Unit-2 Introduction to Mechanics

Vectors & Scalar:

Scalar: The Physical quantities which possess only magnitude and no directions in space are called Scalars!
i.e. temperature, mass, time, volume, speed of Light

Vectors: The Physical quantity which have magnitude and direction both are called vectors
i.e. velocity, acceleration, force etc.

Polar Vector → associated with a linear or directional effect
i.e. force, linear velocity, linear momentum

Axial Vectors → associated with rotation about an axis
i.e. angular ~~Veloc~~ Velocity, angular momentum

☆ Dot Product of two vectors:

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos\theta = AB\cos\theta$$

Application: → Work done on an object:

$$\vec{W} = \vec{F} \cdot \vec{r} = rF\cos\theta$$

→ Electric flux

$$\phi_E = \int \vec{E} \cdot d\vec{l}$$
$$= E \cdot dl \cos\theta$$

Scanned with CamScanner

★ Vectar Product of two Vectars:

$$\vec{A} \times \vec{B} = AB \sin\theta \cdot \vec{n}$$

Application:

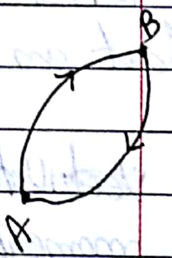
The Targue $\vec{\tau}$ is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= rF \sin\theta \cdot \vec{n}$$

Def^n: A force that tends to cause rotation

★→ Conservative force:

A Conservative force does the same amount of work moving an object from point A to point B, regardless of the path taken.



i.e. Gravitational force, elastic force.

But the work done by a non-conservative force depends on the path.
i.e. friction

Friction does more work as the block if one slides it along the indirect path across the table top.

The longer the path, the more work friction does.

→ Conservative force have Potential Energy but Non-Conservative force ~~dose~~ does not have Potential energy.

Scanned with CamScanner

Numerical :

* Find the torque of a force $7i + 3j - 5k$ among the origin. The force acts on a particle whose position vector is $i - j + k$.

Ans. $2\vec{i} + 12\vec{j} + 10\vec{k}$ ($\sqrt{248}$)

* Find scalar and vector product of two vectors,

$A = (3i - 4j + 5k)$

$B = (-2i + j - 3k)$

⇒ Let the position of the body at any instant be represented by a vector $\vec{r}$ and its standard position, a position at which the potential energy becomes zero i.e. it loses its capacity to do any work by $\vec{r_0}$.

The potential energy at this instant V is given by,

$$V = \int_{r}^{r_0} \vec{F} \cdot d\vec{r} = -\int_{r_0}^{r} \vec{F} \cdot d\vec{r}$$

If the force $\vec{F}$ and displacement $d\vec{r}$ are in the same direction, we have

$$V = -\int_{r_0}^{r} \vec{F} \cdot d\vec{r}$$

Differentiating above relation, we have

$$\frac{dV}{dr} = -F \quad \text{or} \quad \boxed{F = -\frac{dV}{dr}}$$

Hence, the potential energy, V may also be defined as a function of position whose negative gradient gives the force.

We can also define a conservative force as equal to the negative gradient of potential energy V.

★ Central force:

If the force on a body is always towards a fixed point, it is called a central force. Take the fixed point as the origin.

i.e. force due to gravitation.

Scanned with CamScanner

$$F = \frac{d}{dt}\vec{P}$$

Angular Momentum:

It is the rotational motion equivalent to linear momentum.

→ It is a Conserved quantity.

→ ~~Toto~~ The total angular momentum of a system remains constant unless acted on by an external torque.

Relation between ~~Tor~~ Torque and angular momentum

If $\vec{F}$ is the force acting on particle which is moving with a velocity v

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v})$$

multiplying both sides with $\vec{r}$

$$\vec{r} \times \vec{F} = r \times \frac{d}{dt}(m\vec{v})$$

where, $\tau = \vec{r} \times \vec{F}$

$$\therefore \tau = \vec{r} \times \frac{d}{dt}(m\vec{v}) \longrightarrow (1)$$

Angular momentum.

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{P}) = \vec{r} \times \frac{d\vec{P}}{dt} + \frac{d\vec{r}}{dt} \times \vec{P}$$

$$= \left[\vec{r} \times \frac{d}{dt}(m\vec{v})\right] + \left[\vec{v} \times (m\vec{v})\right]$$

$$= \vec{r} \times \vec{F} + 0$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times d/dt\,(m\vec{v}) \longrightarrow (2)$$

Scanned with CamScanner

Date: / /

from Eq^n (1) & (2)

$$\boxed{\tau = \frac{d\vec{L}}{dt}}$$

If the torque is zero

$$\therefore \tau = \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{Constant}$$

The angular momentum is conserved.

* Moment of Inertia:

The measure of the rotational inertia of a body is called moment of Inertia.

→ The Inertia of a body in rotatary motion ~~do~~ depends upon the distribution of the mass in the body about the axis of rotation.

→ For a point mass, the moment of Inertia is mass times the square of distance, which is perpendicular to the rotational axis. $\boxed{I = m r^2} \Rightarrow \boxed{I = mr^2}$

where, m = mass (P.t.)

$r \Rightarrow$ distance perpendicular to rotational axis.

Scanned with CamScanner

Page :
Date : / /

★ Rigid body:

Rigid body is a solid body in which deformation is zero or so small it can be neglected. The distance between any two given points on a rigid body remains constant in time regardless of external forces exerted on it.

A Rigid body is usually considered as continuous distribution of mass.

★ Rigid body Motion:

When force is applied to the body, the body will move in a straight line.
↳ Translational motion

If a force is applied to this body, It will move with respect to the axis. However, the body may be set in rotation about the axis.
↳ called Rotational motion

Angular momentum

$$L = \vec{I} \cdot \vec{w}$$

where, I is the moment of Inertia.

$$v = rw$$

where, w = angular velocity

$$K.E. = \frac{1}{2} m v^2$$

$$L = \vec{I}\,\vec{\omega} \qquad I = m r^2$$

$$= m r^2 \cdot \frac{v}{r} \qquad v = r\omega \Rightarrow \omega = \frac{v}{r}$$

$$= m v r$$

$$= m r^2 \omega$$

Derivation:

$$L = r \cdot m v$$

$$= r \cdot m \cdot r\omega \qquad (v = r\omega)$$

$$= m r^2 \omega$$

$$\boxed{L = \vec{I}\,\vec{\omega}} \qquad (I = m r^2)$$

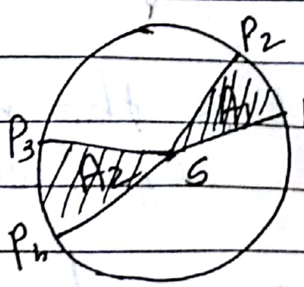
☆ Kepler's Law of Motion:

1st Law: Law of Orbits:

All planets move in elliptical orbits with the sun as one focus.

2nd Law: Law of Areas:

According to this Law a line joinning any planet to the sun sweeps equal areas in equal times.



As shown in fig. let P_1, P_2, P_3 and P_4 represent the positions of a planet at different times in its orbit and S be the position of the sun.

Scanned with CamScanner

Page :
Date : / /

Kepler's Second Law states that if time interval between P_1 and P_2 equals the time interval between P_3 and P_4 then area A_1 and A_2 must be equal.

$V \propto \frac{1}{r} \Rightarrow \frac{V_1}{V_2} = \frac{r_2}{r_1}$

3rd Law : Law of Periods:

The Square of the period of any planet about the sun is proportional to the cube of Planet's mean distance from the Sun.

$$T^2 = K R^3$$

$$\frac{T^2}{R^3} = K =$$ where, K = is Constant.

Scanned with CamScanner

Translational		Rotational
m	$\longleftrightarrow$	$I = mr^2$
$\vec{v}$	$\longleftrightarrow$	$\omega = v/r$
p	$\longleftrightarrow$	$\bar{L} = \bar{r} \times \bar{p}$
$\vec{F}$	$\longleftrightarrow$	$\vec{\tau} = \vec{r} \times \vec{F}$
$K.E = \frac{1}{2}mv^2$	$\longleftrightarrow$	$K.E = \frac{1}{2}I\omega^2$
		$= \frac{1}{2} L \cdot \omega$
		Since, $L = I\omega$

Relation between K.E and L $\rightarrow$

$$\bar{L} = \bar{r} \times \bar{p} \Rightarrow L = |\bar{r} \times \bar{p}| = mvr$$

$$= m\omega r^2$$

as, $I = mr^2$ (moment of inertia)

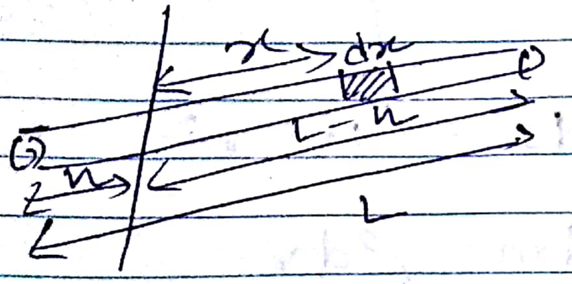
$$L = (mr^2)\omega = I\omega$$

Rotational $K.E = \frac{1}{2} I\omega^2$

$$= \frac{1}{2} L\omega.$$

Scanned with CamScanner

Moment of inertia of a rod of length L and mass M about an axis $\perp^r$ to it $\rightarrow$



$$dI = dm\, x^2$$

$$I = \int dm\, x^2$$

Now $dm = \frac{M}{L} dx$

$$\therefore I = \frac{M}{L}\int_{-h}^{L-h} x^2 dx$$

$$= \frac{1}{3}\frac{M}{L}\left[(L-h)^3 + h^3\right]$$

$$= \frac{1}{3}\frac{M}{L}\left[L^3 - \cancel{h^3} + 3Lh^2 - 3hL^2 + \cancel{h^3}\right]$$

$$= \frac{1}{3} M\left[L^2 + 3h^2 - 3Lh\right]$$

Special case (1) $I \rightarrow$ at one end

$h = 0$

$$\therefore I = \frac{1}{3} ML^2$$

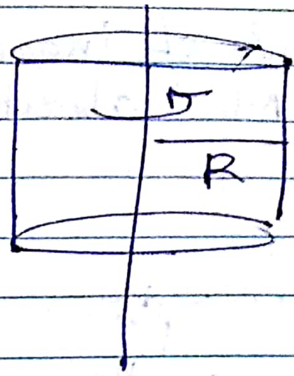
(2) through centre of mass

$h = L/2$

$$I = \frac{1}{12} ML^2$$

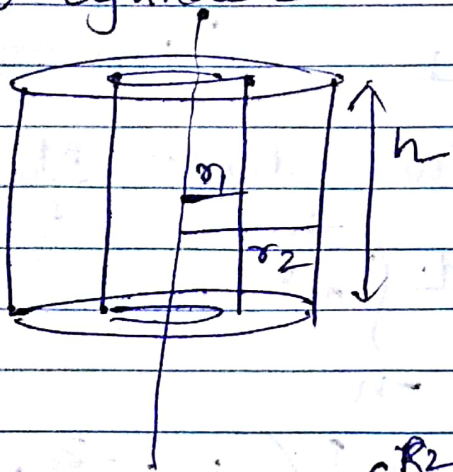
Scanned with CamScanner

(2) Solid Cylinder $\longrightarrow$



$$I = \frac{1}{2}MR^2$$

Hollow cylinder $\longrightarrow$



$$M = \pi(R_2^2 - R_1^2)L \times \rho$$

$$dI = dm\, r^2$$

where $dm = \rho\, dV$

$$dV = 2\pi r L\, dr$$

$$dI = 2\pi\rho L \int_{R_1}^{R_2} r^3\, dr$$

$$= \frac{2}{4}\pi\rho L\,(R_2^4 - R_1^4)$$

$$= \frac{1}{2}\left[\pi\rho(R_2^2 - R_1^2)L\right](R_2^2 + R_1^2)$$

$$= \frac{1}{2}M(R_2^2 + R_1^2)$$

Scanned with CamScanner