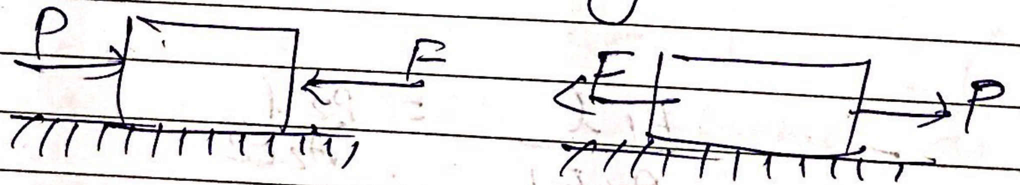
Friction

- **Friction:-** When a body moves or tends to move tangentially with respect to the surface, on which it rests; the interlocking property of the projecting particles opposes the motion. This opposing force, which acts in the opposite direction of the movement of the block is called ...

or When a body slide or tends to slide on a surface on which it is resting, a resisting force opposing the motion is produced at the contact surface. This resisting force is called...



- **Limiting Friction**

The max. friction force, that can be developed at the contact surface, when body is just on the point of moving is called ....

(If external force becomes greater than that friction force, body will move)

Friction

Static
(Friction experienced by a body when it is at rest).

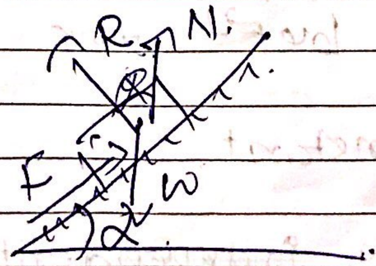
Dynamic.
(Friction experienced by body when it is motion).

- **Sliding** (friction experienced by a body when it slides over another body)
- **Rolling.** (Friction experienced by a body when it rolls over another body.)

* Normal Reaction :-

- Whenever a body, lying on a horizontal or an inclined surface, is in equilibrium, its weight acts vertically downwards through its centre of gravity. The surface, in turn, exerts an upward reaction on the body, called ---.

* Angle of friction



(1) wt of body, acting vertically downwards.
(2) Friction force acting upwards along the plane
(3) Normal Reaction acting at right angle to the plane.

Angle of repose :- with increase in angle of inclined surface, the max^m angle at which, body starts sliding down is called angle of repose.

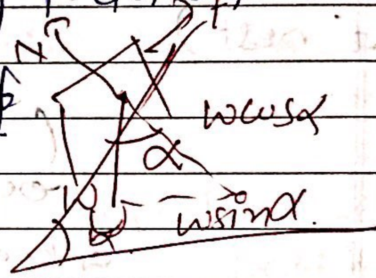
- Angle of inclined plane, at which a body just begins to slide down the plane is called ---.

* **Coefficient of friction**

It is Ratio of limiting friction to the normal reaction.

$$\mu = \frac{F}{R} = \tan\phi$$

or $F = \mu R$



* **Laws of friction**

(1) laws of static friction
(2) law of kinetic friction.

(1) force of friction always acts in dir^n opp. to that in which the body tends to move, if the force of ~~motion~~ friction would have been absent.

(2) magnitude of force of friction is exactly equal to the force, which tends to move the body.

(3) Magnitude of limiting friction bears a constant ratio to the normal reaction bet^n the two surfaces.

$$\frac{F}{R} = \text{Constant.}$$

(4) force of friction is independent of the area of contact bet^n the two surfaces.

Date
Page

(5) The force of friction depends upon the roughness of the surfaces.

* laws of kinetic / dynamic friction

→ The force of friction always acts in a dir^n, opposite to that in which the body is moving.

→ The magnitude of kinetic friction bears a constant ratio to the normal reaction bet^n the two surfaces.

→ For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

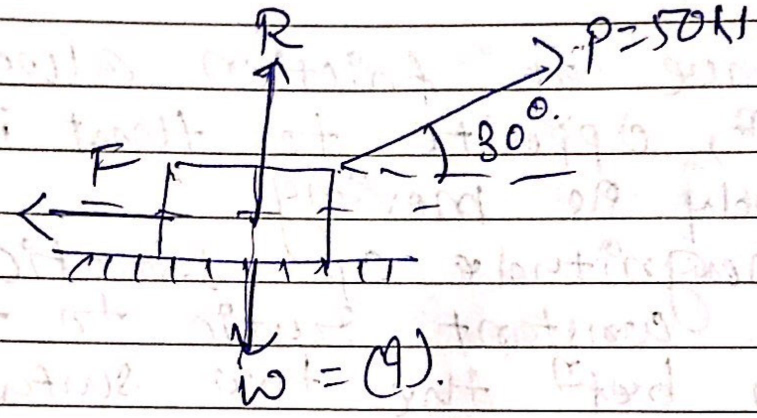
* Equilibrium of a body on a Rough horizontal plane :-

→ We know that body, lying on a rough horizontal plane will remain in equilibrium. but when-ever force is applied on it, the body will tend to move in the dir^n of force. In such cases, eq^m of body is studied first by resolving the forces horizontally & then vertically.

$$F = \underset{\text{coefficient}}{\mu} R \leftarrow \text{Normal Reaction}$$

Date____
Page____

Ex (1) A pull of 50 N inclined at 30° to the horizontal is necessary to move a wooden block on horizontal table. If $\mu = 0.20$. Find wt of wooden block = (?).



$\therefore$ F on P = $50 \cos 30°$,

$\therefore 50 \cos 30 = F$

$\boxed{F = 43.30\ N}$

$\therefore \mu = \frac{F}{R} = \frac{43.30}{R} = 0.20$

$\therefore R = 216.5\ N$

$\therefore R + 50 \sin 30 = W$

$\therefore 216.5 + 25 = W$

$\therefore \boxed{W = 241.5\ N}$

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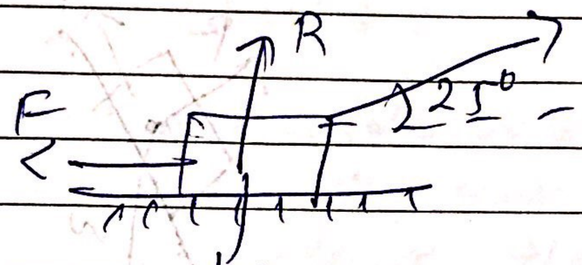
Ex (5). A body of weight ~~150~~ 300 N is lying on a rough horizontal plane having a $\mu = 0.3$. Find the magnitude of force, which can move the body, while acting at an angle of 25° with horizontal.

→



W = 300 N.

$F = P\cos\theta = P\cos 25 = P \times 0.9063$

$\therefore R + P\sin\theta = W$

$\therefore R + P\sin 25 = 300$

$\therefore R = 300 - P\sin 25$

$\therefore F = \mu R$

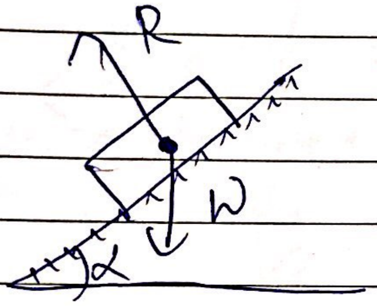
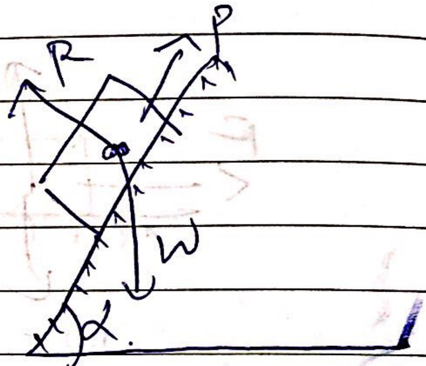
$\therefore P \times 0.9063 = 0.3 \times (300 - P\sin 25)$

$\therefore \boxed{P = 87.1\ N}$

Date
Page

* Equilibrium of a body on a Rough Inclined plane-

→ consider a body of weight W, lying on a rough plane inclined at an angle α with the horizontal.

	
Angle of inclination less than the angle of friction	Angle of inclination more than the angle of friction.
Angle of friction is less, so that body is req^d to be moved upwards or downwards, a corresponding force is req^d for the same.	So, that body will move down. And upward force (P) will be req^d to resist the body from moving down the plane as shown in fig.

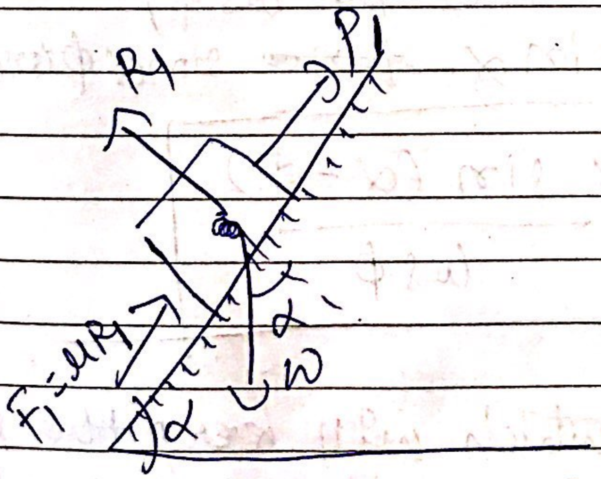
Date______
Page______

★ Equilibrium of a body on a Rough inclined plane sub. to a force acting along the inclined plane.

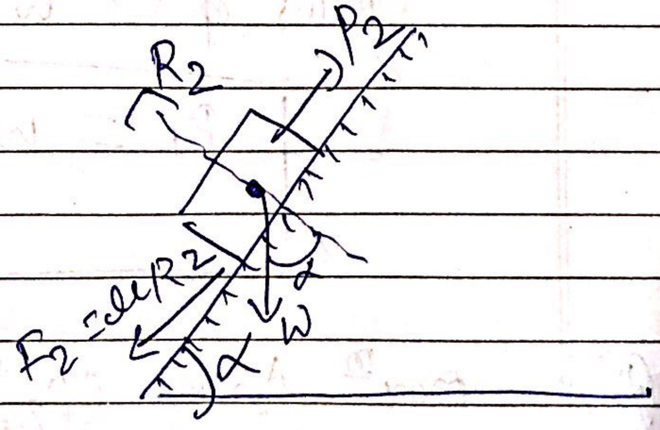
→ Consider a body lying on a rough inclined plane sub. to force acting along the inclined plane.

→ A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the foll^n two cases:

(1) minimum forces (P_1) which will keep the body in eq^m, when it is at the point of sliding downwards.



body at the point of sliding downwards.



body at the point of sliding upwards.

→ In this case, $F_1 = \mu R_1$ will act upwards, as the body is at the point of sliding downwards.

$$P_1 = W\sin\alpha - \mu R_1 \quad \text{(along the plane)} \quad \text{— (1)}$$

$$\therefore R_1 = W\cos\alpha. \quad \text{(perpendicular to the plane).} \quad \text{— (2)}$$

Substituting the value of R_1

$$\therefore P_1 = W\sin\alpha - \mu W\cos\alpha$$
$$= W(\sin\alpha - \mu\cos\alpha)$$

$$\mu = \tan\phi.$$
$$\therefore P_1 = W(\sin\alpha - \tan\phi\cos\alpha)$$

Multiplying both sides by $\cos\phi$,

$$P_1\cos\phi = W(\sin\alpha \, \cancel{\tan\phi} \, \sin\phi\cos\alpha)$$

$$\boxed{P_1 = W \times \frac{\sin(\alpha - \phi)}{\cos\phi}}$$

(2) Max^m forces (P_2) which will keep the body in eq^m, when it is at the point of sliding upwards.

→ In this case, $F_2 = \mu R_2$ will act downwards as the body is at the point of sliding upwards.

Date ____
Page ____

$$P_2 = W\sin\alpha + \mu R_2 \quad \text{--- (1)}$$

$$R_2 = W\cos\alpha \quad \text{--- (2)}$$

$$P_2 = W\sin\alpha + \mu W\cos\alpha$$

$$= W(\sin\alpha + \mu\cos\alpha)$$

$$\mu = \tan\phi$$

$$P_2 = W(\sin\alpha + \tan\phi\cos\alpha)$$

multiplying both sides by $\cos\phi$,

$$P_2\cos\phi = W(\sin\alpha\cos\phi + \sin\phi\cos\alpha)$$

$$P_2 = W\sin(\alpha+\phi)$$

$$\boxed{P_2 = \frac{W \times \sin(\alpha+\phi)}{\cos\phi}}$$

A body of wt 500 N is lying on rough plane which inclined at an angle of 25° with horizontal. It is supported by an effort (p) parallel to plane as shown in fig. Determine the min & max. value of p, for which the eq^m can exist, if ϕ = 20°.

→ min value of P

$$P_1 = \frac{500 \times \sin(25^\circ - 20^\circ)}{\cos 20^\circ}$$

$$\boxed{P_1 = 46.4\ N}$$

→ max^m value of P,

$$P_2 = \frac{500 \times \sin(25^\circ + 20^\circ)}{\cos 20^\circ}$$

$$\boxed{P_2 = 376.2\ N}$$

* Equilibrium of a body on a rough inclined plane sub. to a force acting horizontally.

→ Consider a body lying on a rough inclined plane sub to a force acting horizontally, which keeps it in equilibrium as shown in fig.

→ A little consideration will show that if the force is not there, the body will slide down on the plane.

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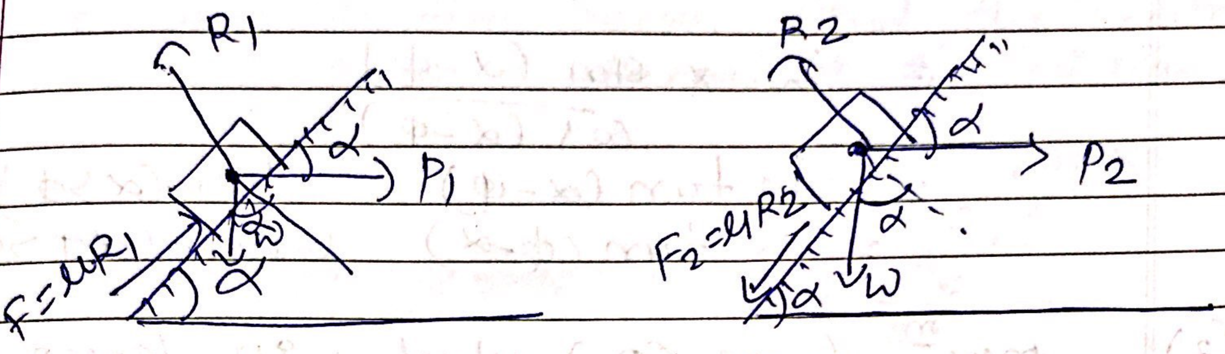
Date
Page

(1) minimum force (P_1) which will keep the body in equilibrium, when it is at the point of sliding downwards.



(a) body at the point of sliding downwards. (b) body at the point of sliding upwards.

→ In this case, $F_1 = \mu R_1$ will act upwards, as the body is at the point of sliding downwards as shown in fig. (Along plane)

$$P_1 \cos\alpha = W \sin\alpha - \mu R_1 \quad \text{—— (1)}$$

$$R_1 = W\cos\alpha + P_1 \sin\alpha \text{ (perpendicular)} \quad \text{—— (2)}$$

$$P_1 \cos\alpha = W\sin\alpha - \mu(W\cos\alpha + P_1\sin\alpha)$$
$$= W\sin\alpha - \mu W\cos\alpha - \mu P_1 \sin\alpha$$

$$\therefore P_1\cos\alpha + \mu P_1 \sin\alpha = W\sin\alpha - \mu W\cos\alpha$$

$$\therefore P_1 = W\frac{(\sin\alpha - \mu\cos\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

$$\mu = \tan\phi,$$

$$P_1 = W \times \frac{(\sin\alpha - \tan\phi\cos\alpha)}{(\cos\alpha + \tan\phi\sin\alpha)}$$

Date
Page

multiplying numerator & denominator by $\cos\phi$,

$$\therefore P_1 = w \times \frac{\sin\alpha\cos\phi - \sin\phi\cos\alpha}{\cos\alpha\cos\phi + \sin\alpha\sin\phi}$$

$$= w \times \frac{\sin(\alpha-\phi)}{\cos(\alpha-\phi)}$$

$$= w\tan(\alpha-\phi) \quad \text{when } (\alpha>\phi)$$

$$= w\tan(\phi-\alpha) \quad \text{when } (\phi>\alpha)$$

(2) max^m force (P_2) which will keep the body in eq^m, when it is at the point of sliding upwards.

→ $F_2 = \mu R_2$ will act downwards, as the body is at the point of sliding upwards. (Along the plane)

$$P_2\cos\alpha = w\sin\alpha + \mu R_2 \quad \text{—— (3)}$$

$$R_2 = w\cos\alpha + P_2\sin\alpha$$

(perpendicular) —— (4)

$$P_2\cos\alpha = w\sin\alpha + \mu(w\cos\alpha + P_2\sin\alpha)$$

$$= w\sin\alpha + \mu w\cos\alpha + \mu P_2\sin\alpha$$

$$\therefore P_2 = w \times \frac{(\sin\alpha + \mu\cos\alpha)}{(\cos\alpha - \mu\sin\alpha)}$$

numerator & denominator by $\cos\phi$,

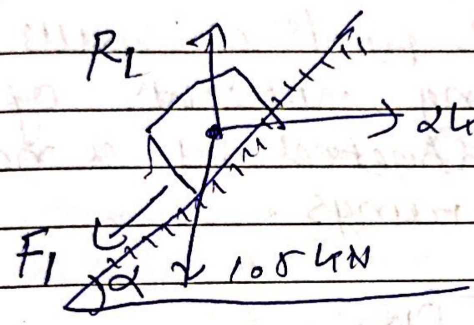
$$P_2 = \frac{w \times \sin\alpha\cos\phi + \sin\phi\cos\alpha}{\cos\alpha\cos\phi - \sin\phi\sin\alpha}$$

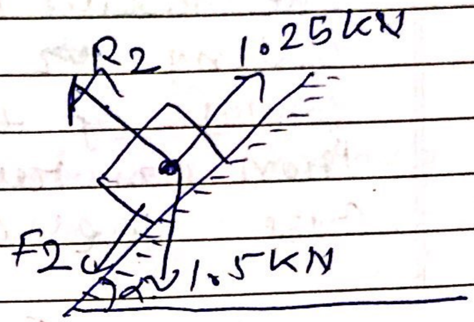
$$= w \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)} = \boxed{w\tan(\alpha+\phi)}$$

Date ______
Page ______

Eg. A load of 1.5 kN, resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane, find the inclination of the plane & coefficient of friction.

→





Horizontal force   force parallel to the plane.

→ We know that when the force is applied horizontally,

(1) $P = W \tan(\alpha+\phi)$. then the magnitude of force, which can move the load up the plane.

$2 = 1.5 \tan(\alpha+\phi)$

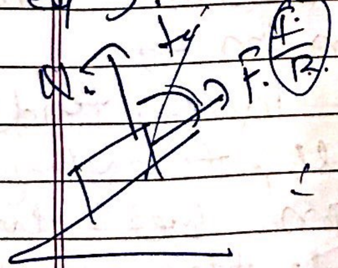
$\tan(\alpha+\phi) = \frac{2}{1.5} = 1.33$

or $(\alpha+\phi) = 53.1^\circ$

(2) $P = W \times \frac{\sin(\alpha+\phi)}{\cos\phi}$ We know that force is applied parallel to the plane, then the magnitude of the force, which can move the load up plane.

(use parallel eq^n).

$1.25 = 1.5 \times \frac{\sin 53.1^\circ}{\cos\phi}$



$= 1.5 \frac{\times 0.8}{\cos\phi}$

$\therefore \cos\phi = 0.96$ or $\phi = 16.3^\circ$

$\therefore \alpha = 53.1^\circ - 16.3^\circ = \boxed{36.8^\circ}$

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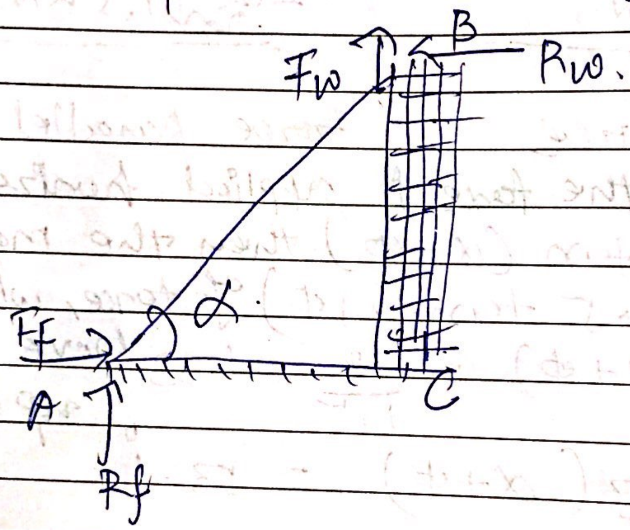
$\mu = \tan\phi$

$$\boxed{\mu = \tan 16.3 = 0.292}$$

* Ladder friction:-

→ The ladder is a device for climbing or scaling ~~on~~ on the roofs or walls. It consists of two long uprights of wood, iron or rope connected by a no. of cross pieces called rungs.



→ Considering ladder AB resting on the rough ground & leaning against a wall,

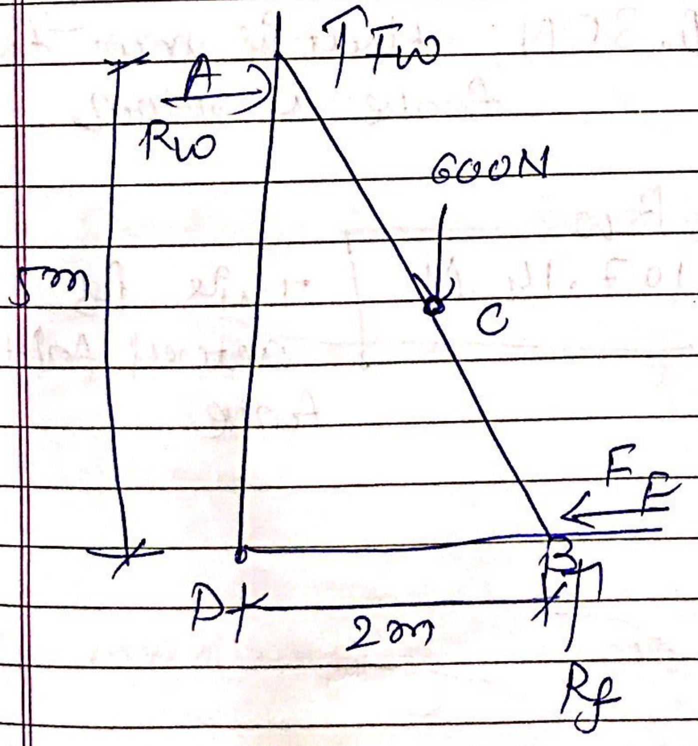
→ As the upper end of the ladder tends to slip downwards, therefore the dirn of force of friction betn the ladder & wall will be upwards as shown in fig.
similarly, as the lower end of ladder tends to slip away from the wall, therefore dirn of the force of friction betn the ladder & floor will be towards the wall as shown in fig.

Date
Page

→ since the system is in equilibrium, therefore the algebric sum of horizontal & vertical components of the forces must also be equal to zero.

(Note) → R_f will act ⊥ of the floor
→ similarly, R_w will also act ⊥ to the wall.

Q. A ladder AB is shown in fig. The μ bet^n ladder and wall and ladder & floor is 0.30. A man weighing 600 N is standing at the mid length of ladder. calculate reactions at floor & wall. Neglect the weight of ladder.



$R_f + F_w = 600$ — (1)

$R_w = F_f$

taking moment @B

$600 \times 1 = R_w \times 5 + F_w \times 2$

but $F_w = \mu_i R_w$
$= 0.3 R_w$

$$600 = 5R_w + (0.3R_w) \times 2$$

$$600 = 5R_w + 0.6R_w$$

$$\boxed{R_w = 107.14\,N}$$

$$F_w = \mu_w R_w$$
$$= 0.3 \times 107.14$$
$$= 32.14\,N$$

$$R_f + F_w = 600$$
$$R_f + 32.14 = 600$$
$$\therefore \boxed{R_f = 567.85\,N}$$

$$F_f = \mu_f . R_f$$
$$= 0.3 \times 567.85$$
$$= 170.35\,N$$ that is maxm friction force at floor.

but, $F_f = R_w$
$$= \boxed{107.14\,N}$$ This is actual friction force.