AC THEORY

Subject Name: Electrical Fundamentals

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INDEX

Advantage of AC over DC.

- AC can be transformed.

A transformer permits voltage to be stepped up or down for the purpose of transmission. Transmission of high voltage (in terms of kV) is that less current is required to produce the same amount of power. Less current permits smaller wires to be used for transmission.

SINUSOIDAL WAVEFORMS

Mathematically

$$v(t) = V_p \cos(wt + \theta)$$

Where

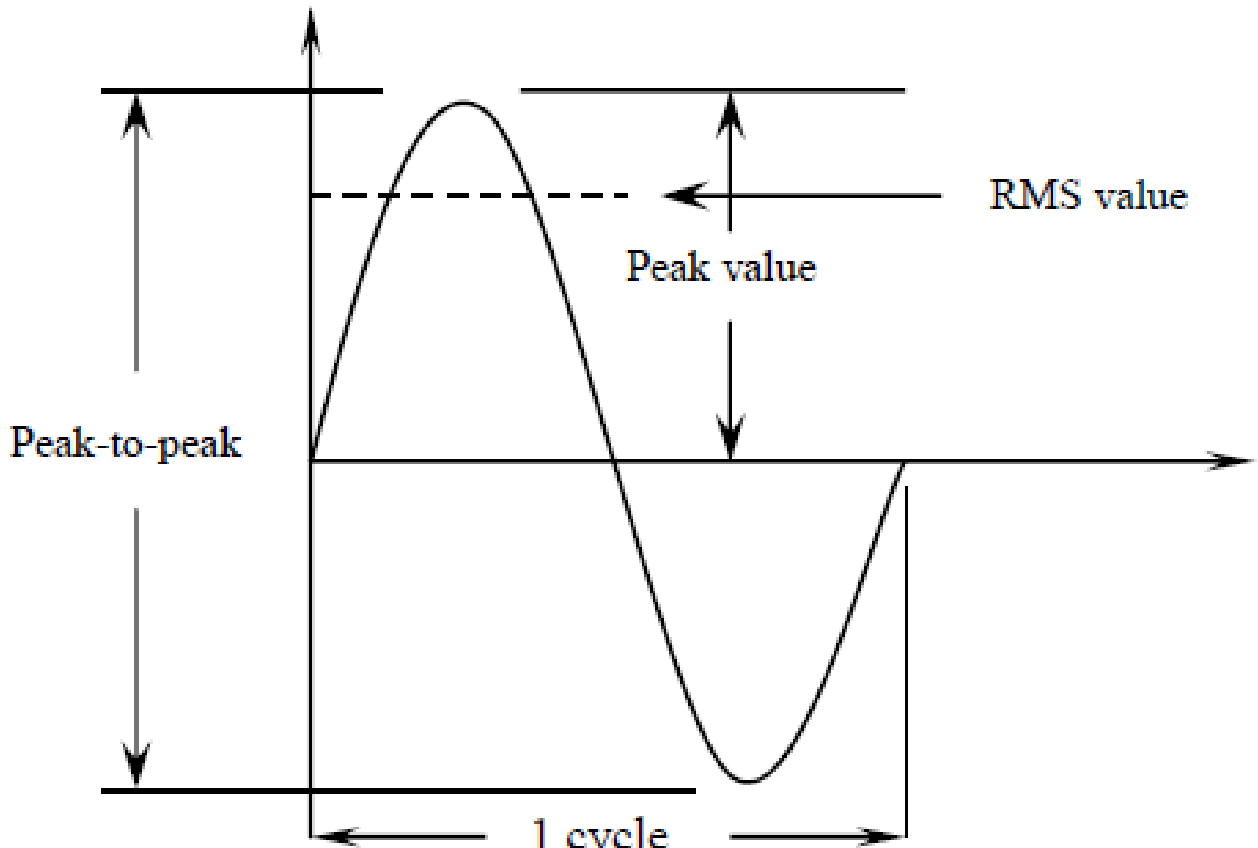
Vp is the peak voltage,

$\omega = 2\pi f$ is the angular speed expressed in radians per second (rad/s),

f is the frequency expressed in Hertz (Hz),

t is the time expressed in second (s), and

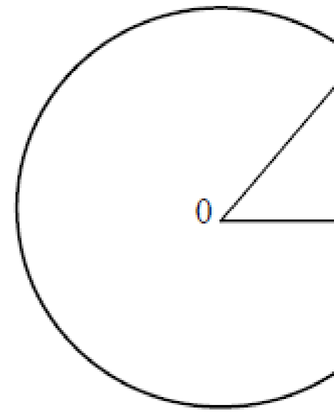
θ is phase of the sinusoid expressed in degrees.


RMS value
Peak value
Peak-to-peak
1 cycle

Radian and Degree

degree is a unit of measurement in degree (its designation is ° or deg), a
rn of a ray by the 1/360 part of the one complete revolution.
he complete revolution of a ray is equal to 360 deg.

radian is defined as the central angle, for which lengths of its arc and
dius are equal (AB = OA).
n arc length is the distance along the arc of a circle from the origin to the
nd of the angle.



$$1\,\text{rad} = \frac{360}{2\pi} \approx 57.3^\circ \qquad 1\,\text{deg} = \frac{2\pi}{360} \approx 0.017453\,\text{rad}$$

ak and Peak-to-Peak Values

peak value is measured from zero to the maximum value obtained in either the ive or negative direction.

difference between the peak positive value and the peak negative value is called peak-to-peak value of the sine wave.

value is twice the maximum or peak value of the sine wave.

peak value is one-half of the peak-to-peak value.

tantaneous Value

nstantaneous value of an AC signal is the value of voltage or current at one cular instant.

rage Value

erage value of an AC current or voltage is the average of all the instantaneous values during one tion. They are actually DC values.
erage value is the amount of voltage that would be indicated by a DC voltmeter if it were connected the load resistor.

rmula for a average voltage is $$V_{av} = 0.636V_{max}$$

Vav is the average voltage for one alteration, and *Vmax* is the maximum or peak voltage.

ive Value

the value of AC signal that will have the same effect on a resistance as a compara of direct voltage or current will have on the same resistance.
ossible to compute the effective value of a sine wave of current to a good degree c acy by taking equally spaced instantaneous values of current along the curve and ting the square root of the average of the sum of the ed values. For this reason, the effective value is often called the "root-meansquare value.

$$I_{eff} = \sqrt{\text{Average of the sum of the squares of } I_{ins}}$$

$$I_{eff} = 0.707 \times I_{max}$$

me that the DC is maintained at 1 A and the resistor temperature at 10
assume that the AC is increased until the temperature of the resistor i
C.

is point it is found that a maximum AC value of 1.414 *A* is required in o
ave the same heating effect as DC.

efore, in the AC circuit the maximum current required is 1.414 times th
ctive current.

's law formula for an AC circuit may be stated as

$$I_{eff} = \frac{V_{eff}}{R}$$

's law, Kirchhoff's law, and the various rules that apply to voltage, curr
power in a DC circuit also apply to the AC circuit.

equency

e number of complete cycles of alternating current or voltage mpleted each second is referred to as the “frequency, *f*” or “event quency”.

equency is always measured and expressed in hertz.

cause there are 2π radians in a full circle, a cycle, the relationshi ween ω, *f*, and period, *T*, can be expressed as

$$\omega = 2\pi f = \frac{2\pi}{T} \text{ radians/second}$$

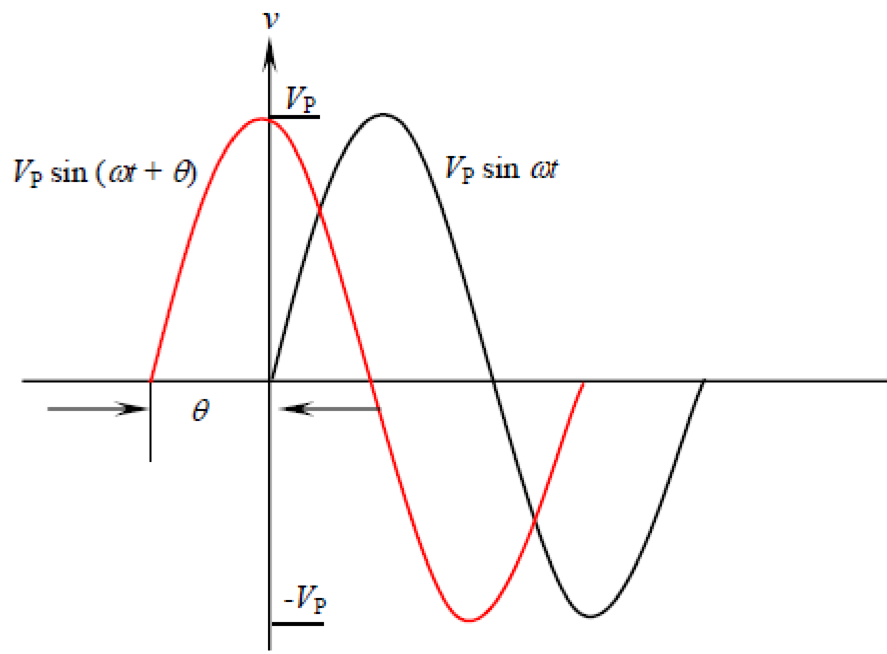
ere ω is the angular velocity in radians per second (rad/s). The dimens requency is reciprocal second.

eriod

e period of a waveform is the time required for completing one full cycle. It is measured in seconds.
e sinusoidal waveform function repeats itself every 2π radians, and its period is therefore 2π radians
e relationship between time (*T*) and frequency (*f*) is indicated by the formulas

$$T = \frac{1}{f}$$

hase



hen two sinusoidal waves are precisely in step with e another, they are said to be in phase. be in phase the two waves must go through their ximum and minimum points at the same ne and in the same direction. To describe the phase ationship between two sinusoidal waves, the ms lead and lag are used. The amount by which e sine wave leads or lags another sine wave is measured degrees. In the figure *V*P sin (ω*t* + θ) leads *V*P sin ω*t* by θ.

vo sine waves that differ in phase by 360° are considered to be in
ase with each other.

xample: To find the phase difference between the two sinusoidal
nctions.

ution

$$v_1 = V_{P1}\cos(10t + 20^\circ)$$

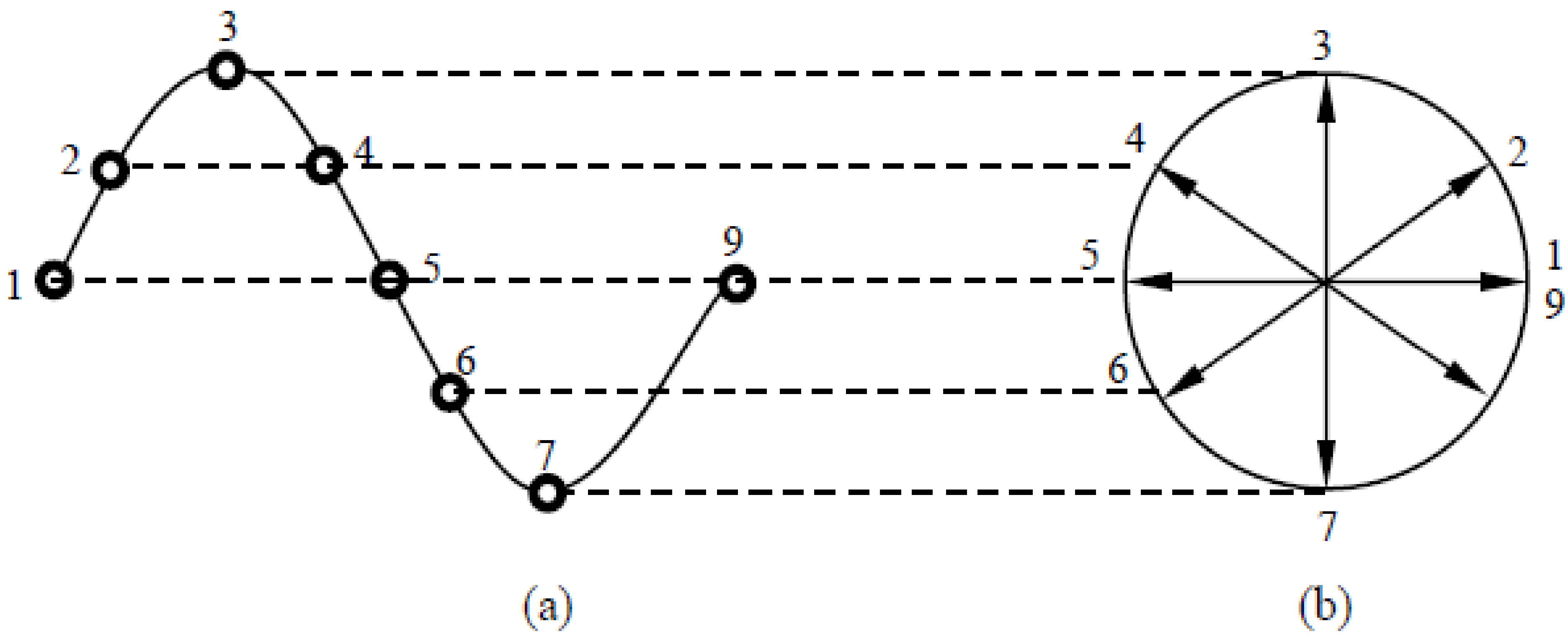
$$= V_{P1}\sin(10t + 90^\circ + 20^\circ)$$

$$= V_{P1}\sin(10t + 110^\circ)$$

$$v_2 = V_{P2}\sin(10t - 40^\circ)$$

$v_1 = V_{P1}\cos(10t + 20^\circ)$ leads $v_2 = V_{P2}\sin(10t - 40^\circ)$

Generation of Sine waveform



(a) Magnitude of a sine wave. (b) A vector with its end fixed at the origin and rotating in a counterclockwise (CCW) direction representing the varying conditions of the AC signal.

HASORS

C sinusoidal quantities are represented by the position of a rotating ector. As the vector rotates it generates an angle. The location of the ector on the plane surface is determined by the magnitude ength) of the vector and by the generated angle. Phase and agnitude defines a phasor (vector) or complex number. The phasor similar to vector that has been studied in mathematics.

ny linear circuit that contains resistors, capacitors, and inductors do ot alter the shape of this signal, nor its frequency. However, the near circuit does change the amplitude of the signal (amplification or ttenuation) and shift its phase (causing the output signal to lead or g the input). The amplitude and phase are the two important uantities that determine the way the circuit affects the signal.

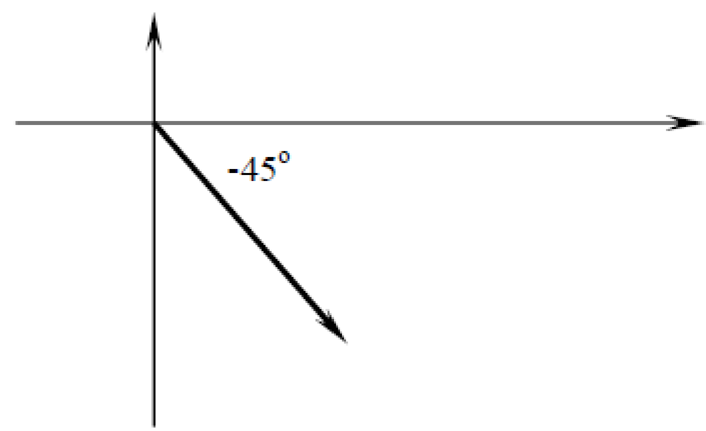
ere are two basic forms of complex number notation: polar and rectangular.

ar Form

ar form is where the length (magnitude) and the angle of its vector denote a complex number.
electrical circuits, a sinusoidal voltage may be represented by

$$V = V_{rms} \angle \theta$$

ndicates that the quantity is a phasor, having both magnitude and phase.
e magnitude is usually RMS.
e phase angle is in degrees. The polarity is very important: + means that the signal leads the
erence; while – means that the signal lags the reference.
e phasor diagram representation is as shown



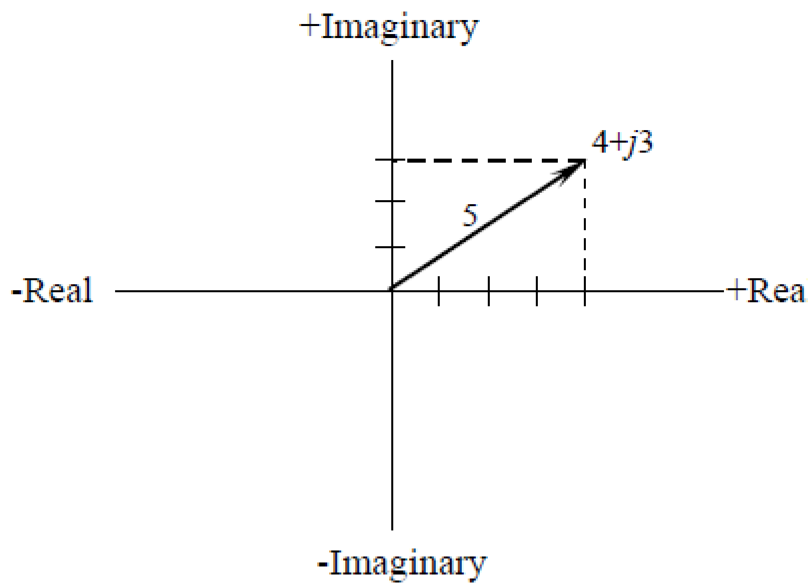
Rectangular Form

The horizontal and vertical components denote a complex number.

The angled vector is taken to be the hypotenuse of a right triangle.

Phasor representation.

Example: 4+*j3*



ransforming Forms

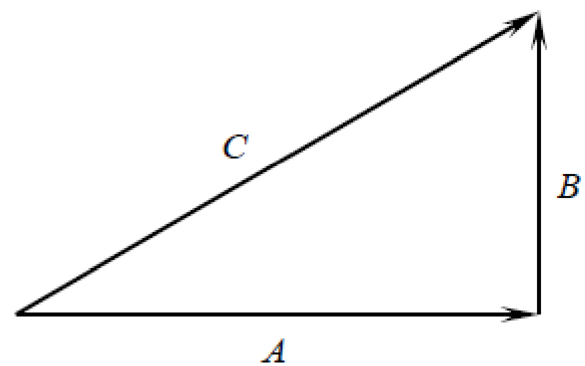
$$V = C\angle\theta = A + jB$$

. Conversion from polar to the rectangular form of a phasor.

onvert *C∠ϑ into A and B.*

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{A}{C}$$

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{B}{C}$$

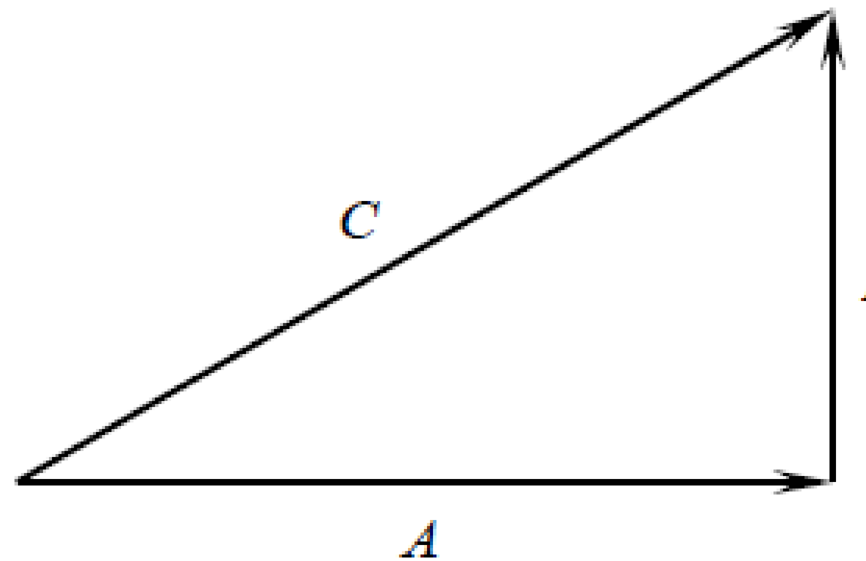


. conversion from rectangular form to polar form

$$C = \sqrt{A^2 + B^2}$$

$$\tan\theta = \frac{B}{A}$$

$$\theta = \tan^{-1}\left(\frac{B}{A}\right)$$



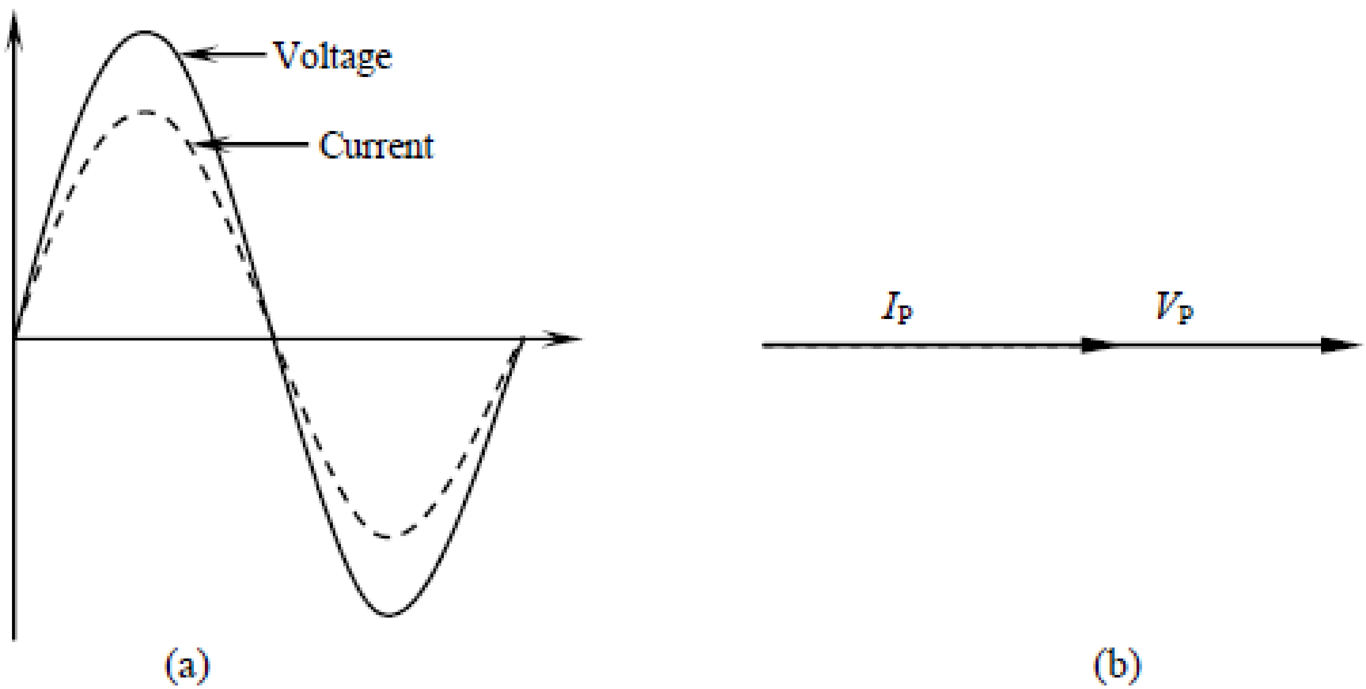
n general, any load in rectangular form may be converted into polar for
s the following

$$Z = R + jX_L$$

$$Z = \sqrt{R^2 + X_L{}^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$

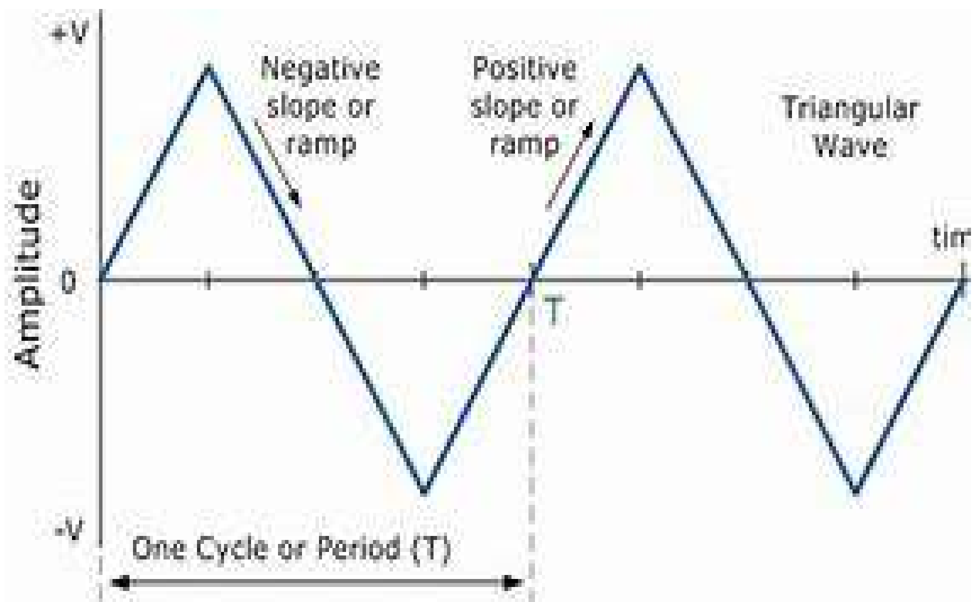
RESISTIVE LOADS

- In a DC circuit, there is one basic type of load, which is resistive.
- AC circuits have three different types of loads: resistive, inductive, and capacitive.
- Voltage divided by current in DC circuits is called resistance.
- In AC circuits it is called impedance.
- The impedance is the opposition an element offers to a sinusoidal current. It is a phasor quantity.
- Any time that a circuit contains resistance, heat will be produced.
- Voltage and current are in phase with each other in a pure resistive circuit as Shown.
- The impedance in AC circuits is defined through Ohm's law. $Z = \frac{V}{I}$

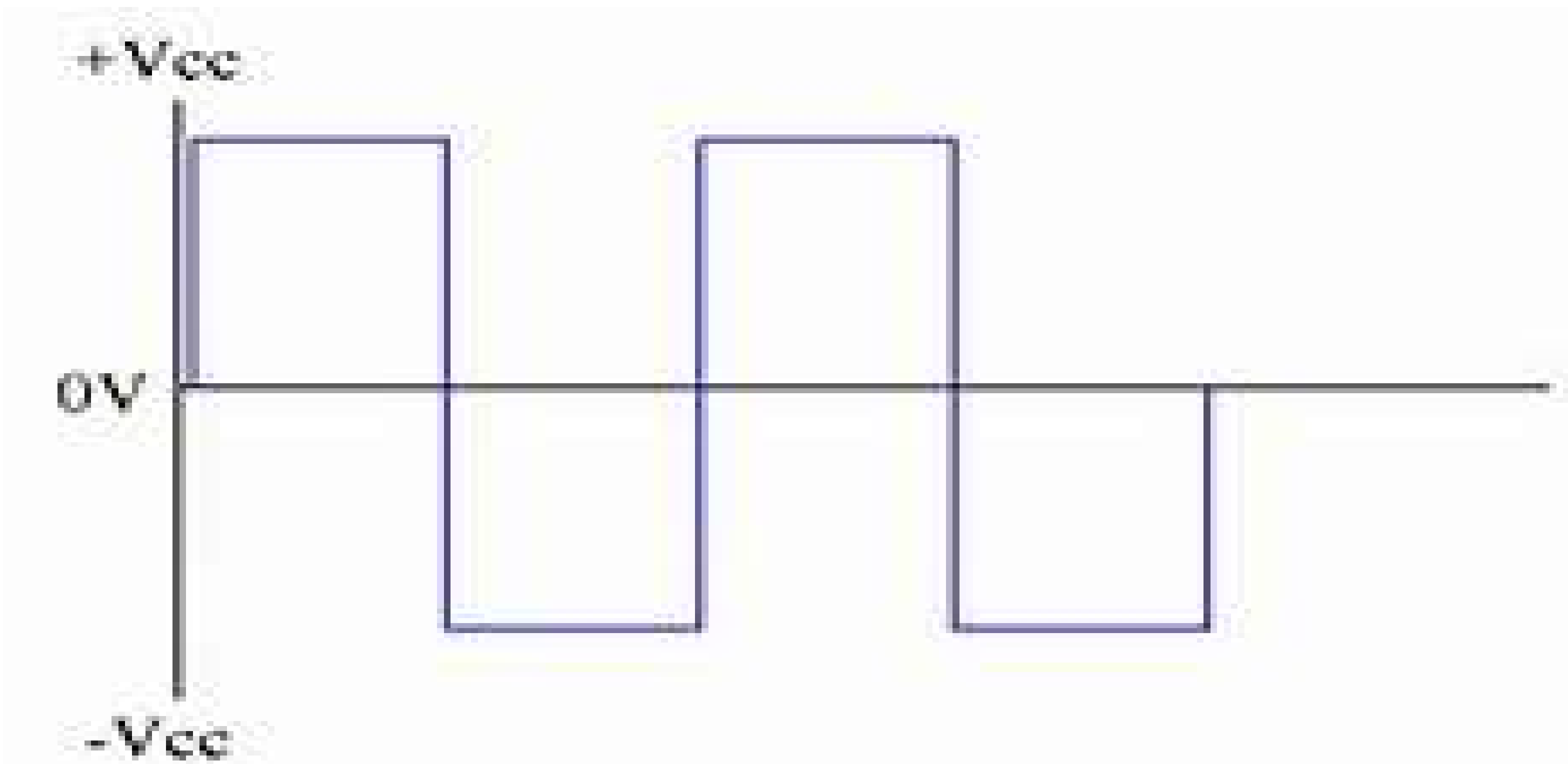


Voltage in phase with current Phase angle between voltage and current is

Triangular wave form



Square Wave form



POWER IN SINGLE PHASE AC CIRCUITS

1. In an AC circuit, the instantaneous values of the voltage, current are constantly changing and therefore power also constantly keeps on changing.
2. AC circuits contain reactance, so there is a power component as a result of the magnetic and/or electric fields created by the components. The result is that unlike a purely resistive component, this power is stored and then returned back to the supply as the sinusoidal waveform goes through one complete periodic cycle.
3. The average power absorbed by a circuit is the sum of the power stored and the power returned over one complete cycle. So a circuits average power consumption will be the average of the instantaneous power over one full cycle.
4. The instantaneous power, **p** defined as the multiplication of the instantaneous voltage, **v** by the instantaneous current, **i**.

As the instantaneous power is the power at any instant of time, then:

$$p = v \times i$$

where:

$$v = V_m \sin(\omega t + \theta_v)$$

$$i = I_m \sin(\omega t + \theta_i)$$

$$p = \left[V_m \sin(\omega t + \theta_v)\right] \times \left[I_m \sin(\omega t + \theta_i)\right]$$

$$\therefore p = V_m I_m \left[\sin(\omega t + \theta_v)\sin(\omega t + \theta_i)\right]$$

Applying the trigonometric product-to-sum identity of:

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

and $\boldsymbol{\theta} = \theta_v - \theta_i$ (the phase difference between the voltage and the current waveforms) into the above equation gives:

$$p = \frac{V_m I_m}{2}\left[\cos\theta - \cos\left(2\omega t + \theta\right)\right]$$

$$\text{As: } \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} \times I_{rms} \ \ (\mathrm{W})$$

Where **V** and **I** are the root-mean-squared (rms) values of the sinusoidal waveforms, *v* and *i* respectively, and θ is the phase difference between the two waveforms. Therefore we can express the instantaneous power as being:

Instantaneous AC Power Equation

$$p = VI\cos\theta - VI\cos(2\omega t + \theta)$$

This equation shows us that the instantaneous AC power has two different parts and is therefore the sum of these two terms.
The first term however is a constant whose value depends only on the phase difference, θ between the voltage, (V) and the current, (I).
The second term is a time varying sinusoid whose frequency is equal to twice the angular frequency of the supply due to the 2ω part of the term.
The average value of the instantaneous power of the sinusoid is given simply as:

$$p = V \times I\cos\theta$$

The AC Power dissipated in a circuit can also be found from the impedance, (Z) of the circuit using the voltage, V_{rms} or the current, I_{rms} flowing through the circuit as shown.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\theta = \cos^{-1}\frac{R}{Z}, \quad \text{or} \quad \sin^{-1}\frac{X_L}{Z}, \quad \text{or} \quad \tan^{-1}\frac{X_L}{R}$$

$$\therefore P = \frac{V^2}{Z}\cos(\theta) \quad \text{or} \quad P = I^2 Z \cos(\theta)$$

Example

The voltage and current values of a 50Hz sinusoidal supply are given as: $v_t = 240 \sin(\omega t + 60^o)$Volts and $i_t = 5 \sin(\omega t - 10^o)$Amps respectively. Find the values of the instantaneous power and the average power absorbed by the circuit.

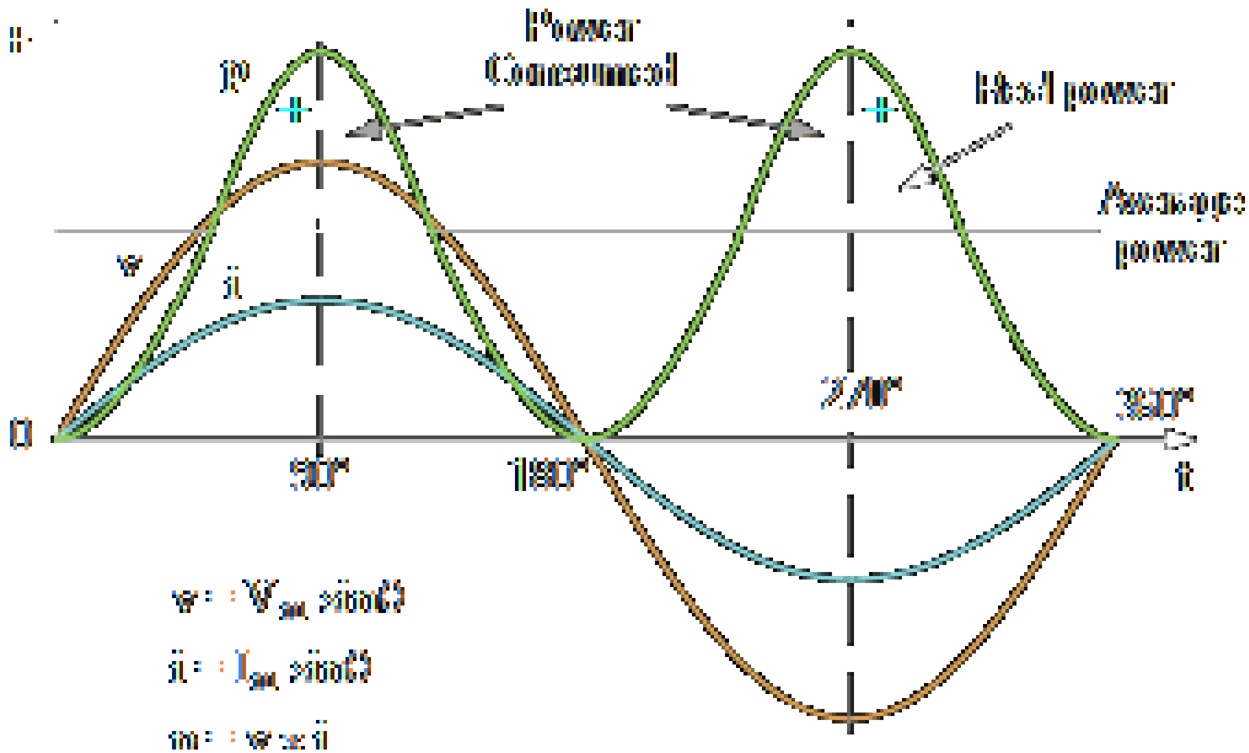
From above, the instantaneous power absorbed by the circuit is given as:

$$p = v \times i = 240\left(\sin\omega t + 60^o\right) \times 5\left(\sin\omega t - 10^o\right)$$

$$p = 240 \times 5\left[\sin\left(314.2t + 60^o\right)\sin\left(314.2t - 10^o\right)\right]$$

$$\therefore p = 1200\left[\sin\left(314.2t + 60^o\right)\sin\left(314.2t - 10^o\right)\right]$$

AC Power Waveforms for a Pure Resistor



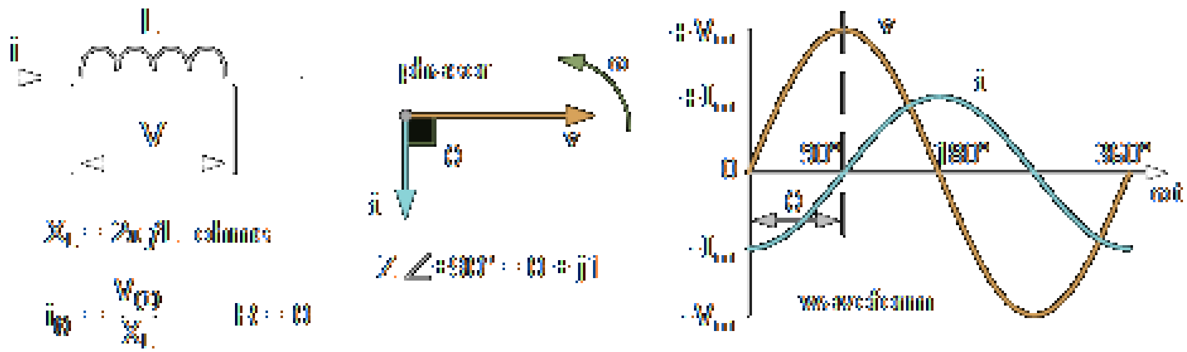
AC Power in a Purely Inductive Circuit

In a purely inductive (that is infinite capacitance, $C = \infty$ and zero resistance, $R = 0$) circuit of L Henries, the voltage and current waveforms are not in-phase.

Whenever a changing voltage is applied to a purely inductive coil, a “back” emf is produced by the coil due to its self-inductance. This self-inductance opposes and limits any changes to the current flowing in the coil.

The effects of this back emf is that the current cannot increase immediately through the coil in-phase with the applied voltage causing the current waveform to reach its peak or maximum value some time after that of the voltage.

The result is that in a purely inductive circuit, the current always “lags” (ELI) behind the voltage by 90^o ($\pi/2$) as shown.



The voltage and current waveforms are "out-of-phase" with each other as the voltage leads the current by 90°. Since the phase difference between the voltage waveform and the current waveform is 90°, then the phase angle resulting in cos 90° = 0.

Real Power in a Pure Inductor

$$P = V \times I\cos\theta$$

$$\cos\left(90^{\circ}\right) = 0$$

$$\therefore P = V \times I \times 0 = 0 \ \text{(watts)}$$

A pure inductor does not consume or dissipate any real or true power

The product of the current and the voltage is imaginary power, commonly called “Reactive Power”, (**Q**) measured in volt-amperes reactive, (VAr), Kilo-voltamperes reactive (KVAr), etc.

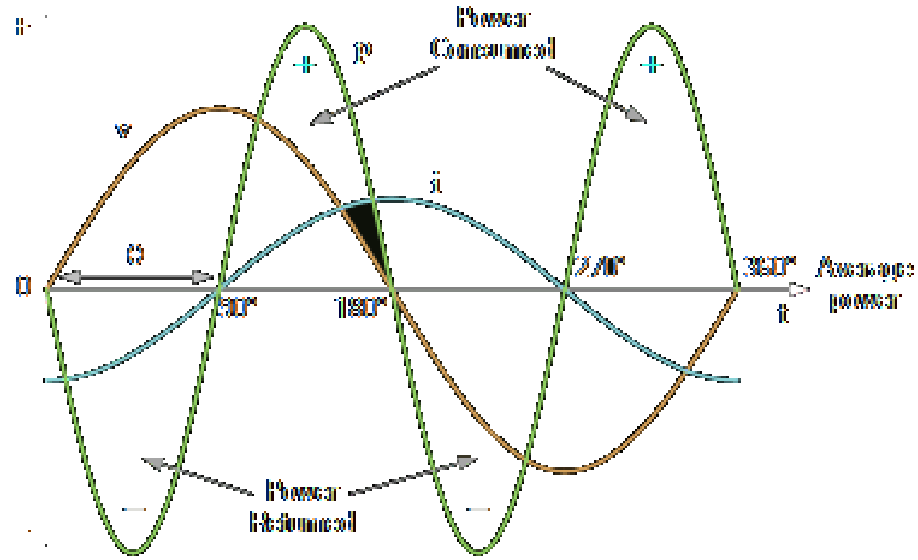
The average reactive power in an inductor becomes:

$$Q_L = V \times I \sin\theta$$

$$\sin\left(+90^{\circ}\right) = +1$$

$$\therefore Q_L = V \times I \times +1 = V \times I \quad (VAr)$$

AC Power Waveforms for a Pure Inductor



This positive power indicates that the coil is consuming electrical energy from the supply. This negative power indicates that the coil is returning the stored electrical energy back to the supply.

The total power taken by a pure inductor over one full-cycle is zero, so an inductors reactive power does not perform any real work.

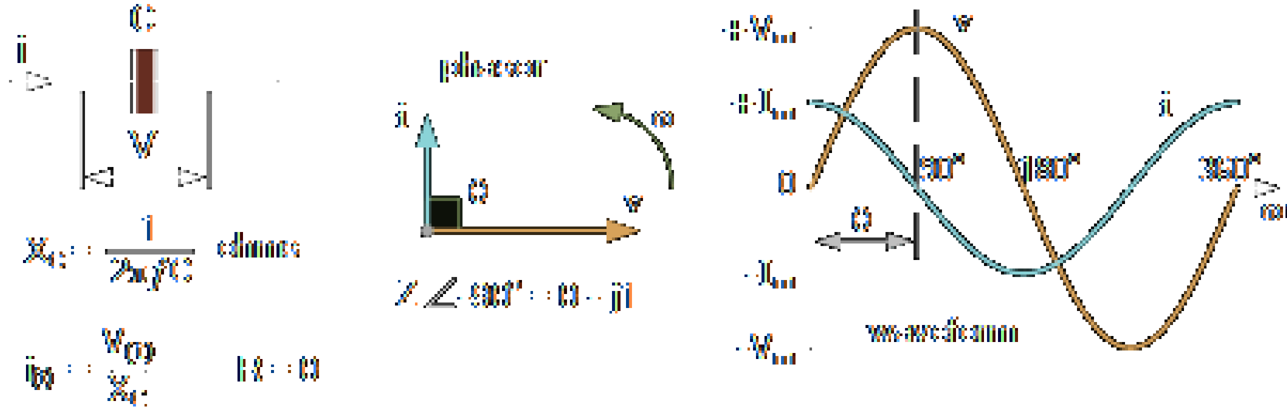
No real power is used up since the power alternately flows to and from the source.

AC Power in a Purely Capacitive Circuit

A purely capacitive (that is zero inductance, $L = 0$ and infinite resistance, $R = \infty$) circuit of C Farads, has the property of delaying changes in the voltage across it. Capacitors store electrical energy in the form of an electric field within the dielectric so a pure capacitor does not dissipate any energy but instead stores it.

In a purely capacitive circuit the voltage cannot increase in-phase with the current as it needs to “charge-up” the capacitors plates first. This causes the voltage waveform to reach its peak or maximum value some time after that of the current. The result is that in a purely capacitive circuit, the current always “leads” (ICE) the voltage by 90^o ($\omega/2$) as shown.

Purely Capacitive Circuit



For a purely capacitive circuit, the phase angle θ = -90° and the equation for the average reactive power in a capacitor becomes

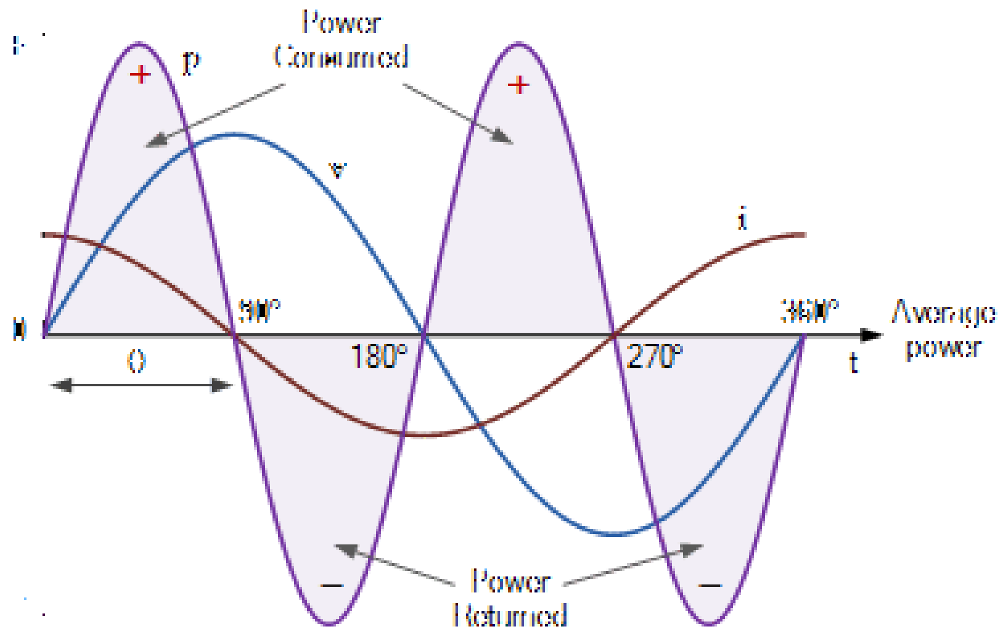
$$Q_C = V \times I \sin\theta$$

$$\sin\left(-90^{\circ}\right) = -1$$

$$\therefore Q_C = V \times I \times -1 = -V \times I \quad (\mathrm{VAr})$$

A pure capacitor does not consume or dissipate any real or true power, P.

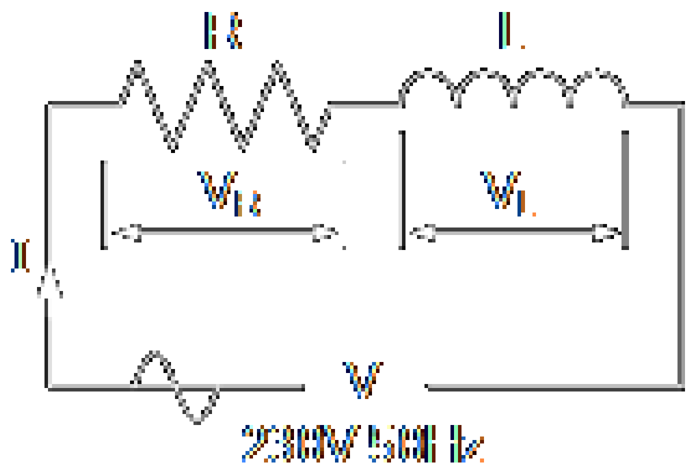
AC Power Waveforms for a Pure Capacitor



Period of positive power indicates that the coil is consuming electrical energy from the supply. Negative power indicates that the coil is returning stored electrical energy back to the supply.

The power delivered from the source to the capacitor is exactly equal to the power returned to the source by the capacitor so no real power is used up since the power alternately flows to and from the source. This means then that the total power taken by a pure capacitor over one full-cycle is zero, so the capacitors reactive power does not perform any real work.

A solenoid coil with a resistance of 30 ohms and an inductance of 200mH is connected to a 230VAC, 50Hz supply. Calculate: (a) the solenoids impedance, (b) the current consumed by the solenoid, (c) the phase angle between the current and the applied voltage, and (d) the average power consumed by the solenoid.



Data given: R = 30Ω, L = 200mH, V = 230V and *f* = 50Hz.

(a) Impedance (Z) of the solenoid coil:

$$R = 30\Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 200 \times 10^{-3} = 62.8\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 62.8^2} = 69.6\Omega$$

(b) Current (I) consumed by the solenoid coil:

$$V = I \times Z$$

$$\therefore I = \frac{V}{Z} = \frac{230}{69.6} = 3.3\,A_{(rms)}$$

(c) The phase angle, θ:

$$\cos\theta = \frac{R}{Z}, \quad \text{or} \quad \sin\theta = \frac{X_L}{Z}, \quad \text{or} \quad \tan\theta = \frac{X_L}{R}$$

$$\therefore \cos\theta = \frac{R}{Z} = \frac{30}{69.6} = 0.431$$

$$\cos^{-1}(0.431) = 64.5^{o} \text{ lagging}$$

(d) Average AC power consumed by the solenoid coil

$$P = V \times I \times \cos\theta$$

$$P = 230 \times 3.3 \times \cos\left(64.5^{o}\right)$$

$$\therefore P = 327 \text{ watts}$$

THREE-PHASE AC CIRCUITS

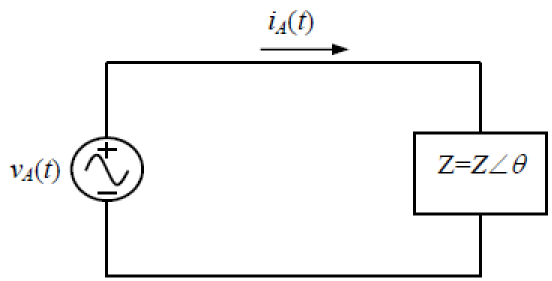
- A three-phase power system consists of three-phase generators, transmission lines, and loads.
- Three-phase systems have two major advantages over single phase systems:

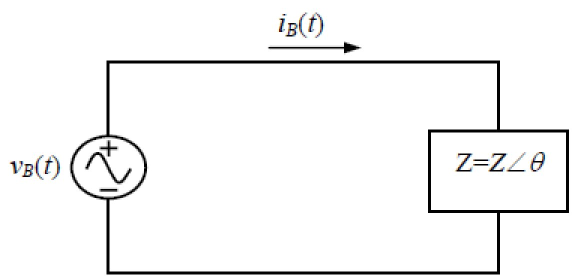
(1) More power is obtained per kilogram of metal from three phase system, and
(2) the power delivered to a three-phase load is constant all the times.

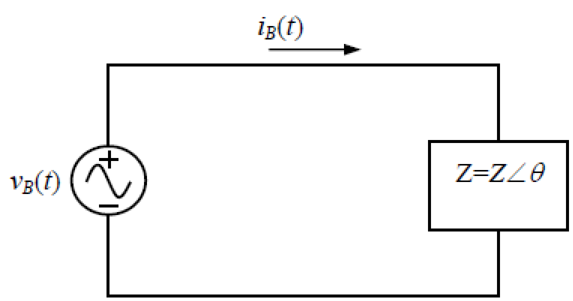
Wye-Connected System

A three-phase system consists of three AC sources, with voltages equal in magnitude but differing in phase angle from the others by 120o, and connected at a common point called neutral as shown in Figure. The current flowing to each load can be found from the equation

$$I = \frac{V}{Z}$$


iA(t)
vA(t)
Z=Z∠θ


iB(t)
vB(t)
Z=Z∠θ

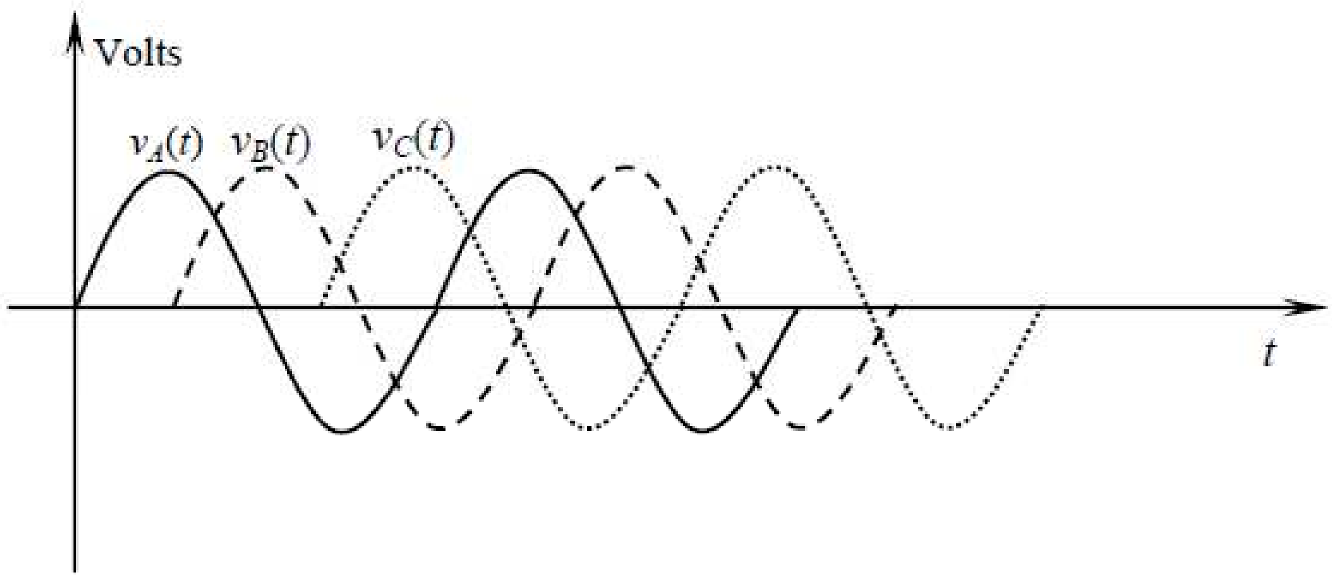

iB(t)
vB(t)
Z=Z∠θ

Accordingly, the currents flowing in the three phases are

$$I_A = \frac{V\angle 0^\circ}{Z\angle\theta} = I\angle -\theta$$

$$I_B = \frac{V\angle -120^\circ}{Z\angle\theta} = I\angle -120^\circ - \theta$$

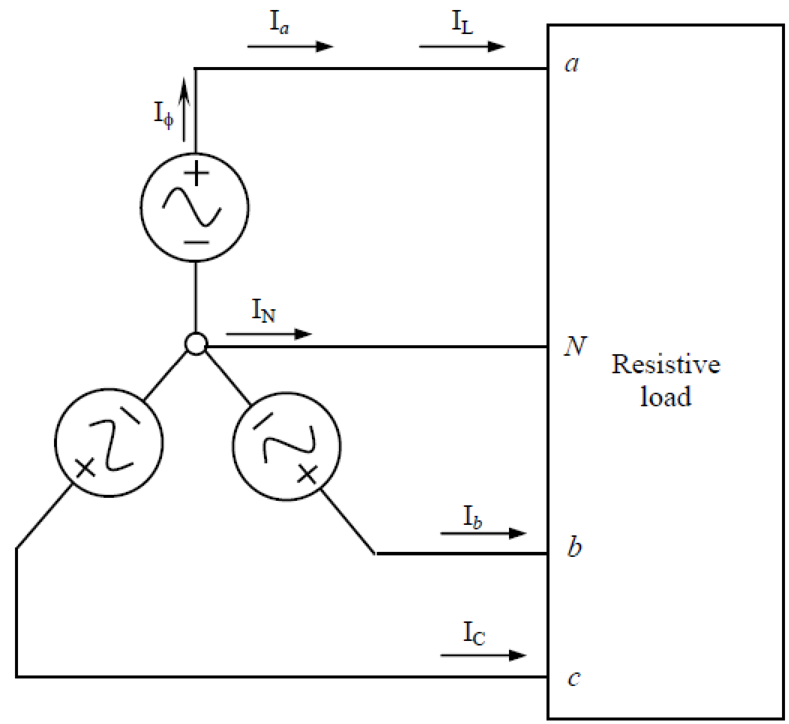
$$I_C = \frac{V\angle -240^\circ}{Z\angle\theta} = I\angle -240^\circ - \theta$$



Voltage waveforms of each phase of the generator.

It is possible to connect the negative ends of these three single-phase generators and loads together, so they share a common neutral. This type of connection is called wye or Y.

In this case four wires are required to supply power from the three generators to resistive load as shown in Figure



Y-connected generator with a resistive load

The voltages between any two line terminals (a, b, or c) are called *line-to-line voltages, and the voltages between any line terminal and the neutral terminal are* called *phase voltages*.

The load connected to this generator is assumed to be resistive, the current in each phase of the generator will be at the same angle as the voltage.

The current in each phase will be given by

$$I_a = I_\phi \angle 0^o$$

$$I_b = I_\phi \angle -120^o$$

$$I_c = I_\phi \angle -240^o$$

For a Y connection

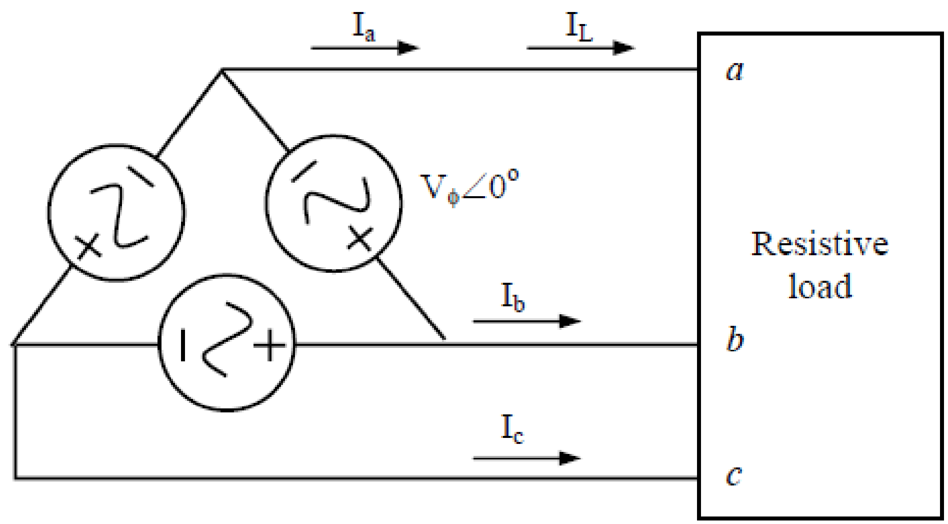
1. The current in any line is the same as the current in the corresponding phase.
2. The relationship between line voltage and phase voltage is given by the following equation

$$V_L = \sqrt{3}\ V_\phi$$

Delta (Δ) Connection

In the delta (Δ) connection the three generators are connected head to tail as shown in Figure.

The Δ connection is possible because the sum of the three voltages VA + VB + VC = 0.



In Δ connection, the line-to-line voltage between any two lines will be the same as the voltage in the corresponding phase.
In a Δ connection, $V_L = V_\varphi$

The relationship between line current and phase is

$$I_L = \sqrt{3} I_\phi$$

What is resonance?

Inductive reactance (X_L = 2 Πf L) and capacitive reactance: (X_C = 1/2 Π f C) are functions of an alternating current frequency.

Decreasing the frequency decreases the ohmic value of the inductive reactance, but increases the capacitive reactance. At some particular frequency, known as the resonant frequency, the reactive effects of a capacitor and an inductor will be equal. Since X_L and X_C are the opposite of one another, they will cancel, leaving only the ohmic value of the resistance to oppose current flow in a circuit. If the value of resistance is small or consists only of the resistance in the conductors, the value of current flow can become very high.