# Centre of gravity

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- Every particle of body is attracted by the earth towards its centre.

* **CG :-** It is defined as a point about which the entire weight of the body is assumed to be concentrated.
- It is related to distribution of mass.
- C.G word is used to represent centre of solid bodies like, cube, cone, sphere etc.

* **Centroid :-** It is defined as a point about which the entire area, or vol^m is assumed to be concentrated.
It have only area, but no mass.

* Methods for centre of gravity

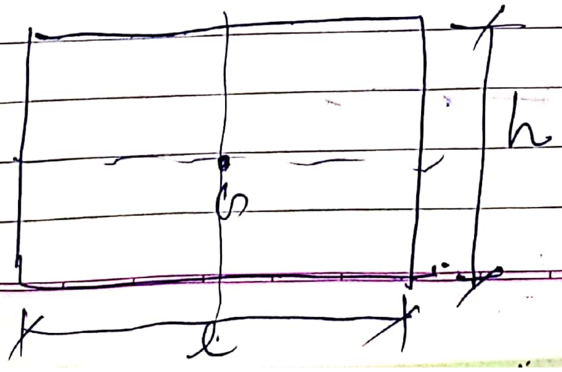
(1) By geometrical considerations.

(2) By moments.

(3) By graphical method.

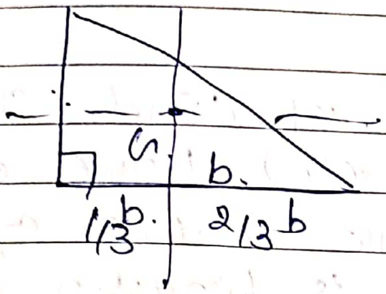
(1) Geometrical

(1)



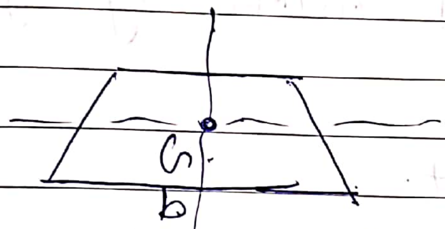

C.G $= (l/2, h/2)$

(2)




$C.G = (1/3\, b, 1/3\, h)$

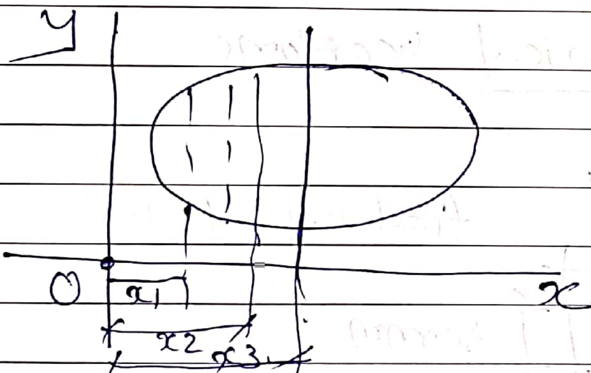
(3)




(Trapezium)

$C.G = \frac{h}{3}\left(\frac{b+2a}{b+a}\right)$

* by moments




Axis of reference

The c.g of body is req^d to be found by with reference to some assumed axis is known as. Axis of reference.

Consider body of mass m whose c.g is req^d to be found out. Divide the body into small masses, whose c.g are known as shown in fig. let $m_1, m_2, m_3$ .. etc be the masses of the particles. & $(x_1, y_1), (x_2, y_2)$ .. be the co-ordinates of the c.g. from O. as shown in fig.

$M\bar{x} = m_1x_1 + m_2x_2 + m_3x_3$

$\bar{x} = \frac{\Sigma mx}{M}$ , $\bar{y} = \frac{\Sigma my}{m}$ , $M = m_1 + m_2 + ...$

## * Cg of plane figures

- plane geometrical figures (such as T, I section, L section etc). have only area but no mass. The centre of area of such figures is known as centroid & coincides with cg. of the fig.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + \dots}{a_1 + a_2 + }$$

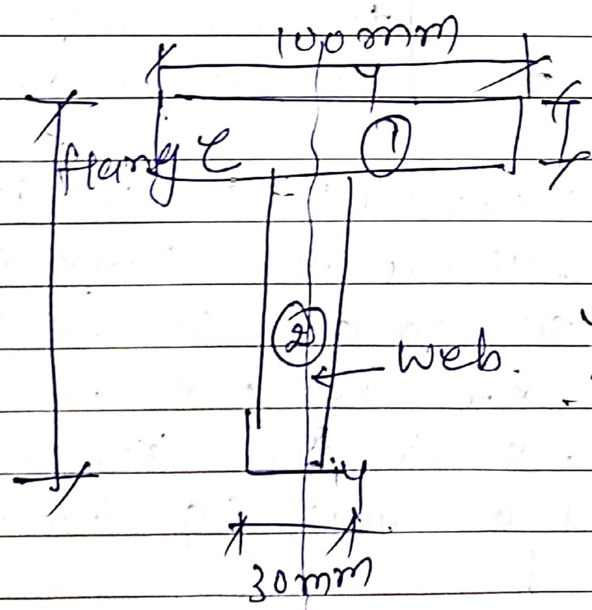
& $$\bar{y} = \frac{a_1y_1 + a_2y_2 + \dots}{a_1 + a_2 + \dots}$$

## * cg of symmetrical sections

T, I, C,

find c.g of 100.

Ex,




symmetrical about Y-Y axis.
∴ c.g will lie on this axis.

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(1) Rect. ABCH,

$a_1 = 100 \times 30 = 3000\ mm^2$.

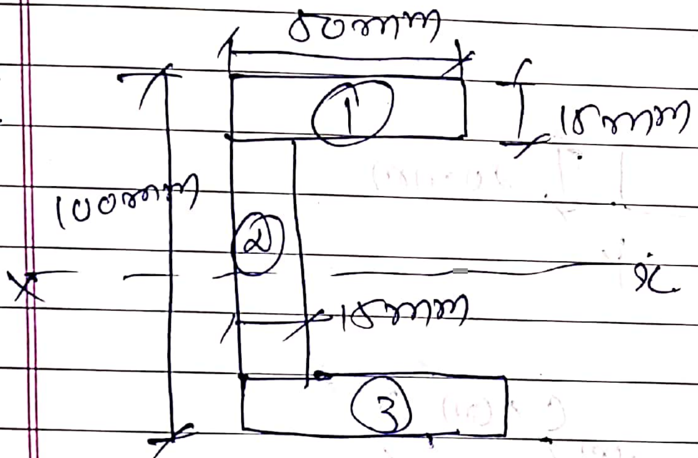
& $y_1 = (150 - 30/2) = 135\ mm$.

(2) DEFG,

$a_2 = 120 \times 30 = 3600\ mm^2$.

$y_2 = 120/2 = 60\ mm$.

$\bar{y} = \dfrac{a_1y_1 + a_2y_2}{a_1+a_2} = 94.1\ mm$.




(1) ABFJ,

$a_1 = 50 \times 15 = 750\ mm^2$

$x_1 = 50/2 = 25\ mm$

(2) EGKJ

$a_2 = (100 - 30) \times 15 = 1050\ mm^2$

$x_2 = 15/2 = 7.5\ mm$

(3) CDHK,

$a_3 = 50 \times 15 = 750\ mm^2$

$x_3 = 50/2 = 25\ mm$.

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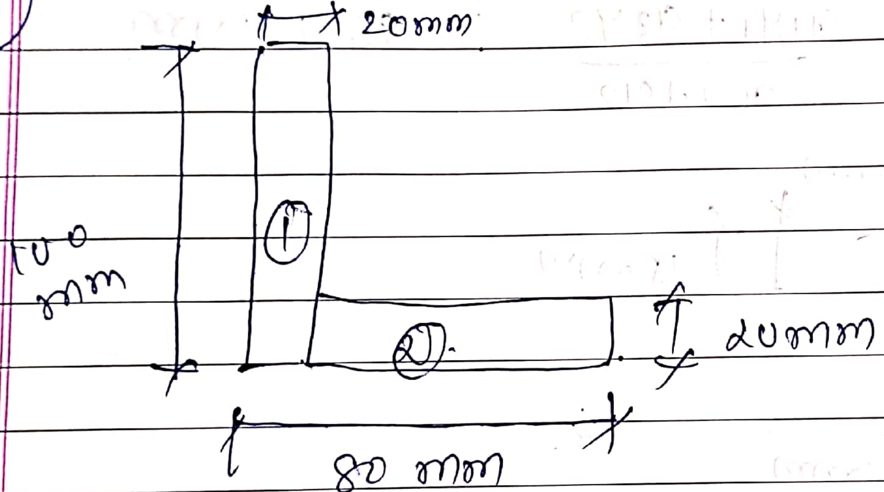
$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1+a_2+a_3}$$

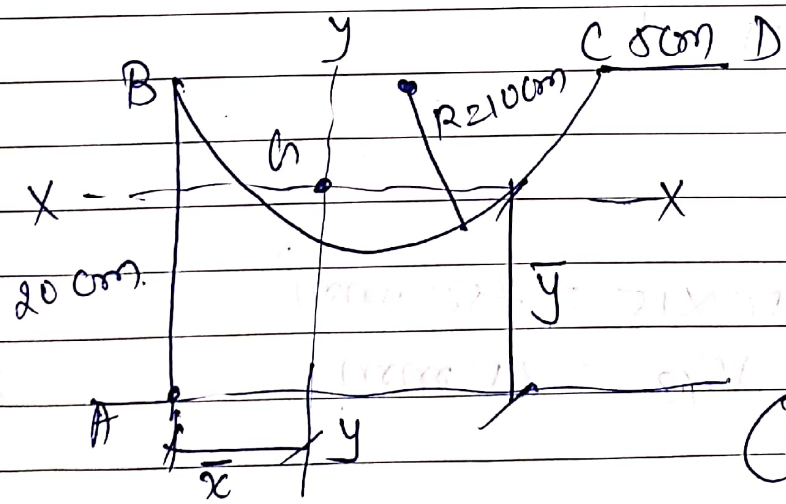
$$= 17.8 \text{ mm}$$

* C.g of unsymmetrical axis.

Ex. find the unequal angle section
① 100 x 80 x 20 mm.




②




① part AB,

$a_1 = 20$ cm, $x_1 = 0$, $y_1 = 10$ cm.

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(2) part BC,

$l_2 = \pi r = \pi \times 10 = 31.4$ cm.
$x_2 = 10$ cm.
$y_2 = 20 - \frac{2r}{\pi} = 13.63$ cm.

(3) part CD,

$l_3 = 5$ cm
$x_3 = 20 + 2.5 = 22.5$ cm.
$y_3 = 20$ cm.

$$\bar{x} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3} = 7.56 \text{ cm}$$

$$\bar{y} = \frac{l_1 y_1 + \dots}{l_1 + l_2 + \dots} = 12.91 \text{ cm}$$

# Cg of solid bodies

- cg of ~~gravity~~ ~~of~~ solid bodies (such as hemisphere, cylinders, solid cones).

(1) Volm of cylinder $= \pi r^2 h$
(2) " of hemisphere $= \frac{2\pi}{3} \times r^3$.
(3) " of right circular cone $= \frac{\pi}{3} r^2 h$.

$$\bar{y} = \frac{v_1 y_1 + v_2 y_2 + \dots}{v_1 + v_2}.$$

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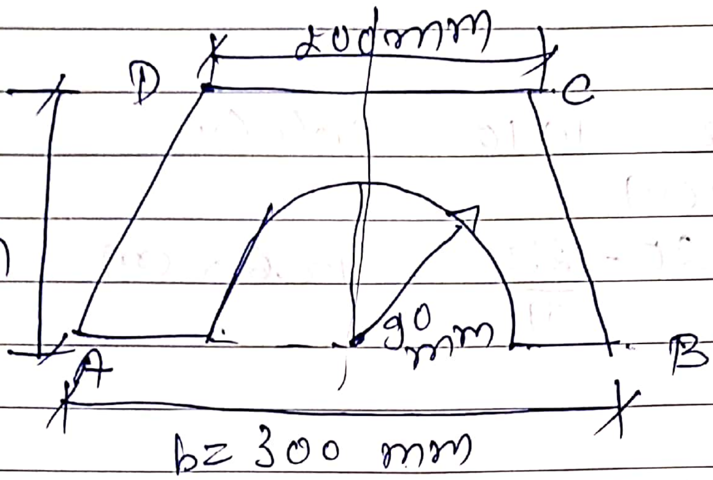
EG




Y-Y symmetrical.

(1) ABCD $\frac{h}{2}(a+b)$

$$a_1 = 120 \times \frac{200+300}{2}$$

$$= 30000 \text{ mm}^2$$

$$y_1 = \frac{h}{3} \times \left(\frac{b+2a}{a+b}\right) = 56 \text{ mm}$$

(2) Semicircle.

$$a_2 = \frac{\pi r^2}{2} = 4050\pi \text{ mm}^2$$

$$y_2 = \frac{4r}{3\pi} = \frac{120}{\pi} \text{ mm.}$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = 69.1 \text{ mm.}$$

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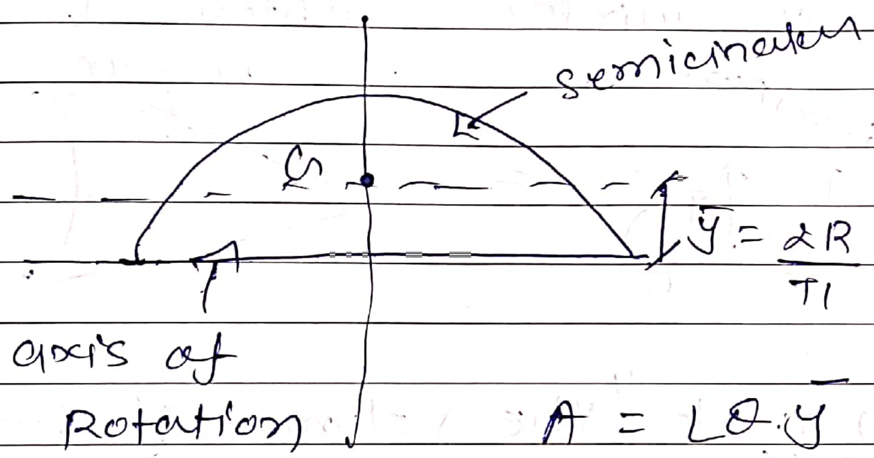
* Theorems of pappus-guldinus.

* Theorem-I : " The area of surface of revolution is equal to the length of the generating curves times the distance travelled by the centroid of the curve while the surface is being generated.




$A = L\theta\bar{y}$

let semicircular arc is rotated at $360^0$ about its base. After one revolution spherical shell is obtained.

surface area of spherical shell = length of generating curve × dist^n travelled by centroid of the generating curve.

$$A = L\theta\bar{y}$$
$$= \pi R \times (\theta \times \bar{y}) \qquad \theta = 360^0 = 2\pi$$
$$= \pi R \times \left(2\pi \times \frac{2R}{\pi}\right)$$
$$= 4\pi R^2$$

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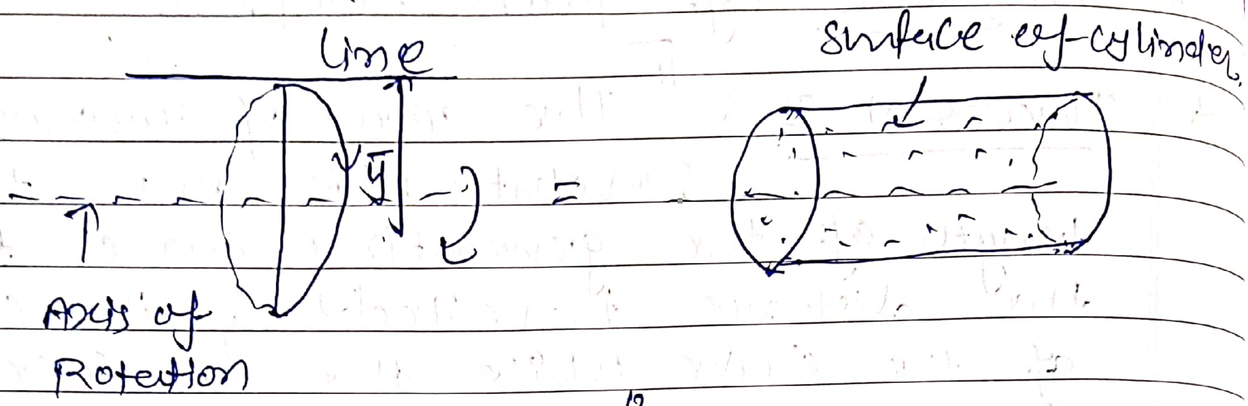
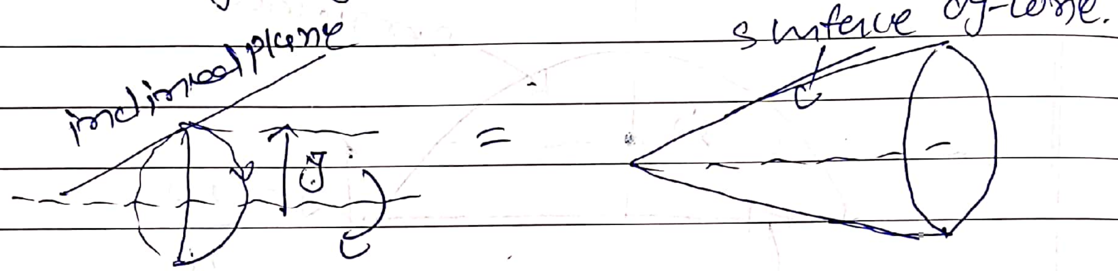
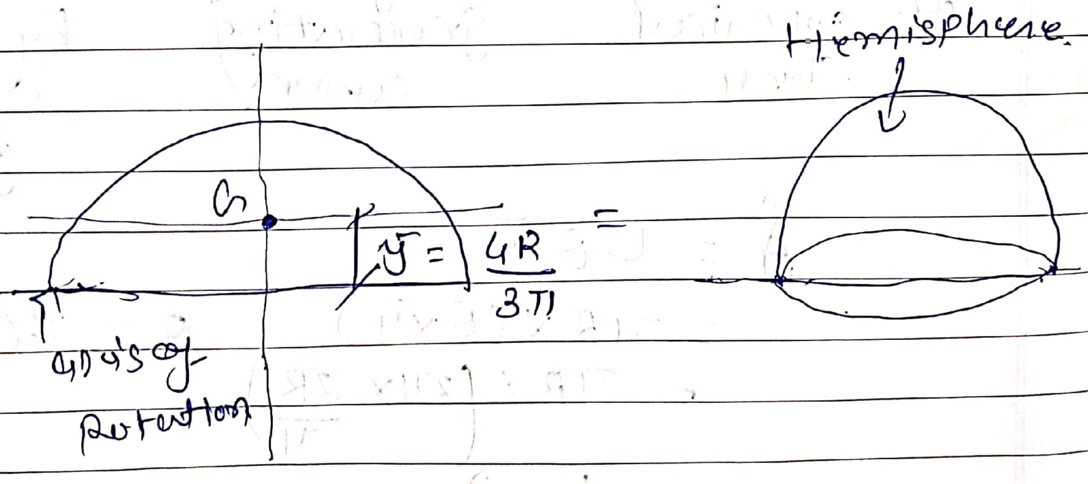



Ex. line is rotated about axis parallel to it, at $360^{\circ}$ surface of cylinder is obtained.




* Theorem II :- "The volume of body of revolution is equal to the generating area times the dist$^n$ travelled by the centroid of the area while the body is being generated."

$$V = A\,\theta\,\bar{y}$$



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let semi-circle is rotated at $180^{\circ}$ about its base. After revolution hemisphere is obtained.

Vol$^{m}$ of hemisphere = Area of generating lamina × dist$^{n}$ travelled by centroid of the generating area.

$$V = A\,\theta\,\bar{y}$$

$$= \frac{\pi R^2}{2} \times \pi \times \frac{4R}{3\pi} \qquad \theta = 180^{\circ} = \pi$$

$$= \frac{2}{3}\pi R^3.$$

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# Moment of Inertia

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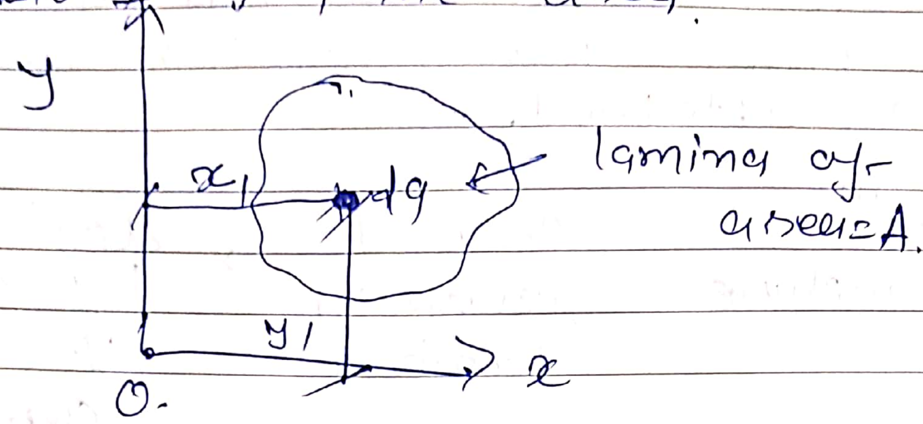
* MO I of plane area.




Consider lamina of area A..
split up the whole area into a no. of small elements.

$a_1, a_2, a_3 \ldots$ = Areas of small elements.
$x_1, x_2 \ldots$ = are the dist^n of small elements from oy axis
$y_1, y_2 \ldots$ = are the dist^n of small elements from ox axis.

moment of small elements about x axis

$I_{xx} = da_1 y_1^2 + \ldots$

$I_{yy} = da_1 x_1^2 + \ldots$

by integrating, whole area.

$I_{xx} = ay^2$

$I_{yy} = ax^2$

Second moment of area is called moment of inertia (I)
unit of M.I = $mm^4$ or $cm^4$.

* Methods for moment of inertia.

1. By integration method.
2. By Routh's method

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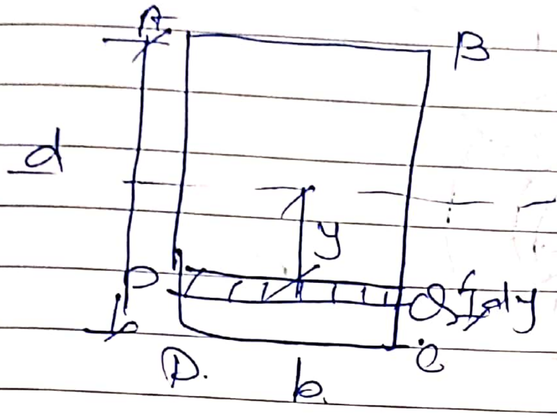
## * MOI of Rect. Section



Consider a strip PQ of thickness dy parallel to x-x axis & dist$^n$ y from it as shown in fig.

Area = $b \times dy$

MOI, about x-x axis,

$= \text{Area} \times y^2 = (b \times dy) \times y^2$

$= by^2 \times dy$

Now, MOI of whole section may be found out by integrating for the whole length section of lamina from $-d/2$ to $d/2$.

$$I_{xx} = \int_{-d/2}^{+d/2} by^2\,dy \therefore b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right]$$

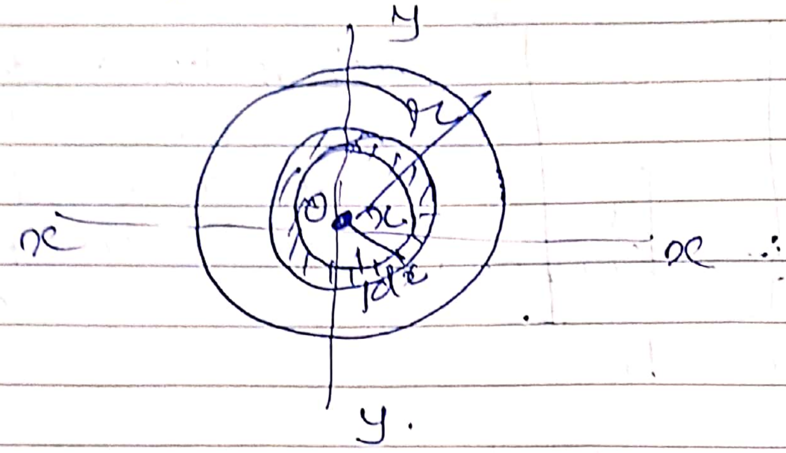
$$= \frac{bd^3}{12}$$

$$I_{yy} = \frac{db^3}{12}$$

## * Hollow Rect. Section

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$ & same $I_{yy} = \ldots$

* MOI of circular section



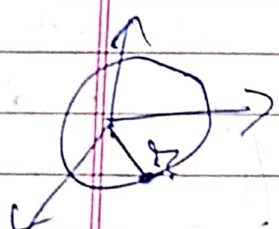

Consider a circular lamina of centre O & radius r.
Consider an elementary ring of radius $x$ & thickness $dx$.
∴ Area of elementary ring

$da = 2\pi x\,dx$

M.I = area × (dist)$^2$

$= 2\pi x\,dx \times (x)^2$

$= 2\pi x^3\,dx$




$\therefore I_{zz} = \int_0^r 2\pi x^3\,dx$

$= 2\pi\left[\frac{x^4}{4}\right]_0^r$

$= \frac{2\pi r^4}{4} = \frac{\pi r^4}{2}$

$= \frac{\pi (d/2)^4}{2} = \frac{\pi}{32}d^4$

& for hollow

$I_{xx} = \frac{\pi}{64}(D^4 - d^4)$

$I_{yy} = \frac{\pi}{64}(D^4 - d^4)$

$I_{zz} = I_{xx} + I_{yy}$ As per perpendicular axis theorem

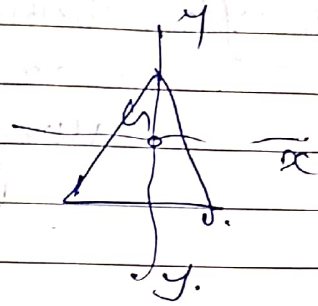
$I_{zz} = 2I_{xx}$

$\frac{\pi}{32}d^4 = 2I_{xx}$ $\therefore I_{xx} = \frac{\pi}{64}d^4$

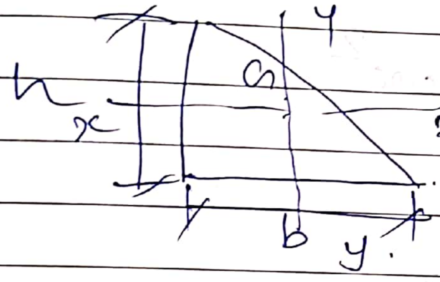
$\therefore I_{yy}$ = same

* For triangle.

$I_{xx} = \frac{bh^3}{36}$ , $I_{yy} = \frac{hb^3}{48}$.



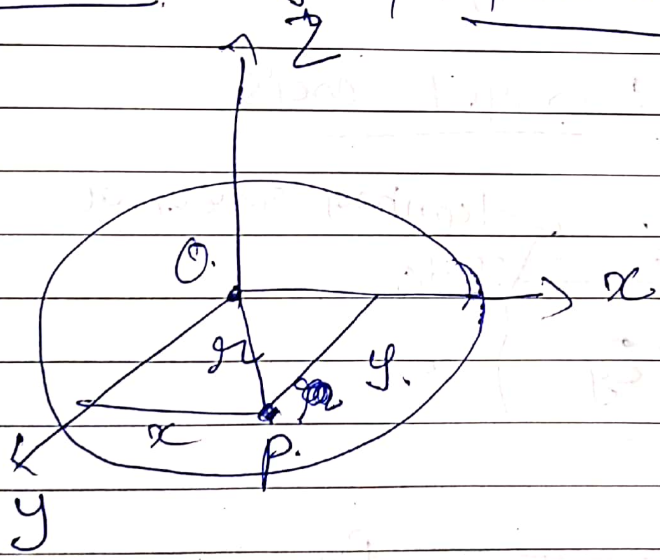

* Right angle triangle.




$I_{xx} = \frac{bh^3}{36}$

$I_{yy} = \frac{hb^3}{36}$.

* Theorem of perpendicular axis.




- It states, If $I_{xx}$ & $I_{yy}$ be the MOI of plane section about two perpendicular axis meeting at O, the MOI $I_{zz}$ about the axis z-z, perpendicular to the plane & passing through the intersection of x-x and y-y is given by.

$$I_{zz} = I_{xx} + I_{yy}$$

proof:- Consider a small lamina (P) of area da. x and y are the co-ordinates of P along ox and oy axes.

$OP = r$.

$r^2 = x^2 + y^2$

M.I of P about x-axis,

$I_{xx} = da\,y^2$

similarly, $I_{yy} = da\,x^2$

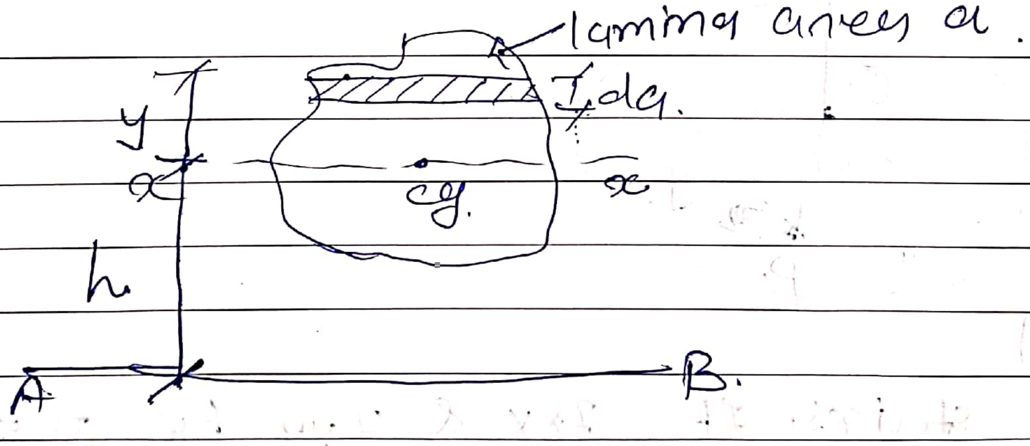
$I_{zz} = da\,r^2$

$= da\,(x^2 + y^2)$

$= da\,x^2 + da\,y^2$

$I_{zz} = I_{xx} + I_{yy}$

* Theory of parallel axis




- If $I_g$ is the moment of inertia of a plane area about an axis passing through its cg then, MOI of the area about axis AB, parallel to the first axis & distn h from cg is given by.

$I_{AB} = I_g + ah^2$

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$I_G$ = MOI of the area about its centre of gravity.
a = area of section
h = distn betn cg of section & axis AB.
$I_{AB}$ = distn betn cg of section & axis AB

Proof:- M.I of strip about c.g of section
$I = da y^2$

M.I of whole section about c.g of section:

$I_g = \Sigma da y^2$.

M.I of whole section about the axis AB,

$I_{AB} = \Sigma da (h+y)^2$
$= \Sigma da (h^2 + 2hy + y^2)$
$= \Sigma h^2 da + \Sigma 2hy da + \Sigma y^2 da$
$= ah^2 + 0 + I_g$
$= I_g + ah^2$

$\Sigma h^2 da = ah^2$
$\Sigma y^2 da = I_g$.
$\Sigma da y = a\bar{y} = 0$.
because $\bar{y} = 0$.

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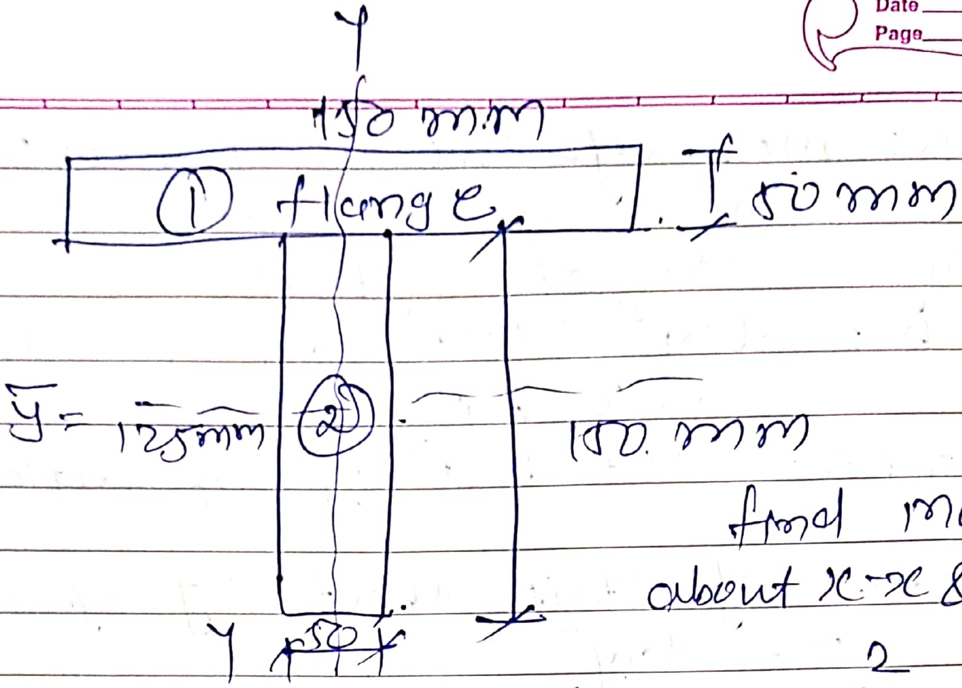
Ex:-




find MOI about x-x & y-y axis.

①
$a_1 = 150 \times 50 = 7500\ mm^2$
$y_1 = 150 + \frac{50}{2} = 175\ mm$

②
$a_1 = 150 \times 50 = 7500\ mm^2$
$y_2 = \frac{150}{2} = 75\ mm$
$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$
$= 125\ mm.$

x-x axis,

$I_{xx_1} = I_g + a_1 h_1^2$ $\quad h_1 = y_1 - \bar{y}$
$= \frac{150 \times 50^3}{12} + a_1 \times h_1^2$

$= 20.3125 \times 10^6\ mm^4.$

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$I_{xx_2} = I_g + a_2 h_2^2$ $\qquad$ $h_2 = \bar{y} - y_2$

$h_2 = 125 - 75 = 50mm$

$= \frac{50 \times (100)^3}{12} + a_2 \times h_2^2$

$= 32.8125 \times 10^6 \ mm^4.$

$I_{xx} = I_{xx_1} + I_{xx_2}$

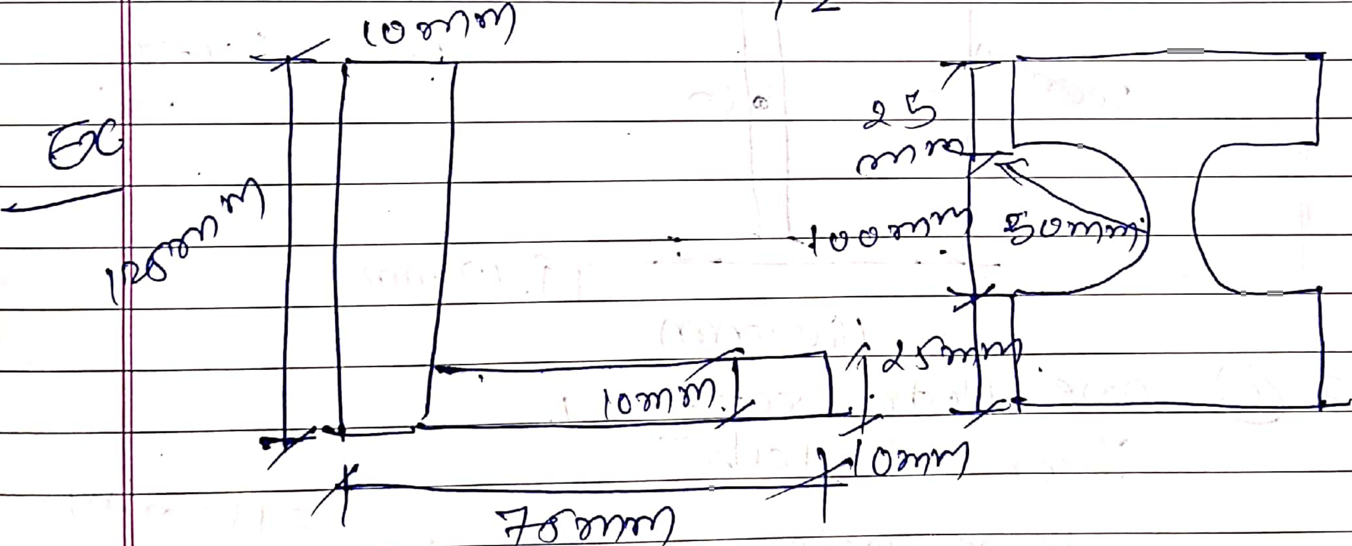
$= 53.125 \times 10^6 \ mm^4.$

Same for Y-Y axis,

$I_{yy_1} = I_g + a_1 h_1^2$ $\qquad$ $\frac{db^3}{12}$ $\qquad$ $h = 0.$

$= \frac{50 \times (100.)^3}{12} + a_1 \times$

Ex




* MOI of built-up section x.

- Built up section consists of a no. of sections such as reet. sections, channel sections, I sections etc. which are made by symmetrically placing & then fixing these sections by welding or riveting.

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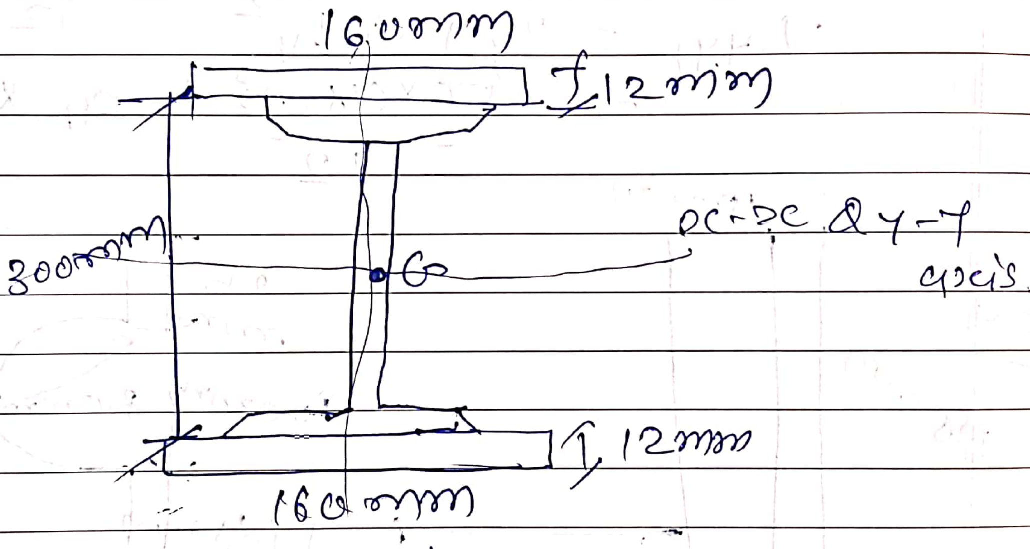
Interesting to know,
– built up section behaves as one unit

Ex A Compound beam is made by welding two steel plates 160 mm x 12 mm one on each flange of an ISLB 300 section as shown in fig.

MOI of ISLB 300 = $73.329 \times 10^6 \, mm^4$
MOI about x-x axis = (?)




MOI @ one plate section,
$I_{xx} = I_G + ah^2$
x-x axis,

$h = 150 + \frac{12}{2}$
$= 156 \, mm$

$= \frac{160 \times (12)^3}{12} + a \times (156)^2$

$= 46.748 \times 10^6 \, mm^4$

MOI of the compound beam section about x-x axis,
$I_{xx}$ = ISLB section + 2 plate section

(given)

$= 73.329 \times 10^6 + 2 \times (46.748 \times 10^6)$

$= 166.825 \times 10^6 \ mm^4$

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