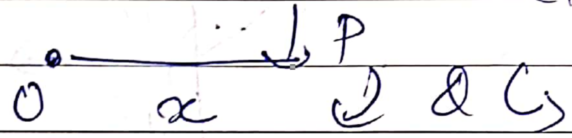
Non-Coplanar forces

- **Moment of force** :- product of force & perpendicular distⁿ of line of action of force from the point about which the moment is reqᵈ.

$$M = P \times x$$



clockwise ⇒ (+)
& anticlockwise moment (−)

$\therefore \; M = P.x$

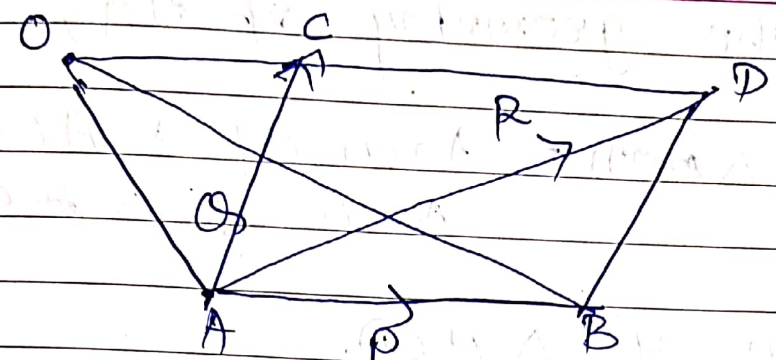
- **Couple**

Two equal & opposite forces whose lines of action are diffⁿ form a couple.

- **Varignon's principle of moments**

or **Law of moments**

- "If a no. of coplanar forces are acting simultaneously on a particle, the algebric sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."



- Consider two forces P & Q acting at a point A represented in magnitude & dir^n by AB & AC shown in fig. below.
- let O be the point, about which the moments are taken. Through O, draw a line CD parallel to the dir^n of force P, to meet the line of action of Q at C. Now with AB & AC as two adjacent sides, complete parallelogram ABDC. Join the diagonal AD of the parallelogram and OA and OB. Diagonal AD represents in magnitude & dir^n the resultant of two forces P and Q.

Now, taking moment of forces P, Q & R about O.

moment of force P, about O,
= 2 x Area of triangle AOB —— (1)

Similarly, moment of force Q, about O,
= 2 x Area of triangle AOC —— (2)

moment of Resultant R, about O,
= 2x Area of triangle AOD —— (3)

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from, the geometry of fig,

Area of ΔAOD = Area of ΔAOC + Area of ΔAOD.

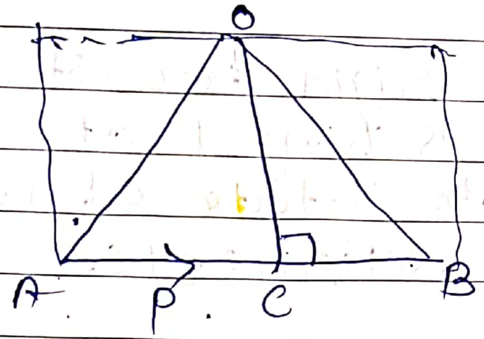
but area of ΔAOD.
= Area of ΔADB
= Area of ΔAOB.
∴ (∵ two triangles AOB & ADB are on the same base AB and bet^n the same parallel lines)

∴ Area of ΔAOD = Area of ΔAOC + Area of ΔAOB.

multiplying both sides by 2,
2x Area of ΔAOD = 2x Area of ΔAOC + 2 x Area of ΔAOB

∴ moment of force R about O
= moment of force P about O
+ moment of force Q about O.

* Graphical Representation of moment.



$(AC \times OG) \times 2$

Consider a force represented, in magnitude & dirⁿ, by the line AB, let O be point, about which the moment of this force is reqᵈ to be found out, as shown in fig. From O, draw OC perpendicular to AB, Join OA & OB.

Now, moment of force P about O,

$= P \times OC = AB \times OG.$

but $AB \times OC$ = twice the area of triangle ABO,

moment of force, about any point = twice the area of triangle, whose base is line to some scale representing the force & whose vertex is the point about which the moment is taken.

* Applications of moments

(1) position of the resultant force

(2) levers.

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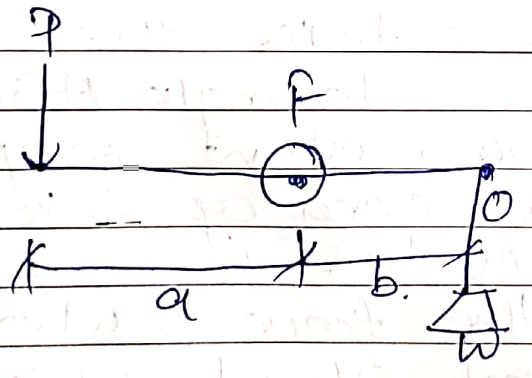
* Levers

- A lever is a rigid bar (straight, curved or bent) & is hinged at one point. It is free to rotate about the hinge
Ex pair of scissors, etc.

* Types of levers

(1) simple levers
(2) Compound levers.

* Simple levers



A lever which is consists of bar having one hinge is known as simple lever.

P = load applied
W = weight lifted
a = length bet^n hinge & effort
b = " " & weight

Now taking moment of the effort & F & equating the same.

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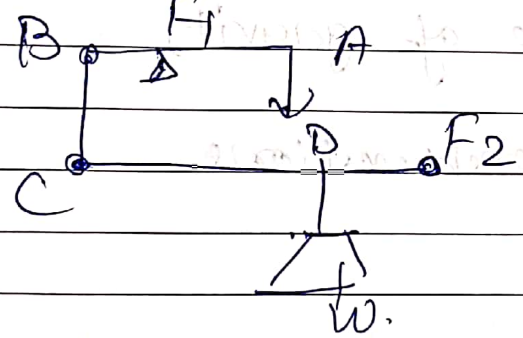
$P.a = W.b$

$\therefore \frac{W}{P} = \frac{a}{b}$ → to lift a heavier load with the app of the same force.

- Term W/P & a/b known as mechanical advantage & leverage.
- If increase the mechanical advantage either length of the lever arm (a) is to be increased or length of the load arm (b) is to reduced.

* Compound levers

- A lever which consists of a no. of simple lever is called Compound lever.



Ex (9) by R.S Khurmi

7/10/19

Sheer Force & Bending Moment.

→ In our day-to day work, we see that whenever we apply a force on a body, it exerts a reaction, eg. when ceiling fan is hung from a girder, it is sub. to the following two forces:-

(1) weight of the fan, acting downwards, and

(2) Reaction on the girder, acting upwards.

Types of Beams:-

(1) Simply supported beam

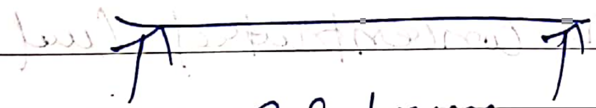
(2) overhanging beam

(3) Cantilever beam

(4) Fixed beam

(5) Continuous beam

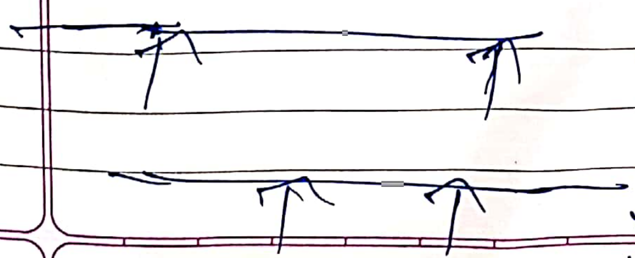
(1) Simply supported beam



S.S. beam

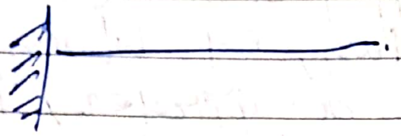
beam supported freely on the walls or columns at its both end is ...

(2) overhanging beam



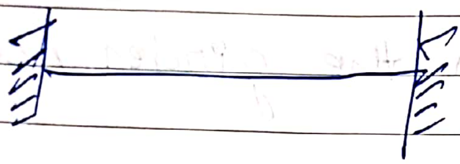
beam is freely supported on two supports. But its one end or both the ends are projected beyond the support.

(3) Cantilever beam



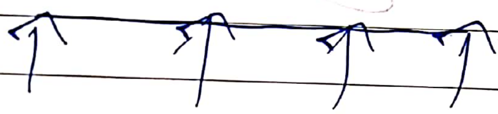
beam fixed at one end and free at the other end is ...

(4) Fixed beam



A beam whose both ends are rigidly fixed or built-in walls is ...

(5) Continuous beam



A beam supported on more than two supports is known ...

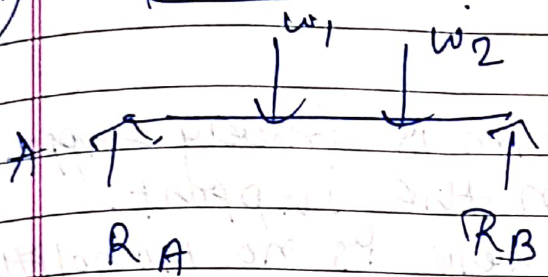
* Types of loads:-

(1) point load or concentrated load

(2) uniformly distributed load (udl)

(3) uniformly varying load (uvl)

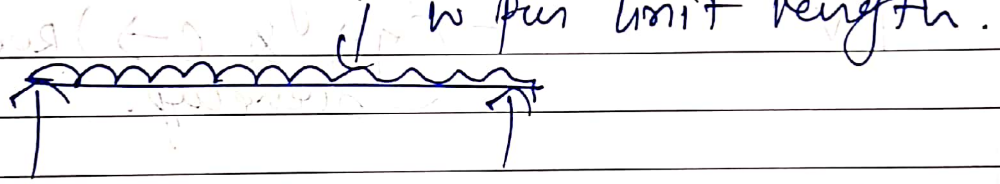
(1) Point load → A load, acting a point on a beam is - - -



→ In actual practice, it is not possible to apply a load at a point as it must have some contact area. But this area being so small, in comparison with the length of the beam, is negligible.

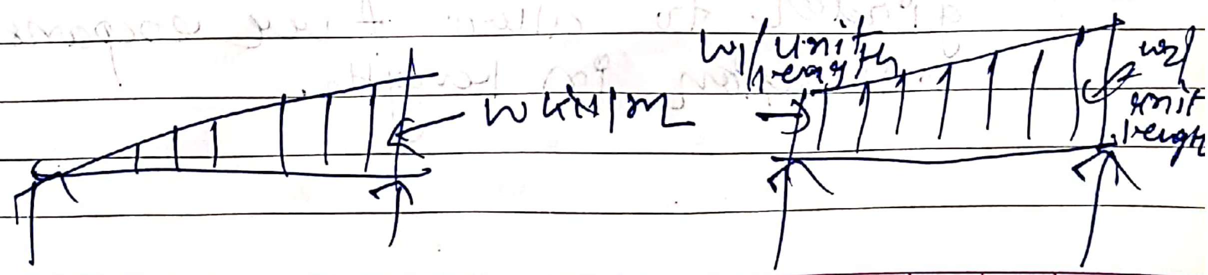
(2) UDL

→ A load, which is spread over a beam, in such a manner that each unit length is loaded to the same extent is called - - -



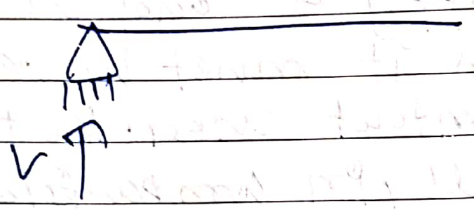
(3) UVL

→ A load, which is spread over a beam, in such a manner that its extent varies uniformly on each unit length. (when load is zero at one end & increases uniformly to the other end, it is called - - -



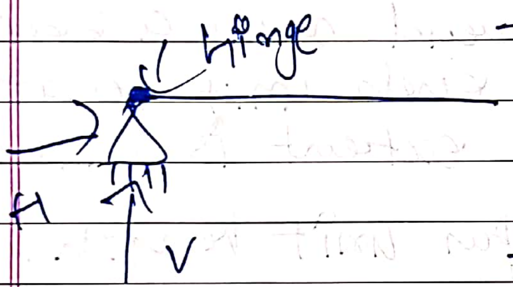
* Types of supports.

(1) Simple support:-



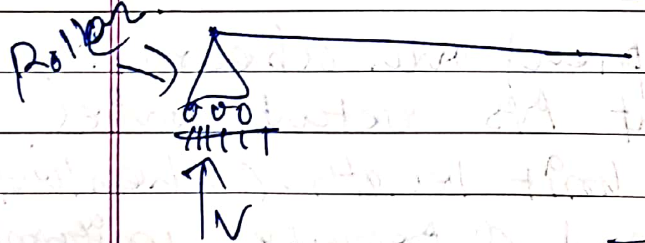
- Beam is freely supported on the support.
- There is no monolithic construction bet^n beam & support.

(2) Hinge support:-



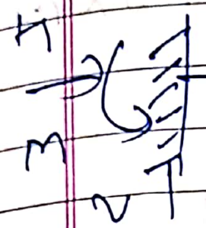
- beam is hinged to the support at end. Beam can rotate about the hinge.
- (↑) & (→) Reaction can develop.

(3) Roller support:-



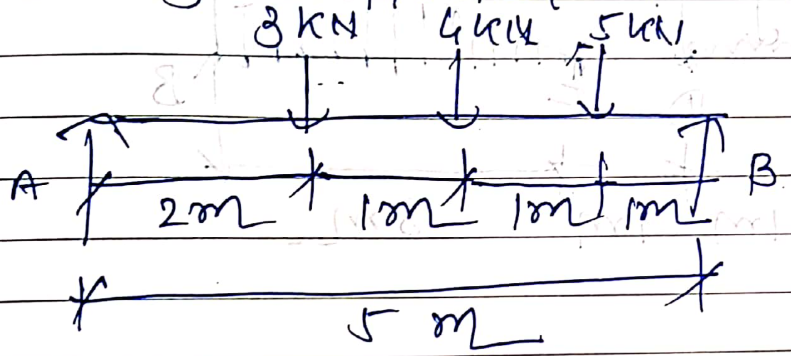
- beam is hinged to the support at the end and rollers are provided below the supports.
- This type of support is provided for bridge girder to allow free expansion & contraction in length.

(4) Fixed support :- - end of beam is rigidly fixed or built in wall. beam can't rotate at the end.



Ex (1) To find out the Reactions.

(simply supported beams).



Taking moment @ A, (anticlockwise moment)

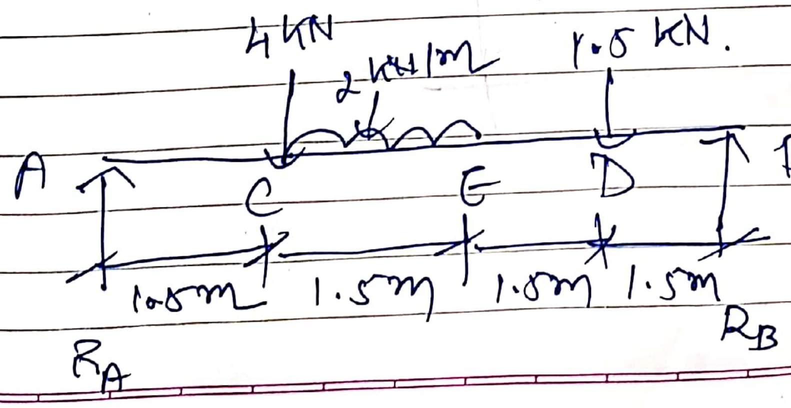
$= R_B \times l = R_B \times 5$ kN.m

$\therefore R_B \times 5 = 3\times 2 + 4\times 3 + 5\times 4$

$\boxed{R_B = 7.6 \text{ kN}}$

$\boxed{R_A = 4.4 \text{ kN}}$ (Total load $(3+4+5-7.6)$)

Ex (2)



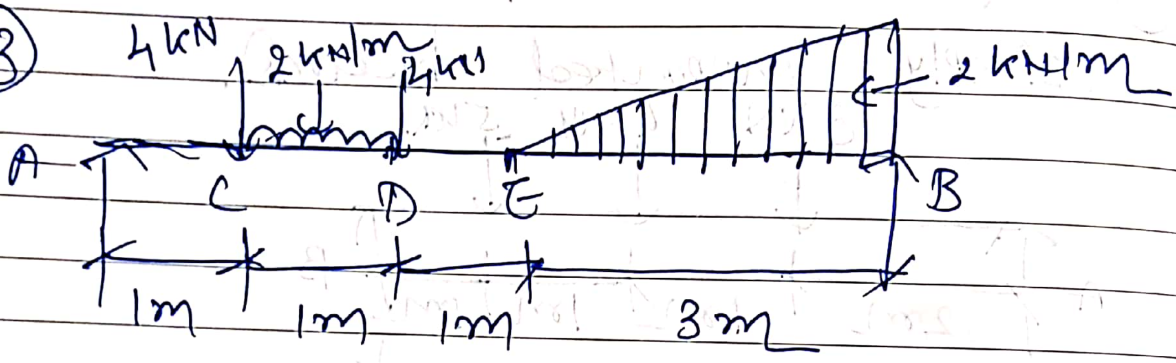
$$R_B \times 6 = 4 \times 1.5 + 2 \times 1.5 \times \left(\underbrace{\tfrac{1.5}{2} + 1.5}_{(2.25)}\right) + 1.5 \times 4.5$$

$$R_B = 3.25 \text{ kN}$$

$$R_A = \left(4 + (2 \times 1.5) + 1.5\right) - 3.25$$

$$= 5.25 \text{ kN}$$

Ex (3)



$$\therefore R_B \times 6 = 4 \times 1 + \left(2 \times 1 \times \left(\tfrac{1}{2} + 1\right)\right) + 4 \times 2 + \frac{(0+2)}{2} \times 3 \times 5 \quad (\text{c.g})$$

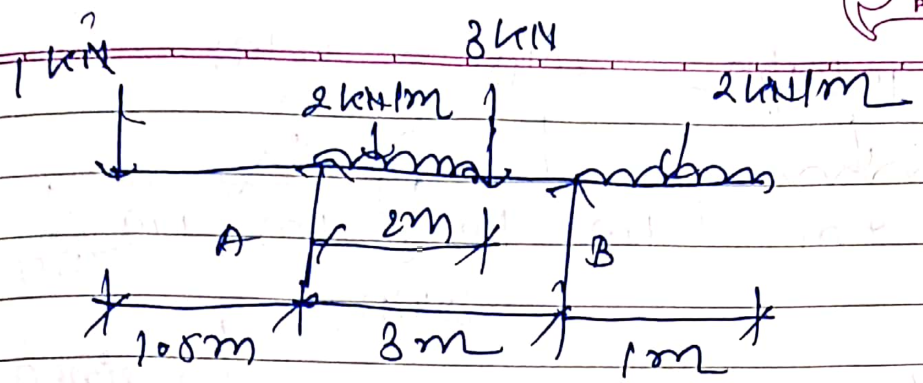
(kN × c.g)

$$\boxed{R_B = 5 \text{ kN}}$$

$$R_A = \left(4 + 2 \times 1 + 4 + 3\right) - 5$$

$$\boxed{R_A = 8 \text{ kN}}$$

Ex (7)

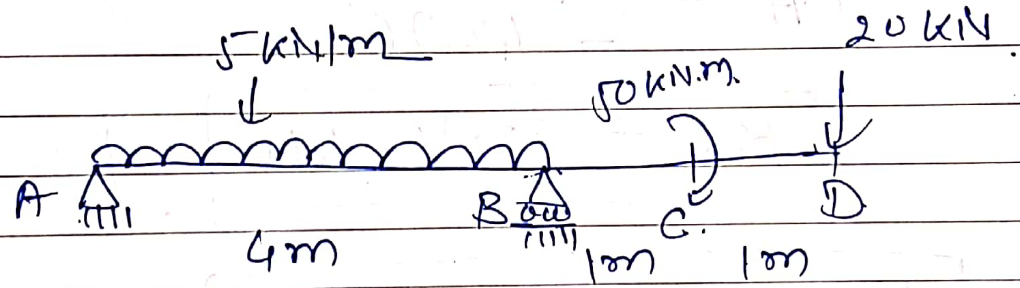


$R_B \times 3 + 1 \times 1.5 = 2 \times 2 \times 1 + 3 \times 2 + 2 \times 1 \times 3.5$

$\boxed{R_B = 4 \text{ kN.}}$

$\boxed{R_A = 5 \text{ kN}}$

Ex (8)



Taking M @ A,

$R_B \times 4 = 5 \times 4 \times 2 + 50 + 20 \times 6$

$\boxed{R_B = 52.5 \text{ kN}}$ ↑

$R_A = (5 \times 4) + 20 - 52.5$

$\boxed{R_A = (-12.5 \text{ kN})}$ ↓

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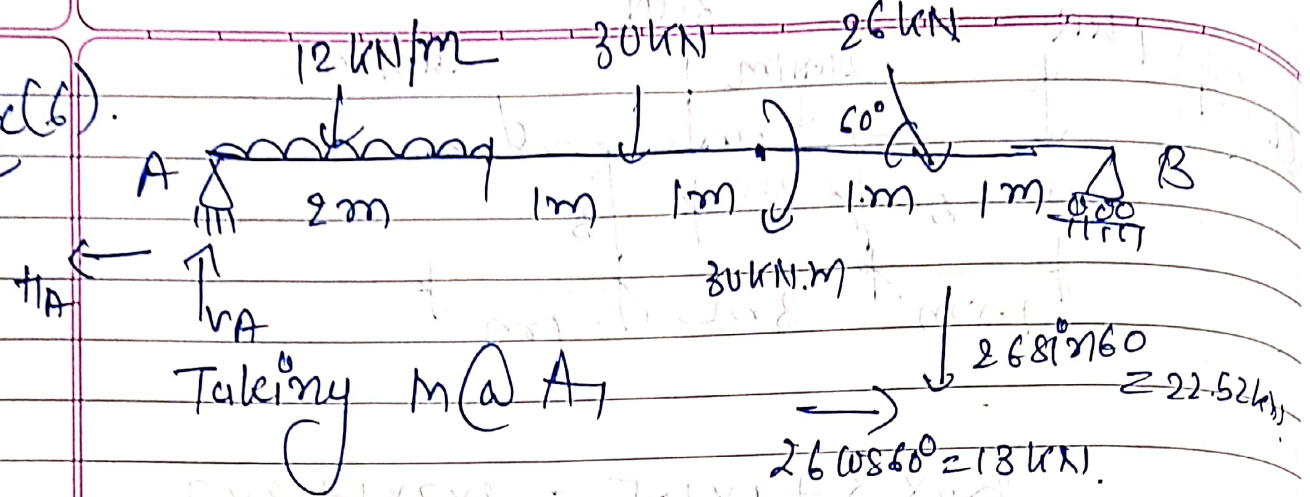
Ex(6).



Taking M @ A

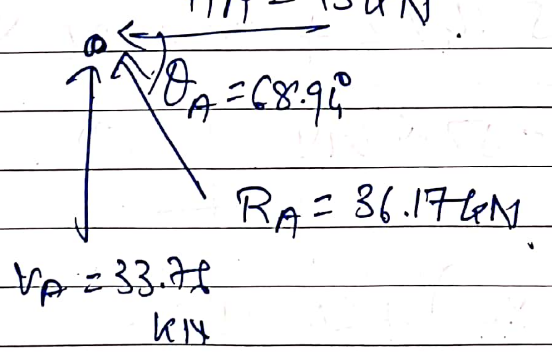
$$R_B \times 6 = (12 \times 2 \times 1) + (30 \times 3) + 30 + 22.52 \times 5.$$

$$\boxed{R_B = 42.76\text{ kN}} \uparrow$$

$$R_A = (24 + 30 + 22.52) - 42.76$$

$$\boxed{R_A = 33.76\text{ kN}} \uparrow$$

$$\boxed{H_A = 26 \cos 60 = 13\text{ kN}} \leftarrow$$



$$R_A = \sqrt{v_A^2 + H_A^2}$$

$$= \sqrt{33.76^2 + 13^2}$$

$$= 36.17\text{ kN}$$

$$\tan\theta_A = \frac{v_A}{H_A} = \frac{33.76}{13} = 2.596$$

$$\theta_A = 68.94^\circ$$