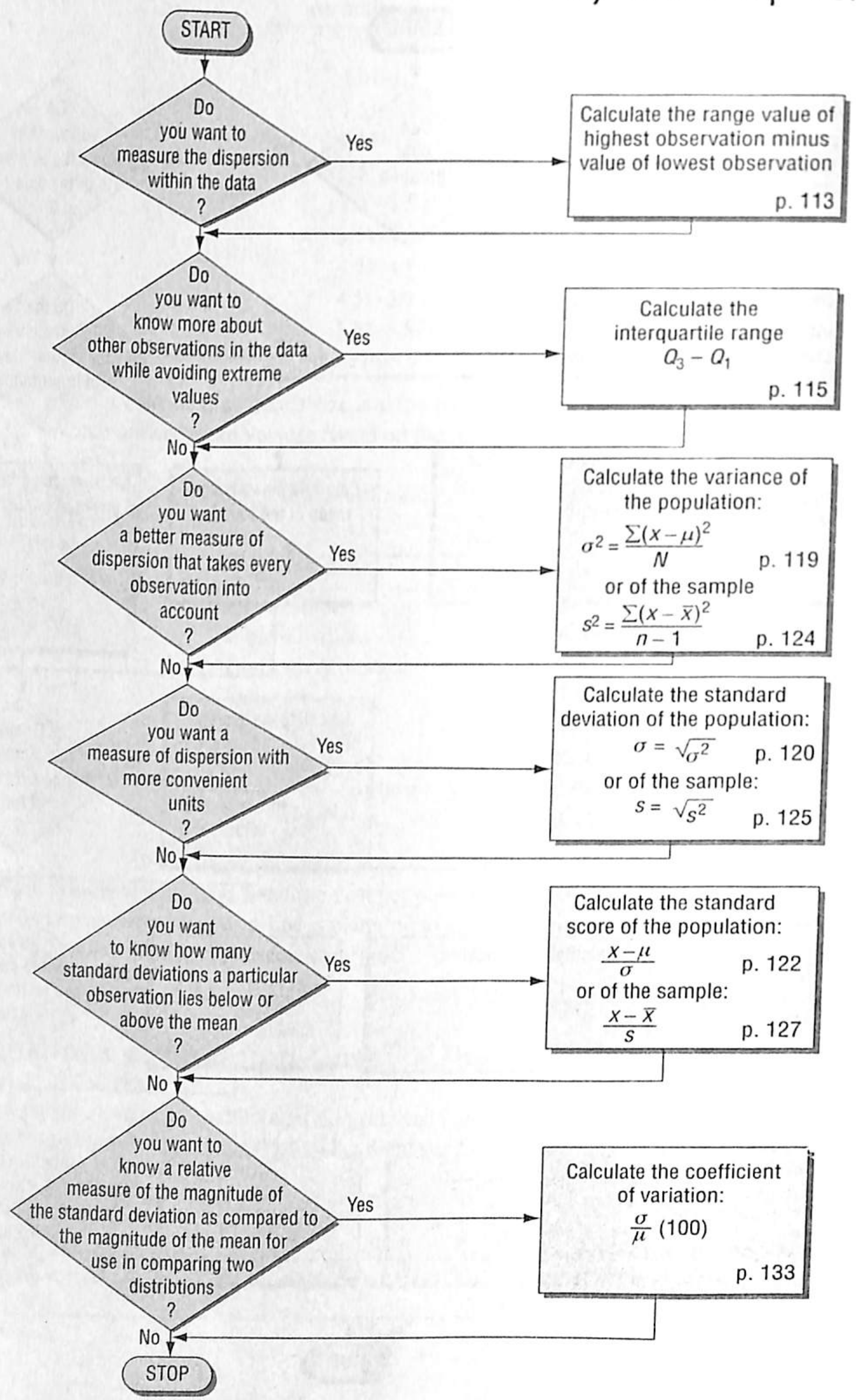
Flow Charts: Measures of Central Tendency and Dispersion



Probability I: Introductory Ideas

LEARNING OBJECTIVES

After reading this chapter, you can understand:

- To examine the use of probability theory in decision making
- To explain the different ways probabilities arise
- To develop rules for calculating different kinds of probabilities
- To use probabilities to take new information into account: the definition and use of Bayes' theorem

CHAPTER CONTENTS

Gamblers have used odds to make bets during most of recorded history. But it wasn't until the seventeenth century that French nobleman Antoine Gombauld (1607–1684) sought a mathematical basis for success at the dice tables. He asked French mathematician Blaise Pascal (1623–1662), "What are the odds of rolling two sixes at least once in twenty-four rolls of a pair of dice?" Pascal solved the problem, having become as interested in the idea of probabilities as was Gombauld. They shared their ideas with the famous mathematician Pierre de Fermat (1601–1665), and the letters written by these three constitute the first academic journal in probability theory. We have no record of the degree of success enjoyed by these gentlemen at the dice tables, but we do know that their curiosity and research introduced many of the concepts we shall study in this chapter and the next. ■

4.1 PROBABILITY: THE STUDY OF ODDS AND ENDS

Early probability theorists

Jacob Bernoulli (1654–1705), Abraham de Moivre (1667–1754), the Reverend Thomas Bayes (1702–1761), and Joseph Lagrange (1736–1813) developed probability formulas and techniques. In the nineteenth century, Pierre Simon, Marquis de Laplace (1749–1827), unified all these early ideas and compiled the first general theory of probability.

Need for probability theory

Probability theory was successfully applied at the gambling tables and, more relevant to our study, eventually to social and economic problems. The insurance industry, which emerged in the nineteenth century, required precise knowledge about the risk of loss in order to calculate premiums. Within 50 years, many learning centers were studying probability as a tool for understanding social phenomena. Today, the mathematical theory of probability is the basis for statistical applications in both social and decision-making research.

Examples of the use of probability theory

Probability is a part of our everyday lives. In personal and managerial decisions, we face uncertainty and use probability theory whether or not we admit the use of something so sophisticated. When we hear a weather forecast of a 70 percent chance of rain, we change our plans from a picnic to a pool game. Playing bridge, we make some probability estimate before attempting a finesse. Managers who deal with inventories of highly styled women's clothing must wonder about the chances that sales will reach or exceed a certain level, and the buyer who stocks up on skateboards considers the probability of the life of this particular fad. Before Muhammad Ali's highly publicized fight with Leon Spinks, Ali was reputed to have said, "I'll give you **odds** I'm still the greatest when it's over." And when you begin to study for the inevitable quiz attached to the use of this book, you may ask yourself, "What are the chances the professor will ask us to recall something about the history of probability theory?"

We live in a world in which we are unable to forecast the future with complete certainty. Our need to cope with uncertainty leads us to the study and use of probability theory. In many instances, we, as concerned citizens, will have some knowledge about the possible outcomes of a decision. By organizing this information and considering it systematically, we will be able to recognize our assumptions, communicate our reasoning to others, and make a sounder decision than we could by using a shot-in-the-dark approach.

EXERCISES 4.1

Applications

4-1 The insurance industry uses probability theory to calculate premium rates, but life insurers know for certain that every policyholder is going to die. Does this mean that probability theory does not apply to the life insurance business? Explain.

4-2 "Use of this product may be hazardous to your health. This product contains saccharin, which has been determined to cause cancer in laboratory animals." How might probability theory have played a part in this statement?

4-3 Is there really any such thing as an "uncalculated risk"? Explain.

4-4 A well-known soft drink company decides to alter the formula of its oldest and most popular product. How might probability theory be involved in such a decision?

4.2 BASIC TERMINOLOGY IN PROBABILITY

In our day-to-day life involving decision-making problems, we encounter two broad types of problems. These problems can be categorized into two types of models: **Deterministic Models** and **Random** or **Probabilistic Models**. **Deterministic Models** cover those situations, where everything related to the situation is known with certainty to the decision-maker, when decision is to be made. Whereas in **Probabilistic Models,** the totality of the outcomes is known but it can not be certain, which particular outcome will appear. So, there is always some uncertainty involved in decision-making.

In Deterministic Models, frequency distribution or descriptive statistics measures are used to arrive at a decision. Similarly, in random situations, probability and probability distributions are used to make decisions. So, probability can also be defined as a measure of uncertainty.

In general, probability is the chance something will happen. Probabilities are expressed as fractions ($\frac{1}{6}$, $\frac{1}{2}$, $\frac{8}{9}$) or as decimals (0.167, 0.500, 0.889) between zero and 1. Assigning a probability of zero means that something can never happen; a probability of 1 indicates that something will always happen.

An event

In probability theory, an *event* is one or more of the possible outcomes of doing something. If we toss a coin, getting a tail would be an *event*, and getting a head would be another event. Similarly, if we are drawing from a deck of cards, selecting the ace of spades would be an event. An example of an event closer to your life, perhaps, is being picked from a class of 100 students to answer a question. When we hear the frightening predictions of highway traffic deaths, we hope not to be one of those events.

An experiment

The activity that produces such an event is referred to in probability theory as an *experiment*. Using this formal language, we could ask the question, "In a coin-toss *experiment*, what is the probability of the event *head*?" And, of course, if it is a fair coin with an equal chance of coming down on either side (and no chance of landing on its edge), we would answer "$\frac{1}{2}$" or "0.5." The set of all possible outcomes of an experiment is called the *sample space* for the experiment. In the coin-toss experiment, the sample space is

$$S = \{\text{head, tail}\}$$

In the card-drawing experiment, the sample space has 52 members: ace of hearts, deuce of hearts, and so on.

Most of us are less excited about coins or cards than we are interested in questions such as "What are the chances of making that plane connection?" or "What are my chances of getting a second job interview?" In short, we are concerned with the chances that an event will happen.

Mutually exclusive events

Events are said to be *mutually exclusive* if one and only one of them can take place at a time. Consider again our example of the coin. We have two possible outcomes, heads and tails. On any toss, either heads or tails may turn up, but not both. As a result, the events heads and tails on a single toss are said to be mutually exclusive. Similarly, you will either pass or fail this course or, before the course is over, you may drop it without a grade. Only one of those three outcomes can happen; they are said to be mutually exclusive events. The crucial question to ask in deciding whether events are really mutually exclusive is, "Can two or more of these events occur at one time?" If the answer is yes, the events are *not* mutually exclusive.

A collectively exhaustive list

When a list of the possible events that can result from an experiment includes every possible outcome, the list is said to be *collectively exhaustive.* In our coin example, the list "head and tail" is collectively exhaustive (unless, of course, the coin stands on its edge when we toss it). In a presidential campaign, the list of outcomes "Democratic candidate and Republican candidate" is *not* a collectively exhaustive list of outcomes, because an independent candidate or the candidate of another party could conceivably win.

Odd in favor and against

Let us consider a situation, total number of possible outcomes related to the situation is "N", out of them "m" are the number of outcomes where the desired event "E" has occurred. So, "N-m" is the number of outcomes where the desired event has not occurred.

Hence, we may define:

Odds in favor of happening of E = m : N-m
Odds against the happening of E = N-m : m

This concept is related to the concept of Probability as:

Probability of happening of the event E = m/N

Ex: A cricket match is to be played between two teams CX Club and TE Club. A cricket analyst has predicated that the odds in favor of CX Club winning the match are 4:3. This prediction is based upon the historical records and upon the current strengths and weaknesses of the two teams. So, if a cricket fan is interested in knowing the chances that CX will win the match, then the desired chances would be $4/7$.

EXERCISES 4.2

Self-Check Exercises

SC 4-1 Give a collectively exhaustive list of the possible outcomes of tossing two dice.

SC 4-2 Give the probability for each of the following totals in the rolling of two dice: 1, 2, 5, 6, 7, 10, and 11.

Basic Concepts

4-5 Which of the following are pairs of mutually exclusive events in the drawing of one card from a standard deck of 52?
(a) A heart and a queen.
(b) A club and a red card.

(c) An even number and a spade.
(d) An ace and an even number.
Which of the following are mutually exclusive outcomes in the rolling of two dice?
(a) A total of 5 points and a 5 on one die.
(b) A total of 7 points and an even number of points on both dice.
(c) A total of 8 points and an odd number of points on both dice.
(d) A total of 9 points and a 2 on one die.
(e) A total of 10 points and a 4 on one die.

4-6 A batter "takes" (does not swing at) each of the pitches he sees. Give the sample space of outcomes for the following experiments in terms of balls and strikes:
(a) Two pitches.
(b) Three pitches.

Applications

4-7 Consider a stack of nine cards, all spades, numbered 2 through 10, and a die. Give a collectively exhaustive list of the possible outcomes of rolling the die and picking one card. How many elements are there in the sample space?

4-8 Consider the stack of cards and the die discussed in Exercise 4-7. Give the probability for each of the following totals in the sum of the roll of the die and the value of the card drawn:

2 3 8 9 12 14 16

4-9 In a recent meeting of union members supporting Joe Royal for union president, Royal's leading supporter said "chances are good" that Royal will defeat the single opponent facing him in the election.
(a) What are the "events" that could take place with regard to the election?
(b) Is your list collectively exhaustive? Are the events in your list mutually exclusive?
(c) Disregarding the supporter's comments and knowing no additional information, what probabilities would you assign to each of your events?

4-10 Southern Bell is considering the distribution of funds for a campaign to increase long-distance calls within North Carolina. The following table lists the markets that the company considers worthy of focused promotions:

Market Segment	Cost of Special Campaign Aimed at Group
Minorities	$350,000
Businesspeople	$550,000
Women	$250,000
Professionals and white-collar workers	$200,000
Blue-collar workers	$250,000

There is up to $800,000 available for these special campaigns.
(a) Are the market segments listed in the table collectively exhaustive? Are they mutually exclusive?
(b) Make a collectively exhaustive and mutually exclusive list of the possible events of the spending decision.
(c) Suppose the company has decided to spend the entire $800,000 on special campaigns. Does this change your answer to part (b)? If so, what is your new answer?

Worked-Out Answers to Self-Check Exercises

SC 4-1 (Die 1, Die 2)

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

SC 4-2 $P(1) = 0/36$, $P(2) = 1/36$, $P(5) = 4/36$, $P(6) = 5/36$, $P(7) = 6/36$, $P(10) = 3/36$, $P(11) = 2/36$.

4.3 THREE TYPES OF PROBABILITY

There are three basic ways of classifying probability. These three represent rather different conceptual approaches to the study of probability theory; in fact, experts disagree about which approach is the proper one to use. Let us begin by defining the

1. Classical approach
2. Relative frequency approach
3. Subjective approach

Classical Probability

Classical probability defined

Classical probability defines the probability that an event will occur as

Probability of an Event

$$\text{Probability of an event} = \frac{\text{number of outcomes where the event occurs}}{\text{total number of possible outcomes}} \qquad [4\text{-}1]$$

It must be emphasized that in order for Eq. 4-1 to be valid, each of the possible outcomes must be equally likely. This is a rather complex way of defining something that may seem intuitively obvious to us, but we can use it to write our coin-toss and dice-rolling examples in symbolic form. First, we would state the question, "What is the probability of getting a head on one toss?" as

$$P(\text{Head})$$

Then, using formal terms, we get

$$P(\text{Head}) = \frac{1}{1+1}$$

Number of outcomes of one toss where the event occurs (in this case, the number hat will produce a head)

Total number of possible outcomes of one toss (a head or a tail)

$$= 0.5 \text{ or } \frac{1}{2}$$

And for the dice-rolling example:

$$P(5) = \frac{1}{1+1+1+1+1+1}$$

Number of outcomes of one roll of the die that will produce a 5

Total number of possible outcomes of one roll of the die (getting a 1, a 2, a 3, a 4, a 5, or a 6)

$$= \frac{1}{6}$$

A priori probability

Classical probability is often called *a priori* probability because if we keep using orderly examples such as fair coins, unbiased dice, and standard decks of cards, we can state the answer in advance (a priori) *without* tossing a coin, rolling a die, or drawing a card. We do not have to perform experiments to make our probability statements about fair coins, standard card decks, and unbiased dice. Instead, we can make statements based on logical reasoning before any experiments take place.

Shortcomings of the classical approach

This approach assumes a number of assumptions, in defining the probability. So, if those assumptions are included then the complete definition should be: Probability of an event may be defined as the ratio of number of outcomes where the event occurs (favorable outcomes) to the total number of possible outcomes, provided these outcomes are equally likely (the chances of happening of all outcomes are equal), exhaustive (the totality of all outcomes are known and defined) and mutually exclusive (happening of one outcome results in non-happening of others). If these assumptions related to the outcomes are not followed, then this approach can not be applied in determining the probability.

This approach to probability is useful when we deal with card games, dice games, coin tosses, and the like, but has serious problems when we try to apply it to the less orderly decision problems we encounter in management. The classical approach to probability assumes a world that does not exist. It assumes away situations that are very unlikely but that could conceivably happen. Such occurrences as a coin landing on its edge, your classroom burning down during a discussion of probabilities, and your eating pizza while on a business trip at the North Pole are all extremely unlikely but not impossible. Nevertheless, the classical approach assumes them all away. Classical probability also assumes a kind of symmetry about the world, and that assumption can get us into trouble. Real-life situations, disorderly and unlikely as they often are, make it useful to define probabilities in other ways.

Relative Frequency of Occurrence

Suppose we begin asking ourselves complex questions such as, "What is the probability that I will live to be 85?" or "What are the chances that I will blow one of my stereo speakers if I turn my 200-watt amplifier up to wide open?" or "What is the probability that the location of a new paper plant on the river near our town will cause a substantial fish kill?" We quickly see that we may not be able to state in advance, without experimentation, what these probabilities are. Other approaches may be more useful.

Probability redefined

In the 1800s, British statisticians, interested in a theoretical foundation for calculating risk of losses in life insurance and commercial insurance, began defining probabilities from statistical data collected on births and deaths. Today, this approach is called the *relative frequency of occurrence*. It defines probability as either:

1. The observed relative frequency of an event in a very large number of trials, or
2. The proportion of times that an event occurs in the long run when conditions are stable.

Using the relative frequency of occurrence approach

This method uses the relative frequencies of past occurrences as probabilities. We determine how often something has happened in the past and use that figure to predict the probability that it will happen again in the future. Let us look at an example. Suppose an insurance company knows from past actuarial data that of all males 40 years old, about 60 out of every 100,000 will die within a 1-year period. Using this method, the company estimates the probability of

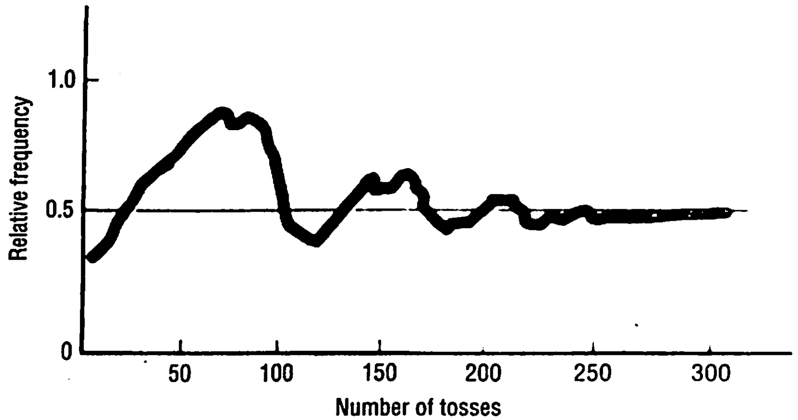


FIGURE 4-1 RELATIVE FREQUENCY OF OCCURRENCE OF HEADS IN 300 TOSSES OF A FAIR COIN

death for that age group as

$$\frac{60}{100,000}, \text{ or } 0.0006$$

More trials, greater accuracy

A second characteristic of probabilities established by the relative frequency of occurrence method can be shown by tossing one of our fair coins 300 times. Figure 4-1 illustrates the outcomes of these 300 tosses. Here we can see that although the proportion of heads was far from 0.5 in the first 100 tosses, it seemed to stabilize and approach 0.5 as the number of tosses increased. In statistical language, we would say that the relative frequency becomes stable as the number of tosses becomes large (if we are tossing the coin under uniform conditions). Thus, when we use the relative frequency approach to establish probabilities, our probability figure will gain accuracy as we increase the number of observations. Of course, this improved accuracy is not free; although more tosses of our coin will produce a more accurate probability of heads occurring, we must bear the time and the cost of additional observations.

Suppose an event is capable of being repeated sufficiently large number of times "N", and the frequency of the desired outcome is "f". Then relative frequency of the outcome is "f/N". The limiting value of the relative frequency can be used to define probability of the outcome.

A limitation of relative frequency

One difficulty with the relative frequency approach is that people often use it without evaluating a sufficient number of outcomes. If you heard someone say, "My aunt and uncle got the flu this year, and they are both over 65, so everyone in that age bracket will probably get the flu," you would know that your friend did not base his assumptions on enough evidence. His observations were insufficient data for establishing a relative frequency of occurrence probability.

This approach of defining probability is better then the Classical Approach, as it is not based on assumptions of mutually exclusive, equally likely and exhaustive. The drawback of using this approach is that it requires the event to be capable of being repeated large number of times. Moreover, one can not be certain that after how many occurrences, the relative frequency may stabilize. In the real and business world, we have to take decisions on those events which occur only once or not so frequent and the environmental conditions related to the situation might change. These factors restrict the use of this approach in real life decision making.

But what about a different kind of estimate, one that seems not to be based on statistics at all? Suppose your school's basketball team lost the first 10 games of the year. You were a loyal fan, however, and bet $100 that your team would beat Indiana's in the eleventh game. To everyone's surprise, you won your bet. We would have difficulty convincing you that you were statistically incorrect. And you would be right to be skeptical about our argument. Perhaps, without knowing that you did so, you may have based your bet on the statistical foundation described in the next approach to establishing probabilities.

Subjective Probabilities

Subjective probability defined

The relative frequency approach can't deal with specific or unique situations, which are typical of the business or management world. So, the probability approach dealing with such unique situations should be based upon some belief or educated guess of the decision maker.

Subjective probabilities are based on the beliefs of the person making the probability assessment. In fact, subjective probability can be defined as the probability assigned to an event by an individual, based on whatever evidence is available. This evidence may be in the form of relative frequency of past occurrences, or it may be just an educated guess. Probably the earliest subjective probability estimate of the likelihood of rain occurred when someone's Aunt Bess said, "My corns hurt; I think we're in for a downpour." Subjective assessments of probability permit the widest flexibility of the three concepts we have discussed. The decision maker can use whatever evidence is available and temper this with personal feelings about the situation.

Subjective probability assignments are often found when events occur only once or at most a very few times. Say that it is your job to interview and select a new social services caseworker. You have narrowed your choice to three people. Each has an attractive appearance, a high level of energy, abounding self-confidence, a record of past accomplishments, and a state of mind that seems to welcome challenges. What are the chances each will relate to clients successfully? Answering this question and choosing among the three will require you to assign a subjective probability to each person's potential.

Using the subjective approach

Here is one more illustration of this kind of probability assignment. A judge is deciding whether to allow the construction of a nuclear power plant on a site where there is some evidence of a geological fault. He must ask himself, "What is the probability of a major nuclear accident at this location?" The fact that there is no relative frequency of occurrence evidence of previous accidents at this location does not excuse him from making a decision. He must use his best judgment in trying to determine the subjective probabilities of a nuclear accident.

Because most higher-level social and managerial decisions are concerned with specific, unique situations, rather than with a long series of identical situations, decision makers at this level make considerable use of subjective probabilities.

The subjective approach to assigning probabilities was introduced in 1926 by Frank Ramsey in his book *The Foundation of Mathematics and Other Logical Essays*. The concept was further developed by Bernard Koopman, Richard Good, and Leonard Savage, names that appeared regularly in advanced work in this field. Professor Savage pointed out that two reasonable people faced with the same evidence could easily come up with quite different subjective probabilities for the same event. The two people who made opposing bets on the outcome of the Indiana basketball game would understand quite well what he meant.

HINTS & ASSUMPTIONS

Warning: In classical probability problems, be sure to check whether the situation is "with replacement" after each draw or "without replacement." The chance of drawing an ace from a 52-card deck on the first draw is $4/52$, or about .077. If you draw one and it is replaced, the odds of drawing an ace on the second draw are the same, $4/52$. However, without replacement, the odds change to $4/51$ if the first card was *not* an ace, or to $3/51$ if the first card *was* an ace. In assigning subjective probabilities, it's normal for two different people to come up with different probabilities for the same event; that's the result of experience and time (we often call this combination "wisdom"). In assigning probabilities using the relative frequency of occurrence method, be sure you have observed an adequate number of outcomes. Just because red hasn't come up in 9 spins of the roulette wheel, you shouldn't bet next semester's tuition on black this spin!

EXERCISES 4.3

Self-Check Exercises

SC 4-3 Union shop steward B. Lou Khollar has drafted a set of wage and benefit demands to be presented to management. To get an idea of worker support for the package, he randomly polls the two largest groups of workers at his plant, the machinists (M) and the inspectors (I). He polls 30 of each group with the following results:

Opinion of Package	M	I
Strongly support	9	10
Mildly support	11	3
Undecided	2	2
Mildly oppose	4	8
Strongly oppose	4	7
	30	30

(a) What is the probability that a machinist randomly selected from the polled group mildly supports the package?
(b) What is the probability that an inspector randomly selected from the polled group is undecided about the package?
(c) What is the probability that a worker (machinist or inspector) randomly selected from the polled group strongly or mildly supports the package?
(d) What types of probability estimates are these?

SC 4-4 Classify the following probability estimates as to their type (classical, relative frequency, or subjective):
(a) The probability of scoring on a penalty shot in ice hockey is 0.47.
(b) The probability that the current mayor will resign is 0.85.
(c) The probability of rolling two sixes with two dice is $1/36$.
(d) The probability that a president elected in a year ending in zero will die in office is $7/10$.
(e) The probability that you will go to Europe this year is 0.14.

Basic Concepts

4-11 Determine the probabilities of the following events in drawing a card from a standard deck of 52 cards:
(a) A seven.
(b) A black card.
(c) An ace or a king.
(d) A black two or a black three.
(e) A red face card (king, queen, or jack).
What type of probability estimates are these?

4-12 During a recent bridge game, once the lead card had been played and the dummy's hand revealed, the declarer took a moment to count up the number of cards in each suit with the results given below:

Suit	We	They
Spades	6	7
Hearts	8	5
Diamonds	4	9
Clubs	8	5
	26	26

(a) What is the probability that a card randomly selected from the We team's hand is a spade?
(b) What is the probability that a card randomly selected from the They team's hand is a club?
(c) What is the probability that a card randomly selected from all the cards is either a spade or heart?
(d) If this type of analysis were repeated for every hand many times, what would be the long-run probability that a card drawn from the We team's hand is a spade?

Applications

4-13 Below is a frequency distribution of annual sales commissions from a survey of 300 media salespeople.

Annual Commission	Frequency
$ 0–4,999	15
5,000–9,999	25
10,000–14,999	35
15,000–19,999	125
20,000–24,999	70
25,000+	30

Based on this information, what is the probability that a media salesperson makes a commission: (a) between $5,000 and $10,000, (b) less than $15,000, (c) more than $20,000, and (d) between $15,000 and $20,000.

4-14 General Buck Turgidson is preparing to make his annual budget presentation to the U.S. Senate and is speculating about his chances of getting all or part of his requested budget approved. From his 20 years of experience in making these requests, he has deduced that his chances of getting between 50 and 74 percent of his budget approved are twice as good as those of getting between 75 and 99 percent approved, and two and one-half times as good as those of getting between 25 and 49 percent approved. Further, the general believes that there is no chance of less than 25 percent of his budget being approved. Finally, the entire budget has been approved only once during the general's tenure, and the general does not expect this pattern to change. What are the probabilities of 0–24 percent, 25–49 percent, 50–74 percent, 75–99 percent, and 100 percent approval, according to the general?

4-15 The office manager of an insurance company has the following data on the functioning of the copiers in the office:

Copier	Days Functioning	Days Out of Service
1	209	51
2	217	43
3	258	2
4	229	31
5	247	13

What is the probability of a copier being out of service based on these data?

4-16 Classify the following probability estimates as classical, relative frequency, or subjective:
(a) The probability the Cubs will win the World Series this year is 0.175.
(b) The probability tuition will increase next year is 0.95.
(c) The probability that you will win the lottery is 0.00062.
(d) The probability a randomly selected flight will arrive on time is 0.875.
(e) The probability of tossing a coin twice and observing two heads is 0.25.
(f) The probability that your car will start on a very cold day is 0.97.

Worked-Out Answers to Self-Check Exercises

SC 4-3 (a) $$\text{P(Machinist mildly supports)} = \frac{\text{number of machinists in "mildly support" class}}{\text{total number of machinists polled}} = 11/30$$

(b) $$\text{P(Inspector undecided)} = \frac{\text{number of inspectors in "undecided" class}}{\text{total number of inspectors polled}} = 2/30 = 1/15$$

(c)

Opinion	Frequency (combined)
SS	19
MS	14
U	4
MO	12
SO	11
	60

P(Strongly or mildly support) = (19 + 14)/60 = 33/60 = 11/20

(d) Relative frequency.

SC 4-4 (a) Relative frequency. (b) Subjective. (c) Classical.
(d) Relative frequency. (e) Subjective.

4.4 PROBABILITY RULES

Most managers who use probabilities are concerned with two conditions:

1. The case where one event *or* another will occur
2. The situation where two or more events will *both* occur

We are interested in the first case when we ask, "What is the probability that today's demand will exceed our inventory?" To illustrate the second situation, we could ask, "What is the probability that today's demand will exceed our inventory *and* that more than 10 percent of our sales force will not report for work?" In the sections to follow, we shall illustrate methods of determining answers to questions such as these under a variety of conditions.

Some Commonly Used Symbols, Definitions, and Rules

Symbol for a Marginal Probability In probability theory, we use symbols to simplify the presentation of ideas. As we discussed earlier in this chapter, the probability of the event A is expressed as

Probability of Event A Happening

P(A) = the probability of event A happening

Marginal or unconditional probability

A *single* probability means that only one event can take place. It is called a *marginal* or *unconditional probability*. To illustrate, let us suppose that 50 members of a school class drew tickets to see which student would get a free trip to the National Rock Festival. Any one of the students could calculate his or her chances of winning as:

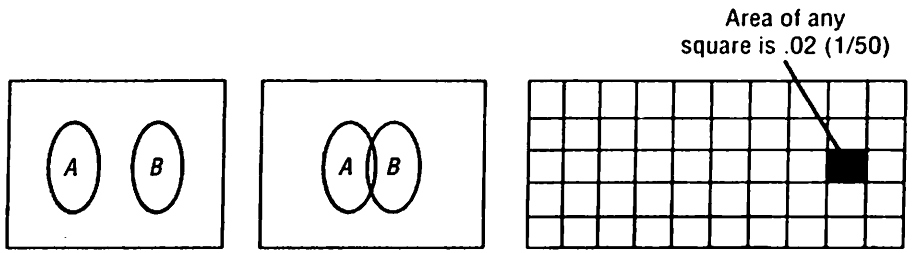
$$\text{P(Winning)} = \frac{1}{50}$$
$$= 0.02$$

In this case, a student's chance is 1 in 50 because we are certain that the possible events are mutually exclusive, that is, only one student can win at a time.

Venn diagrams

There is a nice diagrammatic way to illustrate this example and other probability concepts. We use a pictorial representation called a *Venn diagram*, after the nineteenth-century English mathematician John Venn. In these diagrams, the entire sample space is represented by a rectangle, and events are represented by parts of the rectangle. If two events *are* mutually exclusive, their parts of the rectangle will not overlap each other, as shown in Figure 4-2(a). If two events are *not* mutually exclusive, their parts of the rectangle *will* overlap, as in Figure 4-2(b).

Because probabilities behave a lot like areas, we shall let the rectangle have an area of 1 (because the probability of *something* happening is 1). Then the probability of an event is the area of *its* part of the rectangle. Figure 4-2(c) illustrates this for the National Rock Festival example. There the rectangle is divided into 50 equal, nonoverlapping parts.



Two mutually exclusive events (a) | Two nonexclusive events (b) | National rock festival example (c)

FIGURE 4-2 SOME VENN DIAGRAMS

Probability of one or more events not mutually exclusive

Addition Rule of Probabilistic Events If two events are not mutually exclusive, it is possible for both events to occur. In these cases, our addition rule must be modified. For example, what is the probability of drawing either an ace *or* a heart from a deck of cards? Obviously, the events ace and heart can occur together because we could draw the ace of hearts. Thus, ace and heart are not mutually exclusive events. We must adjust our Equation 4-3 to avoid double counting, that is, we have to *reduce* the probability of drawing either an ace or a heart *by the chance that we could draw both of them together*. As a result, the correct equation for the probability of one or more of two events that are not mutually exclusive is

Addition Rule of Probabilistic Events

Probability of A happening — Probability of A and B happening together

$$P(A \text{ or } B) = P(A) + P(B) - P(AB) \qquad [4\text{-}2]$$

Probability of A or B happening when A and B are not mutually exclusive — Probability of B happening

A Venn diagram illustrating Equation 4-2 is given in Figure 4-3. There, the event A or B is outlined with a heavy line. The event A *and* B is the cross-hatched wedge in the middle. If we add the areas of circles A and B, we *double count* the area of the wedge, and so we must subtract it to make sure it is counted only once.

Using Equation 4-2 to determine the probability of drawing either an ace *or* a heart, we can calculate:

$$P(\text{Ace or Heart}) = P(\text{Ace}) + P(\text{Heart}) - P(\text{Ace and Heart})$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$
$$= \frac{16}{52} \text{ or } \frac{4}{13}$$

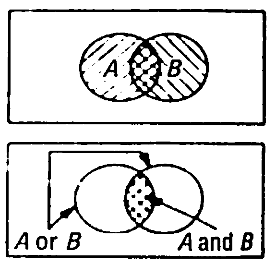


FIGURE 4-3 VENN DIAGRAM FOR THE ADDITION RULE FOR TWO EVENTS NOT MUTUALLY EXCLUSIVE

Let's do a second example. The employees of a certain company have elected five of their number to represent them on the employee-management productivity council. Profiles of the five are as follows:

1.	male	age 30
2.	male	32
3.	female	45
4.	female	20
5.	male	40

This group decides to elect a spokesperson by drawing a name from a hat. Our question is, "What is the probability the spokesperson will be *either* female *or* over 35?" Using Equation 4-2, we can set up the solution to our question like this:

$$P(\text{Female or Over 35}) = P(\text{Female}) + P(\text{Over 35}) - P(\text{Female and Over 35})$$
$$= \frac{2}{5} + \frac{2}{5} - \frac{1}{5}$$
$$= \frac{3}{5}$$

We can check our work by inspection and see that of the five people in the group, three would fit the requirements of being either female or over 35.

Probability of one or more mutually exclusive events

Addition Rule for Mutually Exclusive Events Often, however, we are interested in the probability that one thing *or* another will occur. If these two events are mutually exclusive, we can express this probability using the addition rule for mutually exclusive events. This rule is expressed symbolically as

$P(A \text{ or } B)$ = the **probability** of either A or B happening

and is calculated as follows:

Probability of Either *A* or *B* Happening

$$P(A \text{ or } B) = P(A) + P(B) \qquad [4\text{-}3]$$

This addition rule is illustrated by the Venn diagram in Figure 4-4, where we note that the area in the two circles together (denoting the event A or B) is the sum of the areas of the circle denoting the event A and the circle denoting the event B.

Now to use this formula in an example. Five equally capable students are waiting for a summer job interview with a company that has announced that it will hire only one of the five by random drawing. The group consists of Bill, Helen, John, Sally, and Walter. If our question is, "What is the probability that John will be the candidate?" we can use Equation 4-1 and give the answer.

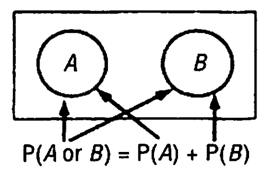


FIGURE 4-4 VENN DIAGRAM FOR THE ADDITION RULE FOR MUTUALLY EXCLUSIVE EVENTS

$$P(\text{John}) = \frac{1}{5}$$
$$= 0.02$$

However, if we ask, "What is the probability that either John *or* Sally will be the candidate?" we would use Equation 4-3:

$$\begin{aligned} P(\text{John or Sally}) &= P(\text{John}) + P(\text{Sally}) \\ &= \frac{1}{5} + \frac{1}{5} \\ &= \frac{2}{5} \\ &= 0.4 \end{aligned}$$

Let's calculate the probability of two or more events happening once more. Table 4-1 contains data on the sizes of families in a certain town. We are interested in the question, "What is the probability that a family chosen at random from this town will have four or more children (that is, four, five, six or more children)?" Using Equation 4-3, we can calculate the answer as

$$\begin{aligned} P(4, 5, 6 \text{ or more}) &= P(4) + P(5) + P(6 \text{ or more}) \\ &= 0.15 + 0.10 + 0.05 \\ &= 0.30 \end{aligned}$$

A special case of Equation 4-3

There is an important special case of Equation 4-3. For any event A, either A happens or it doesn't. So the events A and *not* A are exclusive and exhaustive. Applying Equation 4-3 yields the result

$$P(A) + P(not\ A) = 1$$

or, equivalently,

$$P(A) = 1 - P(not\ A)$$

For example, referring back to Table 4-1, the probability of a family's having five or fewer children is most easily obtained by subtracting from 1 the probability of the family's having six or more children, and thus is seen to be 0.95.

TABLE 4.1 FAMILY-SIZE DATA

NUMBER OF CHILDREN	0	1	2	3	4	5	6 or more
PROPORTION OF FAMILIES HAVING THIS MANY CHILDREN	0.05	0.10	0.30	0.25	0.15	0.10	0.05

HINTS & ASSUMPTIONS

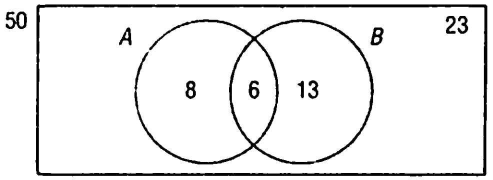
John Venn's diagrams are a useful way to avoid errors when you apply the addition rule for events that are and are not mutually exclusive. The most common error here is double counting. Hint: In applying the addition rule for mutually exclusive events, we're looking for a probability of one event or another and overlap is *not* a problem. However, with non–mutually exclusive events, both can occur together and we need to reduce our probability by the chance that they *could*. Thus, we subtract the overlap or cross-hatched area in the Venn diagram to get the correct value.

EXERCISES 4.4

Self-Check Exercises

SC 4-5 From the following Venn diagram, which indicates the number of outcomes of an experiment corresponding to each event and the number of outcomes that do not correspond to either event, give the probabilities indicated.

Total outcomes = 50



P(A) =
P(B) =
P(A or B) =

SC 4-6 An inspector of the Alaska Pipeline has the task of comparing the reliability of two pumping stations. Each station is susceptible to two kinds of failure: pump failure and leakage. When either (or both) occur, the station must be shut down. The data at hand indicate that the following probabilities prevail:

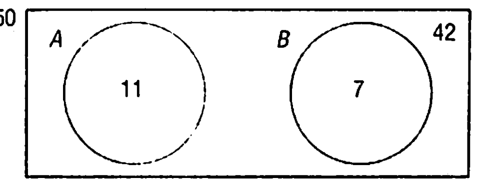
Station	P(Pump Failure)	P(Leakage)	P(Both)
1	0.07	0.10	0
2	0.09	0.12	0.06

Which station has the higher probability of being shut down?

Basic Concepts

4-17 From the following Venn diagram, which indicates the number of outcomes of an experiment corresponding to each event and the number of outcomes that do not correspond to either event, give the probabilities indicated:

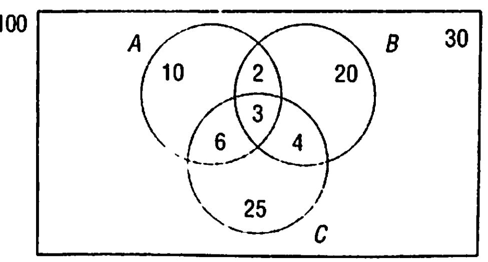
Total outcomes = 50



P(A) =
P(B) =
P(A or B) =

4-18 Using this Venn diagram, give the probabilities indicated:

Total outcomes = 100



P(A) =	P(B) =	P(C) =
P(A or B) =	P(A or C) =	P(B but not (A or C)) =

4-19 In this section, two expressions were developed for the probability of either of two events, A or B, occurring. Referring to Equations 4-2 and 4-3:
(a) What can you say about the probability of A and B occurring simultaneously when A and B are *mutually exclusive*?
(b) Develop an expression for the probability that at least one of three events, A, B, or C, could occur, that is, P(A or B or C). Do not assume that A, B, and C are mutually exclusive of each other.
(c) Rewrite your expression for the case in which A and B are mutually exclusive, but A and C and B and C are not mutually exclusive.
(d) Rewrite your expression for the case in which A and B and A and C are mutually exclusive, but not B and C.
(e) Rewrite your expression for the case in which A, B, and C are mutually exclusive of the others.

Applications

4.20 An employee at Infotech must enter product information into the computer. The employee may use a light pen that transmits the information to the PC along with the keyboard to issue commands, or fill out a bubble sheet and feed it directly into the old mainframe. Historically, we know the following probabilities:

P(Light pen will fail) = 0.025
P(PC keyboard will fail) = 0.15
P(Light pen and PC keyboard will fail) = 0.005
P(Mainframe will fail) = 0.25

Data can be entered into the PC only if both the light pen and keyboard are functioning.
(a) What is the probability that the employee can use the PC to enter data?
(b) What is the probability that either the PC fails or the mainframe fails? Assume they cannot both fail at the same time.

4-21 The HAL Corporation wishes to improve the resistance of its personal computer to disk-drive and keyboard failures. At present, the design of the computer is such that disk-drive failures occur only one-third as often as keyboard failures. The probability of simultaneous disk-drive and keyboard failures is 0.05.
(a) If the computer is 80 percent resistant to disk-drive and/or keyboard failure, how low must the disk-drive failure probability be?
(b) If the keyboard is improved so that it fails only twice as often as the disk-drive (and the simultaneous failure probability is still 0.05), will the disk-drive failure probability from part (a) yield a resistance to disk-drive and/or keyboard failure higher or lower than 90 percent?

4-22 The Herr–McFee Company, which produces nuclear fuel rods, must X-ray and inspect each rod before shipping. Karen Wood, an inspector, has noted that for every 1,000 fuel rods she inspects, 10 have interior flaws, 8 have casing flaws, and 5 have both flaws. In her quarterly report, Karen must include the probability of flaws in fuel rods. What is this probability?

Worked-Out Answers to Self-Check Exercises

SC 4-5 P(A) = 14/50 = 0.28 P(B) = 19/50 = 0.38

$$P(A \text{ or } B) = \frac{14}{50} + \frac{19}{50} - \frac{6}{50} = 0.54$$

SC 4-6 P(Failure) = P(Pump failure or leakage)
Station 1: 0.07 + 0.1 − 0 = 0.17 Station 2: 0.09 + 0.12 − 0.06 = 0.15
Thus, Station 1 has the higher probability of being shut down.

4.5 PROBABILITIES UNDER CONDITIONS OF STATISTICAL INDEPENDENCE

Independence defined

When two events happen, the outcome of the first event may or may not have an effect on the outcome of the second event. That is, the events may be either dependent or independent. In this section, we examine events that are *statistically independent:* The occurrence of one event *has no effect* on the probability of the occurrence of any other event. There are three types of probabilities under statistical independence:

1. Marginal
2. Joint
3. Conditional

Marginal Probabilities under Statistical Independence

Marginal probability of independent events

As we explained previously, a marginal or unconditional probability is the simple probability of the occurrence of an event. In a fair coin toss, P(H) = 0.5, and P(T) = 0.5; that is, the probability of heads equals 0.5 and the probability of tails equals 0.5. This is true for every toss, no matter how many tosses have been made or what their outcomes have been. Every toss stands alone and is in no way connected with any other toss. Thus, the outcome of *each* toss of a fair coin is an event that is statistically independent of the outcomes of *every other* toss of the coin.

Imagine that we have a biased or unfair coin that has been altered in such a way that heads occurs 0.90 of the time and tails 0.10 of the time. On each individual toss, P(H) = 0.90, and P(T) = 0.10. The outcome of any particular toss is completely unrelated to the outcomes of the tosses that may precede or follow it. The outcomes of several tosses of *this* coin are statistically independent events too, even though the coin is biased.

Joint Probabilities under Statistical Independence

Multiplication rule for joint, independent events

The probability of two or more independent events occurring together or in succession is the product of their marginal probabilities. Mathematically, this is stated (for two events):

Joint Probability of Two Independent Events

$$P(AB) = P(A) \times P(B) \qquad [4\text{-}4]$$

where

- $P(AB)$ = probability of events A and B occurring together or in succession; this is known as a *joint probability*
- $P(A)$ = marginal probability of event A occurring
- $P(B)$ = marginal probability of event B occurring

The fair coin example

In terms of the fair coin example, the probability of heads on two successive tosses is the probability of heads on the first toss (which we shall call H_1) times the probability of heads on the second toss (H_2). That is, $P(H_1H_2) = P(H_1) \times P(H_2)$. We have shown that the events are statistically independent, because the probability of any outcome is not affected by any preceding outcome. Therefore, the probability of heads on any toss is 0.5, and $P(H_1H_2) = 0.5 \times 0.5 = 0.25$. Thus, the probability of heads on two successive tosses is 0.25.

Likewise, the probability of getting three heads on three successive tosses is $P(H_1H_2H_3) = 0.5 \times 0.5 \times 0.5 = 0.125$.

Assume next that we are going to toss an unfair coin that has $P(H) = 0.8$ and $P(T) = 0.2$. The events (outcomes) are independent, because the probabilities of all tosses are exactly the same—the individual tosses are completely separate and in no way affected by any other toss or outcome. Suppose our question is, "What is the probability of getting three heads on three successive tosses?" We use Equation 4-4 and discover that:

$$P(H_1H_2H_3) = P(H_1) \times P(H_2) \times P(H_3) = 0.8 \times 0.8 \times 0.8 = 0.512$$

Now let us ask the probability of getting three tails on three successive tosses:

$$P(T_1T_2T_3) = P(T_1) \times P(T_2) \times P(T_3) = 0.2 \times 0.2 \times 0.2 = 0.008$$

Note that these two probabilities do not add up to 1 because the events $H_1H_2H_3$ and $T_1T_2T_3$ do not constitute a collectively exhaustive list. They *are* mutually exclusive, because if one occurs, the other cannot.

Constructing a probability tree

We can make the probabilities of events even more explicit using a *probability tree.* Figure 4-5 is a probability tree showing the possible outcomes and their respective probabilities for one toss of a fair coin.

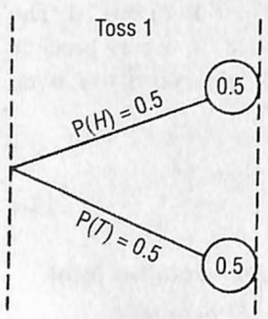


FIGURE 4-5 PROBABILITY TREE OF ONE TOSS

One toss, two possible outcomes

For toss 1, we have two possible outcomes, heads and tails, each with a probability of 0.5. Assume that the outcome of toss 1 is heads. We toss again. The second toss has two possible outcomes, heads and tails, each with a probability of 0.5. In Figure 4-6, we add these two branches of the tree.

Two tosses, four possible outcomes

Next we consider the possibility that the outcome of toss 1 is tails. Then the second toss must stem from the lower branch representing toss 1. Thus, in Figure 4-7, we add two more branches to the tree. Notice that on two tosses, we have four possible outcomes: H_1H_2, H_1T_2, T_1H_2, and T_1T_2 (remember the subscripts indicate the toss number, so that T_2, for example, means tails on toss 2). Thus, after two tosses, we may arrive at any one of four possible points. Because we are going to toss three times, we must add more branches to the tree.

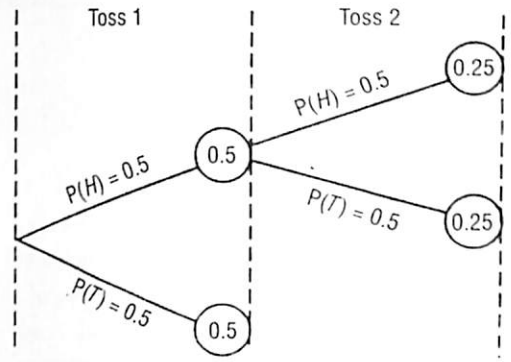


FIGURE 4-6 PROBABILITY TREE OF A PARTIAL SECOND TOSS

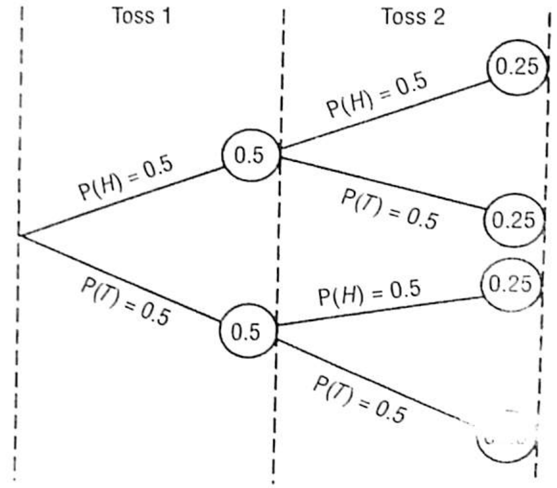


FIGURE 4-7 PROBABILITY TREE OF TWO TOSSES

Three tosses, eight possible outcomes

Assuming that we have had heads on the first two tosses, we are now ready to begin adding branches for the third toss. As before, the two possible outcomes are heads and tails, each with a probability of 0.5. The first step is shown in Figure 4-8. The additional branches are added in exactly the same

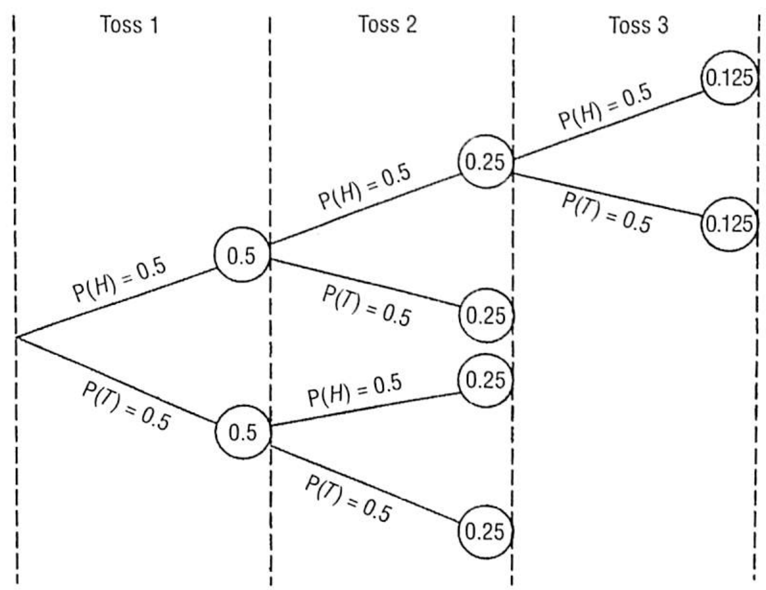


FIGURE 4-8 PROBABILITY TREE OF PARTIAL THIRD TOSS

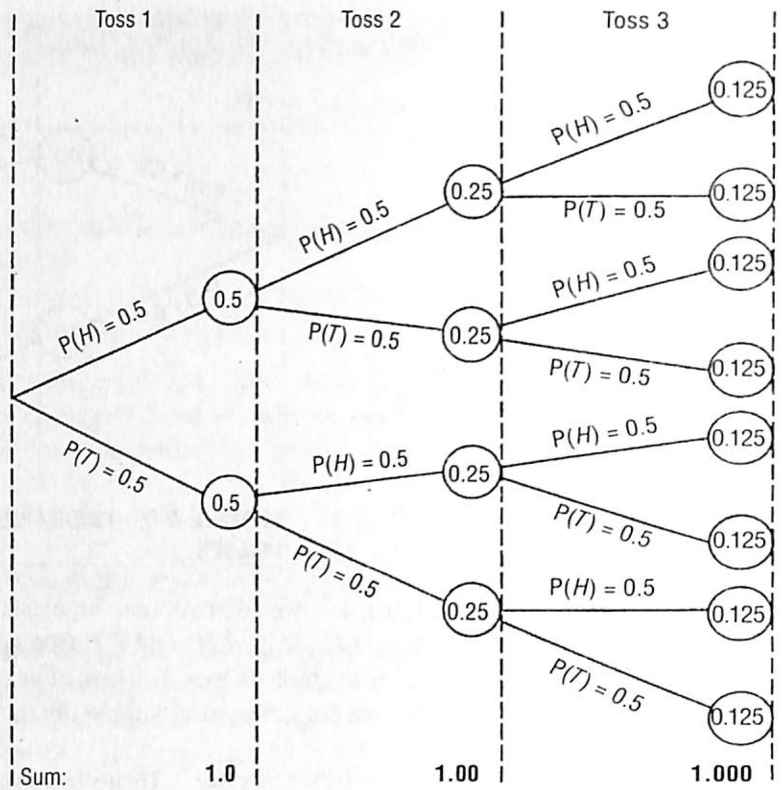


FIGURE 4-9 COMPLETED PROBABILITY TREE

manner. The completed probability tree is shown in Figure 4-9. Notice that both heads and tails have a probability of 0.5 of occurring no matter how far from the origin (first toss) any particular toss may be. **This follows from our definition of independence: No event is affected by the events preceding or following it.**

All tosses are independent

Suppose we are going to toss a fair coin and want to know the probability that all three tosses will result in heads. Expressing the problem symbolically, we want to know $P(H_1H_2H_3)$. From the mathematical definition of the joint probability of independent events, we know that

$$P(H_1H_2H_3) = P(H_1) \times P(H_2) \times P(H_3) = 0.5 \times 0.5 \times 0.5 = 0.125$$

We could have read this answer from the probability tree in Figure 4-9 by following the branches giving $H_1H_2H_3$.

Try solving these problems using the probability tree in Figure 4-9.

Outcomes in a particular order

Example 1 What is the probability of getting tails, heads, tails *in that order* on three successive tosses of a fair coin?

Solution $P(T_1H_2T_3) = P(T_1) \times P(H_2) \times P(T_3) = 0.125$. Following the prescribed path on the probability tree will give us the same answer.

TABLE 4-2 LISTS OF OUTCOMES

1 Toss		2 Tosses		3 Tosses	
Possible Outcomes	**Probability**	**Possible Outcomes**	**Probability**	**Possible Outcomes**	**Probability**
H_1	0.5	H_1H_2	0.25	$H_1H_2H_3$	0.125
T_1	0.5	H_1T_2	0.25	$H_1H_2T_3$	0.125
	1.0	T_1H_2	0.25	$H_1T_2H_3$	0.125
		T_1T_2	0.25	$T_1H_2H_3$	0.125
			1.00	$T_1H_2T_3$	0.125
				$T_1T_2H_3$	0.125
				$T_1T_2T_3$	0.125
					0.125
					1.000

The sum of the probabilities of all the possible outcomes must always equal 1

Example 2 What is the probability of getting tails, tails, heads *in that order* on three successive tosses of a fair coin?

Solution If we follow the branches giving tails on the first toss, tails on the second toss, and heads on the third toss, we arrive at the probability of 0.125. Thus, $P(T_1T_2H_3) = 0.125$.

It is important to notice that the probability of arriving at a given point by a given route is *not* the same as the probability of, say, heads on the third toss. $P(H_1T_2H_3) = 0.125$, but $P(H_3) = 0.5$. The first is a case of *joint probability* that is, the probability of getting heads on the first toss, tails on the second, and heads on the third. The latter, by contrast, is simply the *marginal probability* of getting heads on a particular toss, in this instance toss 3.

Notice that the sum of the probabilities of all the possible outcomes for each toss is 1. This results from the fact that we have mutually exclusive and collectively exhaustive lists of outcomes. These are given in Table 4-2.

Outcomes in terms of "at least"

Example 3 What is the probability of *at least* two heads on three tosses?

Solution Recalling that the probabilities of mutually exclusive events are additive, we can note the possible ways that at least two heads on three tosses can occur, and we can sum their individual probabilities. The outcomes satisfying the requirement are $H_1H_2H_3$, $H_1H_2T_3$, $H_1T_2H_3$, and $T_1H_2H_3$. Because each of these has an individual probability of 0.125, the sum is 0.5. Thus, the probability of at least two heads on three tosses is 0.5.

Example 4 What is the probability of *at least* one tail on three tosses?

Solution There is only one case in which no tails occur, namely $H_1H_2H_3$. Therefore, we can simply subtract for the answer:

$$1 - P(H_1H_2H_3) = 1 - 0.125 = 0.875$$

The probability of at least one tail occurring in three successive tosses is 0.875.

Example 5 What is the probability of *at least* one head on two tosses?

Solution The possible ways at least one head may occur are H_1H_2, H_1T_2, T_1H_2. Each of these has a probability of 0.25. Therefore, the probability of at least one head on two tosses is 0.75. Alternatively, we could consider the case in which no head occurs—namely, $T_1 T_2$—and subtract its probability from 1; that is,

$$1 - P(T_1T_2) = 1 - 0.25 = 0.75$$

Conditional Probabilities under Statistical Independence

Conditional probability

Thus far, we have considered two types of probabilities, marginal (or unconditional) probability and joint probability. Symbolically, marginal probability is $P(A)$ and joint probability is $P(AB)$. Besides these two, there is one other type of probability, known as *conditional probability*. Symbolically, conditional probability is written

$$P(B \mid A)$$

and is read, "the probability of event B given that event A has occurred."

Conditional probability is the probability that a second event (B) will occur *if* a first event (A) has already happened.

Conditional probability of independent events

For statistically independent events, the conditional probability of event B given that event A has occurred is simply the probability of event B:

Conditional Probability for Statistically independent Events

$$P(B \mid A) = P(B) \quad [4\text{-}5]$$

At first glance, this may seem to be contradictory. Remember, however, that by definition, independent events are those whose probabilities are in no way affected by the occurrence of each other. In fact, statistical independence is defined symbolically as the condition in which $P(B \mid A) = P(B)$.

We can understand conditional probability better by solving an illustrative problem. Our question is, "What is the probability that the second toss of a fair coin will result in heads, given that heads resulted on the first toss?" Symbolically, this is written as $P(H_2 \mid H_1)$. Remember that for two independent events, the results of the first toss have absolutely no effect on the results of the second toss. Because the probabilities of heads and tails are identical for every toss, the probability of heads on the second toss is 0.5. Thus, we must say that $P(H_2 \mid H_1) = 0.5$.

Table 4-3 summarizes the three types of probabilities and their mathematical formulas under conditions of statistical independence.

TABLE 4-3 PROBABILITIES UNDER STATISTICAL INDEPENDENCE

Type of Probability	Symbol	Formula
Marginal	$P(A)$	$P(A)$
Joint	$P(AB)$	$P(A) \times P(B)$
Conditional	$P(B \mid A)$	$P(B)$

HINTS & ASSUMPTIONS

Warning: In statistical independence, our assumption is that events are not related. In a series of coin toss examples, this is true, but in a series of business decisions, there may be a relationship among them. At the very least, you learn from the outcome of each decision and that knowledge affects your next decision. Before calculating conditional or joint probabilities in business situations while assuming independence, be careful you have considered some of the ways that experience affects future judgment.

EXERCISES 4.5

Self-Check Exercise

SC 4-7 What is the probability that in selecting two cards one at a time from a deck with replacement, the second card is
(a) A face card, given that the first card was red?
(b) An ace, given that the first card was a face card?
(c) A black jack, given that the first card was a red ace?

SC 4-8 Sol O'Tarry, a prison administrator, has been reviewing the prison records on attempted escapes by inmates. He has data covering the last 45 years that the prison has been open, arranged by seasons. The data are summarized in the table:

Attempted Escapes	Winter	Spring	Summer	Fall
0	3	2	1	0
1–5	15	10	11	12
6–10	15	12	11	16
11–15	5	8	7	7
16–20	3	4	6	5
21–25	2	4	5	3
More than 25	2	5	4	2
	45	**45**	**45**	**45**

(a) What is the probability that in a year selected at random, the number of escapes was between 16 and 20 during the winter?
(b) What is the probability that more than 10 escapes were attempted during a randomly chosen summer season?
(c) What is the probability that between 11 and 20 escapes were attempted during a randomly chosen season? (*Hint:* Group the data together.)

Basic Concepts

4-23 What is the probability that a couple's second child will be
(a) A boy, given that their first child was a girl?
(b) A girl, given that their first child was a girl?

4-24 In rolling two dice, what is the probability of rolling
(a) A total of 7 on the first roll, followed by a total of 11 on the second roll?
(b) A total of 21 on the first two rolls combined?
(c) A total of 6 on the first three rolls combined?

4-25 A bag contains 32 marbles: 4 are red, 9 are black, 12 are blue, 6 are yellow, and 1 is purple. Marbles are drawn one at a time with replacement. What is the probability that
(a) The second marble is yellow given the first one was yellow?
(b) The second marble is yellow given the first one was black?
(c) The third marble is purple given both the first and second were purple?

4-26 George, Richard, Paul, and John play the following game. Each man takes one of four balls numbered 1 through 4 from an urn. The man who draws ball 4 loses. The other three return their balls to the urn and draw again. Now the one who draws ball 3 loses. The other two return their balls to the urn and draw again. The man who draws ball 1 wins the game.
(a) What is the probability that John does not lose in the first two draws?
(b) What is the probability that Paul wins the game?

Applications

4-27 The health department routinely conducts two independent inspections of each restaurant, with the restaurant passing only if both inspectors pass it. Inspector A is very experienced, and, hence, passes only 2 percent of restaurants that actually do have health code violations. Inspector B is less experienced and passes 7 percent of restaurants with violations. What is the probability that
(a) Inspector A passes a restaurant, given that inspector B has found a violation?
(b) Inspector B passes a restaurant with a violation, given that inspector A passes it?
(c) A restaurant with a violation is passed by the health department?

4-28 The four floodgates of a small hydroelectric dam fail and are repaired independently of each other. From experience, it's known that each floodgate is out of order 4 percent of the time.
(a) If floodgate 1 is out of order, what is the probability that floodgates 2 and 3 are out of order?
(b) During a tour of the dam, you are told that the chances of all four floodgates being out of order are less than 1 in 5,000,000. Is this statement true?

4-29 Rob Rales is preparing a report that his employer, the Titre Corporation, will eventually deliver to the Federal Aviation Administration. First, the report must be approved by Rob's group leader, department head, and division chief (in that order). Rob knows from experience that the three managers act independently. Further, he knows that his group leader approves 85 percent of his reports, his department head approves 80 percent of the reports written by Rob that reach him, and his division chief approves 82 percent of Rob's work.
(a) What is the probability that the first version of Rob's report is submitted to the FAA?
(b) What is the probability that the first version of Rob's report is approved by his group leader and department head, but is not approved by his division chief?

4-30 A grocery store is reviewing its restocking policies and has analyzed the number of half-gallon containers of orange juice sold each day for the past month. The data are given below:

Number Sold	Morning	Afternoon	Evening
0–19	3	8	2
20–39	3	4	3
40–59	12	6	4
60–79	4	9	9
80–99	5	3	6
100 or more	3	0	6
	30	**30**	**30**

(a) What is the probability that on a randomly selected day the number of cartons of orange juice sold in the evening is between 80 and 99?
(b) What is the probability that 39 or fewer cartons were sold during a randomly selected afternoon?
(c) What is the probability that either 0–19 or 100 or more cartons were sold in a randomly selected morning?

4-31 Bill Borde, top advertising executive for Grapevine Concepts, has just launched a publicity campaign for a new restaurant in town, The Black Angus. Bill has just installed four billboards on a highway outside of town, and he knows from experience the probabilities that each will be noticed by a randomly chosen motorist. The probability of the first billboard's being noticed by a motorist is 0.75. The probability of the second's being noticed is 0.82, the third has a probability of 0.87 of being noticed, and the probability of the fourth sign's being noticed is 0.9. Assuming that the event that a motorist notices any particular billboard is independent of whether or not he notices the others, what is the probability that
(a) All four billboards will be noticed by a randomly chosen motorist?
(b) The first and fourth, but not the second and third billboards will be noticed?
(c) Exactly one of the billboards will be noticed?
(d) None of the billboards will be noticed?
(e) The third and fourth billboards won't be noticed?

Worked-Out Answers to Self-Check Exercises

SC 4-7 (a) $P(\text{Face}_2 \mid \text{Red}_1) = 12/52 = 3/13$
(b) $P(\text{Ace}_2 \mid \text{Face}_1) = 4/52 = 1/13$
(c) $P(\text{Black jack}_2 \mid \text{Red ace}_1) = 2/52 = 1/26$

SC 4-8 (a) $3/45 = 1/15$
(b) $(7 + 6 + 5 + 4)/45 = 22/45$
(c) $(8 + 12 + 13 + 12)/180 = 45/180 = 1/4$

4.6 PROBABILITIES UNDER CONDITIONS OF STATISTICAL DEPENDENCE

Dependence defined

Statistical dependence exists when the probability of some event is dependent on or affected by the occurrence of some

other event. Just as with independent events, the types of probabilities under statistical dependence are

1. Conditional
2. Joint
3. Marginal

Conditional Probabilities under Statistical Dependence

Conditional and joint probabilities under statistical dependence are more involved than marginal probabilities are. We shall discuss conditional probabilities first, because the concept of joint probabilities is best illustrated by using conditional probabilities as a basis.

Examples of conditional probability of dependent events

Assume that we have one box containing 10 balls distributed as follows:

- Three are colored and dotted.
- One is colored and striped.
- Two are gray and dotted.
- Four are gray and striped.

The probability of drawing any one ball from this box is 0.1, since there are 10 balls, each with equal probability of being drawn. The discussion of the following examples will be facilitated by reference to Table 4-4 and to Figure 4-10, which shows the contents of the box in diagram form.

Example 1 Suppose someone draws a colored ball from the box. What is the probability that it is dotted? What is the probability it is striped?

Solution This question can be expressed symbolically as $P(D|C)$, or "What is the conditional probability that this ball is dotted, *given* that it is colored?"

We have been told that the ball that was drawn is colored. Therefore, to calculate the probability that the ball is dotted, we will ignore *all* the gray balls and concern ourselves with the colored

TABLE 4-4 COLOR AND CONFIGURATION OF 10 BALLS

Event	Probability of Event	
1	0.1	colored and dotted
2	0.1	
3	0.1	
4	0.1	colored and striped
5	0.1	gray and dotted
6	0.1	
7	0.1	gray and striped
8	0.1	
9	0.1	
10	0.1	

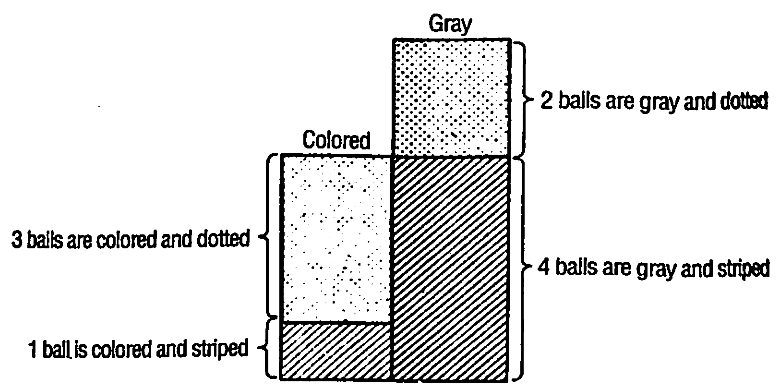


FIGURE 4-10 CONTENTS OF THE BOX

balls only. In diagram form, we consider only what is shown in Figure 4-11.

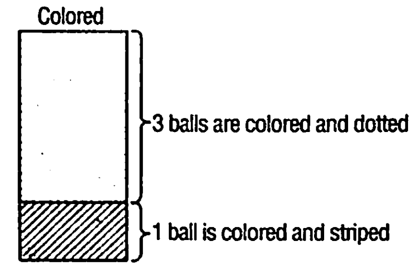


FIGURE 4-11 PROBABILITY OF DOTTED AND STRIPED, GIVEN COLORED

From the statement of the problem, we know that there are four colored balls, three of which are dotted and one of which is striped. Our problem is now to find the simple probabilities of dotted and striped. To do so, we divide the number of balls in each category by the total number of colored balls:

$$P(D|C) = \frac{3}{5} = 0.75$$
$$P(S|C) = \frac{1}{4} = 0.25$$
$$\overline{1.00}$$

In other words, three-fourths of the colored balls are dotted and one-fourth of the colored balls are striped. Thus, the probability of dotted, given that the ball is colored, is 0.75. Likewise, the probability of striped, given that the ball is colored, is 0.25.

Now we can see how our reasoning will enable us to develop the formula for conditional probability under statistical dependence. We can first assure ourselves that these events *are* statistically dependent by observing that the color of the balls determines the probabilities that they are either striped or dotted. For example, a gray ball is more likely to be striped than a colored ball is. Since color affects the probability of striped or dotted, these two events are dependent.

To calculate the probability of dotted given colored, $P(D|C)$, we divided the probability of colored and dotted balls (3 out of 10, or 0.3) by the probability of colored balls (4 out of 10, or 0.4):

$$P(D|C) = \frac{P(DC)}{P(C)}$$

Expressed as a general formula using the letters A and B to represent the two events, the equation is

Conditional Probability for Statistically Dependent Events

$$P(B|A) = \frac{P(BA)}{P(A)} \qquad [4\text{-}6]$$

This is the formula for *conditional probability under statistical dependence.*

Example 2 Continuing with our example of the colored and gray balls, let's answer the questions, "What is $P(D|G)$?" and "What is $P(S|G)$?"

Solution

$$P(D|G) = \frac{P(DG)}{P(G)} = \frac{0.2}{0.6} = \frac{1}{3}$$
$$P(S|G) = \frac{P(SG)}{P(G)} = \frac{0.4}{0.6} = \frac{2}{3}$$
$$\overline{1.0}$$

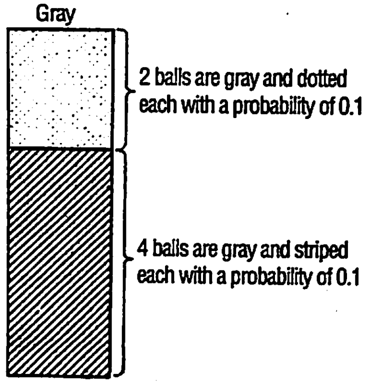


FIGURE 4-12 PROBABILITY OF DOTTED AND STRIPED, GIVEN GRAY

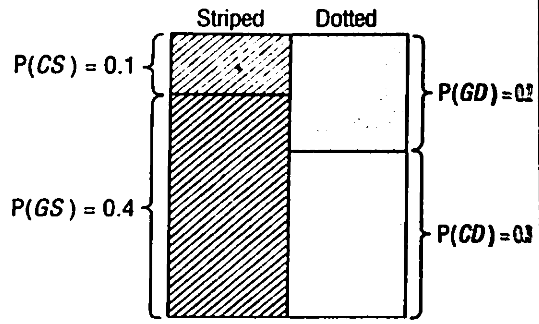


FIGURE 4-13 CONTENTS OF THE BOX ARRANGED BY CONFIGURATION, STRIPED AND DOTTED

The problem is shown diagrammatically in Figure 4-12.

The total probability of gray is 0.6 (6 out of 10 balls). To determine the probability that the ball (which we know is gray) will be dotted, we divide the probability of gray and dotted (0.2) by the probability of gray (0.6), or 0.2/0.6 = 1/3. Similarly, to determine the probability that the ball will be striped, we divide the probability of gray and striped (0.4) by the probability of gray (0.6), or 0.4/0.6 = 2/3.

Example 3 Calculate $P(G|D)$ and $P(C|D)$.

Solution Figure 4-13 shows the contents of the box arranged according to the striped or dotted markings on the balls. Because we have been told that the ball that was drawn is dotted, we can disregard striped and consider only dotted.

Now see Figure 4-14, showing the probabilities of colored and gray, given dotted. Notice that the relative proportions of the two are 0.4 to 0.6. The calculations used to arrive at these proportions were

$$P(G|D) = \frac{P(GD)}{P(D)} = \frac{0.2}{0.5} = 0.4$$

$$P(C|D) = \frac{P(CD)}{P(D)} = \frac{0.3}{0.5} = \underline{0.6}$$

$$\mathbf{1.0}$$

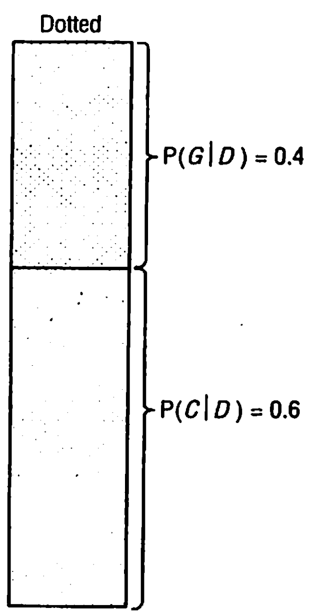


FIGURE 4-14 PROBABILITY OF COLORED AND GRAY, GIVEN DOTTED

Example 4 Calculate $P(C|S)$ and $P(G|S)$.

Solution

$$P(C|S) = \frac{P(CS)}{P(S)} = \frac{0.1}{0.5} = 0.2$$

$$P(G|S) = \frac{P(GS)}{P(S)} = \frac{0.4}{0.5} = \underline{0.8}$$

$$\mathbf{1.0}$$

Joint Probabilities under Statistical Dependence

We have shown that the formula for conditional probability under conditions of statistical dependence is

$$P(B \mid A) = \frac{P(BA)}{P(A)} \qquad [4\text{-}6]$$

If we solve this for $P(BA)$ by cross multiplication, we have the formula for *joint probability under conditions of statistical dependence:*

Joint Probability for Statistically Dependent Events

Joint probability of events B and A happening together or in succession

Probability of event B given that event A has happened

$$P(BA) = P(B \mid A) \times P(A)^{*} \qquad [4\text{-}7]$$

Probability that event A will happen

Notice that this formula is *not* $P(BA) = P(B) \times P(A)$, as it would be under conditions of statistical independence.

Converting the general formula $P(BA) = P(B \mid A) \times P(A)$ to our example and to the terms of colored, gray, dotted, and striped, we have $P(CD) = P(C \mid D) \times P(D)$, or $P(CD) = 0.6 \times 0.5 = 0.3$. Here, 0.6 is the probability of colored, given dotted (computed in Example 3 above) and 0.5 is the probability of dotted (also computed in Example 3).

$P(CD) = 0.3$ can be verified in Table 4-4, where we originally arrived at the probability by inspection: Three balls out of 10 are colored and dotted.

The following joint probabilities are computed in the same manner and can also be substantiated by reference to Table 4-4.

Several examples

$$P(CS) = P(C|S) \times P(S) = 0.2 \times 0.5 = 0.1$$

$$P(GD) = P(G|D) \times P(D) = 0.4 \times 0.5 = 0.2$$

$$P(GS) = P(G|S) \times P(S) = 0.8 \times 0.5 = 0.4$$

Marginal Probabilities under Statistical Dependence

Marginal probabilities under statistical dependence are computed by summing up the probabilities of all the joint events in which the simple event occurs. In the example above, we can compute the marginal probability of the event colored by summing the probabilities of the two joint events in which colored occurred:

$$P(C) = P(CD) + P(CS) = 0.3 + 0.1 = 0.4$$

*To find the joint probability of events A and B, you could also use the formula $P(BA) = P(AB) = P(A \mid B) \times P(B)$. This is because $BA = AB$.

TABLE 4-5 PROBABILITIES UNDER STATISTICAL INDEPENDENCE AND DEPENDENCE

Type of Probability	Symbol	Formula under Statistical Independence	Formula under Statistical Dependence
Marginal	$P(A)$	$P(A)$	Sum of the probabilities of the joint events in which A occurs
Joint	$P(AB)$	$P(A) \times P(B)$	$P(A\|B) \times P(B)$
	or $P(BA)$	$P(B) \times P(A)$	$P(B\|A) \times P(A)$
Conditional	$P(B\|A)$	$P(B)$	$\frac{P(BA)}{P(A)}$
	or $P(A\|B)$	$P(A)$	$\frac{P(AB)}{P(B)}$

Similarly, the marginal probability of the event gray can be computed by summing the probabilities of the two joint events in which gray occurred:

$$P(G) = P(GD) + P(GS) = 0.2 + 0.4 = 0.6$$

In like manner, we can compute the marginal probability of the event dotted by summing the probabilities of the two joint events in which dotted occurred:

$$P(D) = P(CD) + P(GD) = 0.3 + 0.2 = 0.5$$

And, finally, the marginal probability of the event striped can be computed by summing the probabilities of the two joint events in which gray occurred:

$$P(S) = P(CS) + P(GS) = 0.1 + 0.4 = 0.5$$

These four marginal probabilities, $P(C) = 0.4$, $P(G) = 0.6$, $P(D) = 0.5$, and $P(S) = 0.5$, can be verified by inspection of Table 4-4 on page 180.

We have now considered the three types of probability (conditional, joint, and marginal) under conditions of statistical dependence. Table 4-5 provides a résumé of our development of probabilities under both statistical independence and statistical dependence.

Example Department of Social Welfare has recently carried out a socio-economic survey of a village. The information collected is related to the gender of the respondent and level of education (graduation). 1000 respondent were surveyed. The results are presented in the following table:

	Educational Qualification		
Gender	Undergraduate	Graduate	Total
Male	150	450	600
Female	150	250	400
Total	300	700	1000

A respondent has been selected randomly, what are the chances that -

(a) The respondent will be Undergraduate (U)-

$$P(U) = 300/1000 = 0.3$$

(b) The respondent will be Graduate (G),

$$P(G) = 700/1000 = 0.7$$

(c) The respondent will be Female (F),

$$P(F) = 400/1000 = 0.4$$

These are the examples of Unconditional Probability. They are termed as unconditional because no condition is imposed on any event.

(d) The respondent will be Male-Graduate (MG)

$$P(\text{Male \& Graduate}) = P(MG) = 450/1000 = 0.45$$

(e) The respondent will be Undergraduate-Female (UF)

$$P(\text{Undergraduate and Female}) = P(UF) = 150/1000 = 0.15$$

The above two cases (d) and (e) are examples of Joint Probability.

(f) A randomly selected Female will be Graduate (G/F):

Here a condition has been imposed that randomly selected respondent has been Female. So, this is an example of Conditional Probability. In this case, we have to find out the probability of being Graduate, under the condition that the respondent should be Female. Hence consideration should be from a total of 400 (Female respondents only)

So, Probability of the respondent being Graduate, given Female-

$$P(G/F) = 250/400 = 0.625$$

The above concept can also be explained as under:

Probability of the respondent being Female,

$$P(F) = 400/1000 = 0.40$$

Probability of Female-Graduate

$$P(\text{Graduate \& Female}) = P(GF) = 250/1000 = 0.25$$

So,

$$P(G/F) = P(G \text{ and } F) / P(F) = 0.25/0.40 = 0.625$$

(g) A randomly selected Undergraduate will be Male (M/U)):

Probability of the respondent being Male, given Undergraduate,

$$P(M/U) = 150/300 = 0.50$$

Alternatively,

Probability of the respondent being Undergraduate,

$$P(U) = 300/1000 = 0.30$$

Probability of the respondent being Male-Undergraduate-

$$P\text{ (Male \& Undergraduate)} = P\text{ (MU)} = 150/1000 = 0.15$$

So,

$$P\text{ (M/U)} = P\text{ (M and U)} / P\text{ (U)} = 0.15/0.30 = 0.50$$

HINTS & ASSUMPTIONS

Hint: Distinguish between conditional probability and joint probability by careful use of terms *given that* and *both ... and: P(A|B)* is "the probability that A will occur *given* that *B* has occurred" and P(*AB*) is "the probability that *both A and B* will occur." And the *marginal probability* P(A) is the "probability that *A* will occur, whether or not *B* happens."

EXERCISES 4.6

Self-Check Exercises

SC 4-9 According to a survey, the probability that a family owns two cars if its annual income is greater than \$35,000 is 0.75. Of the households surveyed, 60 percent had incomes over \$35,000 and 52 percent had two cars. What is the probability that a family has two cars and an income over \$35,000 a year?

SC 4-10 Friendly's Department Store has been the target of many shoplifters during the past month, but owing to increased security precautions, 250 shoplifters have been caught. Each shoplifter's sex is noted; also noted is whether the perpetrator was a first-time or repeat offender. The data are summarized in the table.

Sex	First-Time Offender	Repeat Offender
Male	60	70
Female	44	76
	104	**146**

Assuming that an apprehended shoplifter is chosen at random, find
(a) The probability that the shoplifter is male.
(b) The probability that the shoplifter is a first-time offender, given that the shoplifter is male.
(c) The probability that the shoplifter is female, given that the shoplifter is a repeat offender.
(d) The probability that the shoplifter is female, given that the shoplifter is a first-time offender.
(e) The probability that the shoplifter is both male and a repeat offender.

Basic Concepts

4-32 Two events, A and B, are statistically dependent. If $P(A) = 0.39$, $P(B) = 0.21$, and $P(A \text{ or } B) = 0.47$, find the probability that
(a) Neither A nor B will occur.
(b) Both A and B will occur.

(c) B will occur, given that A has occurred.
(d) A will occur, given that B has occurred.

4-33 Given that $P(A) = 3/14$, $P(B) = 1/6$, $P(C) = 1/3$, $P(AC) = 1/7$, and $P(B|C) = 5/21$, find the following probabilities: $P(A|C)$, $P(C|A)$, $P(BC)$, $P(C|B)$.

4-34 Assume that for two events A and B, $P(A) = 0.65$, $P(B) = 0.80$, $P(A|B) = P(A)$, and $P(B|A) = 0.85$. Is this a consistent assignment of probabilities? Explain.

Applications

4-35 At a soup kitchen, a social worker gathers the following data. Of those visiting the kitchen, 59 percent are men, 32 percent are alcoholics, and 21 percent are male alcoholics. What is the probability that a random male visitor to the kitchen is an alcoholic?

4-36 During a study of auto accidents, the Highway Safety Council found that 60 percent of all accidents occur at night, 52 percent are alcohol-related, and 37 percent occur at night and are alcohol-related.
(a) What is the probability that an accident was alcohol-related, given that it occurred at night?
(b) What is the probability that an accident occurred at night, given that it was alcohol-related?

4-37 If a hurricane forms in the eastern half of the Gulf of Mexico, there is a 76 percent chance that it will strike the western coast of Florida. From data gathered over the past 50 years, it has been determined that the probability of a hurricane's occurring in this area in any given year is 0.85.
(a) What is the probability that a hurricane will occur in the eastern Gulf of Mexico and strike Florida this year?
(b) If a hurricane in the eastern Gulf of Mexico is seeded (induced to rain by addition of chemicals from aircraft), its probability of striking Florida's west coast is reduced by one-fourth. If it is decided to seed any hurricane in the eastern gulf, what is the new value for the probability in part (a)?

4-38 Al Cascade, president of the Litre Corporation, is studying his company's chances of being awarded an important water purification system contract for the Tennessee Valley Authority. Accordingly, two events are of interest to him. First, Litre's major competitor, WTR, is conducting purification research, which it hopes to complete before the contract award deadline. Second, there are rumors of a TVA investigation of all recent contractors, of which Litre is one and WTR is not. If WTR finishes its research and there is no investigation, then Litre's probability of being awarded the contract is 0.67. If there is an investigation but WTR doesn't finish its research, the probability is 0.72. If both events occur, the probability is 0.58, and if neither occurs, the probability is 0.85. The occurrence of an investigation and WTR's completion of research in time are independent events.
(a) Suppose that Al knows that the probability of WTR's completing its research in time is 0.80. How low must the probability of an investigation be so that the probability of Litre's being awarded the contract is at least 0.65?
(b) Suppose that Al knows that the probability of an investigation is 0.70. How low must the probability of WTR's completing its research on time be so that the probability of Litre's being awarded the contract is at least 0.65?
(c) Suppose that the probability of an investigation is 0.75 and the probability of WTR's completing its research in time is 0.85. What is the probability of Litre's being awarded the contract?

4-39 A company is considering upgrading its computer system, and a major portion of the upgrade is a new operating system. The company has asked an engineer for an evaluation of the operating system. Suppose the probability of a favorable evaluation is 0.65. If the probability the company will upgrade its system given a favorable evaluation is 0.85, what is the probability that the company will upgrade and receive a favorable evaluation?

4-40 The university's library has been randomly surveying patrons over the last month to see who is using the library and what services they have been using. Patrons are classified as undergraduate, graduate, or faculty. Services are classified as reference, periodicals, or books. The data for 350 people are given below. Assume a patron uses only one service per visit.

Patron	Reference	Periodicals	Books
Undergraduate	44	26	72
Graduate	24	61	20
Faculty	16	69	18
	84	156	110

Find the probability that a randomly chosen patron
(a) Is a graduate student.
(b) Visited the periodicals section, given the patron is a graduate student.
(c) Is a faculty member, given a reference section visit.
(d) Is an undergraduate who visited the book section.

4-41 The southeast regional manager of General Express, a private parcel-delivery firm, is worried about the likelihood of strikes by some of his employees. He has learned that the probability of a strike by his pilots is 0.75 and the probability of a strike by his drivers is 0.65. Further, he knows that if the drivers strike, there is a 90 percent chance that the pilots will strike in sympathy.
(a) What is the probability of both groups' striking?
(b) If the pilots strike, what is the probability that the drivers will strike in sympathy?

4-42 National Horticulture Board has been entrusted with the responsibility of sending good quality mangoes to overseas. For this purpose, an inspection is conducted on 10,000 boxes of mangoes from Malihabad and Hyderabad for exports. The inspection of boxes gave the following information:-

	Number of Boxes with		
	Number of Boxes	Damaged Fruit	Overripe Fruit
Malihabad	6000	200	840
Hyderabad	4000	365	295

(a) What are the chances that a selected box will contain damaged or overripe fruit?
(b) A randomly selected box contains overripe fruit, what is the probability that it has came from Hyderabad?

4-43 Fragnance Soaps Pvt Ltd is a leading soap manufacturing company in India. "Active" is a well known brand of the company. Company conducted a survey to find out preference for this brand. The marketing research responses are as shown in the following table:

Prefer	Ahmedabad	Gwalior	Raipur	Lucknow
Yes	55	40	80	75
No	40	30	20	90
No Opinion	5	10	20	35

If a customer is selected at random, what is the probability?
(a) That he or she prefers active?
(b) The consumer prefers Active and is from Ahmadabad?
(c) The consumer prefers Active given he is from Lucknow?
(d) That he is from Raipur and has no opinion?

Worked-Out Answers to Self-Check Exercises

SC 4-9 Let I = income > \$35,000 C = 2 cars.

$$P(C \text{ and } I) = P(C|I)P(I) = (0.75)(0.6) = 0.45$$

SC 4-10 M/W = shoplifter is male/female; F/R = shoplifter is first-time/repeat offender.
(a) $P(M) = (60 + 70)/250 = 0.520$
(b) $P(F|M) = P(F \text{ and } M)/P(M) = (60/250)/(130/250) = 0.462$
(c) $P(W|R) = P(W \text{ and } R)/P(R) = (76/250)/(146/250) = 0.521$
(d) $P(W|F) = P(W \text{ and } F)/P(F) = (44/250)/(104/250) = 0.423$
(e) $P(M \text{ and } R) = 70/250 = 0.280$

4.7 REVISING PRIOR ESTIMATES OF PROBABILITIES: BAYES' THEOREM

At the beginning of the baseball season, the fans of last year's pennant winner thought their team had a good chance of winning again. As the season progressed, however, injuries side lined the shortstop and the team's chief rival drafted a terrific home run hitter. The team began to lose. Late in the season, the fans realized that they must alter their prior probabilities of winning.

A similar situation often occurs in business. If a manager of a boutique finds that most of the purple and chartreuse ski jackets that she thought would sell so well are hanging on the rack, she must revise her prior probabilities and order a different color combination or have a sale.

Posterior probabilities defined

In both these cases, certain probabilities were altered after the people involved got additional information. The new probabilities are known as revised, or *posterior*, probabilities. Because probabilities can be revised as more information is gained, probability theory is of great value in managerial decision making.

Bayes' theorem

The origin of the concept of obtaining posterior probabilities with limited information is attributable to the Reverend Thomas Bayes (1702–1761), and the basic formula for conditional probability under dependence

$$P(B|A) = \frac{P(BA)}{P(A)} \quad [4\text{-}6]$$

is called *Bayes' Theorem.*

Bayes, an Englishman, was a Presbyterian minister and a competent mathematician. He pondered how he might prove the existence of God by examining whatever evidence the world about him provided. Attempting to show "that the Principal End of the Divine Providence . . . is the Happiness of His Creatures," the Reverend Bayes used mathematics to study God. Unfortunately, the theological implications of his findings so alarmed the good Reverend Bayes that he refused to permit publication of his work during his lifetime. Nevertheless, his work outlived him, and modern decision theory is often called Bayesian decision theory in his honor.

Value of Bayes' theorem

Bayes' theorem offers a powerful statistical method of evaluating new information and revising our prior estimates (based upon limited information only) of the probability that things are in one state or another. **If correctly used, it makes it unnecessary to gather masses of data over long periods of time in order to make good decisions based on probabilities.**

Calculating Posterior Probabilities

Finding a new posterior estimate

Revising probabilities based on one outcome

Assume, as a first example of revising prior probabilities, that we have equal numbers of two types of deformed (biased or weighted) dice in a bowl. On half of them, ace (or one dot) comes up 40 percent of the time; therefore P(ace) = 0.4. On the other half, ace comes up 70 percent of the time; P(ace) = 0.7. Let us call the former type 1 and the latter type 2. One die is drawn, rolled once, and comes up ace. What is the probability that it is a type 1 die? Knowing the bowl contains the same number of both types of dice, we might incorrectly answer that the probability is one-half; but we can do better than this. To answer the question correctly, we set up Table 4-6.

The sum of the probabilities of the elementary events (drawing either a type 1 or a type 2 die) is 1.0 because there are only two types of dice. The probability of each type is 0.5. The two types constitute a mutually exclusive and collectively exhaustive list.

The sum of the P(ace | elementary event) column does *not* equal 1.0. The figures 0.4 and 0.7 simply represent the conditional probabilities of getting an ace, given type 1 and type 2 dice, respectively.

The fourth column shows the joint probability of ace and type 1 occurring together ($0.4 \times 0.5 = 0.20$), and the joint probability of ace and type 2 occurring together ($0.7 \times 0.5 = 0.35$). The sum of these joint probabilities (0.55) is the marginal probability of getting an ace. Notice that in each case, the joint probability was obtained by using the formula

$$P(AB) = P(A|B) \times P(B) \qquad [4\text{-}7]$$

TABLE 4-6 FINDING THE MARGINAL PROBABILITY OF GETTING AN ACE

Elementary Event	Probability of Elementary Event	P(Ace| Elementary Event)	P(Ace, Elementary Event)*
Type 1	0.5	0.4	$0.4 \times 0.5 = 0.20$
Type 2	0.5	0.7	$0.7 \times 0.5 = 0.35$
	1.0		**P(ace) = 0.55**

*A comma is used to separate joint events. We can join individual letters to indicate joint events without confusion (*AB*, for example), but joining whole words in this way could produce strange looking events (aceelementaryevent) in this table, and they could be confusing.

To find the probability that the die we have drawn is type 1, we use the formula for conditional probability under statistical dependence:

$$P(B|A) = \frac{P(BA)}{P(A)} \qquad [4\text{-}6]$$

Converting to our problem, we have

$$P(\text{type } 1|\text{ace}) = \frac{P(\text{type } 1, \text{ace})}{P(\text{ace})}$$

or

$$P(\text{type } 1|\text{ace}) = \frac{0.20}{0.55} = 0.364$$

Thus, the probability that we have drawn a type 1 die is 0.364.

Let us compute the probability that the die is type 2:

$$P(\text{type } 2|\text{ace}) \frac{P(\text{type } 2, \text{ace})}{P(\text{ace})} = \frac{0.35}{0.55} = 0.636$$

Conclusion after one roll

What have we accomplished with one additional piece of information made available to us? What inferences have we been able to draw from one roll of the die? Before we rolled this die, the best we could say was that there is a 0.5 chance it is a type 1 die and a 0.5 chance it is a type 2 die. However, after rolling the die, we have been able to *alter*, or revise, *our prior probability estimate.* Our new posterior estimate is that there is a higher probability (0.636) that the die we have in our hand is a type 2 than that it is a type 1 (only 0.364).

Posterior Probabilities with More Information

Finding a new posterior estimate with more information

We may feel that one roll of the die is not sufficient to indicate its characteristics (whether it is type 1 or type 2). In this case, we can obtain additional information by rolling the die again. (Obtaining more information in most decision-making situations, of course, is more complicated and time-consuming.) Assume that the same die is rolled a second time and again comes up ace. What is the further revised probability that the die is type 1? To determine this answer, see Table 4-7.

TABLE 4-7 FINDING THE MARGINAL PROBABILITY OF TWO ACES ON TWO SUCCESSIVE ROLLS

Elementary Event	Probability of Elementary Event	P(Ace| Elementary Event)	P(2Aces| Elementary Event)	P(2 Aces, Elementary Event)
Type 1	0.5	0.4	0.16	$0.16 \times 0.5 = 0.080$
Type 2	0.5	0.7	0.49	$0.49 \times 0.5 = 0.245$
	1.0			**P(2 aces) = 0.325**

We have one new column in this table. P(2 aces | elementary event). This column gives the *joint* probability of two aces on two successive rolls if the die is type 1 and if it is type 2: P(2 aces | type 1) = 0.4 × 0.4 = 0.16, and P(2 aces | type 2) = 0.7 × 0.7 = 0.49. In the last column, we see the joint probabilities of two aces on two successive rolls and the elementary events (type 1 and type 2). That is, P(2 aces, type 1) is equal to P(2 aces | type 1) times the probability of type 1, or 0.16 × 0.5 = 0.080, and P(2 aces, type 2) is equal to P(2 aces | type 2) times the probability of type 2, or 0.49 × 0.5 = 0.245. The sum of these (0.325) is the marginal probability of two aces on two successive rolls.

We are now ready to compute the probability that the die we have drawn is type 1, given an ace on each of two successive rolls. Using the same general formula as before, we convert to

$$P(\text{type } 1|2 \text{ aces}) = \frac{P(\text{type } 1, 2 \text{ aces})}{P(2 \text{ aces})} = \frac{0.080}{0.325} = 0.246$$

Similarly,

$$P(\text{type } 2|2 \text{ aces}) = \frac{P(\text{type } 2, 2 \text{ aces})}{P(2 \text{ aces})} = \frac{0.245}{0.325} = 0.754$$

What have we accomplished with two rolls? When we first drew the die, all we knew was that there was a probability of 0.5 that it was type 1 and a probability of 0.5 that it was type 2. In other words, there was a 50–50 chance that it was either type 1 or type 2. After rolling the die once and getting an ace, we revised these original probabilities to the following:

Probability that it is type 1, given that an ace was rolled = 0.364

Probability that it is type 2, given that an ace was rolled = 0.636

After the second roll (another ace), we revised the probabilities again:

Probability that it is type 1, given that two aces were rolled = 0.246

Probability that it is type 2, given that two aces were rolled = 0.754

We have thus changed the original probabilities from 0.5 for each type to 0.246 for type 1 and 0.754 for type 2. This means that if a die turns up ace on two successive rolls, we can now assign a probability of 0.754 that it is type 2.

In both these experiments, we gained new information free of charge. We were able to roll the die twice, observe its behavior, and draw inferences from the behavior without any monetary cost. Obviously, there are few situations in which this is true, and managers must not only understand how to use new information to revise prior probabilities, but also be able to determine *how much that information is worth* to them before the fact. In many cases, the value of the information obtained may be considerably less than its cost.

A Problem with Three Pieces of Information

Example of posterior probability based on three trials

Consider the problem of a Little League baseball team that has been using an automatic pitching machine. If the machine is correctly set up—that is, properly adjusted—it will pitch strikes 85 percent of the time. If it is incorrectly set up, it will pitch strikes only 35 percent of the time. Past experience indicates that 75 percent of the setups of the machine are correctly done. After the machine has been set up at batting practice one day, it throws three strikes on the first three pitches. What is the revised probability that the setup has been done correctly? Table 4-8 illustrates how we can answer this question.

TABLE 4-8 POSTERIOR PROBABILITIES WITH THREE TRIALS

Event	P(Event) (1)	P(1 Strike|Event) (2)	P(3 Strikes|Event) (3)	P(Event, 3 Strikes) (4)
Correct	0.75	0.85	0.6141	0.6141 × 0.75 = 0.4606
Incorrect	0.25	0.35	0.0429	0.429 × 0.25 = 0.0107
	1.00			P(3 strikes) = 0.4713

We can interpret the numbered table headings in Table 4-8 as follows:

1. P(event) describes the individual probabilities of correct and incorrect. P(correct) = 0.75 is given in the problem. Thus, we can compute

   $$P(\text{incorrect}) = 1.00 - P(\text{correct}) = 1.00 - 0.75 = 0.25$$

2. P(1 strike | event) represents the probability of a strike given that the setup is correct or incorrect. These probabilities are given in the problem.
3. P(3 strikes | event) is the probability of getting three strikes on three successive pitches, given the event, that is, given a correct or incorrect setup. The probabilities are computed as follows:

   $$P(3 \text{ strikes} \mid \text{correct}) = 0.85 \times 0.85 \times 0.85 = 0.6141$$

   $$P(3 \text{ strikes} \mid \text{incorrect}) = 0.35 \times 0.35 \times 0.35 = 0.0429$$

4. P(event, 3 strikes) is the probability of the joint occurrence of the event (correct or incorrect) and three strikes. We can compute the probability in the problem as follows:

   $$P(\text{correct, 3 strikes}) = 0.6141 \times 0.75 = 0.4606$$

   $$P(\text{incorrect, 3 strikes}) = 0.0429 \times 0.25 = 0.0107$$

Notice that if A = event and S = strikes, these last two probabilities conform to the general mathematical formula for joint probabilities under conditions of dependence: $P(AS) = P(SA) = P(S \mid A) \times P(A)$, Equation 4-7.

After finishing the computation in Table 4-8, we are ready to determine the revised probability that the machine is correctly set up. We use the general formula

$$P(A|S) = \frac{P(AS)}{P(S)} \qquad [4\text{-}6]$$

and convert it to the terms and numbers in this problem:

$$P(\text{correct} \mid 3 \text{ strikes}) = \frac{P(\text{correct, 3 strikes})}{P(3 \text{ strikes})} = \frac{0.4606}{0.4713} = 0.9773$$

The *posterior probability* that the machine is correctly set up is 0.9773, or 97.73 percent. We have thus revised our original probability of a correct setup from 75 to 97.73 percent, based on three strikes being thrown in three pitches.

TABLE 4-9 POSTERIOR PROBABILITIES WITH INCONSISTENT OUTCOMES

Event	P (Event)	P (S|Event)	P(SBSSS|Event)	P(Event, SBSSS)
Correct	0.75	0.85	$0.85 \times 0.15 \times 0.85 \times 0.85 \times 0.85 = 0.07830$	$0.07830 \times 0.75 = 0.05873$
Incorrect	0.25	0.35	$0.35 \times 0.65 \times 0.35 \times 0.35 \times 0.35 = 0.00975$	$0.00975 \times 0.25 = 0.00244$
	1.00			P(SBSSS) = 0.06117

$$P(\text{correct setup}|SBSSS) = \frac{P(\text{correct setup}, SBSSS)}{P(SBSSS)} = \frac{0.05873}{0.06117} = 0.9601$$

Posterior Probabilities with Inconsistent Outcomes

An example with inconsistent outcomes

In each of our problems so far, the behavior of the experiment was consistent: the die came up ace on two successive rolls, and the automatic machine three strikes on each of the first three pitches. In most situations, we would expect a less consistent distribution of outcomes. In the case of the pitching machine, for example, we might find the five pitches to be: strike, ball, strike, strike, strike. Calculating our posterior probability that the machine is correctly set up in this case is really no more difficult than it was with a set of perfectly consistent outcomes. Using the notation S = strike and B = ball, we have solved this example in Table 4-9.

HINTS & ASSUMPTIONS

Posteriori Probabilities under Bayes Theorem has an application utility, they provided revised estimates of priori probabilities (chances) to the decision maker using the additional information presented. This helps in more effective decision-making. So estimates of the probability, based on historical information, are revised using additional information.

Bayes' theorem is a formal procedure that lets decision makers combine classical probability theory with their best intuitive sense about what is likely to happen. Warning: The real value of Bayes' theorem is not in the algebra, but rather in the ability of informed managers to make good guesses about the future. Hint: In all situations in which Bayes' theorem will be used, first use all the historical data available to you, and then (and only then) add your own intuitive judgment to the process. Intuition used to make guesses about things that are already statistically well-described is misdirected.

EXERCISES 4.7

Self-Check Exercises

SC 4-11 Given: The probabilities of three events, A, B, and C, occurring are $P(A) = 0.35$, $P(B) = 0.45$, and $P(C) = 0.2$. Assuming that A, B, or C has occurred, the probabilities of another event, X, occurring are $P(X \mid A) = 0.8$, $P(X \mid B) = 0.65$, and $P(X \mid C) = 0.3$. Find $P(A \mid X)$, $P(B \mid X)$, and $P(C \mid X)$.

SC 4-12 A doctor has decided to prescribe two new drugs to 200 heart patients as follows: 50 get drug A, 50 get drug B, and 100 get both. The 200 patients were chosen so that each had an 80 percent chance of having a heart attack if given neither drug. Drug A reduces the probability of a heart attack by 35 percent, drug B reduces the probability by 20 percent, and the two drugs, when taken together, work independently. If a randomly selected patient in the program has a heart attack, what is the probability that the patient was given both drugs?

Basic Concept

4-44 Two related experiments are performed. The first has three possible, mutually exclusive outcomes: A, B, and C. The second has two possible, mutually exclusive outcomes: X and Y. We know $P(A) = 0.2$ and $P(B) = 0.65$. We also know the following conditional probabilities if the result of the second experiment is X: $P(X \mid A) = 0.75$, $P(X \mid B) = 0.60$, and $P(X|C) = 0.40$. Find $P(A \mid X)$, $P(B \mid X)$, and $P(C \mid X)$. What is the probability that the result of the second experiment is Y?

Applications

4-45 Martin Coleman, credit manager for Beck's, knows that the company uses three methods to encourage collection of delinquent accounts. From past collection records, he learns that 70 percent of the accounts are called on personally, 20 percent are phoned, and 10 percent are sent a letter. The probabilities of collecting an overdue amount from an account with the three methods are 0.75, 0.60, and 0.65 respectively. Mr. Coleman has just received payment from a past-due account. What is the probability that this account
(a) Was called on personally?
(b) Received a phone call?
(c) Received a letter?

4-46 A public-interest group was planning to make a court challenge to auto insurance rates in one of three cities: Atlanta, Baltimore, or Cleveland. The probability that it would choose Atlanta was 0.40; Baltimore, 0.35; and Cleveland, 0.25. The group also knew that it had a 60 percent chance of a favorable ruling if it chose Baltimore, 45 percent if it chose Atlanta, and 35 percent if it chose Cleveland. If the group did receive a favorable ruling, which city did it most likely choose?

4-47 EconOcon is planning its company picnic. The only thing that will cancel the picnic is a thunderstorm. The Weather Service has predicted dry conditions with probability 0.2, moist conditions with probability 0.45, and wet conditions with probability 0.35. If the probability of a thunderstorm given dry conditions is 0.3, given moist conditions is 0.6, and given wet conditions is 0.8, what is the probability of a thunderstorm? If we know the picnic was indeed canceled, what is the probability moist conditions were in effect?

4-48 An independent research group has been studying the chances that an accident at a nuclear power plant will result in radiation leakage. The group considers that the only possible types of accidents at a reactor are fire, mechanical failure, and human error, and that two or more accidents never occur together. It has performed studies that indicate that if there were a fire, a radiation leak would occur 20 percent of the time; if there were a mechanical failure, a radiation leak would occur 50 percent of the time; and if there were a human error,

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- A fire and a radiation leak occurring together is 0.0010.
- A mechanical failure and a radiation leak occurring together is 0.0015.
- A human error and a radiation leak occurring together is 0.0012.

(a) What are the respective probabilities of a fire, mechanical failure, and human error?
(b) What are the respective probabilities that a radiation leak was caused by a fire, mechanical failure, and human error?
(c) What is the probability of a radiation leak?

4-49 A physical therapist at Enormous State University knows that the football team will play 40 percent of its games on artificial turf this season. He also knows that a football player's chances of incurring a knee injury are 50 percent higher if he is playing on artificial turf instead of grass. If a player's probability of knee injury on artificial turf is 0.42, what is the probability that
(a) A randomly selected football player incurs a knee injury?
(b) A randomly selected football player with a knee injury incurred the injury playing on grass?

4-50 The physical therapist from Exercise 4-48 is also interested in studying the relationship between foot injuries and position played. His data, gathered over a 3-year period, are summarized in the following table:

	Offensive Line	Defensive Line	Offensive Backfield	Defensive Backfield
Number of players	45	56	24	20
Number injured	32	38	11	9

Given that a randomly selected player incurred a foot injury, what is the probability that he plays in the (a) offensive line, (b) defensive line, (c) offensive backfield, and (d) defensive backfield?

4-51 A state Democratic official has decided that changes in the state unemployment rate will have a major effect on her party's chance of gaining or losing seats in the state senate. She has determined that if unemployment rises by 2 percent or more, the respective probabilities of losing more than 10 seats, losing 6 to 10 seats, gaining or losing 5 or fewer seats, gaining 6 to 10 seats, and gaining more than 10 seats are 0.25, 0.35, 0.15, 0.15, and 0.10, respectively. If unemployment changes by less than 2 percent, the respective probabilities are 0.10, 0.10, 0.15, 0.35, and 0.30. If unemployment falls by 2 percent or more, the respective probabilities are 0.05, 0.10, 0.10, 0.40, and 0.35. Currently this official believes that unemployment will rise by 2 percent or more with probability 0.25, change by less than 2 percent with probability 0.45, and fall by 2 percent or more with probability 0.30.
(a) If the Democrats gained seven seats, what is the probability that unemployment fell by 2 percent or more?
(b) If the Democrats lost one seat, what is the probability that unemployment changed by less than 2 percent?

4-52 T. C. Fox, marketing director for Metro-Goldmine Motion Pictures, believes that the studio's upcoming release has a 60 percent chance of being a hit, a 25 percent chance of being a moderate success, and a 15 percent chance of being a flop. To test the accuracy of his opinion, T. C. has scheduled two test screenings. After each screening, the audience rates the film on a scale of 1 to 10, 10 being best. From his long experience in the industry, T. C. knows that 60 percent of the time, a hit picture will receive a rating of 7 or higher; 30 percent of the time, it will receive a rating of 4, 5, or 6; and 10 percent of the time, it will receive a rating of 3 or lower. For a moderately successful picture, the respective probabilities are 0.30, 0.45, and 0.25; for a flop, the respective probabilities are 0.15, 0.35, and 0.50.
(a) If the first test screening produces a score of 6, what is the probability that the film will be a hit?
(b) If the first test screening produces a score of 6 and the second screening yields a score of 2, what is the probability that the film will be a flop (assuming that the screening results are independent of each other)?

Worked-Out Answers to Self-Check Exercises

SC 4-11

Event	P(Event)	P(X | Event)	P(X and Event)	P(Event | X)
A	0.35	0.80	0.2800	0.2800/0.6325 = 0.4427
B	0.45	0.65	0.2925	0.2925/0.6325 = 0.4625
C	0.20	0.30	0.0600	0.0600/0.6325 = 0.0949
			P(X) = 0.6325	

Thus, $P(A|X) = 0.4427$, $P(B|X) = 0.4625$, and $P(C|X) = 0.0949$.

SC 4-12 H = heart attack.

Event	P(Event)	P(H | Event)	P(H and Event)	P(Event | H)
A	0.25	(0.8)(0.65) = 0.520	0.130	0.130/0.498 = 0.2610
B	0.25	(0.8)(0.80) = 0.640	0.160	0.160/0.498 = 0.3213
$A\&B$	0.50	(0.8)(0.65)(0.80) = 0.416	0.208	0.208/0.498 = 0.4177
			P(X) = 0.498	

Thus, $P(A\&B \mid H) = 0.4177$.

STATISTICS AT WORK

Loveland Computers

Case 4: Probability "Aren't you going to congratulate me, Uncle Walter?" Lee Azko asked the CEO of Loveland Computers as they waved goodbye to their new-found investment bankers who were boarding their corporate jet.

"Sure, Lee, it was pretty enough stuff. But you'll find out that in business, there's more to life than gathering data. You have to make decisions, too— and often you don't have all the data you'd like because you're trying to guess what *will* happen in the future, not what *did* happen in the past. Get in the car and I'll explain.

"When we first started Loveland Computers, it was pretty much a wholesaling business. We'd bring in the computers from Taiwan, Korea, or wherever, and just ship 'em out the door with a label on them. Now that still works for some of the low-end products, but the higher-end stuff needs to be customized, so we run an assembly line here. Now I won't call it a factory, because there isn't a single thing that we 'make' here. We buy the cases from one place, the hard drives from somewhere else, and so on. Then we run the assembly line to make the machines just the way customers want them."

"Why don't you just have all the gizmos loaded on all the PCs, uncle?"

"Not a bad question, but here's the reason we can't do that. In this game, price is very important. And if you load a machine with something that a customer is never going to use—for example, going to the expense of adding a very large hard drive to a machine that's going to be used in a local area network, where most of the data will be kept on a file server—you end up pricing yourself out of the market, or selling at a loss. We can't afford to do either of those things. When we get back to the office, I want you to see Nancy Rainwater—she's the head of Production. She needs some help figuring out this month's schedule. This should give you some experience with real decision making."

Nancy Rainwater had worked for Loveland Computers for 5 years. Although Nancy was short on book learning, growing up on a farm nearby, she had learned some important practical skills about managing a workforce and getting work done on time. Her rise through the ranks to Production Supervisor had been rapid. Nancy explained her problem to Lee as follows.

"We have to decide whether to close the production line on Martin Luther King Day on the 20th of the month. Most of the workers on the line have children who will be off school that day. Your uncle, Mr. Azko, won't make it a paid vacation. But he might be open to closing the production line and letting people take the day off without pay if we can put in enough work days by the end of the month to meet our target production."

"Well, that shouldn't be too difficult to figure out—just count up the number of PCs produced on a typical day and divide that into the production target and see how many workdays you'll need," replied Lee with confidence.

"Well, I've already got that far. Not counting today, there are 19 workdays left until the end of the month, and I'll need 17 days to complete the target production."

"So let the workers take Martin Luther King Day off," Lee concluded.

"But there's more to it than that," Nancy continued. "This is 'colds and flu' season. If too many people call in sick—and believe me that happens when there's a 'bug' going around—I have to close the line for the day. I have records going back for a couple of years since I've been supervisor, and on an average winter day, there's a 1 in 30 chance that we'll have to close the line because of too many sick calls.

"And there's always a chance that we'll get a bad snowstorm—maybe even two—between now and the end of the month. Two years ago, two of the staff were in a terrible car wreck, trying to come to work on a day when the weather was real bad. So the company lawyer has told us to have a very tight 'snow day' policy. If the roads are dangerous, we close the line and lose that day's production. I'm not allowed to schedule weekend work to make up—that costs us time-and-a-half on wages and costs get out of line.

"I'd feel a lot better about closing the line for the holiday if I could be reasonably certain that we'd get in enough workdays by the end of the month. But I guess you don't have a crystal ball."

"Well, not a crystal ball, exactly. But I do have some ideas," Lee said, walking back toward the administrative offices, sketching something on a notepad. "By the way," said the younger Azko, turning back toward Nancy Rainwater, "What's *your* definition of 'reasonably certain?'"

Study Questions: What was Lee sketching on the notepad? What type of calculation will Lee make and what additional information will be needed? What difference will it make if Nancy's definition of "reasonably certain" means to meet the required production goal "75 percent of the time" or "99 percent of the time"?

CHAPTER REVIEW

Terms Introduced in Chapter 4

A Priori Probability Probability estimate made prior to receiving new information.

Bayes' Theorem The formula for conditional probability under statistical dependence.

Classical Probability The number of outcomes favorable to the occurrence of an event divided by the total number of possible outcomes.

Collectively Exhaustive Events A list of events that represents all the possible outcomes of an experiment.

Conditional Probability The probability of one event occurring, given that another event has occurred.

Event One or more of the possible outcomes of doing something, or one of the possible outcomes from conducting an experiment.

Experiment The activity that results in, or produces, an event.

Joint Probability The probability of two events occurring together or in succession.

Marginal Probability The unconditional probability of one event occurring; the probability of a single event.

Mutually Exclusive Events Events that cannot happen together.

Posterior Probability A probability that has been revised after additional information was obtained.

Probability The chance that something will happen.

Probability Tree A graphical representation showing the possible outcomes of a series of experiments and their respective probabilities.

Relative Frequency of Occurrence The proportion of times that an event occurs in the long run when conditions are stable, or the observed relative frequency of an event in a very large number of trials.

Sample Space The set of all possible outcomes of an experiment.

Statistical Dependence The condition when the probability of some event is dependent on, or affected by, the occurrence of some other event.

Statistical Independence The condition when the occurrence of one event has no effect on the probability of occurrence of another event.

Subjective Probability Probabilities based on the personal beliefs of the person making the probability estimate.

Venn Diagram A pictorial representation of probability concepts in which the sample space is represented as a rectangle and the events in the sample space as portions of that rectangle.

Equations Introduced in Chapter 4

4-1 $$\text{Probability of an event} = \frac{\text{number of outcomes where the event occurs}}{\text{total number of possible outcomes}}$$ p. 158

This is the definition of the *classical* probability that an event will occur.

$$P(A) = \text{probability of event } A \text{ happening}$$ p. 165

A single probability refers to the probability of one particular event occurring, and it is called *marginal* probability.

$P(A \text{ or } B)$ = probability of *either* A or B happening p. 167

This notation represents the probability that one event *or* the other will occur.

4-2 $$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$ p. 166

The addition rule for events that are not mutually exclusive shows that the probability of A or B happening when A and B are not mutually exclusive is equal to the probability of event A happening plus the probability of event B happening minus the probability of A and B happening together, symbolized $P(AB)$.

4-3 $$P(A \text{ or } B) = P(A) + P(B)$$ p. 167

The probability of either A or B happening when A and B are mutually exclusive equals the sum of the probability of event A happening and the probability of event B happening. This is the *addition ride for mutually exclusive events.*

4-4 $$P(AB) = P(A) \times P(B)$$ p. 172

where

- $P(AB)$ = joint probability of events A and B occurring together or in succession
- $P(A)$ = marginal probability of event A happening
- $P(B)$ = marginal probability of event B happening

The *joint* probability of two or more *independent* events occurring together or in succession is the product of their marginal probabilities.

$P(B \mid A)$ = probability of event B, *given* that event A has happened p. 176

This notation shows *conditional* probability, the probability that a second event (B) will occur if a first event (A) has already happened.

4-5 $$P(B \mid A) = P(B)$$ p. 176

For *statistically independent* events, the *conditional* probability of event B, given that event A has occurred, is simply the probability of event B. Independent events are those whose probabilities are in no way affected by the occurrence of each other.

4-6 $$P(B|A) = \frac{P(BA)}{P(A)}$$

and

$$P(A|B) = \frac{P(AB)}{P(B)}$$ p. 181

For statistically *dependent* events, the *conditional* probability of event B, given that event A has occurred, is equal to the joint probability of events A and B divided by the marginal probability of event A.

4-7 $$P(AB) = P(A|B) \times P(B)$$

and

$$P(BA) = P(B \mid A) \times P(A)$$ p. 183

Under conditions of statistical *dependence,* the *joint* probability of events A and B happening together or in succession is equal to the probability of event A, given that event B has already happened, multiplied by the probability that event B will happen.

Review and Application Exercises

4-53 Life insurance premiums are higher for older people, but auto insurance premiums are generally higher for younger people. What does this suggest about the risks and probabilities associated with these two areas of the insurance business?

4-54 "The chance of rain today is 80 percent." Which of the following best explains this statement?
(a) It will rain 80 percent of the day today.
(b) It will rain in 80 percent of the area to which this forecast applies today.
(c) In the past, weather conditions of this sort have produced rain in this area 80 percent of the time.

4-55 "There is a 0.25 probability that a restaurant in the United States will go out of business this year." When researchers make such statements, how have they arrived at their conclusions?

4-56 Using probability theory, explain the success of gambling and poker establishments.

4-57 Studies have shown that the chance of a new car being a "lemon" (one with multiple warranty problems) is greater for cars manufactured on Mondays and Fridays. Most consumers don't know on which day their car was manufactured. Assuming a 5-day production week, for a consumer taking a car at random from a dealer's lot,
(a) What is the chance of getting a car made on a Monday?
(b) What is the chance of getting a car made on Monday or Friday?
(c) What is the chance of getting a car made on Tuesday through Thursday?
(d) What type of probability estimates are these?

4-58 Isaac T. Olduso, an engineer for Atlantic Aircraft, disagrees with his supervisor about the likelihood of landing-gear failure on the company's new airliner. Isaac contends that the probability of landing-gear failure is 0.12, while his supervisor maintains that the probability is only 0.03. The two agree that if the landing gear fails, the airplane will crash with probability 0.55. Otherwise, the probability of a crash is only 0.06. A test flight is conducted, and the airplane crashes.
(a) Using Isaac's figure, what is the probability that the airplane's landing gear failed?
(b) Repeat part (a) using the supervisor's figure.

4-59 Congressman Bob Forehead has been thinking about the upcoming midterm elections and has prepared the following list of possible developments in his career during the midterm elections:

- He wins his party's nomination for reelection.
- He returns to his law practice.
- He is nominated for vice president.
- He loses his party's nomination for reelection.
- He wins reelection.

(a) Is each item on this list an "event" in the category of "Midterm Election Career Developments?"
(b) Are all of the items qualifying as "events" in part (a) mutually exclusive? If not, are any mutually exclusive?
(c) Are the events on the list collectively exhaustive?

4-60 Which of the following pairs of events are mutually exclusive?
(a) A defense department contractor loses a major contract, and the same contractor increases its work force by 50 percent.
(b) A man is older than his uncle, and he is younger than his cousins.

(c) A baseball team loses its last game of the year, and it wins the World Series.
(d) A bank manager discovers that a teller has been embezzling, and she promotes the same teller.

4-61 The scheduling officer for a local police department is trying to decide whether to schedule additional patrol units in each of two neighborhoods. She knows that on any given day during the past year, the probabilities of major crimes and minor crimes being committed in the northern neighborhood were 0.478 and 0.602, respectively, and that the corresponding probabilities in the southern neighborhood were 0.350 and 0.523. Assume that major and minor crimes occur independently of each other and likewise that crimes in the two neighborhoods are independent of each other.
(a) What is the probability that no crime of either type is committed in the northern neighborhood on a given day?
(b) What is the probability that a crime of either type is committed in the southern neighborhood on a given day?
(c) What is the probability that no crime of either type is committed in either neighborhood on a given day?

4-62 The Environmental Protection Agency is trying to assess the pollution effect of a paper mill that is to be built near Spokane, Washington. In studies of six similar plants built during the last year, the EPA determined the following pollution factors:

Plant	1	2	3	4	5	6
Sulfur dioxide emission in parts per million (ppm)	15	12	18	16	11	19

EPA defines excessive pollution as a sulfur dioxide emission of 18 ppm or greater.
(a) Calculate the probability that the new plant will be an excessive sulfur dioxide polluter.
(b) Classify this probability according to the three types discussed in the chapter: classical, relative frequency, and subjective.
(c) How would you judge the accuracy of your result?

4-63 The American Cancer Society is planning to mail out questionnaires concerning breast cancer. From past experience with questionnaires, the Cancer Society knows that only 15 percent of the people receiving questionnaires will respond. It also knows that 1.3 percent of the questionnaires mailed out will have a mistake in address and never be delivered, that 2.8 percent will be lost or destroyed by the post office, that 19 percent will be mailed to people who have moved, and that only 48 percent of those who move leave a forwarding address.
(a) Do the percentages in the problem represent classical, relative frequency, or subjective probability estimates?
(b) Find the probability that the Cancer Society will get a reply from a given questionnaire.

4-64 McCormick and Tryon, Inc., is a "shark watcher," hired by firms fearing takeover by larger companies. This firm has found that one of its clients, Pare and Oyd Co., is being considered for takeover by two firms. The first, Engulf and Devour, considered 20 such companies last year and took over 7. The second, R. A. Venus Corp., considered 15 such companies last year and took over 6. What is the probability of Pare and Oyd's being taken over this year, assuming that
(a) The acquisition rates of both Engulf and Devour and R. A. Venus are the same this year as they were last year?
(b) This year's acquisition rates are independent of last year's?
In each case, assume that only one firm may take over Pare and Oyd.

4-65 As the administrator of a hospital, Cindy Turner wants to know what the probability is that a person checking into the hospital will require X-ray treatment and will also have hospital insurance that will cover the X-ray treatment. She knows that during the past 5 years, 23 percent of the people entering the hospital required X-rays, and that during the same period, 72 percent of the people checking into the hospital had insurance that covered X-ray treatments. What is the correct probability? Do any additional assumptions need to be made?

4-66 An air traffic controller at Dulles Airport must obey regulations that require her to divert one of two airplanes if the probability of the aircraft's colliding exceeds 0.025. The controller has two inbound aircraft scheduled to arrive 10 minutes apart on the same runway. She knows that Flight 100, scheduled to arrive first, has a history of being on time, 5 minutes late, and 10 minutes late 95, 3, and 2 percent of the time, respectively. Further, she knows that Flight 200, scheduled to arrive second, has a history of being on time, 5 minutes early, and 10 minutes early 97, 2, and 1 percent of the time, respectively. The flights' timings are independent of each other.
(a) Must the controller divert one of the planes, based on this information?
(b) If she finds out that Flight 100 definitely will be 5 minutes late, must the controller divert one of the airplanes?
(c) If the controller finds out that Flight 200 definitely will be 5 minutes early, must she divert one of the airplanes?

4-67 In a staff meeting called to address the problem of returned checks at the supermarket where you are interning as a financial analyst, the bank reports that 12 percent of all checks are returned for insufficient funds, and of those, in 50 percent of cases, there was cash given back to the customer. Overall, 10 percent of customers ask for cash back at the end of their transaction with the store. For 1,000 customer visits, how many transactions will involve:
(a) Insufficient funds?
(b) Cash back to the customer?
(c) Both insufficient funds and cash back?
(d) Either insufficient funds or cash back?

4-68 Which of the following pairs of events are statistically independent?
(a) The times until failure of a calculator and of a second calculator marketed by a different firm.
(b) The life-spans of the current U.S. and Russian presidents.
(c) The amounts of settlements in asbestos poisoning cases in Maryland and New York.
(d) The takeover of a company and a rise in the price of its stock.
(e) The frequency of organ donation in a community and the predominant religious orientation of that community.

4-69 F. Liam Laytor, supervisor of customer relations for GLF Airlines, is studying his company's overbooking problem. He is concentrating on three late-night flights out of LaGuardia Airport in New York City. In the last year, 7, 8, and 5 percent of the passengers on the Atlanta, Kansas City, and Detroit flights, respectively, have been bumped. Further, 55, 20, and 25 percent of the late-night GLF passengers at LaGuardia take the Atlanta, Kansas City, and Detroit flights, respectively. What is the probability that a bumped passenger was scheduled to be on the
(a) Atlanta flight?
(b) Kansas City flight?
(c) Detroit flight?

4-70 An electronics manufacturer is considering expansion of its plant in the next 4 years. The decision depends on the increased production that will occur if either government or consumer sales increase. Specifically, the plant will be expanded if either (1) consumer sales increase 50 percent over the present sales level or (2) a major government contract is obtained. The company also believes that both these events will not happen in the same year. The planning director has obtained the following estimates:

- The probability of consumer sales increasing by 50 percent within 1, 2, 3, and 4 years is 0.05, 0.08, 0.12, and 0.16, respectively.
- The probability of obtaining a major government contract within 1, 2, 3, and 4 years is 0.08, 0.15, 0.25, and 0.32, respectively.

What is the probability that the plant will expand
(a) Within the next year (in year 1)?
(b) Between 1 and 2 years from now (in year 2)?
(c) Between 2 and 3 years from now (in year 3)?
(d) Between 3 and 4 years from now (in year 4)?
(e) At all in the next 4 years (assume at most one expansion)?

4-71 Draw Venn diagrams to represent the following situations involving three events, *A*, *B*, and *C*, which are part of a sample space of events but do not include the whole sample space.
(a) Each pair of events (*A* and *B*, *A* and *C*, and *B* and *C*) may occur together, but all three may not occur together.
(b) *A* and *B* are mutually exclusive, but not *A* and *C* nor *B* and *C*.
(c) *A*, *B*, and *C* are all mutually exclusive of one another.
(d) *A* and *B* are mutually exclusive, *B* and *C* are mutually exclusive, but *A* and *C* are not mutually exclusive.

4-72 Cartoonist Barry Bludeau sends his comics to his publisher via Union Postal Delivery. UPD uses rail and truck transportation in Mr. Bludeau's part of the country. In UPD's 20 years of operation, only 2 percent of the packages carried by rail and only 3.5 percent of the packages carried by truck have been lost. Mr. Bludeau calls the claims manager to inform him that a package containing a week of comics has been lost. If UPD sends 60 percent of the packages in that area by rail, which mode of transportation was more likely used to carry the lost comics? How does the solution change if UPD loses only 2 percent of its packages, regardless of the mode of transportation?

4-73 Determine the probability that
(a) Both engines on a small airplane fail, given that each engine fails with probability 0.05 and that an engine is twice as likely to fail when it is the only engine working.
(b) An automobile is recalled for brake failure and has steering problems, given that 15 percent of that model were recalled for brake failure and 2 percent had steering problems.
(c) A citizen files his or her tax return and cheats on it, given that 70 percent of all citizens file returns and 25 percent of those who file cheat.

4-74 Two-fifths of clients at Show Me Realty come from an out-of-town referral network, the rest are local. The chances of selling a home on each showing are 0.075 and 0.053 for out-of-town and local clients, respectively. If a salesperson walks into Show Me's office and announces "It's a deal!" was the agent more likely to have conducted a showing for an out-of-town or local client?

4-75 A senior North Carolina senator knows he will soon vote on a controversial bill. To learn his constituents' attitudes about the bill, he met with groups in three cities in his state. An aide jotted down the opinions of 15 attendees at each meeting:

Opinion	City		
	Chapel Hill	Raleigh	Lumberton
Strongly oppose	2	2	4
Slightly oppose	2	4	3
Neutral	3	3	5
Slightly support	2	3	2
Strongly support	6	3	1
Total	**15**	**15**	**15**

(a) What is the probability that someone from Chapel Hill is neutral about the bill? Strongly opposed?
(b) What is the probability that someone in the three city groups strongly supports the bill?
(c) What is the probability that someone from the Raleigh or Lumberton groups is neutral or slightly opposed?

4-76 The breakdown by political party of the 435 members of the U.S. House of Representatives before and after the 1992 Congressional elections was

	House Seats	
	Old	New
Democrats	268	259
Republicans	166	175
Independents	1	1

(a) Determine the probability that a member selected at random before the 1992 election would be a Republican.
(b) Determine the probability that a member selected at random after that election would not be a Republican.
(c) Is it fair to conclude that the probability that a randomly selected Democratic incumbent was not re-elected was 9/268? Explain.

4-77 A produce shipper has 10,000 boxes of bananas from Ecuador and Honduras. An inspection has determined the following information:

		# of Boxes with	
	# of Boxes	Damaged Fruit	Overripe Fruit
Ecuadoran	6,000	200	840
Honduran	4,000	365	295

(a) What is the probability that a box selected at random will contain damaged fruit? Overripe fruit?
(b) What is the probability that a randomly selected box is from Ecuador or Honduras?
(c) Given that a randomly selected box contains overripe fruit, what is the probability that it came from Honduras?
(d) If damaged fruit and overripe fruit are mutually exclusive, what is the probability that a box contains damaged or overripe fruit? What if they are not mutually exclusive?

4-78 Marcia Lerner will graduate in 3 months with a master's degree in business administration. Her school's placement office indicates that the probability of receiving a job offer as the result of any given on-campus interview is about 0.07 and is statistically independent from interview to interview.
(a) What is the probability that Marcia will not get a job offer in any of her next three interviews?

(b) If she has three interviews per month, what is the probability that she will have at least one job offer by the time she finishes school?
(c) What is the probability that in her next five interviews she will get job offers on the third and fifth interviews only?

4-79 A standard set of pool balls contains 15 balls numbered from 1 to 15. Pegleg Woodhull, the famous blind poolplayer, is playing a game of 8-ball, in which the 8-ball must be, the last one hit into a pocket. He is allowed to touch the balls to determine their positions before taking a shot, but he does not know their numbers. Every shot Woodhull takes is successful.
(a) What is the probability that he hits the 8-ball into a pocket, on his first shot, thus losing the game?
(b) What is the probability that the 8-ball is one of the first three balls he hits?
(c) What is the probability that Pegleg wins the game, that is, that the 8-ball is the last ball hit into a pocket?

4-80 BMT, Inc., is trying to decide which of two oil pumps to use in its new race car engine. One pump produces 75 pounds of pressure and the other 100. BMT knows the following probabilities associated with the pumps:

	Probability of Engine Failure Due to	
	Seized Bearings	**Ruptured Head Gasket**
Pump A	0.08	0.03
Pump B	0.02	0.11

(a) If seized bearings and ruptured head gaskets are mutually exclusive, which pump should BMT use?
(b) If BMT devises a greatly improved "rupture-proof" head gasket, should it change its decision?

4-81 Sandy Irick is the public relations director for a large pharmaceutical firm that has been attacked in the popular press for distributing an allegedly unsafe vaccine. The vaccine protects against a virulent contagious disease that has a 0.04 probability of killing an infected person. Twenty-five percent of the population has been vaccinated.

A researcher has told her the following: The probability of any unvaccinated individual acquiring the disease is 0.30. Once vaccinated, the probability of acquiring the disease through normal means is zero. However, 2 percent of vaccinated people will show symptoms of the disease, and 3 percent of that group will die from it. Of people who are vaccinated and show no symptoms from the vaccination, 0.05 percent will die. Irick must draw some conclusions from these data for a staff meeting in 1 hour and a news conference later in the day.
(a) If a person is vaccinated, what is the probability of dying from the vaccine? If he was not vaccinated, what is the probability of dying?
(b) What is the probability of a randomly selected person dying from either the vaccine or the normally contracted disease?

4-82 The pressroom supervisor for a daily newspaper is being pressured to find ways to print the paper closer to distribution time, thus giving the editorial staff more leeway for last-minute changes. She has the option of running the presses at "normal" speed or at 110 percent of normal—"fast" speed. She estimates that they will run at the higher speed 60 percent of the time. The roll of paper (the newsprint "web") is twice as likely to tear at the higher speed, which would mean temporarily stopping the presses,
(a) If the web on a randomly selected printing run has a probability of 0.112 of tearing, what is the probability that the web will not tear at normal speed?

(b) If the probability of tearing on fast speed is 0.20, what is the probability that a randomly selected torn web occurred on normal speed?

4-83 Refer to Exercise 4-83. The supervisor has noted that the web tore during each of the last four runs and that the speed of the press was not changed during these four runs. If the probabilities of tearing at fast and slow speeds were 0.14 and 0.07, respectively, what is the revised probability that the press was operating at fast speed during the last four runs?

4-84 A restaurant is experiencing discontentment among its customers. Historically it is known that there are three factors responsible for discontent amongst the customers viz. food quality, services quality, and interior décor. By conducting an analysis, it assesses the probabilities of discontentment with the three factors as 0.40, 0.35 and 0.25, respectively. By conducting a survey among customers, it also evaluates the probabilities of a customer going away discontented on account of these factors as 0.6, 0.8 and 0.5, respectively. The restaurant manager knows that a customer is discontented, what is the probability that it is due to service quality?

4-85 An economist believes that the chances of the Indian Rupee appreciating during period of high economic growth is 0.70, during moderate economic growth the chances of appreciation is 0.40, and during low economic growth it is 0.20. During any given time period the probability of high and moderate economic growth is 0.30 and 0.50 respectively. According to the RBI report the Rupee has been appreciating during the present period. What is the probability that the economy is experiencing a period of low economic growth?

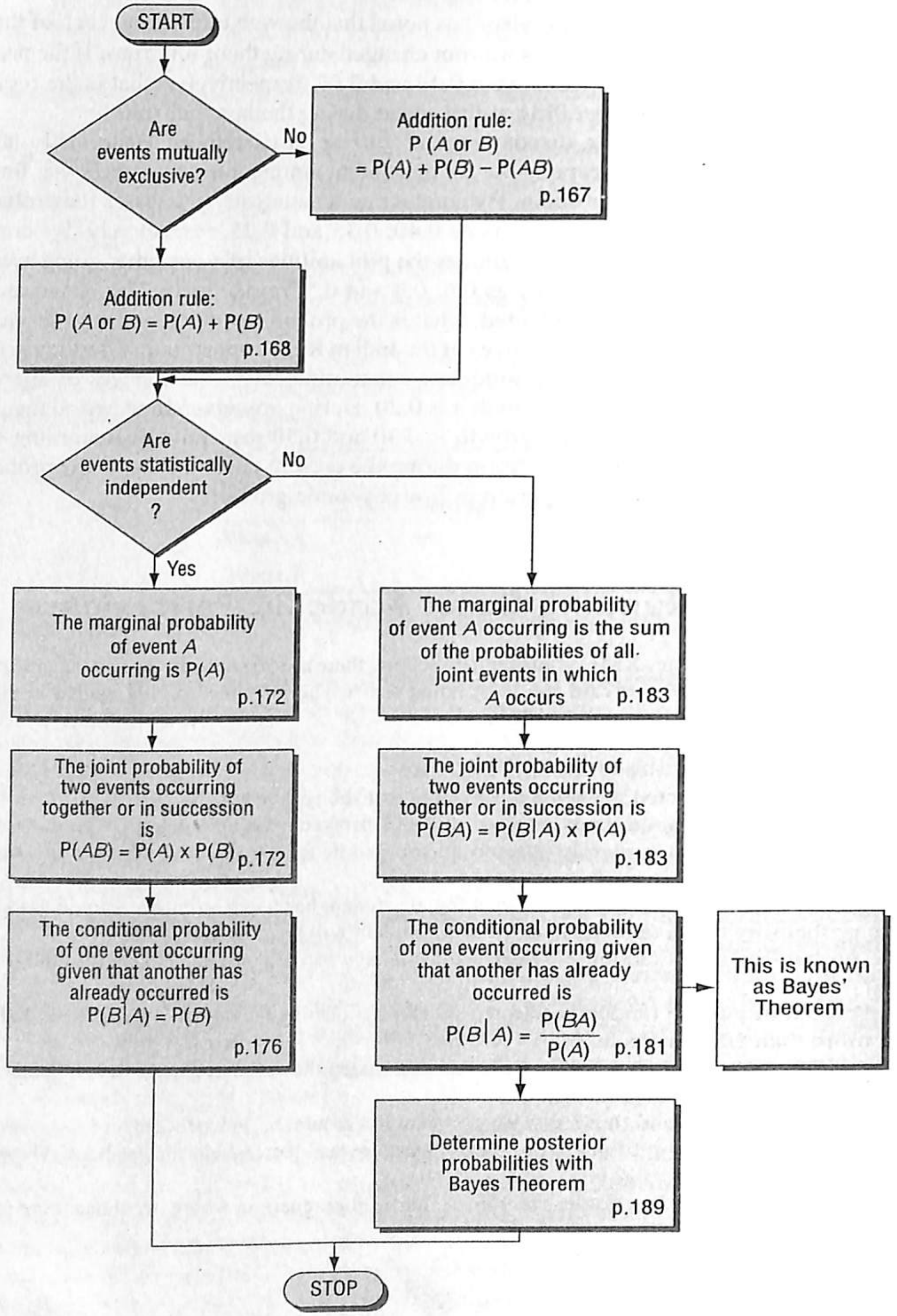
CASE Questions on Running Case: Academic Performance

In the MBA-I Trimester of a college, XML Management School, there are 50 students. Their academic performance along with their gender and subject-stream has been noted down. The information is presented in the data sheet provide in Disk (Case_Academic Performance-Data.xls)

Answer the following questions:

1. If a student is randomly selected, what are the chances that she will be a male?
2. What is the probability that a randomly selected student has taken commerce stream in graduation?
3. What is the probability that a randomly selected student will be female and have taken professional stream in graduation?
4. What is the probability that a randomly selected female student has taken science stream in graduation?
5. What is the probability that a randomly selected arts student will be male?
6. What is the probability that a randomly selected student have secured at least 75% marks both in XII and graduation?
7. What is the probability that a randomly selected student has obtained less than 70% marks in XII provided he/she has more than 80% marks in X?
8. What is the probability that a randomly selected female student have secured more than 80 percentile in CAT if she has above 75% marks in graduation?
9. Are the events 'being male' and 'having science stream in graduation' independent?
10. A randomly selected student is found to be female, what are the chances that she has her CAT percentile in between 75 and 90?
11. A randomly selected student has got less than 65% marks in graduation, which event has more probability that the student would be a male or female?

Flow Chart: Probability I: Introductory Ideas



5 Probability Distributions

LEARNING OBJECTIVES

After reading this chapter, you can understand:

- To introduce the probability distributions most commonly used in decision making
- To use the concept of expected value to make decisions
- To show which probability distribution to use and how to find its values
- To understand the limitations of each of the probability distributions you use

CHAPTER CONTENTS