in any structure is termed as the '**PRESSURE LINE or THRUST LINE**'.

- The concept is very useful in understanding the load-carrying mechanism of a prestressed concrete section.

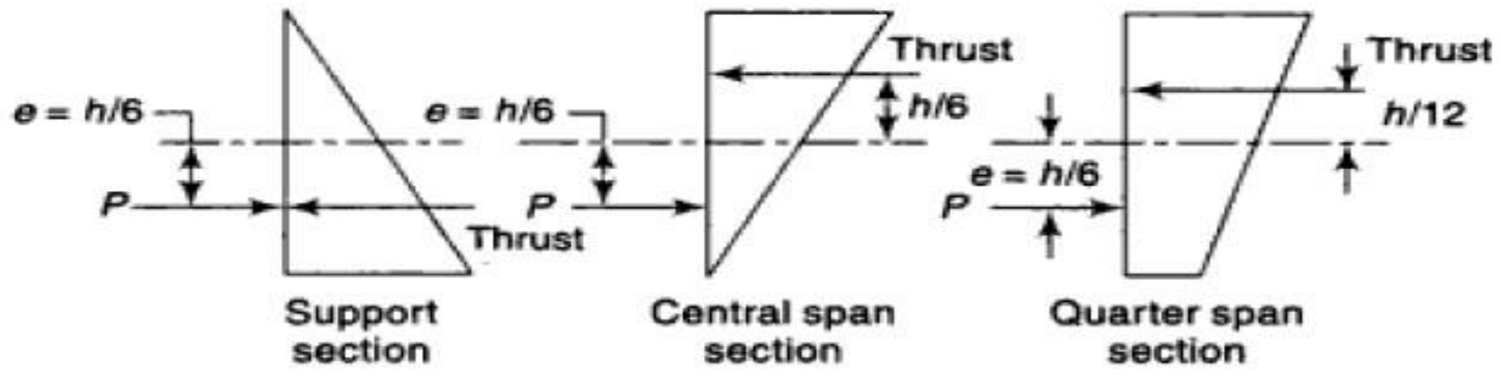
- For prestressed concrete members, the location of pressure line depends upon the magnitude and direction of moments applied at the c/s and the magnitude, distribution of stress due to prestressing force.

- Consider a concrete beam as shown in figure below, which is prestresed by force P acting at an eccentricity e.
- The beam supports a uniformly distributed load (including self-weight) of intensity q per unit length.



Beam with Eccentric Tendons

- The load is of such magnitude that the bottom fiber stress at the central span section of the beam is zero.
- Figure below shows the resultant stress distribution at support, center and quarter span section of the beam.



Distribution of Stresses at Various Sections along the Span

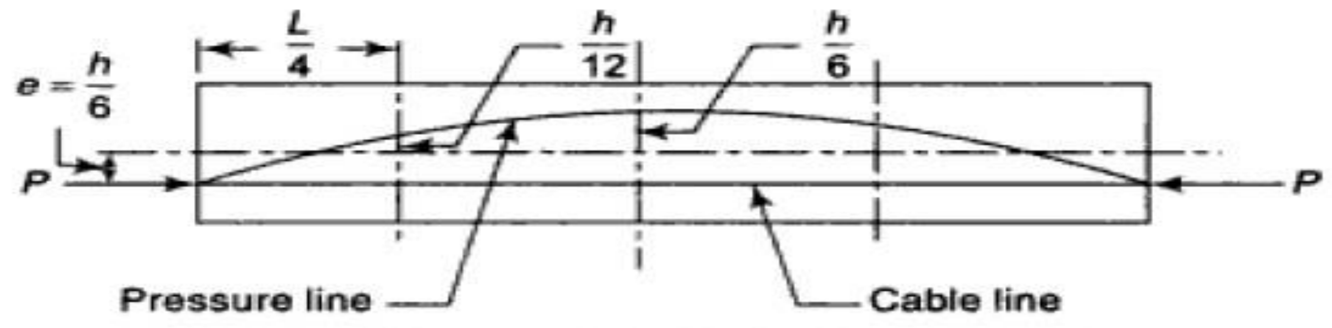
At support, x = 0

$$\frac{P}{A} - \frac{M}{I} y = 0$$

$$\frac{(P\,e)\,6}{b\,d^2}\left(\frac{d}{2}\right) = \frac{P}{A}$$

$$\Rightarrow e = \frac{d}{6}$$

- External moment at the quarter span section being smaller in magnitude, the shift in the pressure line is also correspondingly smaller, being equal to h/4 from the initial position.
- Similarly, it can be shown that a larger UDL on the beam would result in the pressure line being shifted even higher at the center and quarter span sections.

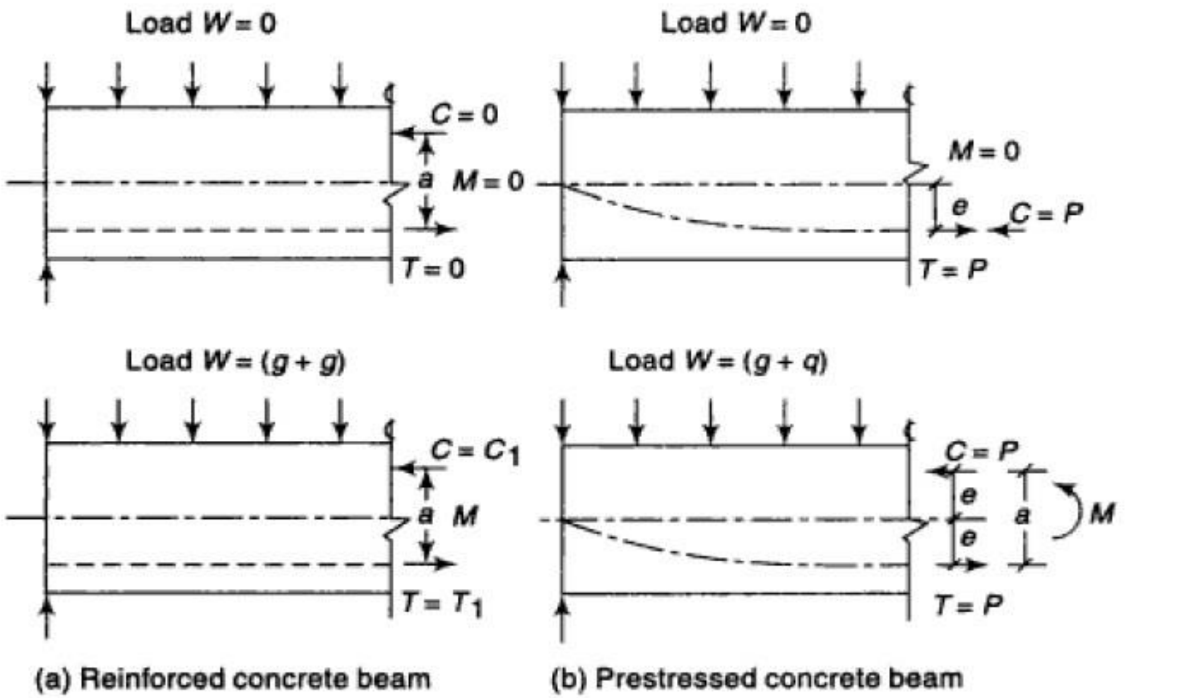


Location of Pressure Line in the Prestressed Beam

- ***Above observations lead to the following principle:***

"A CHANGE IN THE EXTERNAL MOMENTS IN THE ELASTIC RANGE OF A PRESTRESSED CONCRETE BEAM RESULTS IN SHIFT OF PRESSURE LINE RATHER THAN IN AN INCREASE IN THE RESULTANT FORCE IN THE BEAM".

- This is in contrast to RC beam, where an increase in the external moment result in a corresponding increase in the tensile force and compressive force.
- The increase in the resultant forces are due to a more or less constant lever arm between the forces, characterized by the properties of the composite section.
- For prestressed concrete sections load carrying mechanism is comprised of a constant force with a changing lever arm.
- For RC constant lever arm with changing forces.
- Behaviour of Cracked prestressed member = RC member

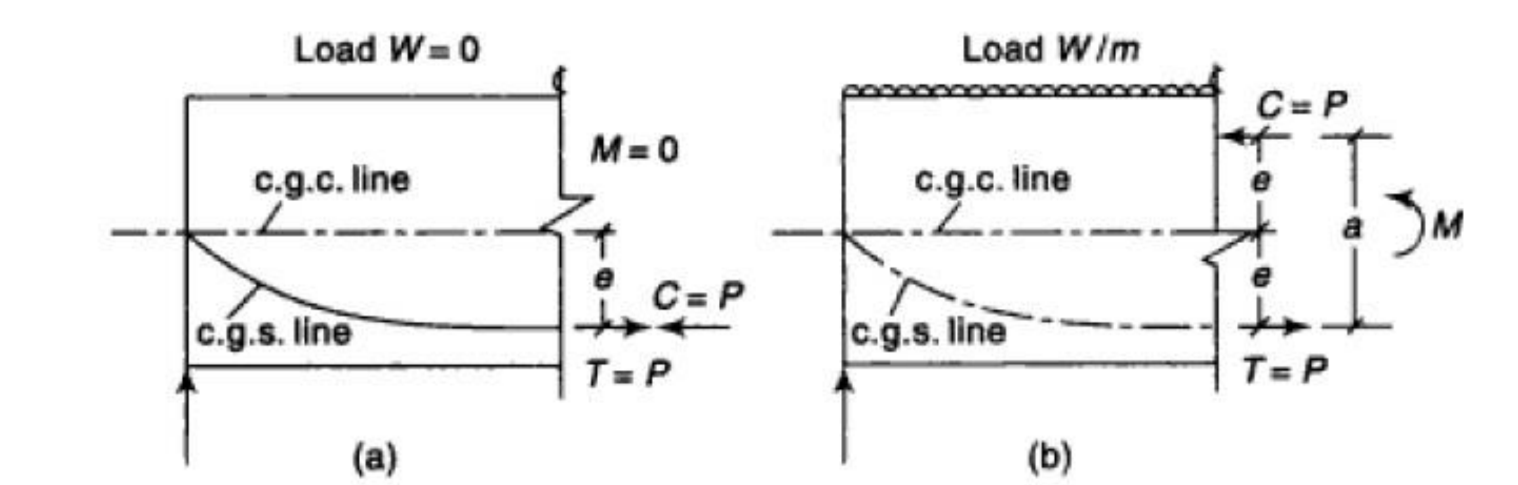


Load-Carrying Mechanism of Reinforced Concrete and Prestressed Concrete Beam Sections

"Pressure line concept can also be used to evaluate the stresses"

METHOD → INTERNAL RESISTING COUPLE METHOD OR C-LINE METHOD

- The prestressed beam is analysed as a plain concrete elastic beam using principles of statics.
- The prestressing force is considered as an external compressive force with a constant tensile force T in the tendon throughout the span.
- So, at any section of loaded prestressed beam, equilibrium is maintained by satisfying the equations, H=0 and M=0.



Free-Body Diagram of Forces and Moments at a Section of Prestressed Concrete Beam

- Figure shows the FBD of segment of a beam without and with transverse loads respectively.
- When gravity loads are zero, the C and T lines coincide as there is no moment in the section.
- Under transverse loads, the C-line or center of pressure or thrust line, is at a varying distance a from the T-line.

If M = BM at the section due to dead and live loads

e = Eccentricity of the tendon

T = P = Prestressing force in the tendon

- Moments equilibrium yields the relation,

$$M = Ca = Ta = Pa \text{ and } a = (M/P)$$

- ✓ The shift of pressure line measured from the centroidal axis is obtained as

$$e' = (a - e) = (M/P) - e$$

The resultant stresses at the top and bottom fibers of the section are expressed as,

$$f_{sup} = (P / A) + (Pe' / Z_t)$$

$$f_{inf} = (P / A) - (Pe' / Z_b)$$

EX: **A prestressed concrete beam with a rectangular section 120 mm wide by 300 mm deep supports a UDL of 4 kN/m, which includes self wt. of the beam. The beam is concentrically prestressed by a cable carrying a force of 180kN. Locate the position of pressure line in the beam.**

- Pre-stressing force, P = 180 kN
- Eccentricity, e = 0
- A = 36E3 mm^2
- $Z_t = Z_b = I/Y_{max}$ = 18E5 mm^3
- BM at the center of the span = $4 \times 6^2 / 8 = 18$ kNm
- Direct stress = P/A = 180E3 / 36E3 = 5 N/mm^2
- Bending stress = M/ Z = 18E6 / 18E5 = 10 N/mm^2
- Resultant stress at the center of the span section:

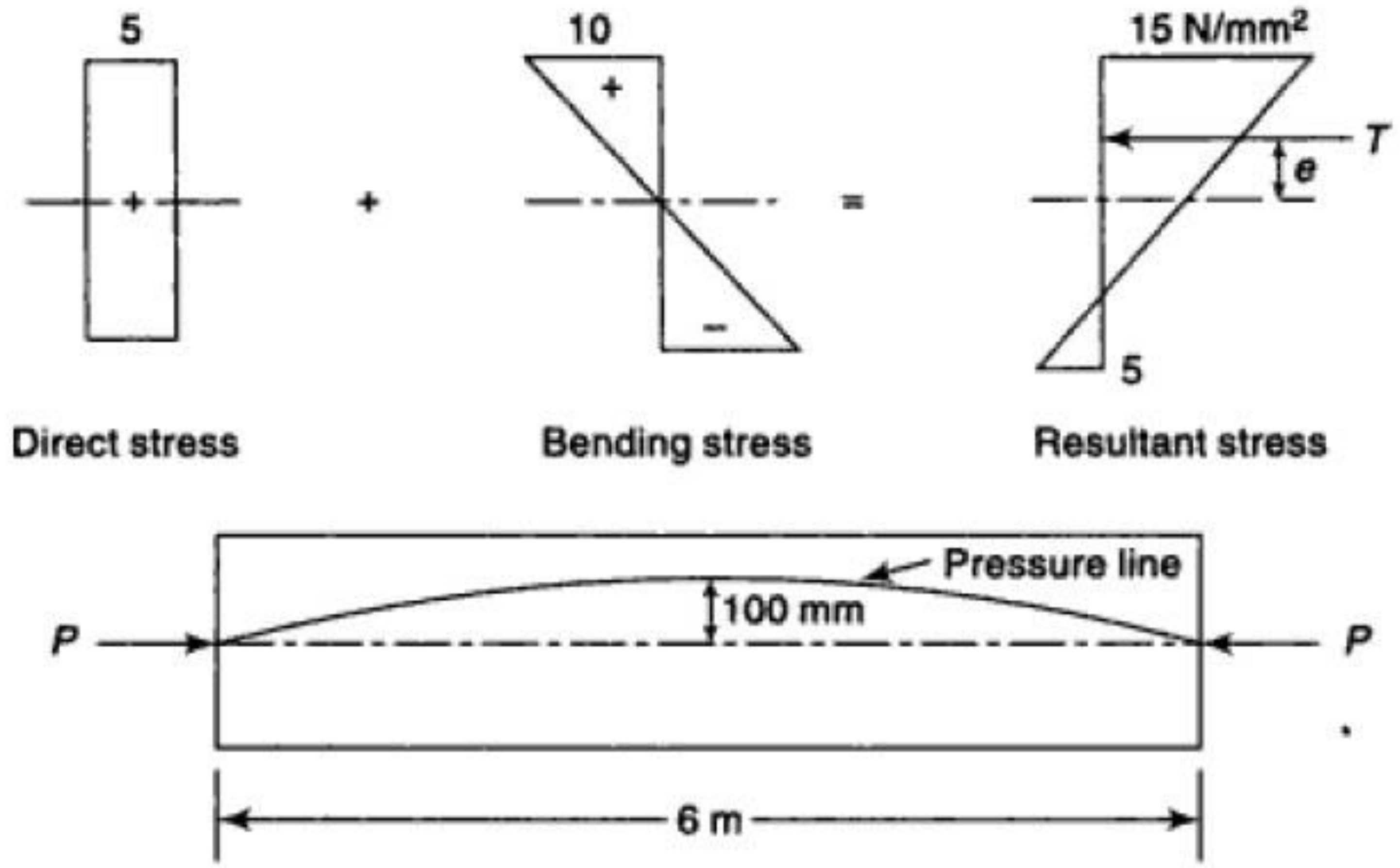
Top = 5 + 10 = 15N/mm^2 | Bottom = 5 – 10 = -5 N/mm^2

Assuming N is resultant thrust in the section, e is corresponding eccentricity (shift of pressure line) then,

- N/A + Ne/Z = 15
- Here, N = 180E3 N , A = 36E3 mm^2 & Z = 18E5 mm^3

➔ e = 100 mm

Resultant stress distribution diagram and the pressure line is shown in figure:



Distribution of Stresses and Location of Pressure Line in Prestressed Beam

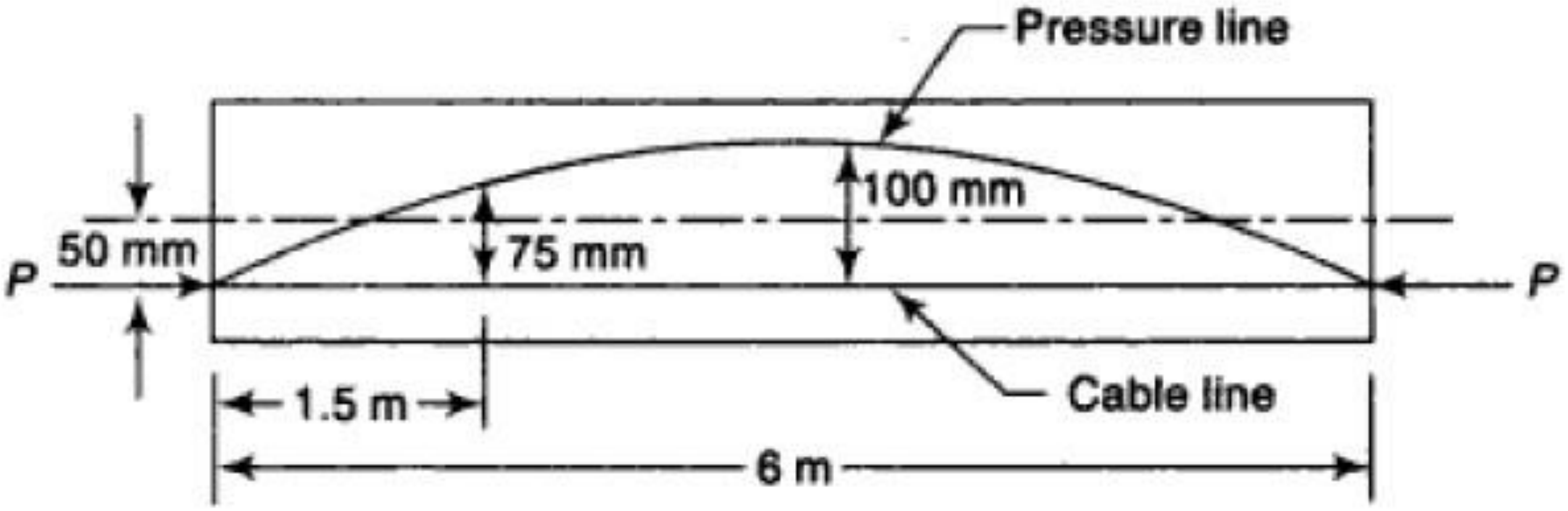
EX: A prestressed concrete beam of section 120 mm by 300 mm is used over an effective span of 6 m to support a UDL of 4 kN/m, which includes self weight of the beam. The beam is prestressed by a straight cable carrying a force of 180 kN and located at an eccentricity of 50 mm. Determine the location of thrust line in the beam and plot its position at quarter and central span.

- P = 180 kN, e = 50 mm, A = 36E3 mm^2, Z = 18E5 mm^3
- P/A = 5 N/mm^2 & Pe/Z = (180E3 x 50 / 18E3) = 5 N/mm^2
- BM at center span = 18 kNm
- Bending stress at top and bottom = ± 10 N/mm^2
- Resultant stresses at central section:

  Top = (5 - 5 +10) = 10 N/mm^2 | Bottom = (5 + 5 -10) = 0 N/mm^2
- Shift of pressure line from cable line = M / P = (18E6 / 18E4) = 100 mm
- BM at quarter span section = (3/32) qL^2 = (3/32) x 4 x 6^2 = 13.5 kNm
- Bending stress at top and bottom = ± 7.5 N/mm^2
- Resultant stresses at quarter span section:

  Top = (5 - 5 + 7.5) = 7.5 N/mm^2 | Bottom = (5 + 5 - 7.5) = 2.5 N/mm^2
- Shift of pressure line from cable line = M / P = (13.5E6 / 18E4) = 75mm

The location of pressure line is shown in figure.

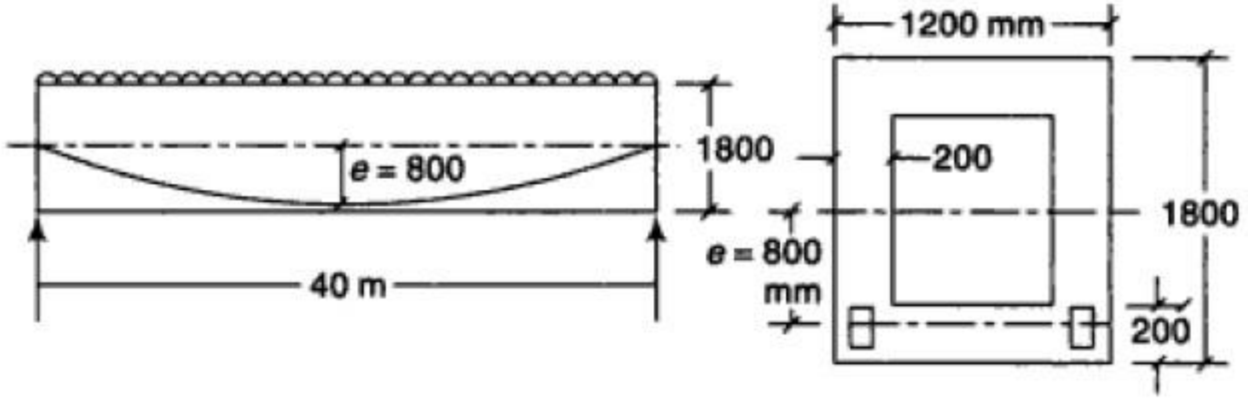


Location of Pressure Line in the Prestressed Beam

EX: A box girder of prestressed concrete bridge of span 40 m has overall dimensions of 1200 mm by 1800 mm. The uniform thickness of the walls is 200 mm. The live load analysis indicates a max live load moment of 2000 kNm at the center of the span. The beam is prestressed by parabolic cables with an effective force of 7000 kN. The cables which are concentric at support have an eccentricity of 800 mm at the center span section. Compute the resultant stresses at the center of span section using the internal resisting couple method.

- $A = (1.2 \times 1.8) - (0.8 \times 1.4) = 1.04\ m^2$
- $g = (1.04 \times 24) = 25\ kN/m$
- P = 7000 kN, e = 800 mm, L = 40 m
- $I = (1/12)\ [\ (1200 \times 1800^3) - (800 \times 1400^3)\ ]$

  $= 40 \times 10^{10}\ mm^4$



Prestressed Concrete Box Section Bridge Girder

$Z_b = Z_t = Z = (40 \times 10^{10}) / 900 = 400 \times 10E6\ mm^3$

$Mg = (0.125 \times 25 \times 40^2) = 5000\ kNm$

$Mq = 2000\ kNm$

$M = (Mg + Mq) = 7000\ kNm$

Lever arm, $a = (M/P) = (7000E3 / 7000) = 1000\ mm$

Shift of pressure line, $e' = (a – e) = (1000 – 800) = 200\ mm$

The resultant stresses are obtained as,

$f_{sup} = [\ P/A + Pe'/Z_t]$

$= (7000E3/1.04E6) + (7000E3 \times 200 / 444E6)$

$= 9.88\ N/mm^2$

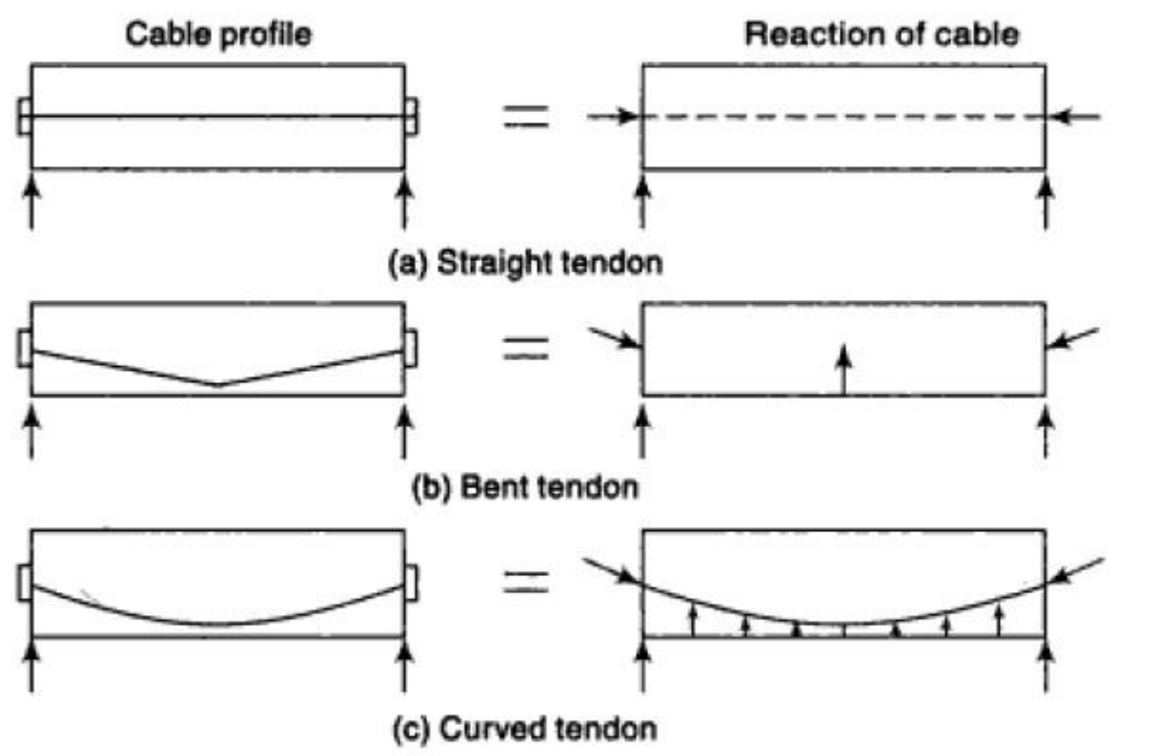
$f_{inf} = [\ P/A - Pe'/Z_b] = (7000E3/1.04E6) - (7000E3 \times 200 / 444E6)$

$= 3.58\ N/mm^2$

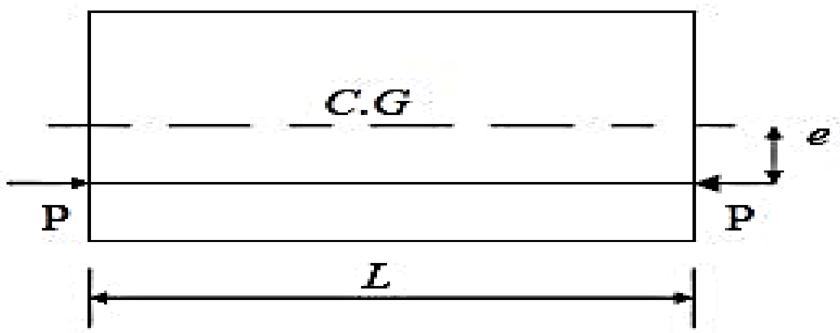
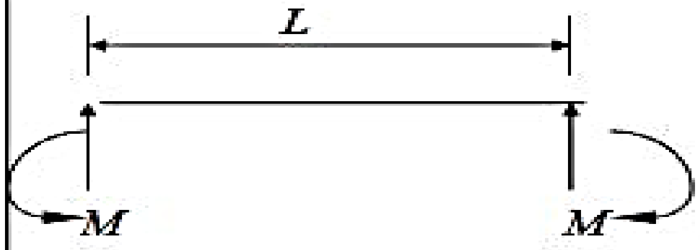
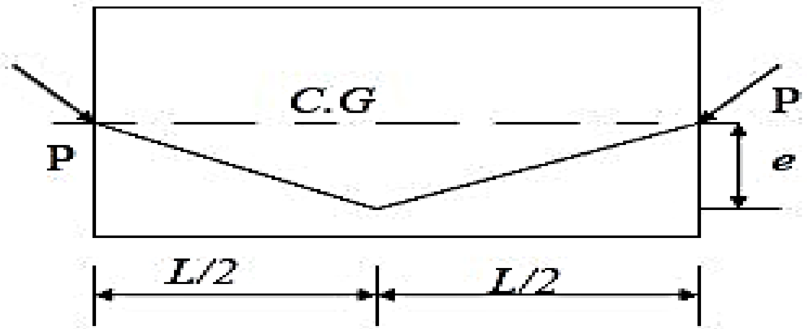
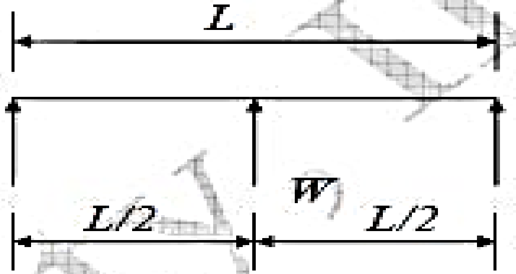
LOAD BALANCING

Load Balancing

- It is possible to select suitable cable profiles in a prestressed concrete member such that the transverse component of the cable force balances the given type of external loads.
- Various types of reactions of a cable upon a concrete member depend upon the shape of the cable profile.
- Straight profile of the cable do not induce any reactions except at ends, while curved cables result in UDL.
- Sharp angles in a cable induce concentrated loads.
- The concept of load balancing is useful in selecting the tendon profile.
- This requirement will be satisfied if the cable profile in a Prestressed member corresponds to the shape of the BMD resulting from the external loads.



Tendon Profiles and Equivalent Loads in Prestressed Concrete Beams

Tendon profile	Equivalent moment or Load	Equivalent loading	Camber
	$M = pe$		$\frac{ML^2}{8EI}$
	$W = \frac{4Pe}{L}$		$\frac{WL^3}{48EI}$

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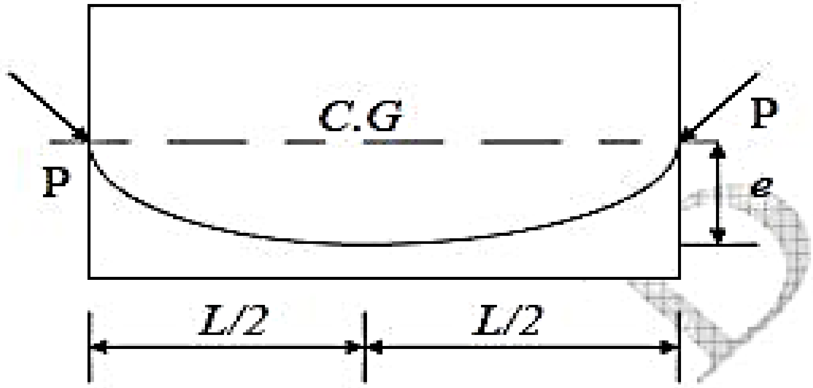
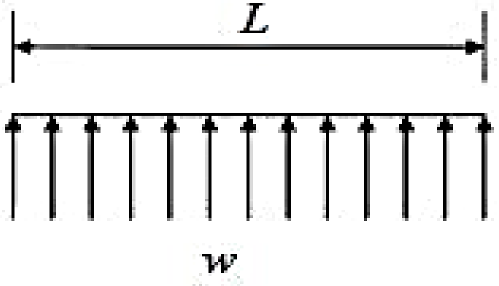
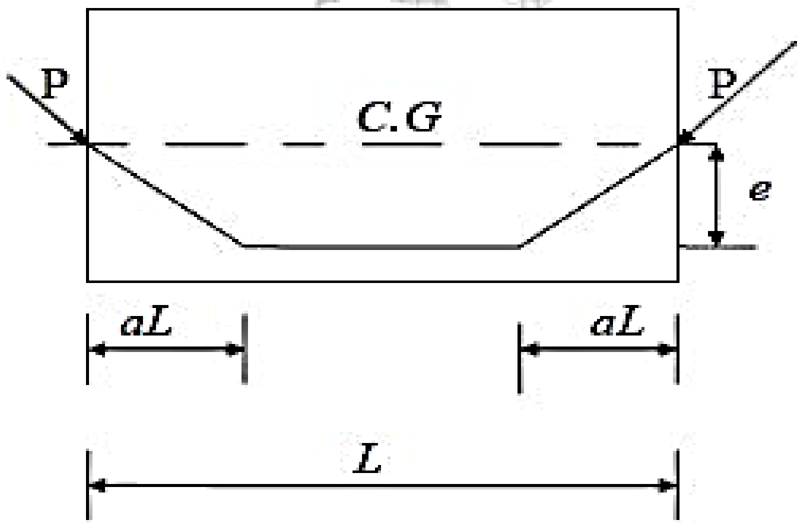
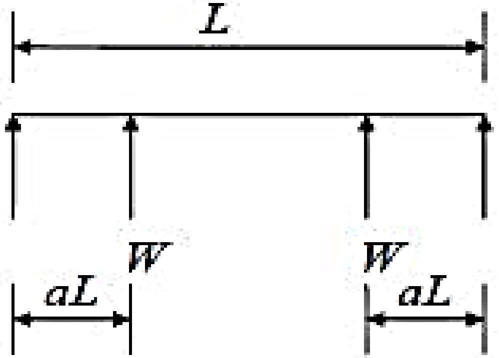
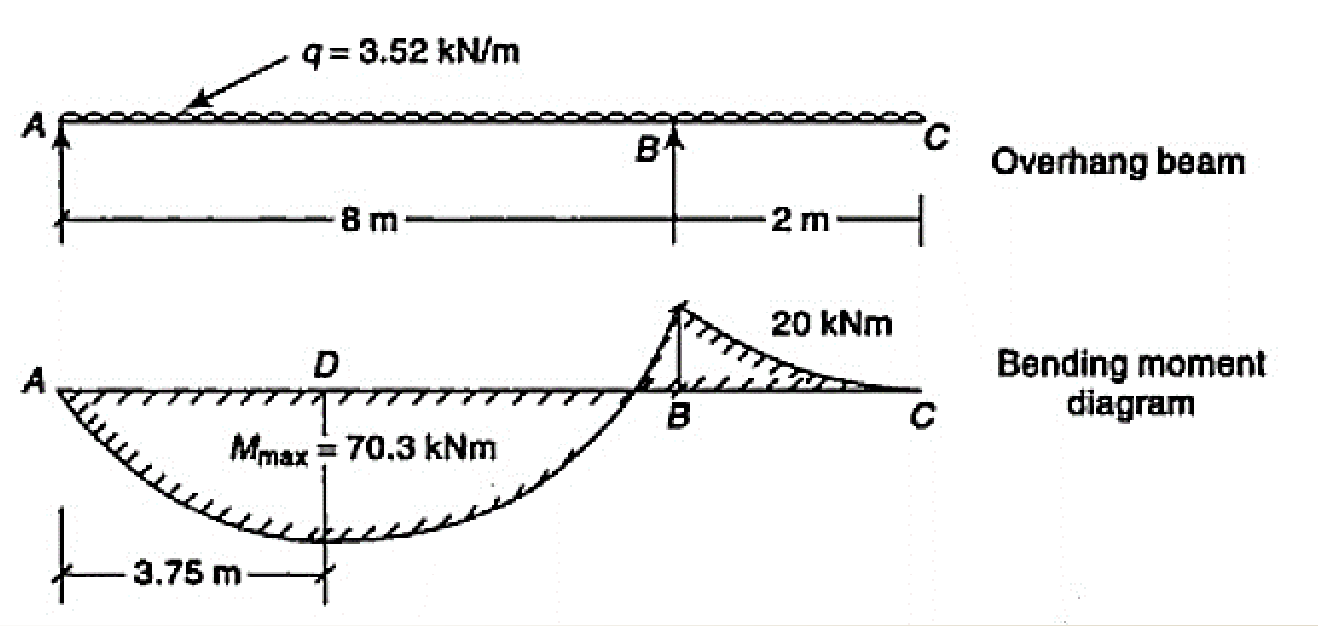
	$w = \frac{8Pe}{L^2}$		$\frac{5wL^4}{384EI}$
	$W = \frac{Pe}{aL}$		$\frac{a\left(3-4a^2\right)WL^3}{24EI}$

EX: A concrete beam with a single overhang is simply supported at *A* & *B* over a span of 8 m & the overhang *BC* is 2 m. The beam is of rectangular section 300 mm wide and 900 mm deep & supports a uniformly distributed live load of 3.52 kN/m over the entire length in addition to its self-weight. Determine the profile of the prestressing cable with an effective force of 500 kN which can balance the dead & live loads on the beam. Sketch the profile of the cable along the length of the beam.

- Span = 8 m, overhang = 2 m
- Pre-stressing force in the cable, *P* = 500 kN
- Self weight of the beam = (0.3 × 0.9 × 24)

  = 6.48 kN/m
- Live load on the beam = 3.52 kN/m
- Thus, the total load = 10.00 kN/m



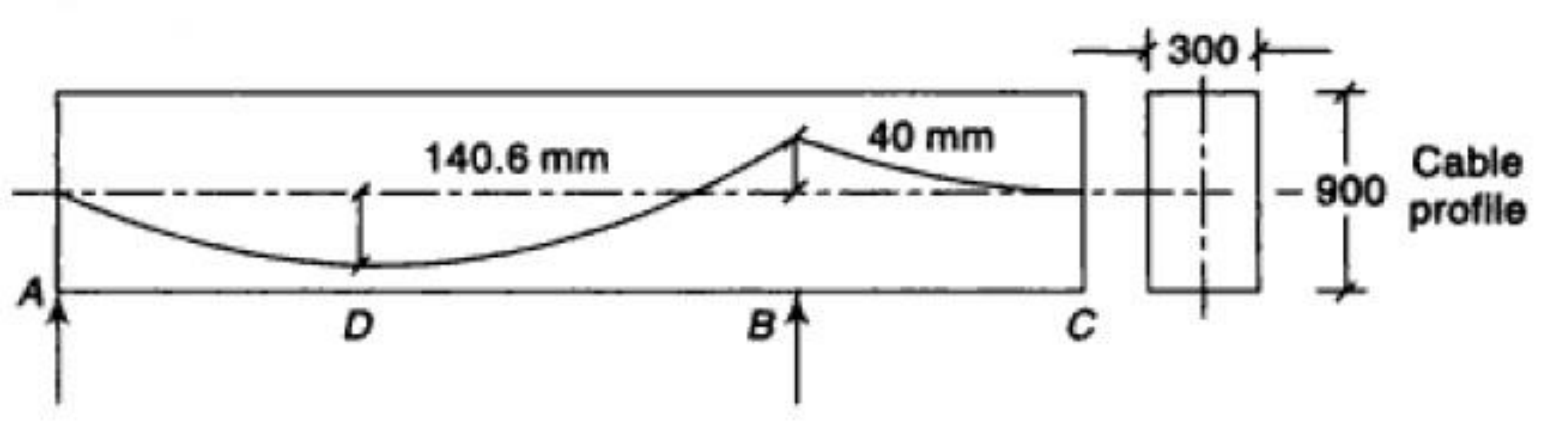
Reactions at A and B are

- $R_a = 37.5$ kN; $R_b = 62.5$ kN
- $M_b = 0.5 \times 10 \times 2^2 = 20$ kNm

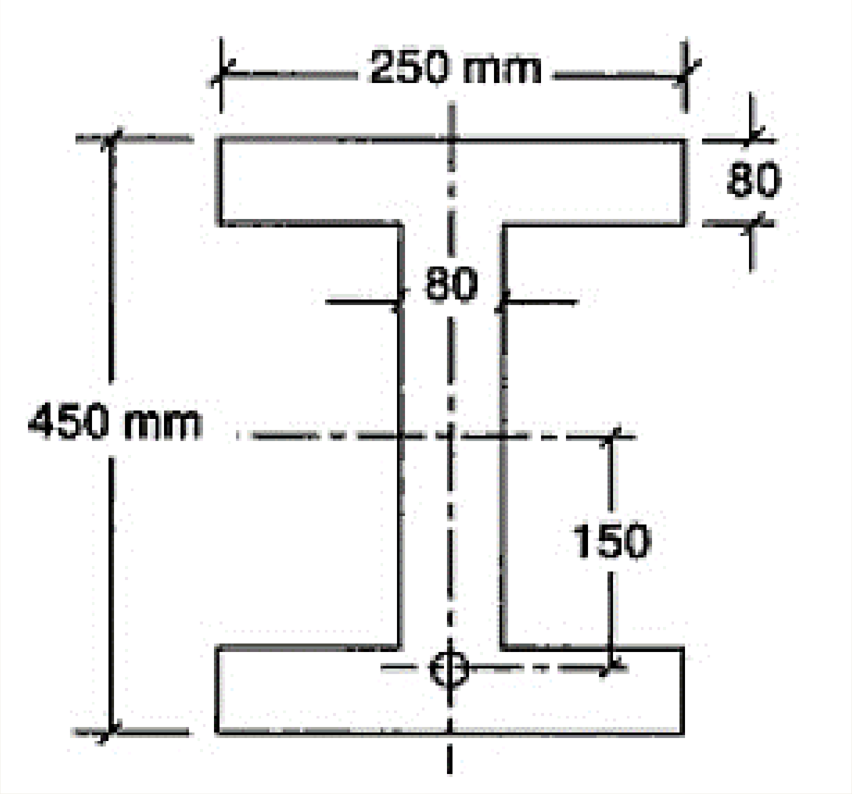
- Bending moment at a distance x from A is $M_x = 37.5x - 0.5 \times 10x^2$
- For maximum BM $(dM_x/dx) = 0 \Rightarrow 37.5 - 10x = 0 \rightarrow x = 3.75$ m
- Hence, maximum BM = $37.5 \times 3.75 - 0.5 \times 10 \times 3.75^2 = 70.3$ kNm (Assume point D)
- $M_x = 0$, at x = 7.5 m [As, $5x^2 - 37.5x = 0$]

The eccentricity of the cable at the position of maximum BM is

- $e = M_{max} / P$ = 70.3E6 / 500E3 = 140.6 mm
- Eccentricity at B,= M_B / P = 20E6 / 500E3 = 40 mm
- Since the bending moment at point A and C are zero, the cable is concentric at these points.
- The cable profile is parabolic with eccentricity of 140.6 mm below the centroidal axis at D and and 40 mm above the centroidal axis at support section B.



Ex: A beam of symmetrical I-section spanning 8 m has a flange width of 250 mm & flange thickness of 80 mm respectively. The overall depth of the beam is 450 mm. Thickness of the web is 80 mm. The beam is prestressed by a parabolic cable with an eccentricity of 150 mm at the centre of the span & zero at the supports. The LL on the beam is 2.5 kN/m.
(a) Determine the effective force in the cable for balancing the DL & LL on the beams.
(b) Sketch the distribution of resultant stress at the centre of span section for the above case.
(c) Calculate the shift of the pressure line from the tendon–centre–line.



The properties of the I-section are as follows:

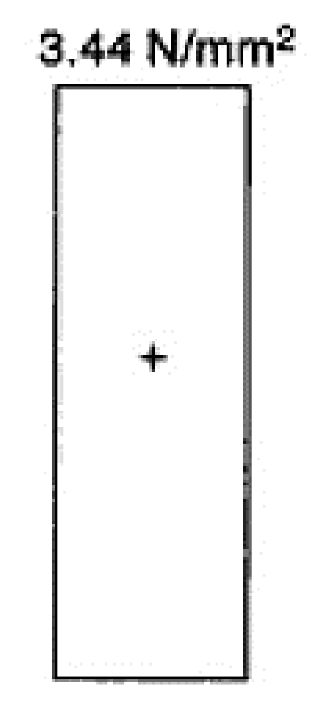
- Area of the section, A = 20.63 m^2
- Moment of inertia, I = 1.553E9 mm^4
- Section modulus, Z = 6.9E6 mm^3
- Eccentricity, e = 150 mm
- Span, L = 8 m
- Live load, w_q = 2.5 kN/m
- Dead load, w_g = 0.63 ×25 = 1.57 kN/m [Assuming unit weight of concrete as 25 kN/m^3]
- The BM at the centre of the span due to DL, M_g = (0.125 × 1.57 × 8^2) = 12.56 kNm
- The BM at the centre of the span due to LL, M_q = (0.125 × 2.5 × 8^2) = 20 kNm
- Total moment, M = (M_g + M_q) = 12.56 + 20 = 32.56 kNm

If , P = tendon force, For load balancing we have

$$P = (M/e) = 217 \text{ kN}$$

- The center–of-span section is subjected to a direct stress of intensity, (P/A) = (217E3 / 0.063E6) = 3.44 N/mm^2
- Shift of pressure line = (M / P) = 32.56E6 / 217E3 =150 mm

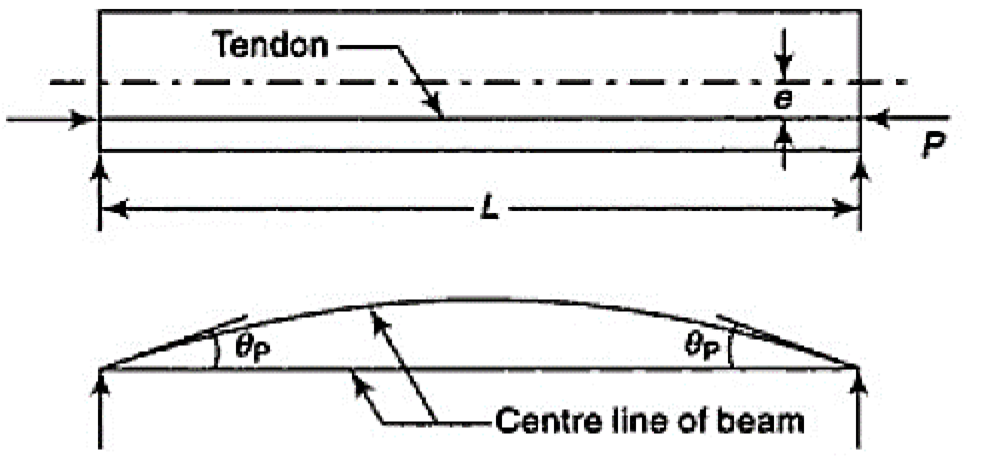
The pressure line coincides with the centroidal axis of the beam.



STRESSES IN TENDONS

Effect of Loading on Tensile Stresses in Tendons

- A pre-stressed member undergoes deformation due to the action of the prestressing force and transverse loads acting on the member.
- Curvature of the cable changes, which results in a slight variation of stresses in the tendons.

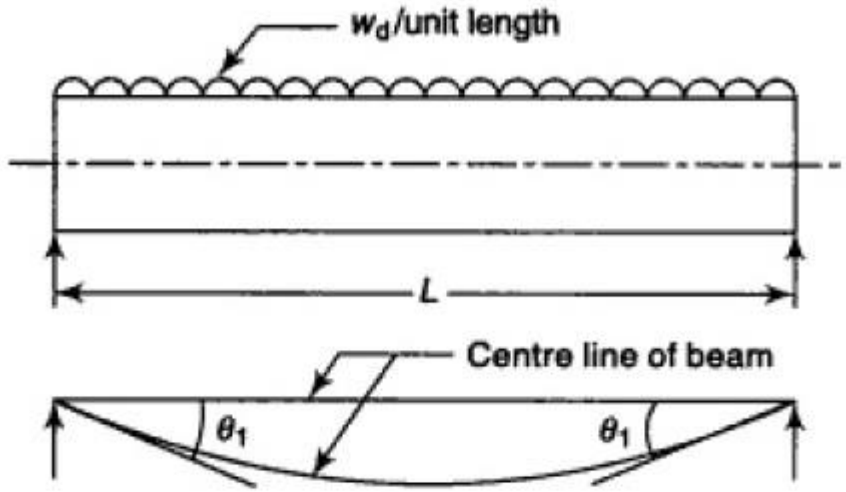


Effect of Prestressing Force on Rotation of Concrete Beam

- Consider a concrete beam of span L as shown in figure. The beam is prestressed by a cable carrying an effective force P at an eccentricity e, the rotation θ_p at the supports due to hogging of the beam is obtained by applying Mohr's theorem as,

$$\theta_p = \left(\frac{\text{Area of BMD}}{\text{Flexural Rigidity}} \right) = \frac{PeL}{2EI}$$

Where, EI = Flexural rigidity of the beam

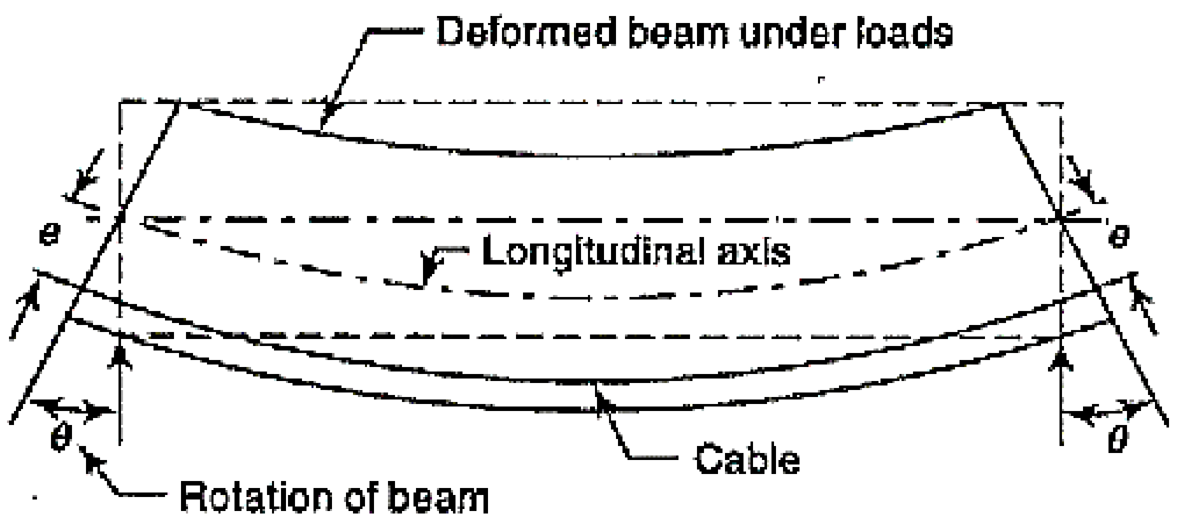


Effect of Transverse Loads on Rotation of Concrete Beam

- If the beam supports a total UDL of W_d per unit length, the rotation θ_1 at the supports due to sagging of the beam is evaluated from above fig.

$$\theta_1 = \left(\frac{\frac{1}{2} X \frac{2}{3} X\ L\ X\ Wd \frac{L^2}{8}}{EI}\right) = \left(\frac{W_d L^3}{24EI}\right)$$

- If the rotation due to loads is greater than that due to the prestressing force, the net rotation θ is given by, $\theta = (\theta_1 - \theta_p)$



Rotation of Beam under the Action of Loads

Considering above figure,

- Total elongation of the cable = 2eθ
- Strain in the cable = (2eθ/L)
- Increase in the stress due to loading = (E_s 2eθ/L)

Generally in the elastic range, any increase of loading on a pre-stressed member does not result in any significant change in the steel stress.

EX: A pre-stressed concrete beam spanning over 6 m have the c/s of 100 mm wide by 300 mm deep. The initial stress in the tendons located at a constant eccentricity of 50 mm is 1000 N/mm². The c/s area of the tendons is 100 mm². Find the percentage increase in stress in the wires when beam supports a Live Load of 4 kN/m. Dc = 24 kN/m³, Ec = 36 kN/mm², Es = 210 kN/mm².

- Pre-stressing force, P = (1000 x 100) = 100 kN
- $I = 100 \times 300^3 / 12 = 225E6\ mm^4$
- Rotation due to prestress, $\theta_p = \left(\frac{PeL}{2EI}\right) = \left(\frac{100\ X\ 50\ X\ 6E3}{2\ X\ 36\ X\ 225E6}\right)$ = 0.001858 radians
- Self weight of the beam, g = (0.1 x 0.3 x 24) = 0.72 kN/m
- LL = 4 kN/m
- Total Load = 4.72 kN/m
- W_d = 0.00472 kN/mm
- Rotation due to loads, $\theta_1 = \left(\frac{W_d L^3}{24EI}\right)$
- Net rotation = 0.00525 – 0.001858 = 0.0034 radians
- Elongation of the cable = 2 x 50 x 0.0034 = 0.34 mm

Increase in the stress due to loading = 12 N/mm²
Initial stress in the cable = 1000 N/mm²
% increase in stress = (12 x 100 / 1000) = 1.2%

Variation of Steel Stress in Bonded and Unbonded Members

- The rate of increase of stress in the tendons of prestressed concrete member under loads depends upon the degree of bond between the high tensile steel wires and the surrounding concrete.

- For bonded members such as pretensioned elements or post-tensioned grouted members, the composite action between steel and concrete prevails and the stresses in steel are computed using the theory of composite sections up to the stage of cracking.

- For unbonded beams, the tendons are free to elongate independently throughout their length under the action of transverse loads on the beam.

- The increase of stress in steel depends on the average strain in concrete at the level of steel.

Bonded Beams

- If M = Moment at the section due to loads

  E_s = Modulus of elasticity of steel

  E_c = Modulus of elasticity of concrete

  α_e = modular ratio

  y = position of steel from the centroidal axis

  f = Stress in concrete at level y from the centroidal axis

  I = Second moment of area of the concrete section

Stress in steel = Modular ratio x Stress in concrete

$= \alpha_e f$

$= \alpha_e (M y / I)$

Unbonded Beams

- If M = Bending Moment at the cross-section

  E_s = Modulus of elasticity of steel

  E_c = Modulus of elasticity of concrete

  α_e = modular ratio

  y = position of steel from the centroidal axis

  f = Stress in concrete at level y from the centroidal axis

  I = Second moment of area of the concrete section

  δL = Total elongation of the cable at a distance y from the centroidal axis

- Strain in concrete at the level of steel = $(M\ y/E_c\ I)$
- Total elongation of fiber of concrete at the level of steel = δL = $\int_0^L \left(\frac{M\,y}{E_c\,I}\right) dx$
- Average Strain = $\left(\frac{\delta L}{L}\right) = \frac{y}{E_c\,I\,L}\int_0^L M dx$

- Stress in steel = $\frac{E_s}{E_c}\left(\frac{y}{IL}\right)\int_0^L Mdx$

  = $\left(\frac{\alpha_e y}{IL}\right)\int_0^L Mdx$

If A= area of the BMD under a system of loads,

A = $\int_0^L Mdx$

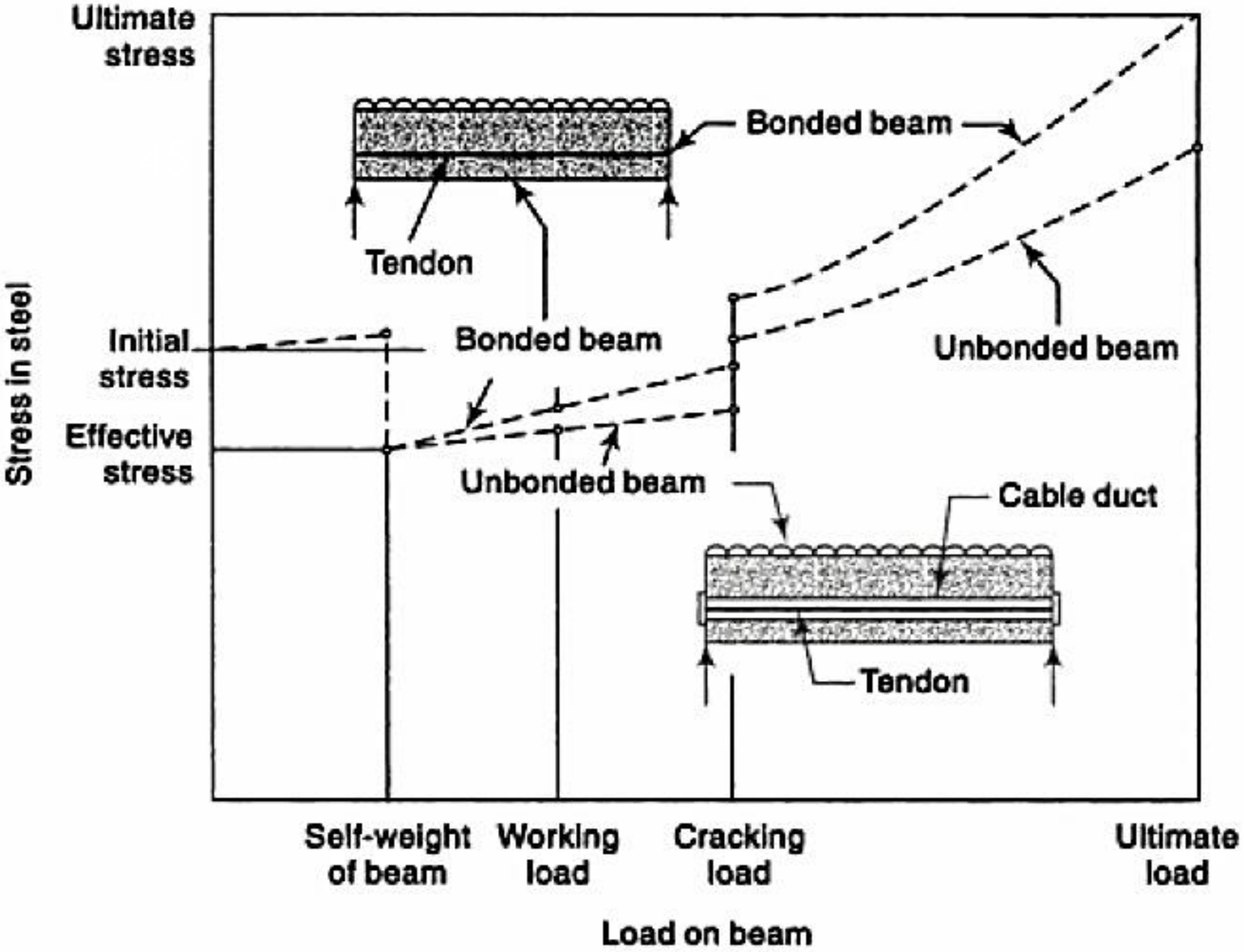
→ stress in steel = $\left(\frac{\alpha_e yA}{IL}\right)$

→ If the beam supports only a UDL of w_d per unit length,

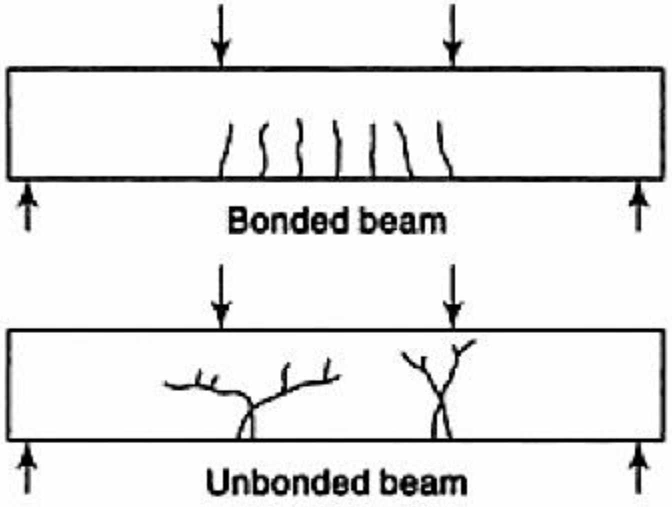
Then A = $\int_0^L Mdx$ = $\left[\left(\frac{2}{3}\right) L\, wd \left(\frac{L^2}{8}\right)\right]$

= $\left[w_d\left(\frac{L^3}{12}\right)\right]$

Increase of stress in steel = $\left(\frac{\alpha_e\ y\, W_d\ L^2}{12\ I}\right)$



Variation of Stress in Steel in Bonded and Unbonded Beams



Typical Crack Patterns of Bonded and Unbonded Beams

Variation of stresses in steel in Bonded and Unbonded beams

- The rate of increase of stress is larger in the case of bonded beams than in unbonded beams both in the pre-cracking and post-cracking stages.
- After the crack, the stress in steel increases at a faster rate in both types of beams.
- Since the steel does not reach its ultimate strength in the case of unbonded beams, the ultimate load supported by the beam is smaller than that of a bonded beam in which the steel attains its ultimate strength at the failure stage of the member.
- In the post-cracking stage, while the bonded beams are characterised by small cracks, which are well distributed in the zone of larger moments, unbonded beams develop only a few cracks, which are localized at weaker sections and the crack widths are correspondingly larger in comparison with the bonded beams.
- In general, bonded beams are preferable due to their higher flexural strength and predictable deformation characteristics.

EX: A pre-stressed concrete beam 200 mm wide by 300 mm deep supports a Live Load of 2.56 kN/m over an effective span of 10 m. The tendons housed in ducts are located at an eccentricity of 100 mm. Calculate the increase in steel stress due to loading when: a) The ducts are grouted so that the strains in the steel and adjacent concrete are equal

and

b) The ducts are ungrouted so that the tendons can move in ducts without friction. D_c = 24 kN/m^3, E_c = 35 kN/mm^2, Es = 210 kN/mm^2.

- Self weight of the beam, g = (0.2 x 0.3 x 24) = 1.44 kN/m
- LL, q = 2.56 kN/m
- w_d = 4 kN/m
- I = 200 x 300^3 /12 = 45E7 mm^4
- Modular ratio , $\alpha_e = (E_s / E_c) = 6$
- BM at center span = 0.125 x 4 x 10^2 = 50 kNm

a) **Bonded beam**

Stress in concrete = (My / I) = (50E6 x 100 / 45E7) = 11.1 N/mm^2

Stress in steel = (α_e) (Stress in concrete) = (6 x 11.1) = 66.6 N/mm^2

b) Unbonded beam

Stress in steel = $\left(\frac{\alpha_e \, y \, W_d \, L^2}{12 \, I}\right)$

$$= \left[\frac{6 \, X \, 100 \, X \, 4 \, X \, (10 \, X \, 1000)^2}{12 \, X \, 45 \, X \, 107}\right]$$

= 44.4 N/mm^2

Cracking Moment

- **Bending Moment at which visible cracks develop in Pre-stressed concrete members is referred as *Cracking Moment*.**
- **After the transfer of pre-stress to concrete, the soffit of the beam will be under compression.**
- **Gradually, these comp stresses are balanced by the tensile stresses developed due to the transverse loads on the beam, so that the resultant stresses at the bottom fibre is zero.**
- **A further increase in loading results in development of tensile stress at the soffit of the beam.**
- **As concrete is weak in tension, micro cracks develop as soon as the tensile stain in concrete exceeds 80 – 100E-6 units and if the loads are further increased, visible cracks appear in tension zone.**
- **At this stage, it is estimated that the crack widths are of order of 0.01 – 0.02 mm.**

- **Tensile stress developed when cracks become visible at the soffit of beams depend upon the type and distribution of steel reinforcement and the quality of concrete in the beam.**
- **It is generally considered that visible cracks appear when the tensile stresses at the soffit are approximately equal to the modulus of rupture of the material.**
- **The widths of the cracks are highly influenced by the degree of bond developed between concrete and steel.**

EX: A rectangular prestressed concrete beam of c/s 120 mm wide by 300 mm deep is pre-stressed by a straight cable carrying an effective force of 180 kN at an eccentricity of 50 mm. The beam supports an imposed load of 3.14 kN/m over a span of 6m. If the modulus of rupture of concrete is 5 N/mm^2, evaluate the load factor against cracking assuming self weight of concrete as 24 kN/m^3.

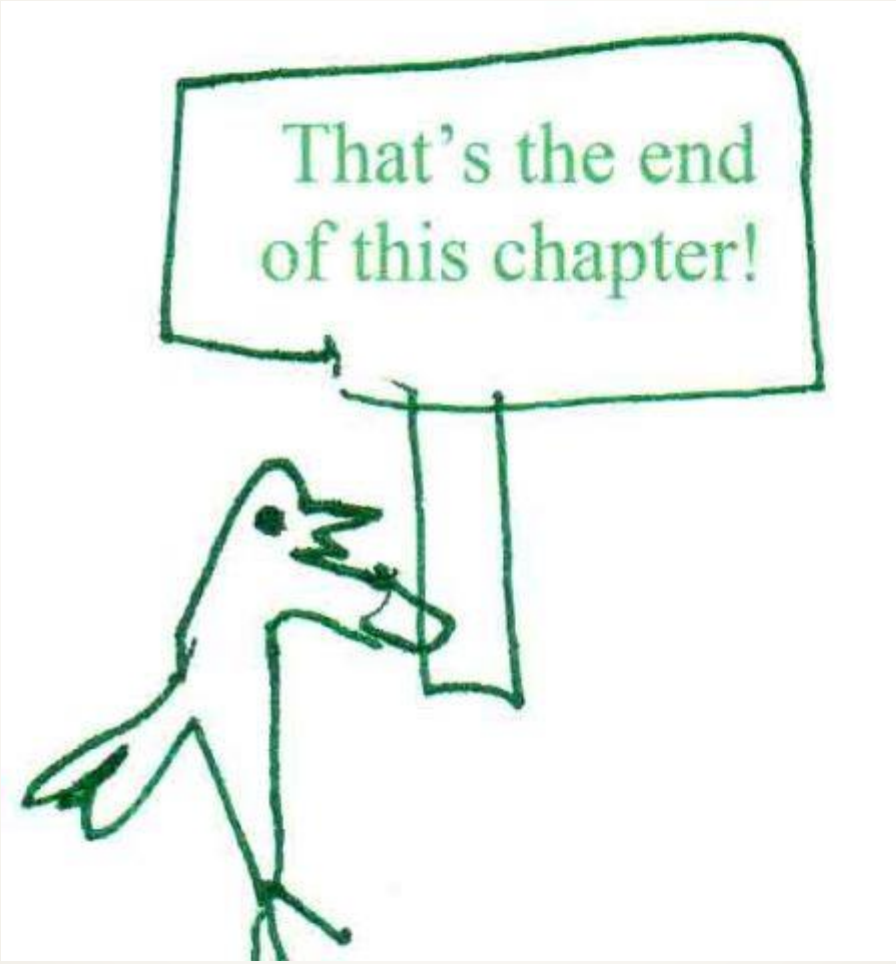
- P = 180 kN   I = 27E7 mm^4 e = 50 mm
- Z = 18E5 mm^3 A = 36E3 mm^2 g = (0.12 x 0.3 x 24) = 0.86 kN/m
- Total load = 4 kN/m

Stresses due to prestress

P/A = 5 N/mm^2 Pe/Z = 5 N/mm^2

Stresses due to loads:

- Max. Working moment = 0.125 x 4 x 62 = 18 kNm   ➔ (M/Z) = 10 N/mm^2
- Stress at the bottom fiber at working load = (5 + 5 - 10) = 0 N/mm^2
- Stress corresponding to cracking moment at bottom fiber = 5 N/mm^2
- Extra moment required to create this stress = 5 x 18E5 = 9 kNm
- Cracking moment = (18 + 9) = 27 kNm
- Load factor against cracking = (cracking moment / working moment) = 27 / 18 = 1.5


That's the end
of this chapter!