Antenna & Wave Propagation

(EC0602)

Unit-3

B.Tech. (Electronics and Communication)

Semester-VI

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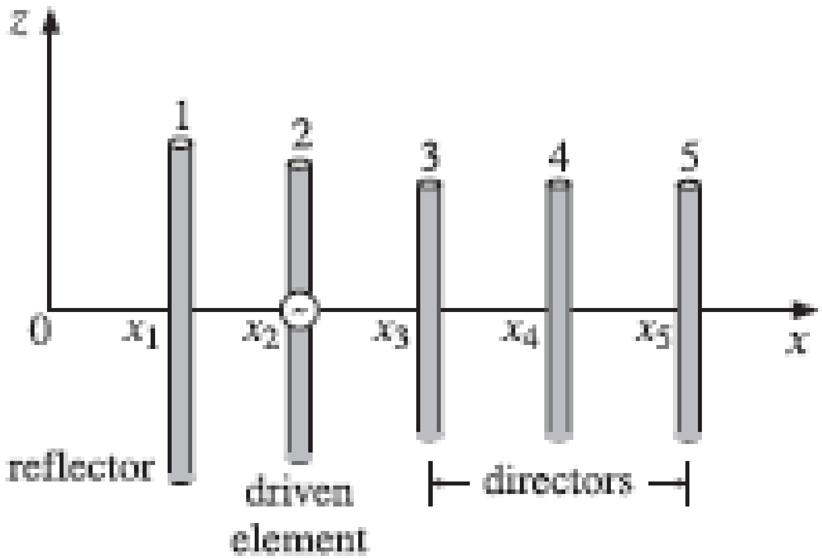
Arrays of Antennas

Examples of Arrays of antennas

- Sectoral array: used at mobile base station antenna for mobile communication
- Triangular array consisting of 12 dipoles, with 4 dipoles on each side of triangle, to cover an angular sector of 120 deg.

- Yagi Uda Array: used for TV and radio signal reception


z
1
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reflector
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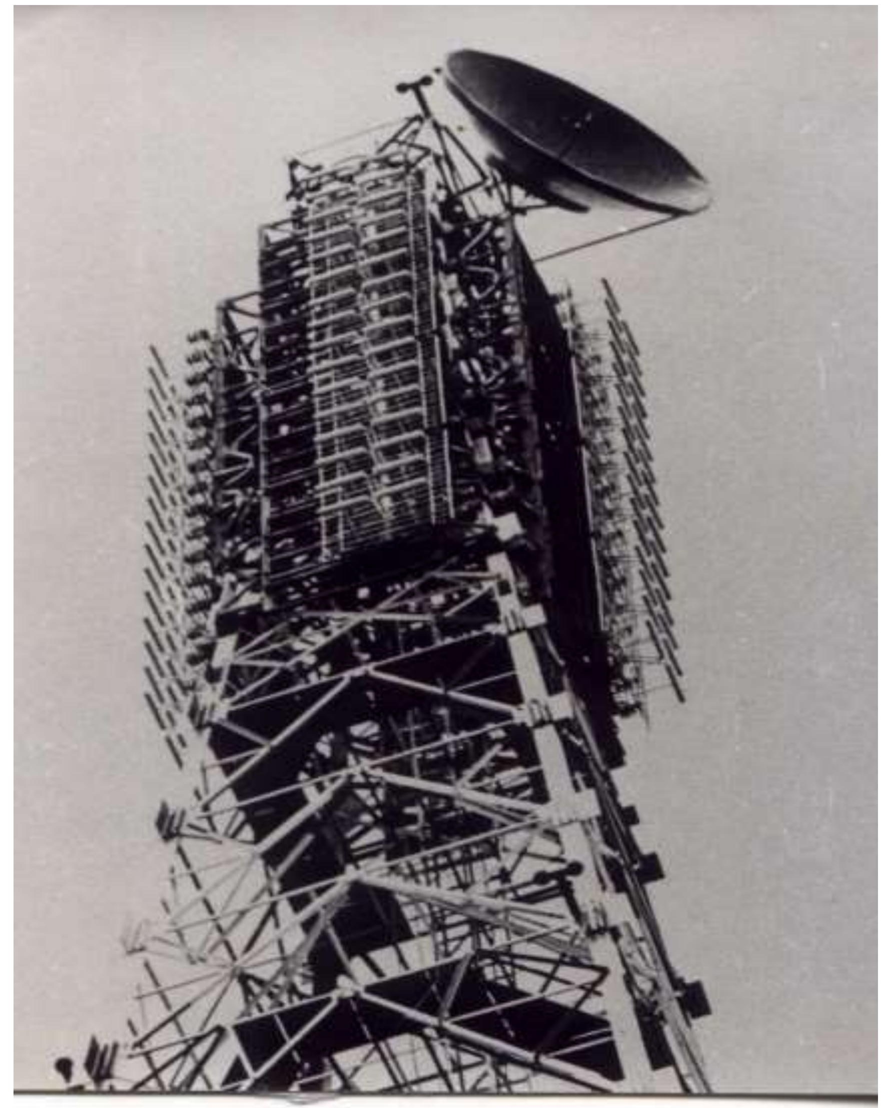
directors



Arrays

- Usually the radiation pattern of a single element is relatively wide, and each element provides low values of directivity. In many applications it is necessary to design antennas with very directive characteristics to meet the demands of long distance communication. This can only be accomplished by increasing the electrical size of the antenna.
- Enlarging the dimensions of single elements often leads to more directive characteristics. Another way to enlarge the dimensions of the antenna, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration. This new antenna, formed by multi-elements, is referred to as an *array.*

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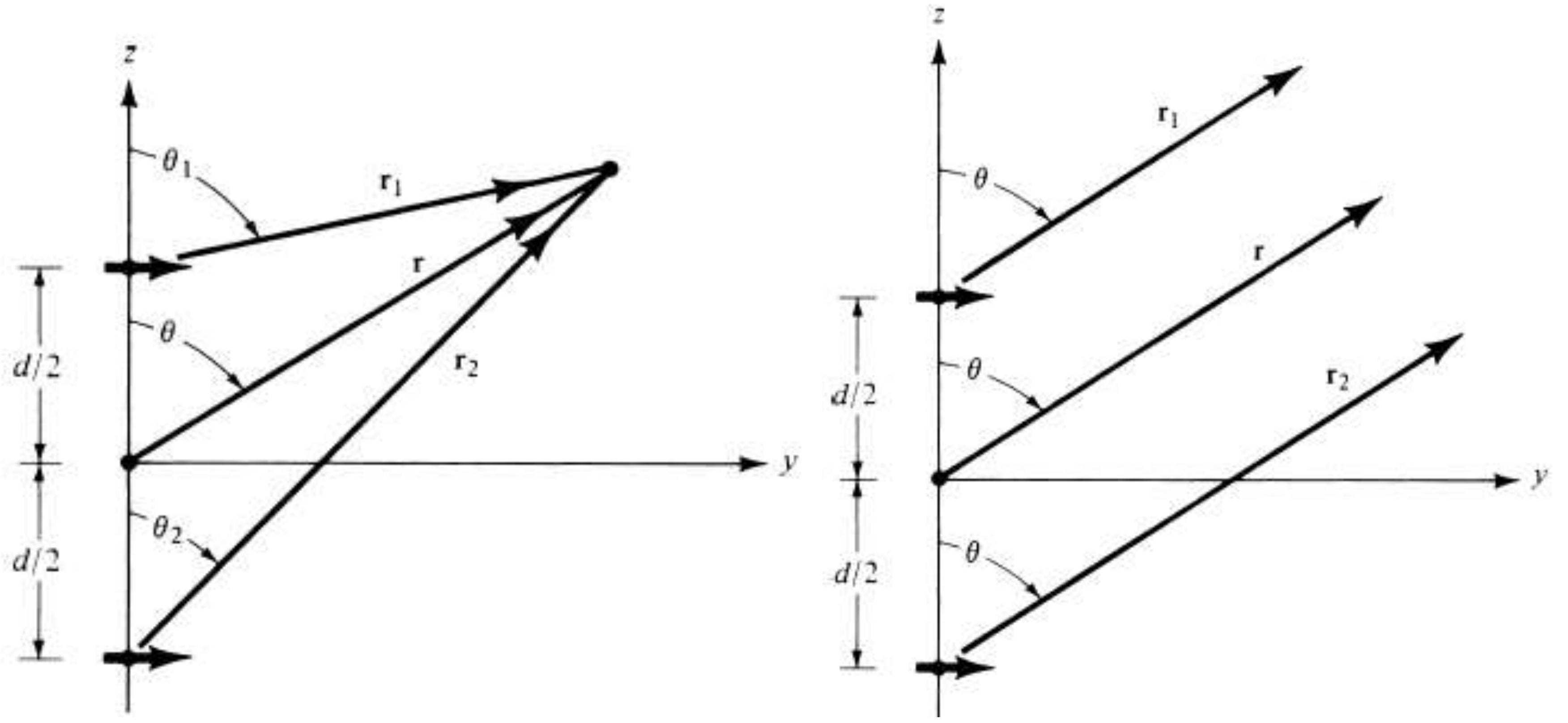
- The total field of the array is determined by the vector addition of the fields radiated by the individual elements.
- In an array of identical elements, there are at least five controls that can be used to shape the overall pattern of the antenna.
- These are:

1. the geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)
2. the relative displacement between the elements
3. the excitation amplitude of the individual elements
4. the excitation phase of the individual elements
5. the relative pattern of the individual elements

TWO-ELEMENT ARRAY

- Let us assume that the antenna under investigation is an array of two infinitesimal horizontal dipoles positioned along the z-axis, as shown in Figure.



$$\mathbf{E}_t = \mathbf{E}_1 + \mathbf{E}_2 = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l}{4\pi}\left\{\frac{e^{-j[kr_1-(\beta/2)]}}{r_1}\cos\theta_1 + \frac{e^{-j[kr_2+(\beta/2)]}}{r_2}\cos\theta_2\right\}$$

- where β is the difference in phase excitation between the elements. The magnitude excitation of the radiators is identical. Assuming far-field observations,

$$\theta_1 \simeq \theta_2 \simeq \theta$$

$$\left.\begin{aligned} r_1 &\simeq r - \frac{d}{2}\cos\theta \\ r_2 &\simeq r + \frac{d}{2}\cos\theta \end{aligned}\right\} \text{ for phase variations}$$

$$r_1 \simeq r_2 \simeq r \qquad \text{for amplitude variations}$$

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos\theta [e^{+j(kd\cos\theta+\beta)/2} + e^{-j(kd\cos\theta+\beta)/2}]$$

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos\theta \left\{2\cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]\right\}$$

- The above equation indicate that the total field of the array is equal to the field of a single element positioned at the origin multiplied by a factor which is widely referred to as the array factor.
- Thus for the two-element array of constant amplitude, the array factor is given by,

$$\mathrm{AF} = 2\cos[\tfrac{1}{2}(kd\cos\theta + \beta)]$$

- The array factor is a function of the geometry of the array and the excitation phase. By varying the separation d and/or the phase β between the elements, the characteristics of the array factor and of the total field of the array can be controlled.

- It has been illustrated that the far-zone field of a uniform two-element array of identical elements is equal to the product of the field of a single element, at a selected reference point (usually the origin), and the array factor of that array.

$$\mathbf{E}(\text{total}) = [\mathbf{E}(\text{single element at reference point})] \times [\text{array factor}]$$

- This is referred to as **pattern multiplication for arrays of identical elements.**
- Each array has its own array factor. The array factor, in general, is a function of the number of elements, their geometrical arrangement, their relative magnitudes, their relative phases, and their spacing.

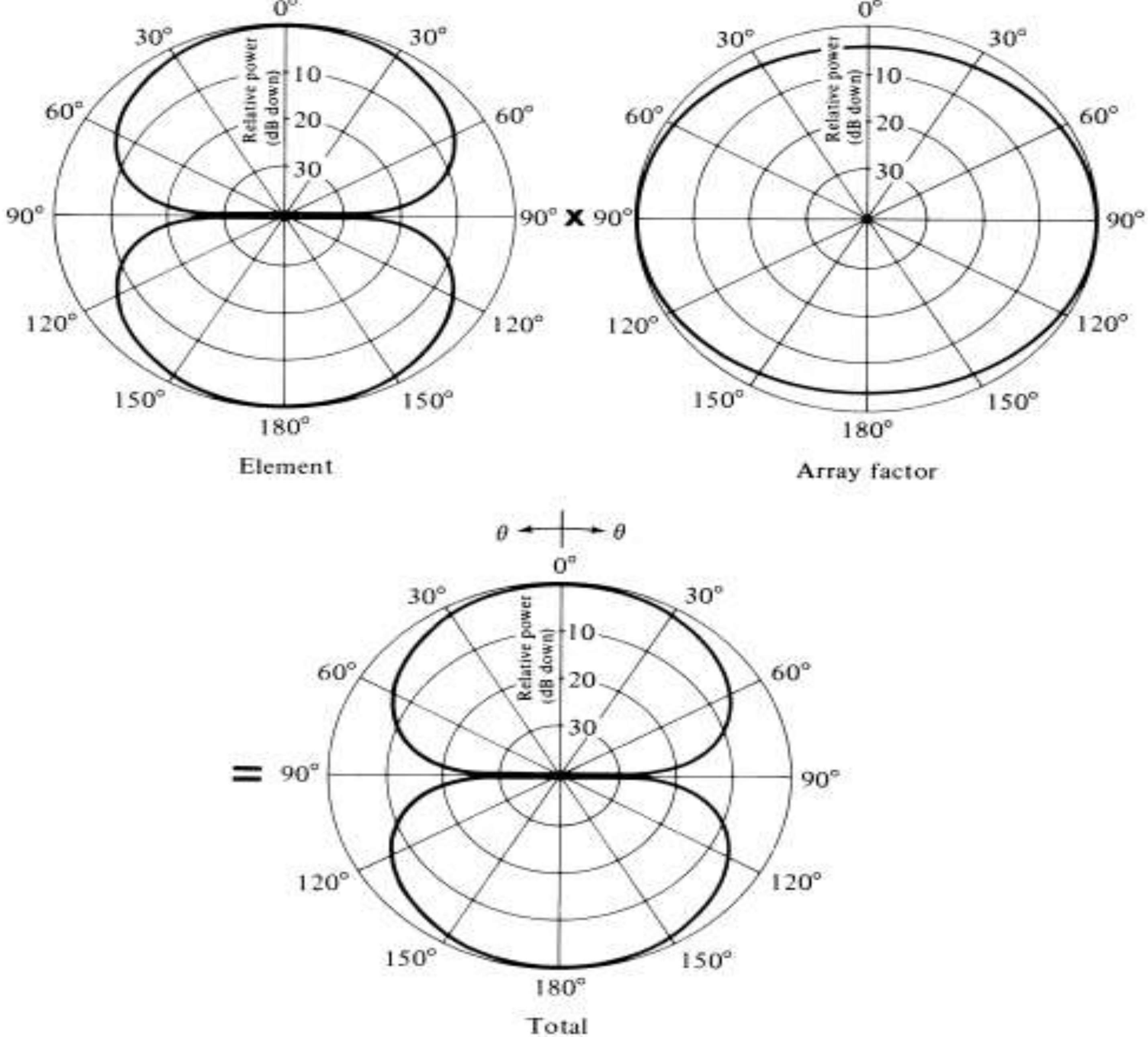


Figure 6.3 Element, array factor, and total field patterns of a two-element array of infinitesimal horizontal dipoles with identical phase excitation ($\beta = 0^\circ$, $d = \lambda/4$).

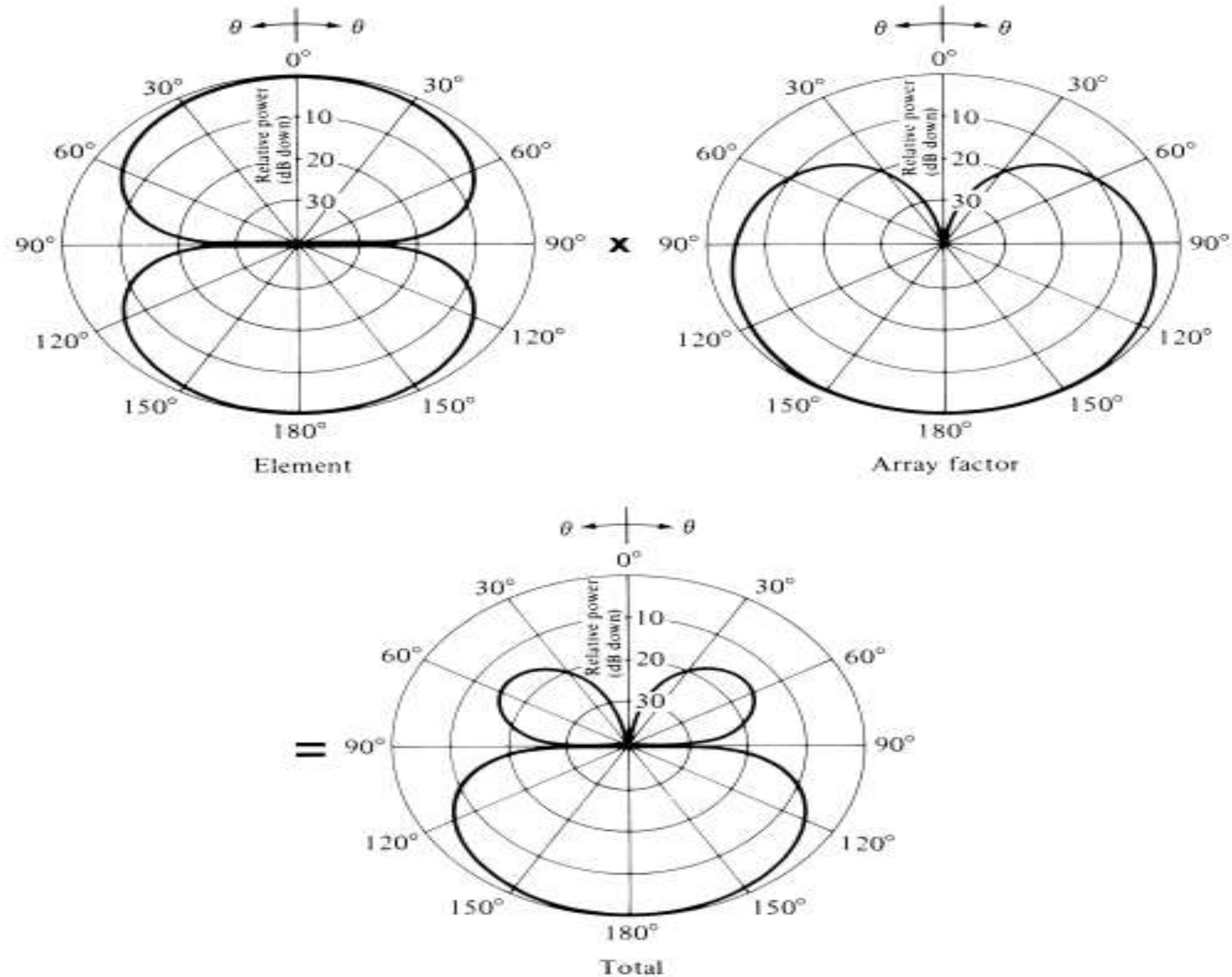


Figure 6.4 Pattern multiplication of element, array factor, and total array patterns of a two-element array of infinitesimal horizontal dipoles with (a) $\beta = +90^\circ$, $d = \lambda/4$.

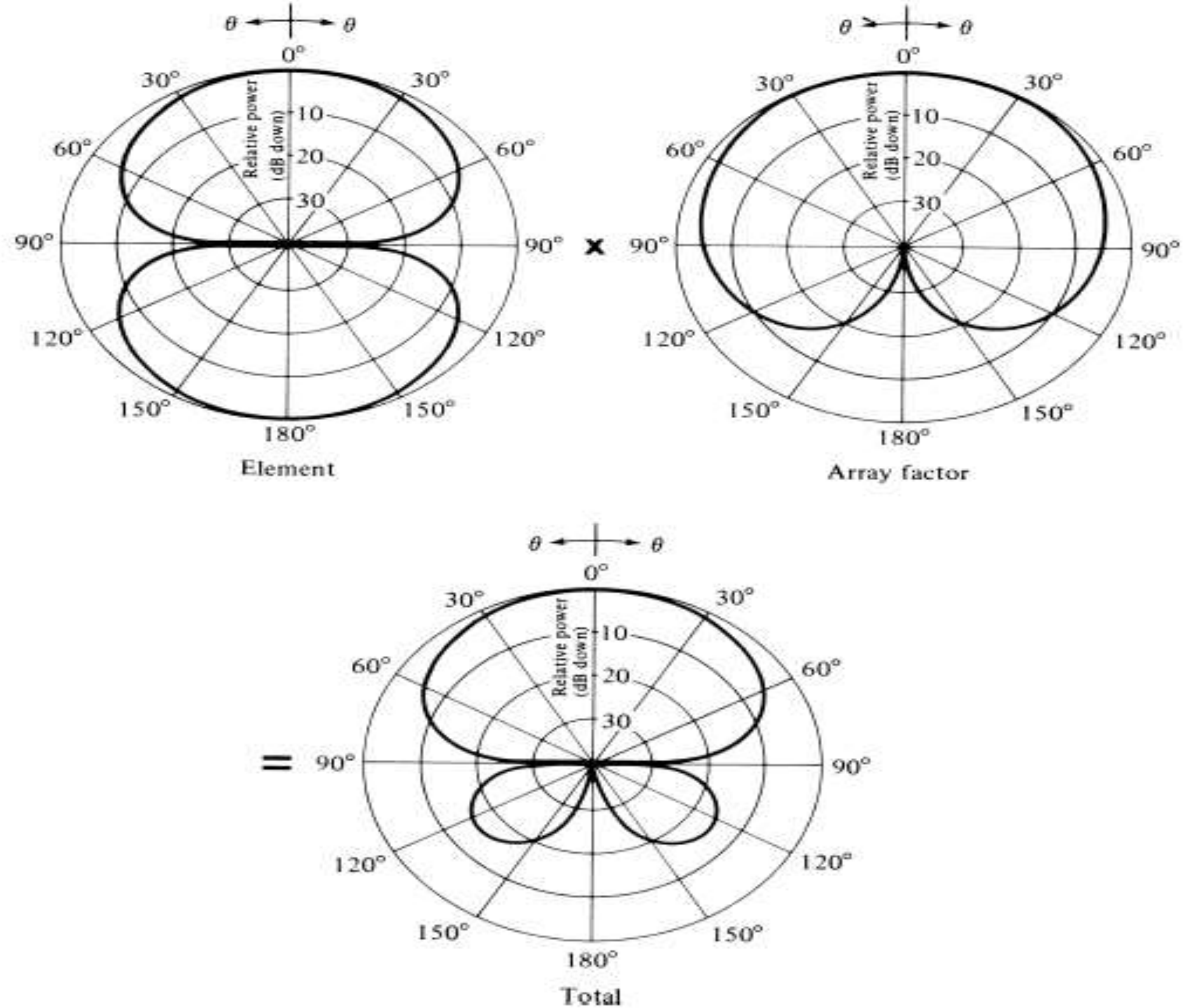


Figure 6.4 (b) $\beta = -90^\circ$, $d = \lambda/4$ (*continued*).

N-Element Linear Array: Uniform Amplitude and Spacing

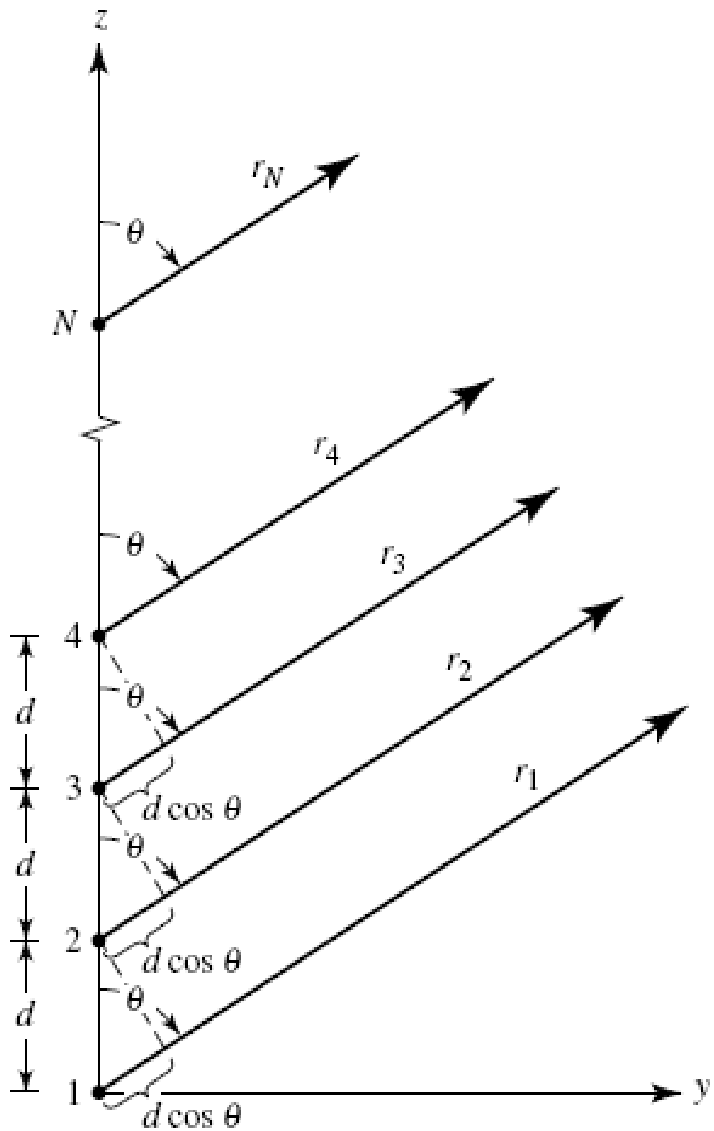
- Let us assume that all the elements have identical amplitudes but each succeeding element has a β progressive phase lead current excitation relative to the preceding one.
- The array factor is given by,

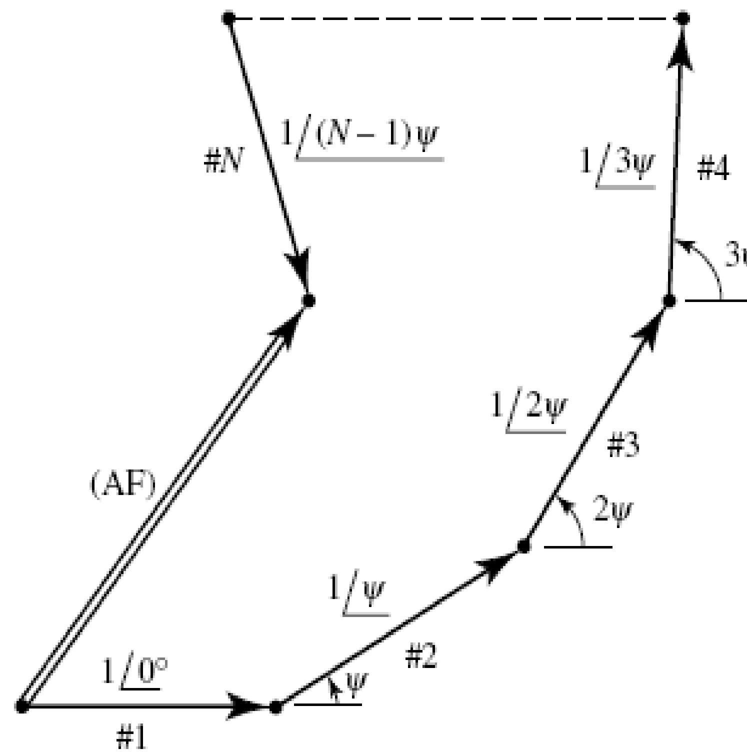
$$\mathrm{AF} = 1 + e^{+j(kd\cos\theta+\beta)} + e^{+j2(kd\cos\theta+\beta)} + \cdots + e^{j(N-1)(kd\cos\theta+\beta)}$$

$$\mathrm{AF} = \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta+\beta)}$$

$$\mathrm{AF} = \sum_{n=1}^{N} e^{j(n-1)\psi}$$

where $\psi = kd\cos\theta + \beta$


z
r_N
θ
N
r_4
θ
r_3
r_2
r_1
4
3
2
1
d
d
d
θ
θ
θ
d cos θ
d cos θ
d cos θ
y


#N
1/(N − 1)ψ
1/3ψ
#4
3ψ
(AF)
1/2ψ
#3
2ψ
1/ψ
#2
1/0°
ψ
#1

- Multiplying both sides by $e^{j\varphi}$

$$(\text{AF})e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(N-1)\psi} + e^{jN\psi}$$

- Subtract both the equations,

$$\text{AF}(e^{j\psi} - 1) = (-1 + e^{jN\psi})$$

$$\text{AF} = \left[\frac{e^{jN\psi} - 1}{e^{j\psi} - 1}\right] = e^{j[(N-1)/2]\psi}\left[\frac{e^{j(N/2)\psi} - e^{-j(N/2)\psi}}{e^{j(1/2)\psi} - e^{-j(1/2)\psi}}\right]$$

$$= e^{j[(N-1)/2]\psi}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

- If the reference point is the physical center of the array, the array factor is reduces to

$$\mathrm{AF} = \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)} \right]$$

- For small values of ψ,

$$\mathrm{AF} \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{\psi}{2}} \right]$$

- The maximum value of AF is equal to N. To normalize the array factors so that the maximum value of each is equal to unity

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

- To find the nulls of the array, the equ. is set equal to zero

$$\sin\left(\frac{N}{2}\psi\right) = 0 \Rightarrow \frac{N}{2}\psi|_{\theta=\theta_n} = \pm n\pi \Rightarrow \theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

$$n = 1, 2, 3, \ldots$$

$$n \neq N, 2N, 3N, \ldots$$

- The maximum values

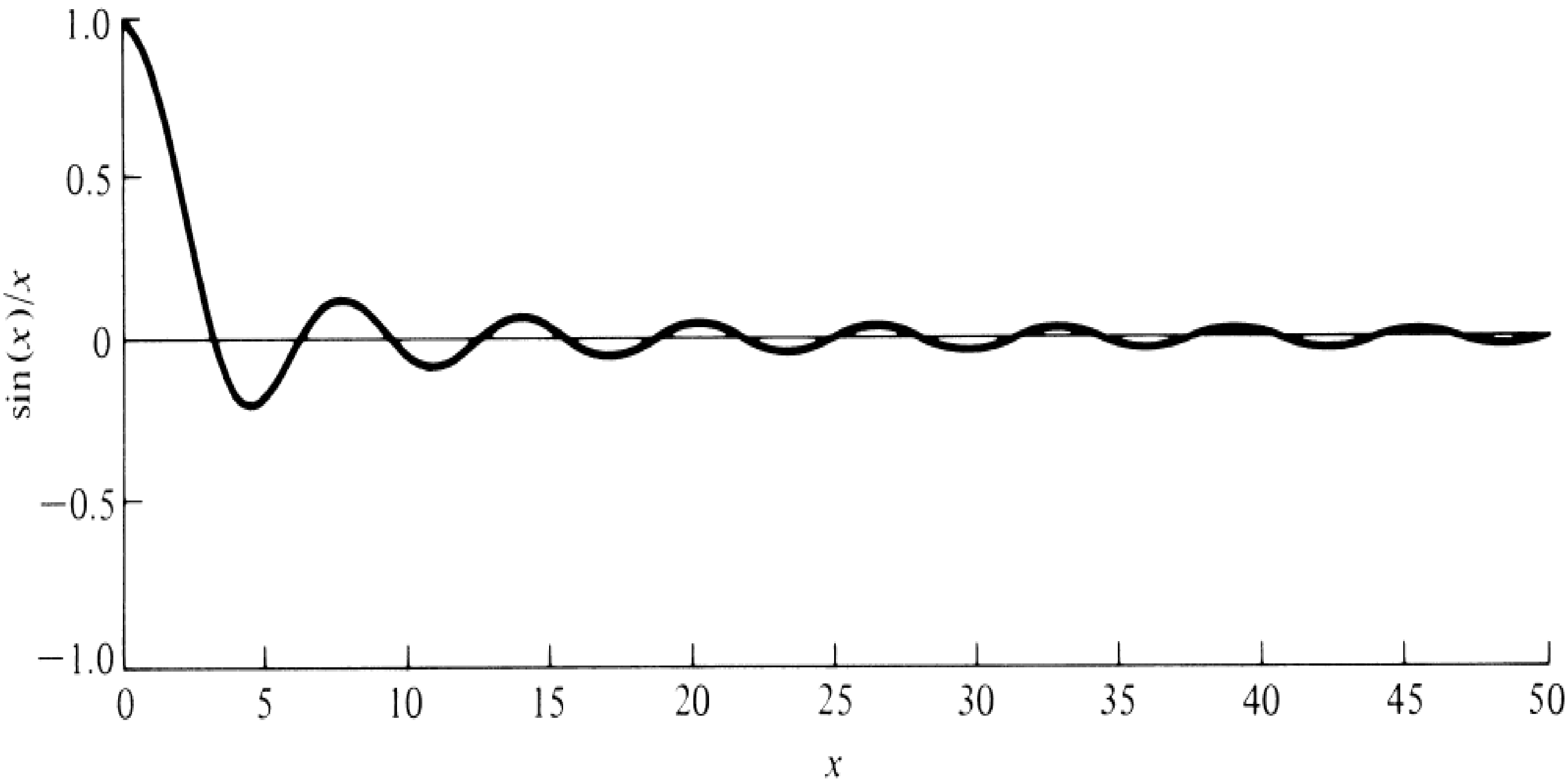
$$\frac{\psi}{2} = \frac{1}{2}(kd\cos\theta + \beta)|_{\theta=\theta_m} = \pm m\pi \Rightarrow \theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right]$$

$$m = 0, 1, 2, \ldots$$

- The 3-dB point for the array factor can occurs when

$$\frac{N}{2}\psi = \frac{N}{2}(kd\cos\theta + \beta)|_{\theta=\theta_h} = \pm 1.391$$

$$\Rightarrow \theta_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$$



$$\theta_h = \frac{\pi}{2} - \sin^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$$

- For large values of d(d >> λ), it reduces to

$$\theta_h \simeq \left[\frac{\pi}{2} - \frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$$

- The half-power beamwidth can be found once the angles of the first maximum (θm) and the half-power point (θh) are determined.

$$\Theta_h = 2|\theta_m - \theta_h|$$

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Types of Arrays

- The two sources in the case 1(in phase) produce a pattern with maximum field normal to the line joining the sources, the two sources for this case may be described as a simple "**broadside**" type of array.
- The two sources in the case 2(opposite phase) produce a pattern with maximum field in the same direction to the line joining the sources, the two sources for this case may be described as a simple "**end-fire**" type of array.

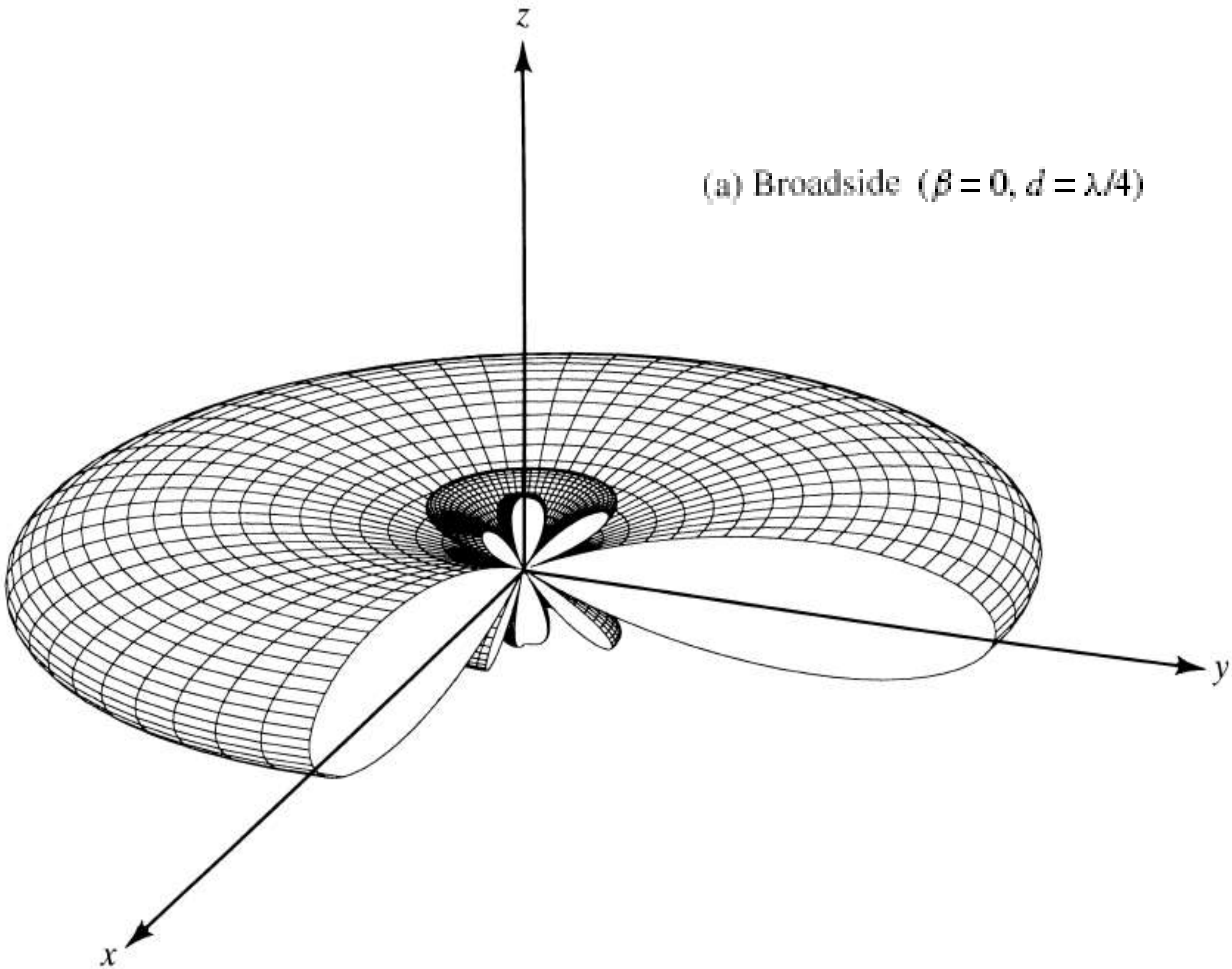
- **Broadside Array:** In many applications it is desirable to have the maximum radiation of an array directed normal to the axis of the array.
- To optimize the design, the maxima of the single element and of the array factor should both be directed toward $\vartheta = 90^\circ$.
- The requirements of the single elements can be accomplished by the appropriate choice of the radiators, and those of the array factor by the proper separation and excitation of the individual radiators.
- The first maximum of the array factor occurs when

$$\psi = kd\cos\theta + \beta = 0$$

- Since it is desired to have the first maximum directed toward $\theta = 90^\circ$

$$\psi = kd\cos\theta + \beta|_{\theta=90^\circ} = \beta = 0$$

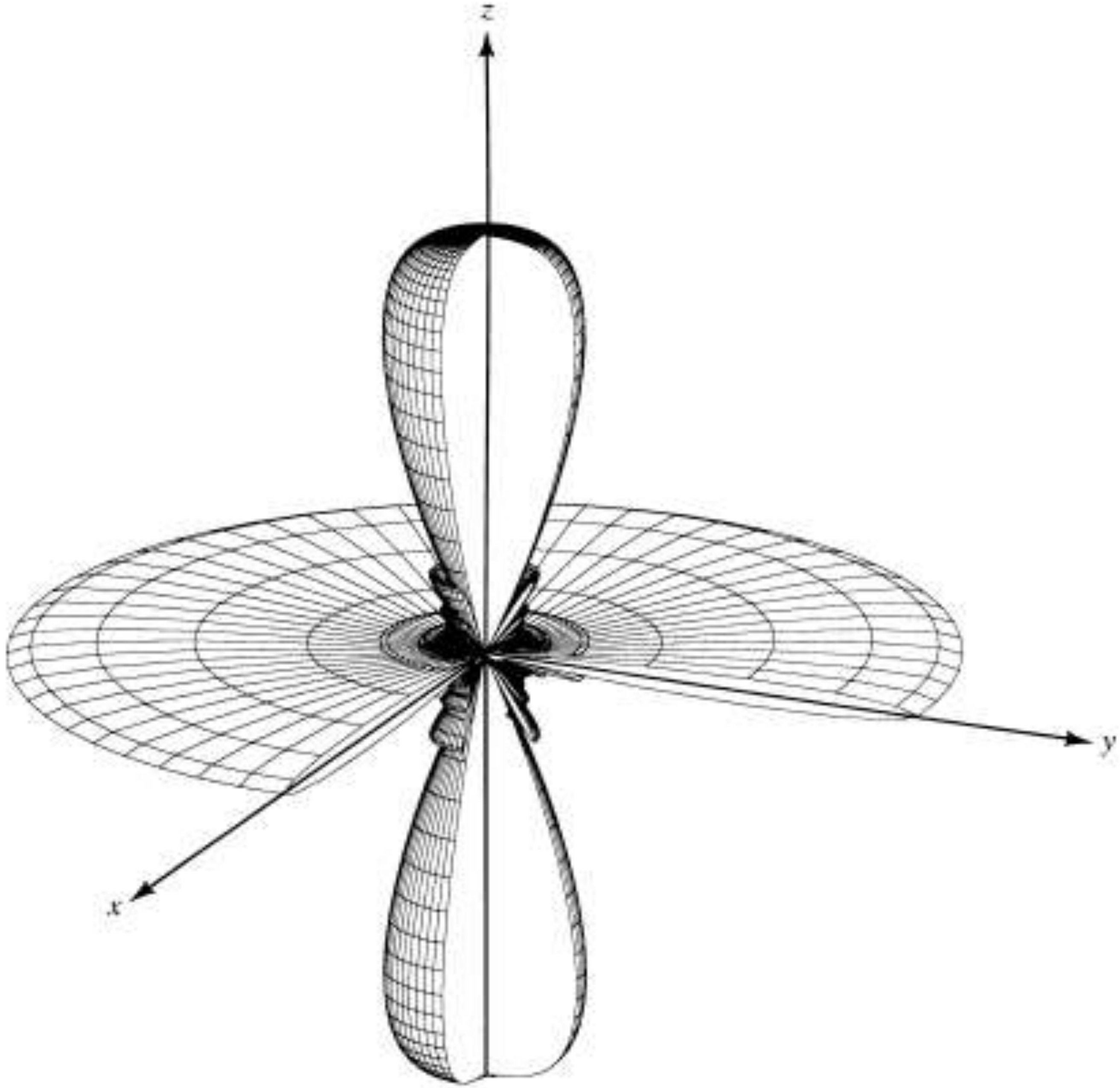
- Thus to have the maximum of the array factor of a uniform linear array directed broadside to the axis of the array, it is necessary that all the elements have the same phase excitation.


z
(a) Broadside ($\beta = 0$, $d = \lambda/4$)
y
x

- To ensure that there are no principal maxima in other directions, which are referred to as grating lobes, the separation between the elements should not be equal to multiples of a wavelength when β = 0.
- If d = nλ, n = 1, 2, 3, . . . and β = 0, then

$$\psi = kd\cos\theta + \beta\Big|_{\substack{d=n\lambda \\ \beta=0 \\ n=1,2,3,\ldots}} = 2\pi n\cos\theta\Big|_{\theta=0^\circ,180^\circ} = \pm 2n\pi$$

- Thus for a uniform array with β = 0 an d = nλ, in addition to having the maxima of the array factor directed broadside ($\theta_0 = 90^\circ$) to the axis of the array, there are additional maxima directed along the axis ($\theta_0 = 0^\circ$, 180°) of the array.
- **To avoid any grating lobe, the largest spacing between the elements should be less than one wavelength ($d_{max} < \lambda$).**



(b) Broadside/end-fire ($\beta = 0$, $d = \lambda$)

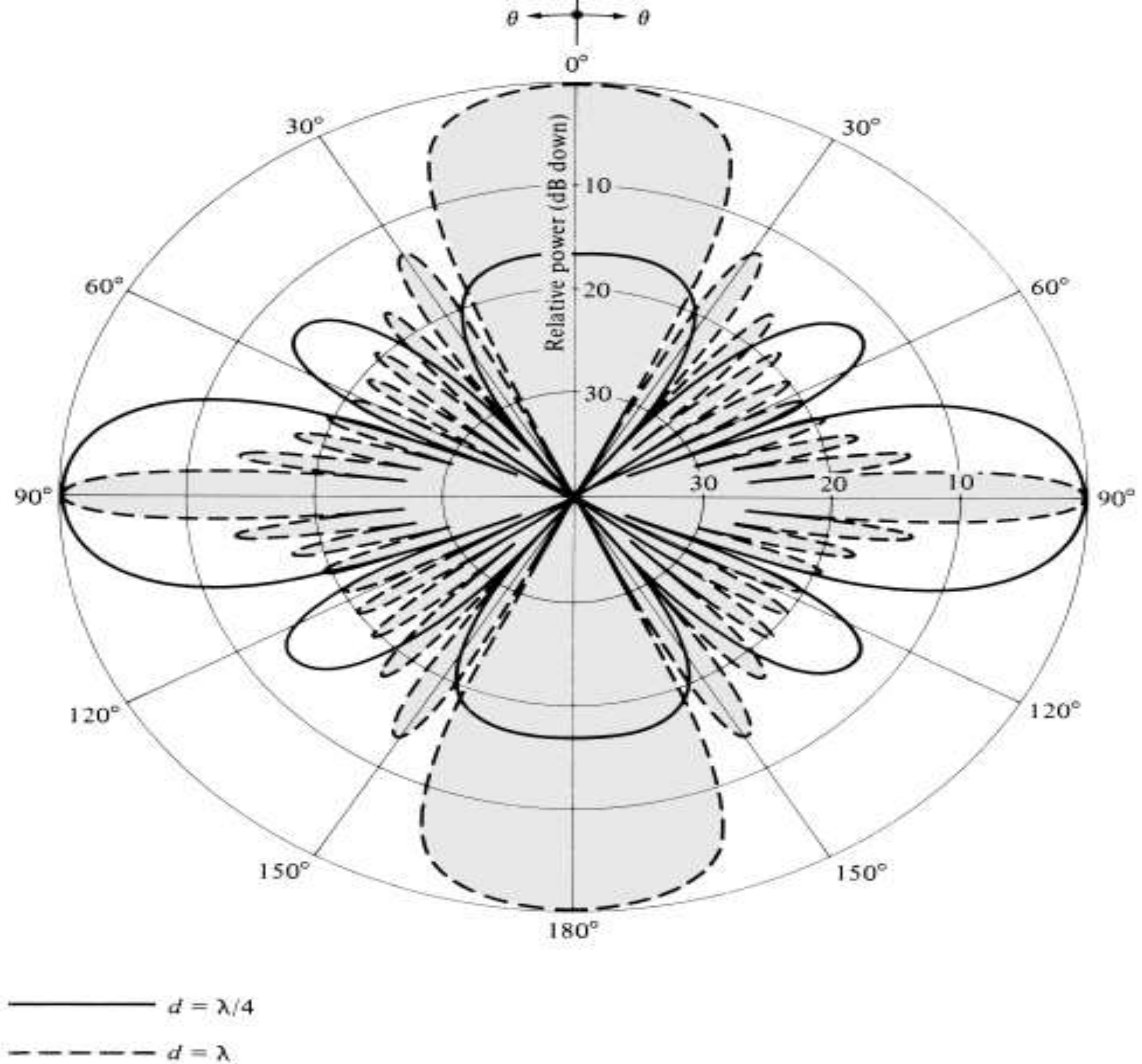


Figure 6.7 Array factor patterns of a 10-element uniform amplitude broadside array ($N = 10, \beta = 0$).

TABLE 6.1 Nulls, Maxima, Half-Power Points for Uniform Amplitude Broadside Arrays

NULLS	$\theta_n = \cos^{-1}\left(\pm\frac{n}{N}\frac{\lambda}{d}\right)$ $n = 1, 2, 3, \ldots$ $n \neq N, 2N, 3N, \ldots$
MAXIMA	$\theta_m = \cos^{-1}\left(\pm\frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \ldots$
HALF-POWER POINTS	$\theta_h \simeq \cos^{-1}\left(\pm\frac{1.391\lambda}{\pi N d}\right)$ $\pi d/\lambda \ll 1$

- **End-Fire Array:** Instead of having the maximum radiation broadside to the axis of the array, it may be desirable to direct it along the axis of the array (end-fire).
- To direct the first maximum toward $\theta_0 = 0^\circ$,

$$\psi = kd\cos\theta + \beta|_{\theta=0^\circ} = kd + \beta = 0 \Rightarrow \beta = -kd$$

- If the first maximum is desired toward $\theta_0 = 180^\circ$, then

$$\psi = kd\cos\theta + \beta|_{\theta=180^\circ} = -kd + \beta = 0 \Rightarrow \beta = kd$$

- Thus end-fire radiation is accomplished when $\beta = -kd$ (for $\theta_0 = 0^\circ$) or $\beta = kd$ (for $\theta_0 = 180^\circ$).

- If the element separation is $d = \lambda/2$, end-fire radiation exists simultaneously in both directions ($\theta_0 = 0^\circ$ and $\theta_0 = 180^\circ$).
- If the element spacing is a multiple of a wavelength ($d = n\lambda$, $n = 1, 2, 3, \ldots$), then in addition to having end-fire radiation in both directions, there also exist maxima in the broadside directions. Thus for $d = n\lambda$, $n = 1, 2, 3, \ldots$ there exist four maxima; two in the broadside directions and two along the axis of the array.
- To have only one end-fire maximum and to avoid any grating lobes, the maximum spacing between the elements should be less than **$d_{max} < \lambda/2$**.

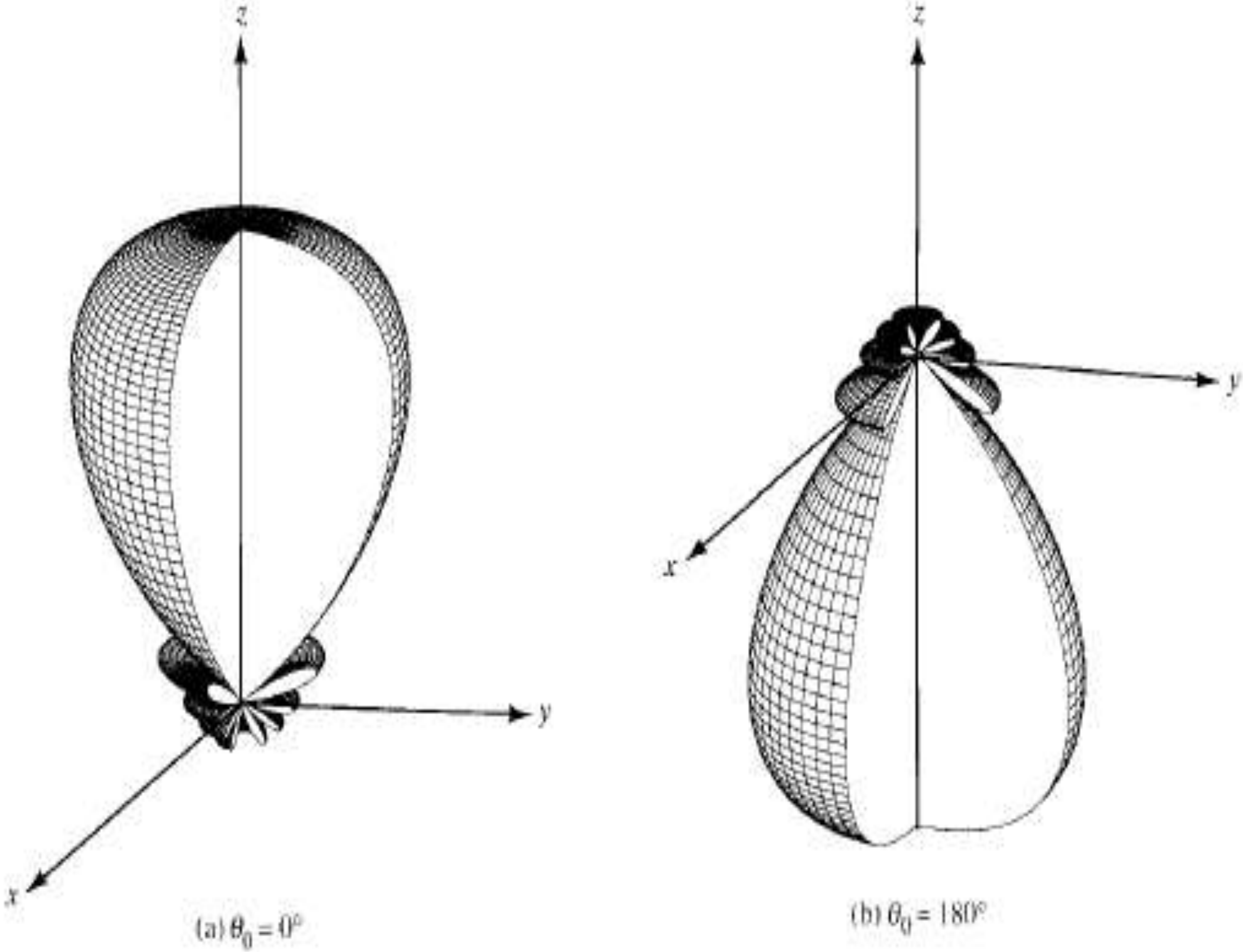


Figure 6.8 Three-dimensional amplitude patterns for end-fire arrays toward $\theta_0 = 0°$ and $180°$ ($N = 10$, $d = \lambda/4$).

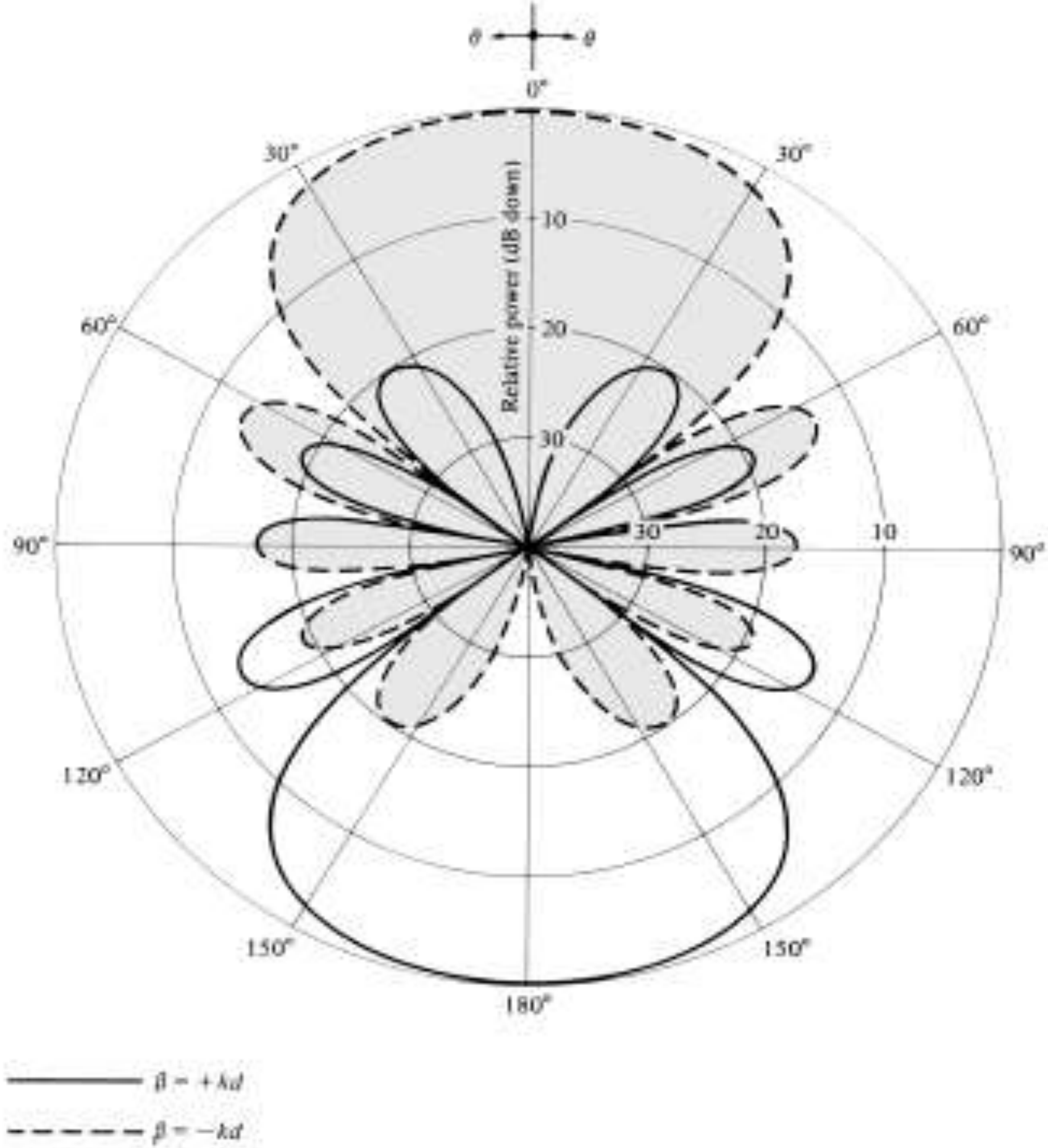


Figure 6.9 Array factor patterns of a 10-element uniform amplitude end-fire array ($N = 10$, $d = \lambda/4$).

TABLE 6.3 Nulls, Maxima, Half-Power Points, for Uniform Amplitude Ordinary End-Fire Arrays

NULLS	$\theta_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right)$ $n = 1, 2, 3, \ldots$ $n \neq N, 2N, 3N, \ldots$
MAXIMA	$\theta_m = \cos^{-1}\left(1 - \frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \ldots$
HALF-POWER POINTS	$\theta_h \simeq \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right)$ $\pi d/\lambda \ll 1$

- **Phased Array:** It is then logical to assume that the maximum radiation can be oriented in any direction to form a scanning array.
- Let us assume that the maximum radiation of the array is required to be oriented at an angle θ0(0◦ ≤ θ0 ≤ 180◦).
- To accomplish this, the phase excitation β between the elements must be adjusted so that

$$\psi = kd\cos\theta + \beta|_{\theta=\theta_0} = kd\cos\theta_0 + \beta = 0 \Rightarrow \beta = -kd\cos\theta_0$$

- Thus by controlling the progressive phase difference between the elements, the maximum radiation can be squinted in any desired direction to form a scanning array.

Array with Uniform Spacing but Non-Uniform Amplitude

- In this section there are two types of arrays:

1. Binomial Array
2. Dolph- Tchebyscheff broadside array

- Of the three distributions (uniform, binomial, and Dolph- Tchebyscheff), a uniform amplitude array yields the smallest half-power beamwidth. It is followed, in order, by the Dolph-Tchebyscheff and binomial arrays. In contrast, binomial arrays usually possess the smallest side lobes followed, in order, by the Dolph-Tchebyscheff and uniform arrays.
- As a matter of fact, binomial arrays with element spacing equal or less than $\lambda/2$ have no side lobes.

- **Array Factor:** An array of an even number of isotropic elements 2M (where M is an integer) is positioned symmetrically along the z-axis, as shown in Figure. The separation between the elements is d, and M elements are placed on each side of the origin.
- The array factor for a non-uniform amplitude broadside array with even number of element can be written as,

$$
\begin{aligned}
(\mathrm{AF})_{2M} = a_1 e^{+j(1/2)kd\cos\theta} &+ a_2 e^{+j(3/2)kd\cos\theta} + \ldots \\
&+ a_M e^{+j[(2M-1)/2]kd\cos\theta} \\
&+ a_1 e^{-j(1/2)kd\cos\theta} + a_2 e^{-j(3/2)kd\cos\theta} + \ldots \\
&+ a_M e^{-j[(2M-1)/2]kd\cos\theta}
\end{aligned}
$$

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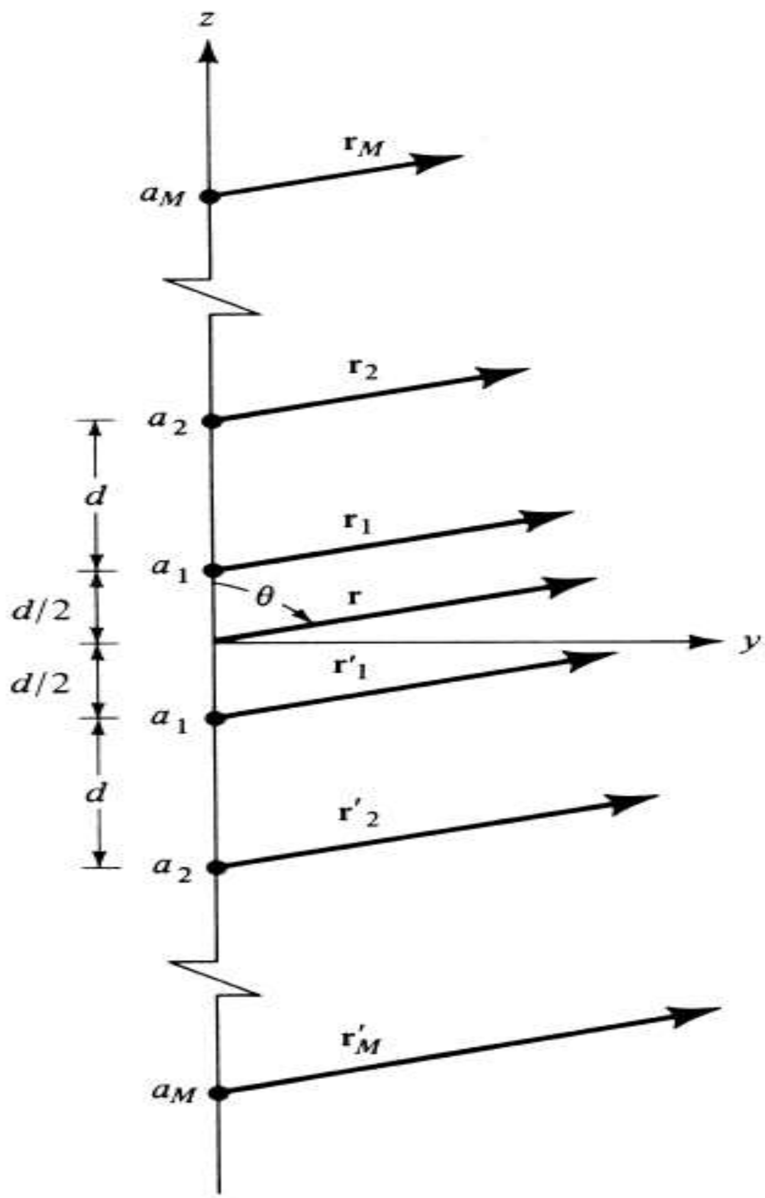
z
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a_M
r_2
a_2
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r
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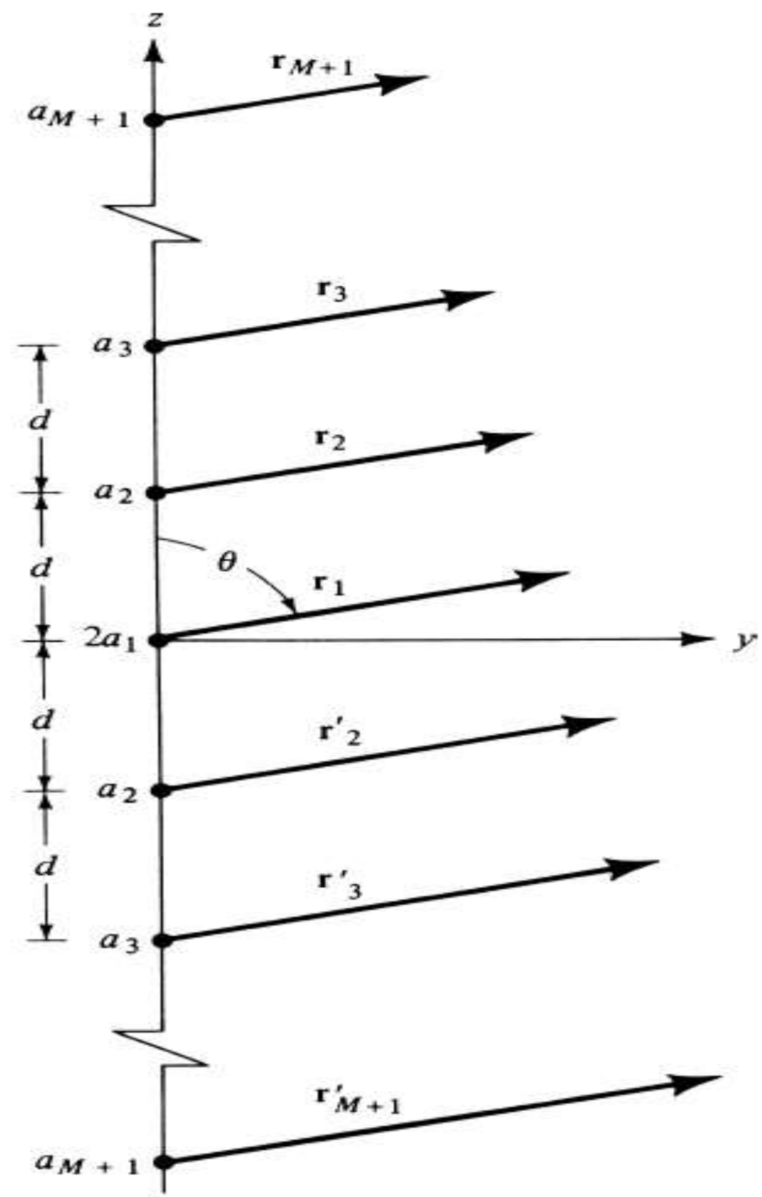
d/2
r'_1
a_1
d
r'_2
a_2
r'_M
a_M


z
r_{M+1}
a_{M+1}
r_3
a_3
d
r_2
a_2
d
θ
r_1
$2a_1$
y
d
r'_2
a_2
d
r'_3
a_3
r'_{M+1}
a_{M+1}

$$(\mathrm{AF})_{2M} = 2\sum_{n=1}^{M} a_n \cos\left[\frac{(2n-1)}{2}kd\cos\theta\right]$$

- Which is written in normalized form as, $$(\mathrm{AF})_{2M} = \sum_{n=1}^{M} a_n \cos\left[\frac{(2n-1)}{2}kd\cos\theta\right]$$
- The array factor for a non-uniform amplitude broadside array with odd number of element can be written as,

$$\begin{aligned}(\mathrm{AF})_{2M+1} = 2a_1 &+ a_2 e^{+jkd\cos\theta} + a_3 e^{j2kd\cos\theta} + \cdots + a_{M+1} e^{jMkd\cos\theta} \\ &+ a_2 e^{-jkd\cos\theta} + a_3 e^{-j2kd\cos\theta} + \cdots + a_{M+1} e^{-jMkd\cos\theta}\end{aligned}$$

$$(\mathrm{AF})_{2M+1} = 2\sum_{n=1}^{M+1} a_n \cos[(n-1)kd\cos\theta]$$

- Which is written in normalized form as, $(\mathrm{AF})_{2M+1} = \sum_{n=1}^{M+1} a_n \cos[(n-1)kd\cos\theta]$

- In general we can write,

$$(\mathrm{AF})_{2M}(\text{even}) = \sum_{n=1}^{M} a_n \cos[(2n-1)u]$$

$$(\mathrm{AF})_{2M+1}(\text{odd}) = \sum_{n=1}^{M+1} a_n \cos[2(n-1)u]$$

where

$$u = \frac{\pi d}{\lambda}\cos\theta$$

- **Binomial Array :** The array factor for the binomial array is represented by the above equations.
- To determine the excitation coefficients of a binomial array, the binomial expansion is used, which is given by,

$$(1+x)^{m-1} = 1 + (m-1)x + \frac{(m-1)(m-2)}{2!}x^2 + \frac{(m-1)(m-2)(m-3)}{3!}x^3 + \ldots$$

- For two elements (2M=2), $a_1 = 1$.
- For three elements (2M+1= 3), $2a_1 = 2$ $a_1 = 1$, $a_2 = 1$.

- For three elements (2M= 4), a1= 3, a2= 1.
- The coefficients for other arrays can be determined in a similar manner.
- The HPBW of the binomial array is given by, HPBW $= \frac{1.06}{\sqrt{N-1}}$
- The maximum directivity is given by, $D_0 = 1.77\sqrt{N}$

- **Dolph-Tschebyscheff Array:** It is primarily a compromise between uniform and binomial arrays.
- Its excitation coefficients are related to Tschebyscheff polynomials.
- The array factor of an array of even or odd number of elements with symmetric amplitude excitation is nothing more than a summation of M or M + 1 cosine terms.
- Each cosine term, whose argument is an integer times a fundamental frequency, can be rewritten as a series of cosine functions with the fundamental frequency as the argument.

$$
\begin{aligned}
m &= 0 \quad \cos(mu) = 1 \\
m &= 1 \quad \cos(mu) = \cos u \\
m &= 2 \quad \cos(mu) = \cos(2u) = 2\cos^2 u - 1 \\
m &= 3 \quad \cos(mu) = \cos(3u) = 4\cos^3 u - 3\cos u \\
m &= 4 \quad \cos(mu) = \cos(4u) = 8\cos^4 u - 8\cos^2 u + 1
\end{aligned}
$$

$$z = \cos u$$

$$
\begin{aligned}
m &= 0 \quad \cos(mu) = 1 = T_0(z) \\
m &= 1 \quad \cos(mu) = z = T_1(z) \\
m &= 2 \quad \cos(mu) = 2z^2 - 1 = T_2(z) \\
m &= 3 \quad \cos(mu) = 4z^3 - 3z = T_3(z) \\
m &= 4 \quad \cos(mu) = 8z^4 - 8z^2 + 1 = T_4(z)
\end{aligned}
$$

and each is related to a Tschebyscheff (Chebyshev) polynomial Tm(z).

These relation between the cosine functions and the Tschebyscheff polynomials are valid only in the $-1 \le z \le +1$ range.

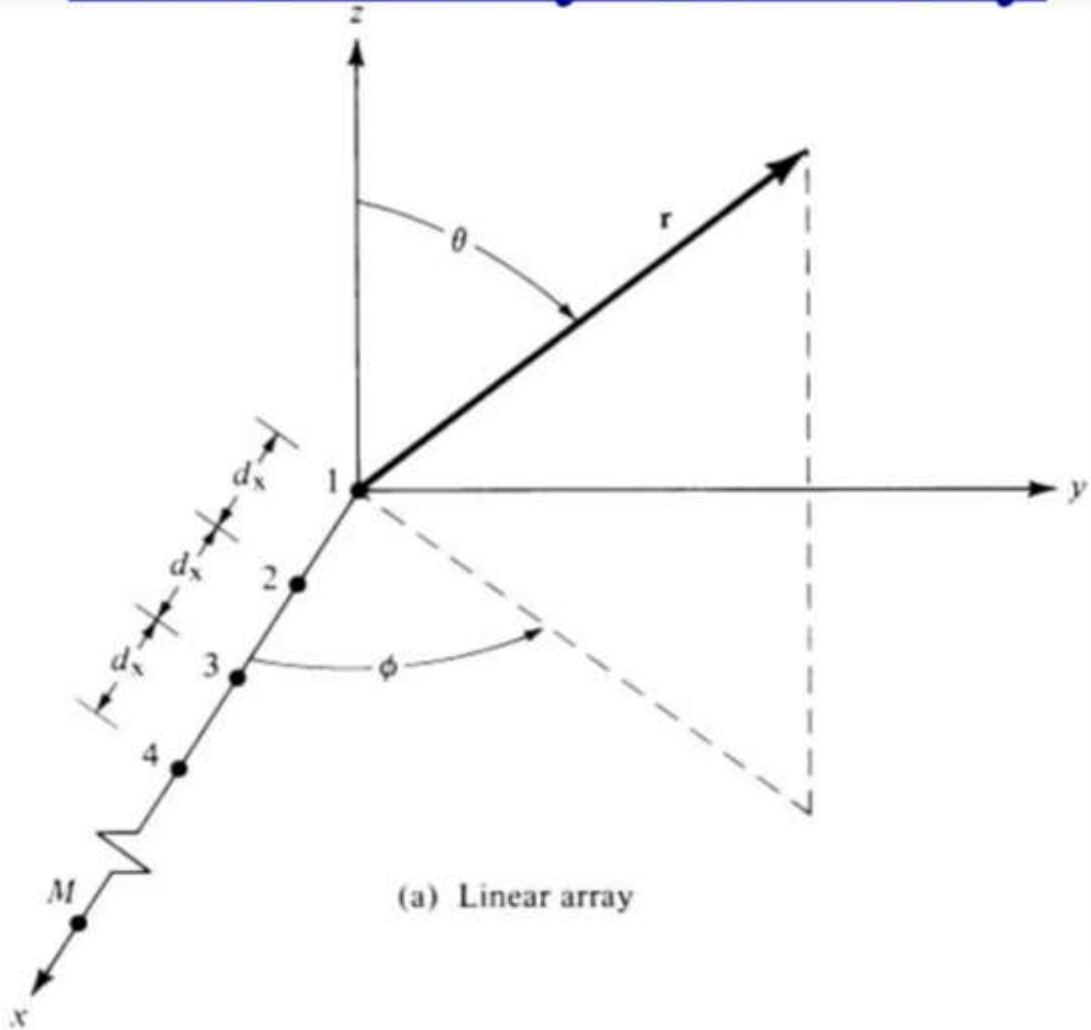
Planar Arrays

1. Linear arrays (one-dimensional) can scan the beam only in one plane.
2. To scan the beam in any direction, two-dimensional array geometries are needed, such as elements placed along a circle and planar, cubical, cylindrical, spherical, etc., surfaces.

PLANNAR ARRAY

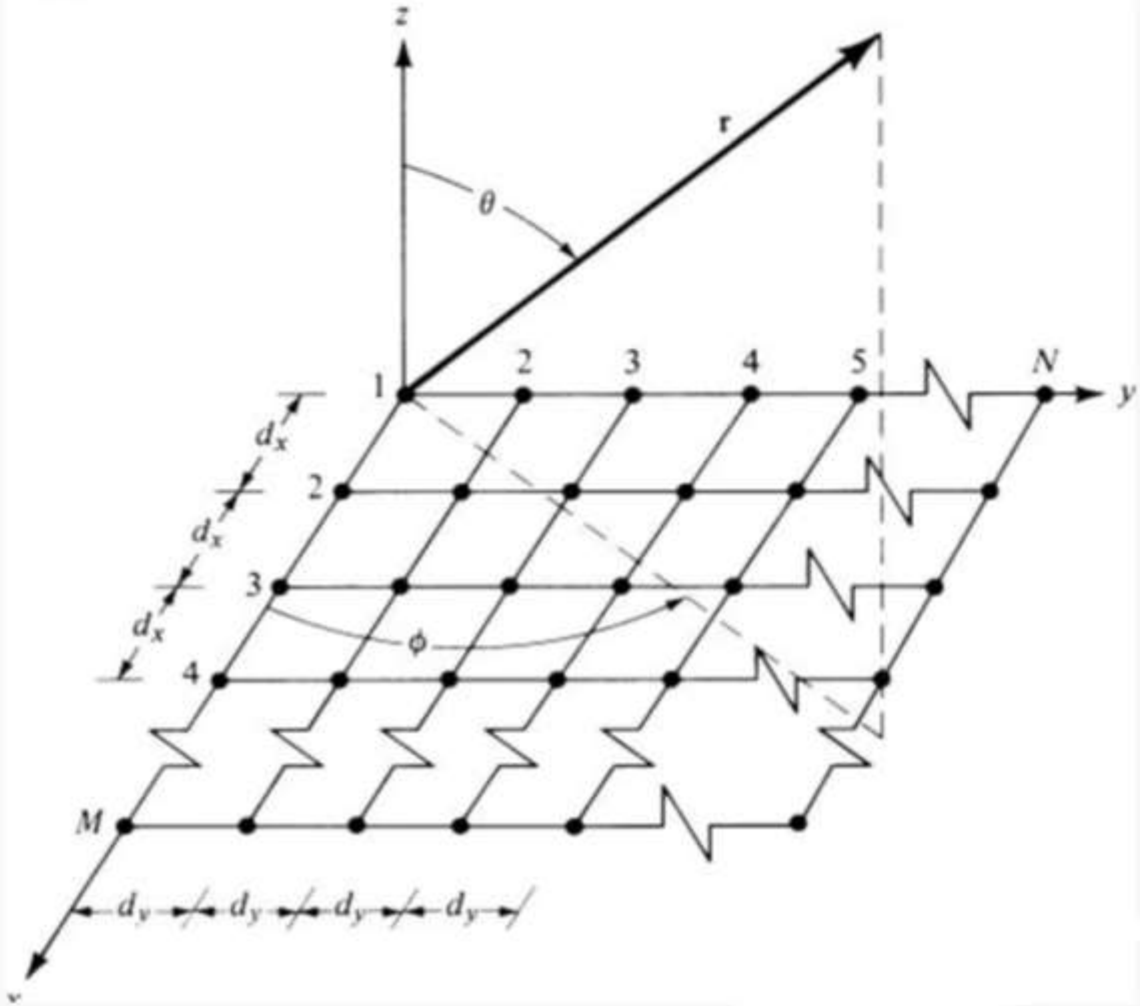
- The individual elements can be positioned along the rectangular grid.
- It provides additional variables which can be used to control and shape the pattern of array
- More versatile and can provide more symmetrical patterns with low side lobes.
- They can be used to scan the main beam of antenna toward any point in space.
- Applications: Tracking radar, search radar, remote sensing etc.

Linear Array Geometry



(a) Linear array

Rectangular Planar Array Geometry



Array Factor: Uniform

$$|AF| = \begin{cases} \left|\sum_{n=1}^{N} e^{j(n-1)\psi}\right| \\ \dfrac{1}{N}\left|\dfrac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)}\right| \end{cases}$$

$$\psi = kd\cos\gamma + \beta$$

Axis	$(\psi = kd\cos\gamma + \beta)$
z:	$\psi = kd\cos\theta + \beta$
x:	$\psi = kd\sin\theta\cos\phi + \beta$
y:	$\psi = kd\sin\theta\sin\phi + \beta$

Nonuniform Linear/Planar Array

Linear :

$$(AF)_x = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)}$$

Planar :

$$(AF)_{xy} = \sum_{n=1}^{N}\left\{\sum_{m=1}^{M}\left[I_{m1} e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)}\right]\right\} \times\left[I_{n1} e^{j(n-1)(kd_y \sin\theta\sin\phi + \beta_y)}\right]$$

Uniform Planar Array

If $I_{mn} = I_{m1} I_{1n} = I_o = \text{Constant}$

$$(AF)_{xy} = I_o \left[\sum_{m=1}^{M} e^{j(m-1)(kd_x \sin\theta \cos\phi + \beta_x)} \right] \times \left[\sum_{n=1}^{N} e^{j(n-1)(kd_y \sin\theta \sin\phi + \beta_y)} \right]$$

Array Factor (Uniform Array)

$$(AF)_n = \left[\frac{1}{M} \frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)} \right] \left[\frac{1}{N} \frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)} \right]$$

$\psi_x = kd_x \sin\theta \cos\phi + \beta_x$

$\psi_y = kd_y \sin\theta \sin\phi + \beta_y$

M = number of elements in x direction

N = number of elements in y direction

Thus the principal maximum ($m=n=0$) and the grating lobes can be located by

$$kd_x(\sin\theta\cos\phi - \sin\theta_o\cos\phi_o) = \pm 2m\pi, \quad m = 0, 1, 2, \ldots$$

$$kd_y(\sin\theta\sin\phi - \sin\theta_o\sin\phi_o) = \pm 2n\pi, \quad n = 0, 1, 2, \ldots$$

P.S. θ_o, ϕ_o *represent main maximum;*
θ, ϕ *represent other maxima* (*grating lobes*)

The principal maximum ($m = n = 0$) and the grating lobes can be located by solving the following two equations simultaneously, or

$$\sin\theta\cos\phi - \sin\theta_o\cos\phi_o = \pm\frac{m\lambda}{d_x}, \quad m = 0, 1, 2, \ldots$$

$$\sin\theta\sin\phi - \sin\theta_o\sin\phi_o = \pm\frac{n\lambda}{d_y}, \quad n = 0, 1, 2, \ldots$$

Maxima

$$\psi_x = kd_x \sin\theta\cos\phi + \beta_x$$
$$= \pm 2m\pi, \quad m = 0,1,2,...$$

$$\psi_y = kd_y \sin\theta\sin\phi + \beta_y$$
$$= \pm 2n\pi, \quad n = 0,1,2,...$$

First main maximum

(m=0, n=0) @ $\theta = \theta_o$, $\phi = \phi_o$

$$\psi_x = kd_x \sin\theta_o \cos\phi_o + \beta_x = 0$$
$$\Rightarrow \beta_x = -kd_x \sin\theta_o \cos\phi_o$$
$$\psi_y = kd_y \sin\theta_o \sin\phi +_o \beta_y = 0$$
$$\Rightarrow \beta_y = -kd_y \sin\theta_o \sin\phi_o$$

When solved simultaneously:

$$\tan\phi_o = \frac{\beta_y d_x}{\beta_x d_y}$$

$$\sin^2\theta_o = \left(\beta_x / kd_x\right)^2 + \left(\beta_y / kd_y\right)^2$$

When solved simultaneously:

$$\phi = \tan^{-1}\left[\frac{\sin\theta_o \sin\phi_o \pm n\lambda / d_y}{\sin\theta_o \cos\phi_o \pm m\lambda / d_x}\right]$$

$$\theta = \sin^{-1}\left[\frac{\sin\theta_o \cos\phi_o \pm m\lambda / d_x}{\cos\phi}\right]$$

$$= \sin^{-1}\left[\frac{\sin\theta_o \sin\phi_o \pm n\lambda / d_y}{\sin\phi}\right]$$

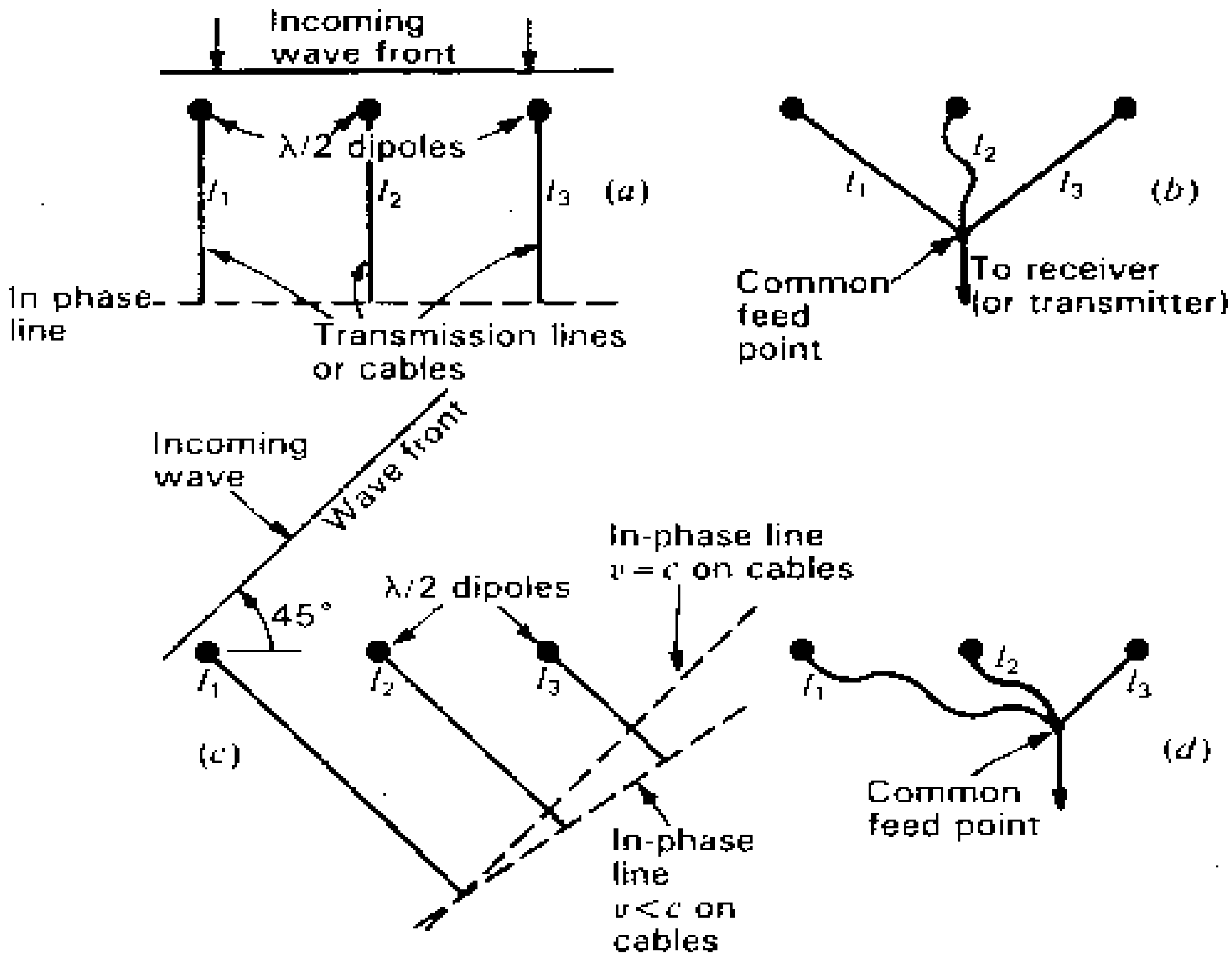
P.S. Both of the forms of (6-94b) must be satisfied simultaneously (*lead to same* θ)

Phased Arrays

- The phased array means as array of many elements with the phase of each element being a variable providing control of the beam direction and pattern shape including side lobes.
- The main phased arrays are, the frequency scanning array, the retroarray and the adaptive array.
- In scanning array, phase change is accomplished by varying the frequency, thus called frequency scanning arrays.
- A retroarray is one which automatically reflects an incoming signal back toward its source.
- Adaptive array have an awareness of their environment and adjust to it in a desired fashion. Such an array may be called smart antenna.

…….Continue

- An objective of a phased array is to accomplish beam steering without the mechanical and inertial problem of rotating the entire array.
- The beam of a rotatable array maintains its shape with change in direction whereas a phased array beam may not.
- Another objective of the phased array is to provide beam control at a fixed frequency or at any number of frequencies within a certain bandwidth in a frequency-independent manner.
- Consider the 3-element λ/2 dipole array.
- Suppose an incoming wave arriving broadside as shown in figure will induce voltages in the transmission lines in the same phase so that if all cables are of the same length ($l_1=l_2=l_3$) the voltages will be in phase at the line.



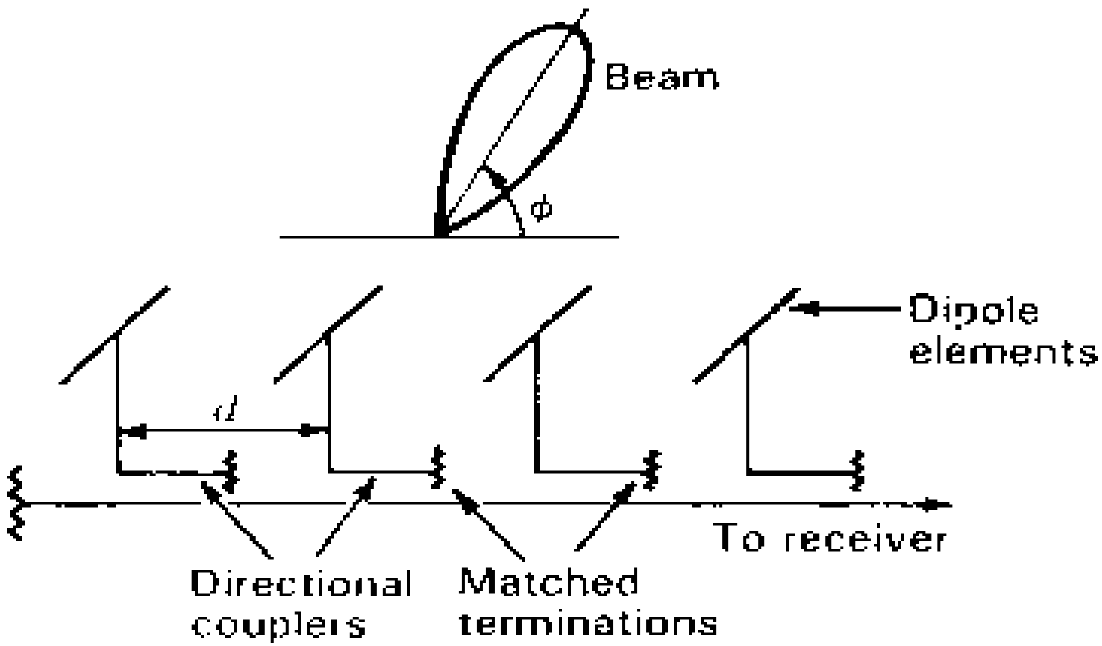
Incoming
wave front
λ/2 dipoles
l_1
l_2
l_3
(a)
In phase
line
Transmission lines
or cables
l_1
l_2
l_3
(b)
Common
feed
point
To receiver
(or transmitter)
Incoming
wave
Wave front
45°
λ/2 dipoles
In-phase line
v = c on cables
l_1
l_2
l_3
(c)
In-phase
line
v<c on
cables
l_1
l_2
l_3
(d)
Common
feed point

…….Continue

- By bringing all 3 cables to a common point, the 3-elements array will operate as a broadside array.
- Now consider a wave arriving at an angle of 45 deg from broadside. If the wave velocity ($v=c$) on the cables, the in-phase line is parallel to the wave front of the incoming wave.
- However, if ($v<c$), the lengths l_2 and l_3 must be increased as suggested in order for all phases to be the same.
- Then, if cables of these length are joined, the 3 element array will have its beam 45 deg from broadside.

Frequency-Scanning Array

- Consider a line-fed array of uniformly spaced elements with a receiver connected at the right end of the line.



…….Continue

- In this design beam sweeping or scanning done by changing the frequency.
- Consider $cos\emptyset = \frac{1}{p} + \frac{m}{(\frac{d}{\lambda_0})}$
- Where ,

Φ = beam angle from array axis

p = phase velocity on transmission line = (v/c)

m = mode number

d = element spacing

λ_0 = free-space wavelength at center frequency of array operation.

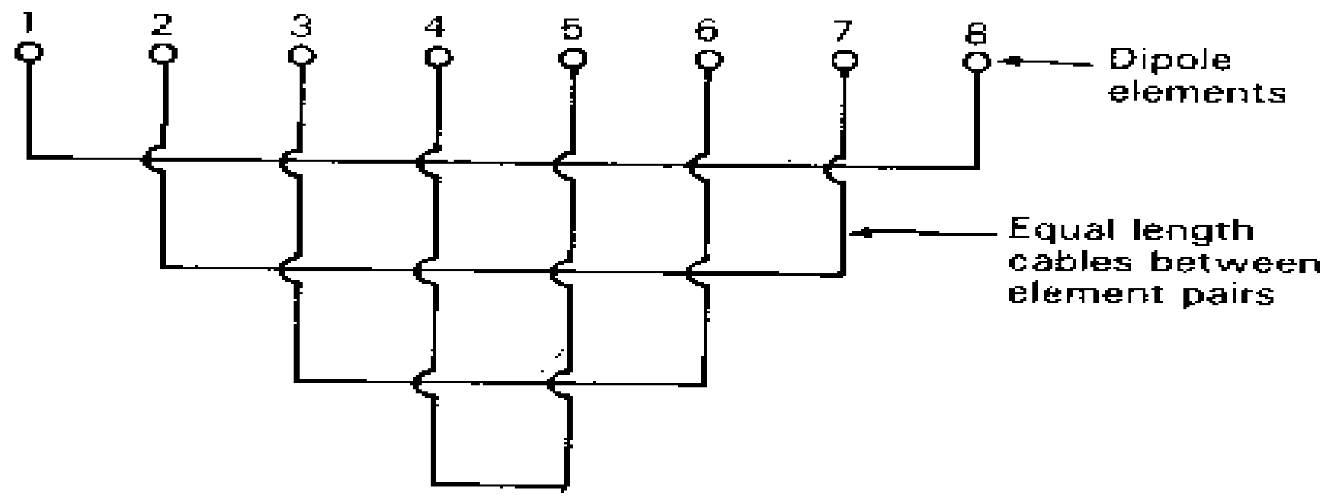
- For p = 1 and m = 0 , Φ = 0 deg, the beam fixed at end-fire.
- Consider now the situation for p = 1, m = -1, d = 1 m and λ_0 = 1 m, so cos Φ = 1-1 = 0, Φ = 90 deg, beam is broadside.

…….Continue

- Suppose, the frequency is increased so that the wavelength is 0.9 λ_0 or 0.9 m, then cos Φ = 1-0.9 = 0.1, Φ = 84.3 deg or 5.7 deg right of broadside.
- Now suppose the frequency shifting to a lower frequency so that the wavelength is 1.1 λ_0 or 1.1 m, then cos Φ = 1-1.1 = -0.1, Φ = 95.7 deg or 5.7 deg left of broadside.
- Thus, a ± 10 % shift in wavelength swings the beam ± 5.7 deg from broadside.
- The larger frequency shift resulting in larger scan angles.
- This frequency-scanning array has no moving parts, no phase shifter and no switches, making it one of the simplest types of phased arrays.

Retroarrays (Van Atta Array)

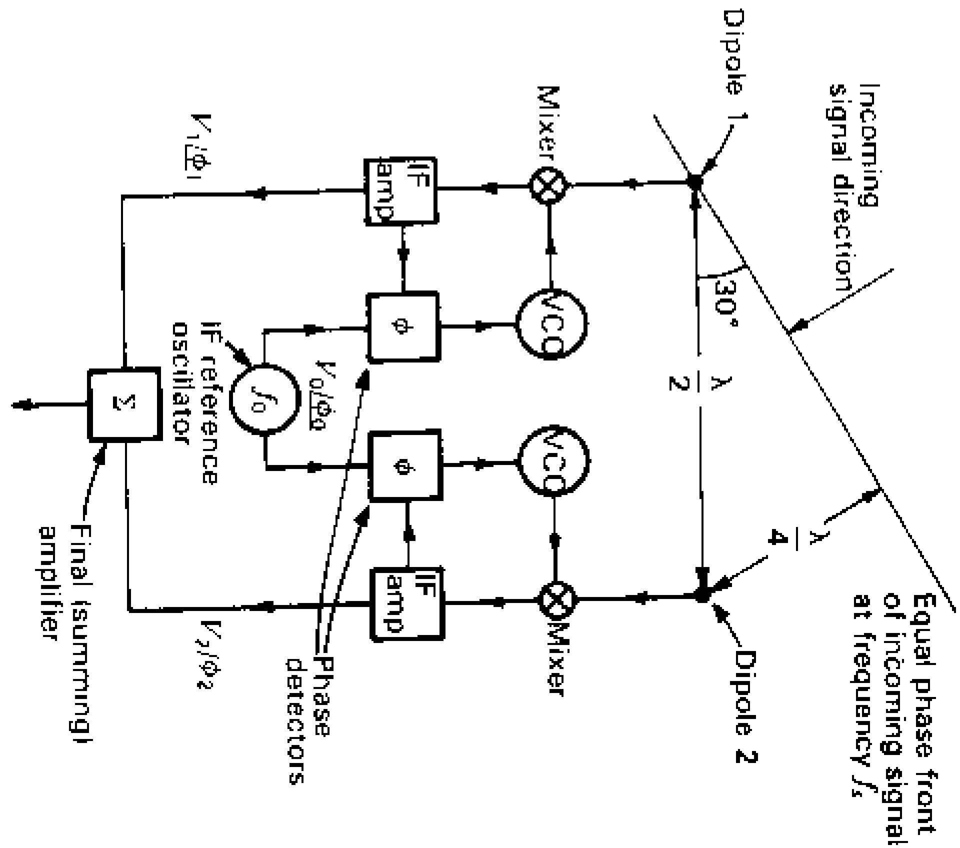
- If wave incident on an array is received and transmitted back in the same direction, the array acts as a retroarray.
- In general, each element of a retroarray reradiates a signal which is the conjugate of the received signal.
- The 8-elements array is shown in the figure, with element pairs connected by identical equal-length cables.



Adaptive Array (Smart Antenna)

- This array processing the signals from individual elements, react intelligently to its environment, steering its beam to ward a desired signal while simultaneously steering a null toward an undesired , interfering signal and thereby maximizing the signal to noise ratio of the desired signal. The term adaptive array is applied to this kind of antenna.
- In addition, by appropriate sampling and digitizing the signals at the terminals of each element and processing them with a computer, a very intelligent or smart antenna can be built.
- An adaptive array of a simple 2-elements is shown in figure.
- With elements operating in phase, the beam is broadside.

……Continue

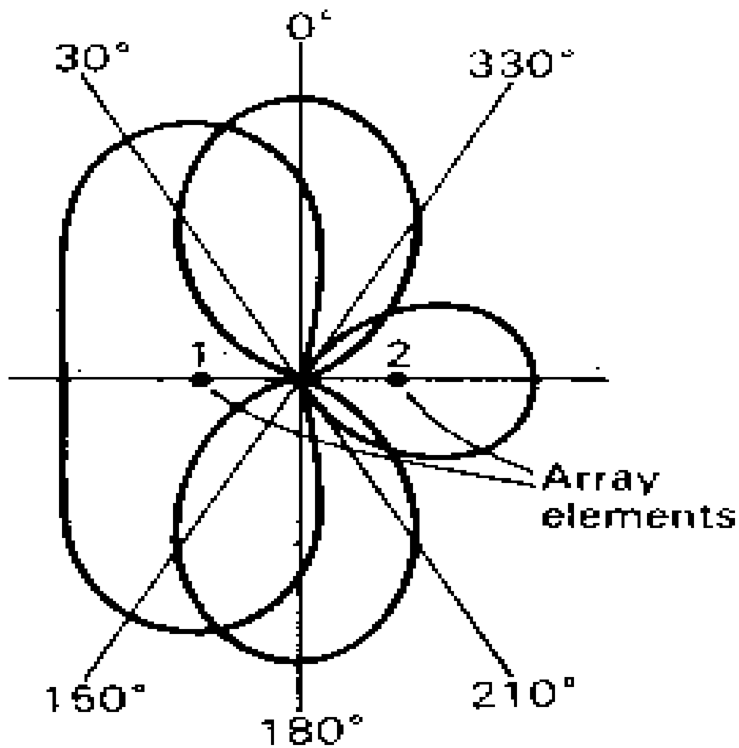


Incoming signal direction
Dipole 1
Mixer
IF amp
VCO
φ
30°
λ/2
λ/4
Equal phase front of incoming signal at frequency f_s
Dipole 2
Mixer
IF amp
VCO
φ
Phase detectors
V_0/ϕ_0
IF reference oscillator
f_0
Σ
Final (summing) amplifier
V_1/ϕ_1
V_2/ϕ_2

…….Continue

- Consider the case of a signal at 30 deg from broadside as shown in figure. So that the wave arriving at element 2 travels λ/4 farther that the element 1, thus retarding the phase of the signal by 90 deg at element 2.
- Each elements equipped with its own mixer, VCO, intermediate frequency amplifier, phase detector.
- The phase detector compares the phase of the downshifted signal with the phase of the reference oscillator and produces a voltage, in turn advances or retards the phase of the VCO outputs so as to reduce the phase deference to zero.
- So $\Phi_1 = \Phi_2 = \Phi_0$
- With equal gain from both IF amplifiers the voltages V_1 and V_2 from both elements should be equal so that $V_1\angle\Phi_1 = V_2\angle\Phi_2$.

- Making the voltage from the summing amplifier proportional to 2 V_1 and maximizing the response of the array to the incoming signal by steering the beam onto the incoming signal.



Wave Propagation

- Wave propagation means the way in which wave travel from one point to another.
- In Electromagnetic , wave propagation is the way by which EM waves are **transmitted or propagated from one point on the earth to another or into various parts of the atmosphere.**

Radio Spectrum

Symbol	Frequency range	Wavelength, λ	Comments
ELF	< 300 Hz	> 1000 km	**<u>Earth-ionosphere waveguide propagation</u>**
ULF	300 Hz – 3 kHz	1000 – 100 km	
VLF	3 kHz – 30 kHz	100 – 10 km	
LF	30 – 300 kHz	10 – 1 km	**<u>Ground wave propagation</u>**
MF	300 kHz – 3 MHz	1 km – 100 m	
HF	3 – 30 MHz	100 – 10 m	**<u>Ionospheric sky-wave propagation</u>**
VHF	30 – 300 MHz	10 – 1 m	**<u>(LOS propagation) Space waves,</u>** scattering by objects similarly sized to, or bigger than, a free-space wavelength, increasingly affected by tropospheric phenomena
UHF	300 MHz – 3 GHz	1 m – 100 mm	
SHF	3 – 30 GHz	100 – 10 mm	
EHF	30 – 300 GHz	10 – 1 mm	

$$c = f \cdot \lambda; \qquad c = 3 \cdot 10^{8}\,\mathrm{ms}^{-1}$$

study of atmosphere

- In radio wave propagation, **the media** between the transmitting and receiving antenna plays an important role.
- Since the medium is free space, the study of atmosphere becomes necessary for us.
- Our atmosphere has three main regions:

1. Troposphere
2. Stratosphere
3. Ionosphere

TROPOSPHERE

- It is the nearest region, around **10 to 20km above the earth surface.**
- Height varies with poles and equator. Minimum height is at poles and maximum height is at the equator.
- Gas components almost remains constant while water vapor components decreases with height.
- Temperature decreases with increase in height.

STRATOSPHERE

- The region above troposphere is called stratosphere.
- It extends **upto 70km in space.**
- It is **isothermal region** because temperature remains constant to -50 °C irrespective of height.

Ionosphere

- It has practical importance because, among other functions, it influences radio propagation to distant places on the Earth.
- Because of the high energy from the Sun and from cosmic rays, the atoms in this area have been stripped of one or more of their electrons, or **"ionized**," and are therefore positively charged. The ionized electrons behave as free particles.

  During the night, without interference from the Sun, cosmic rays ionize the ionosphere, though not nearly as strongly as the Sun.

- **50 km to over 500 km, most of the molecules of the atmosphere are ionised by radiation from the Sun. This region is called the ionosphere.**
- The ionosphere is the region of the upper atmosphere where the Sun's ultraviolet radiation can ionize oxygen molecules to create a positive ion and a free electron.
- The ionosphere protects us from excessive ultraviolet radiation.
- HF radio waves are returned from the F-layer(100 to 300 miles in altitude) of the ionosphere by a form of refraction.
- Due to variations in properties like pressure, temperature, density, etc., different layers are formed.

Some terms:

Ionization:

- The process of upsetting electrical neutrality of atom.

Recombination:

- Reverse process of ionization
- Free electron collide with positive ion and positive ion return to its original neutral state

- There are four main types of layers.

1. D layer
2. E layer
3. F1 layer
4. F2 layer

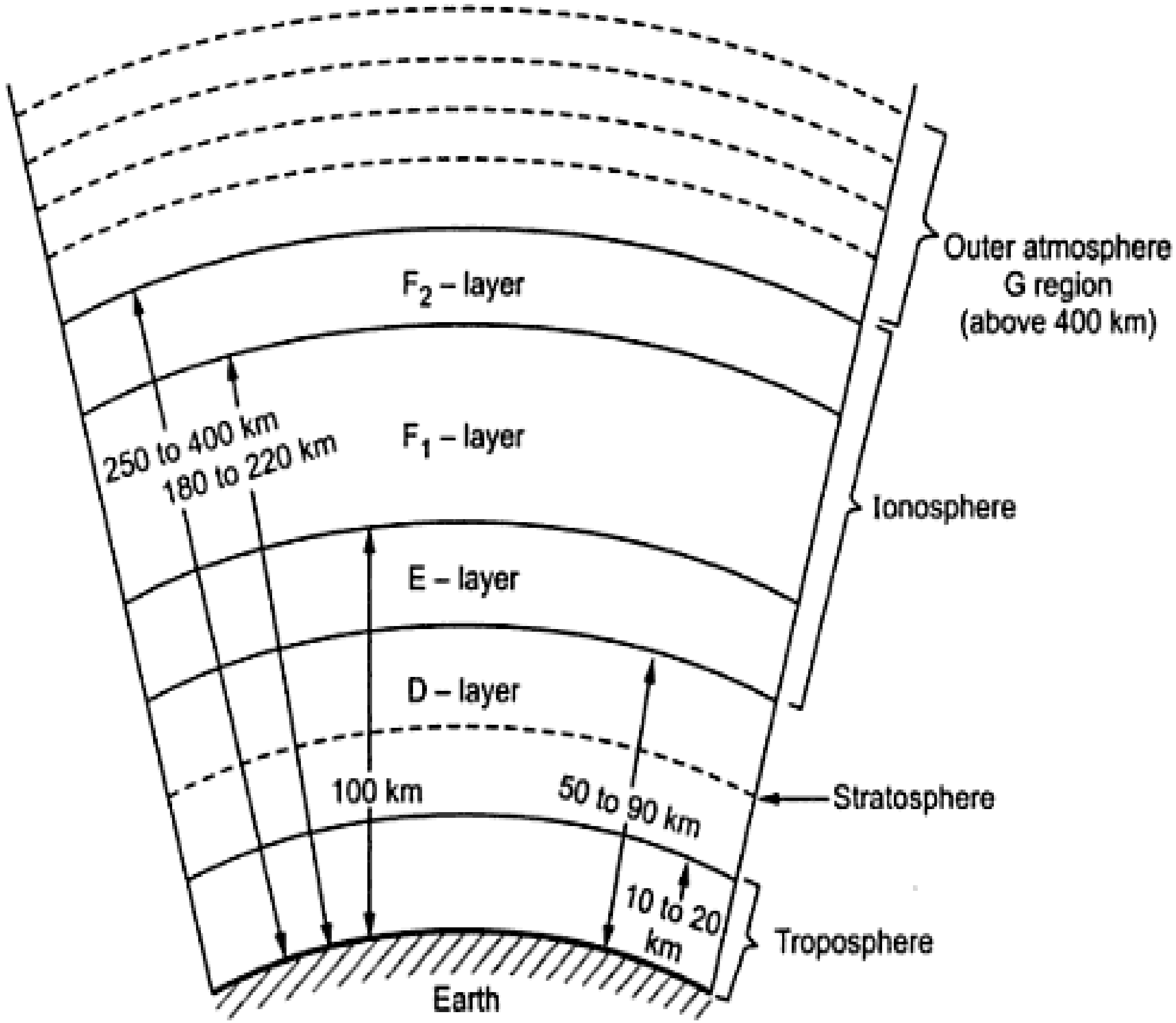


Fig. 3.9 Region in the earth's atmosphere

D LAYER

- Nearest region at **height of 70km.**
- Exists during day and disappears at night due **to higher recombination rate.**
- Electron density decreases with time as degree of ionization depends on altitude of sun.
- Reflects VLF and LF waves.
- Absorbs MF and HF waves.
- Insufficient electron density at night.
- So waves do not bend and suffer attenuation.

E LAYER

- At **100km above** earth surface.
- Appears during day with maximum electron density.
- Electron recombination at night.
- Reflects some HF waves during day.
- Another layer with high ionization density exists called **sporadic E layer** and it occurs in form of cloud.
- Provides better reception at night.

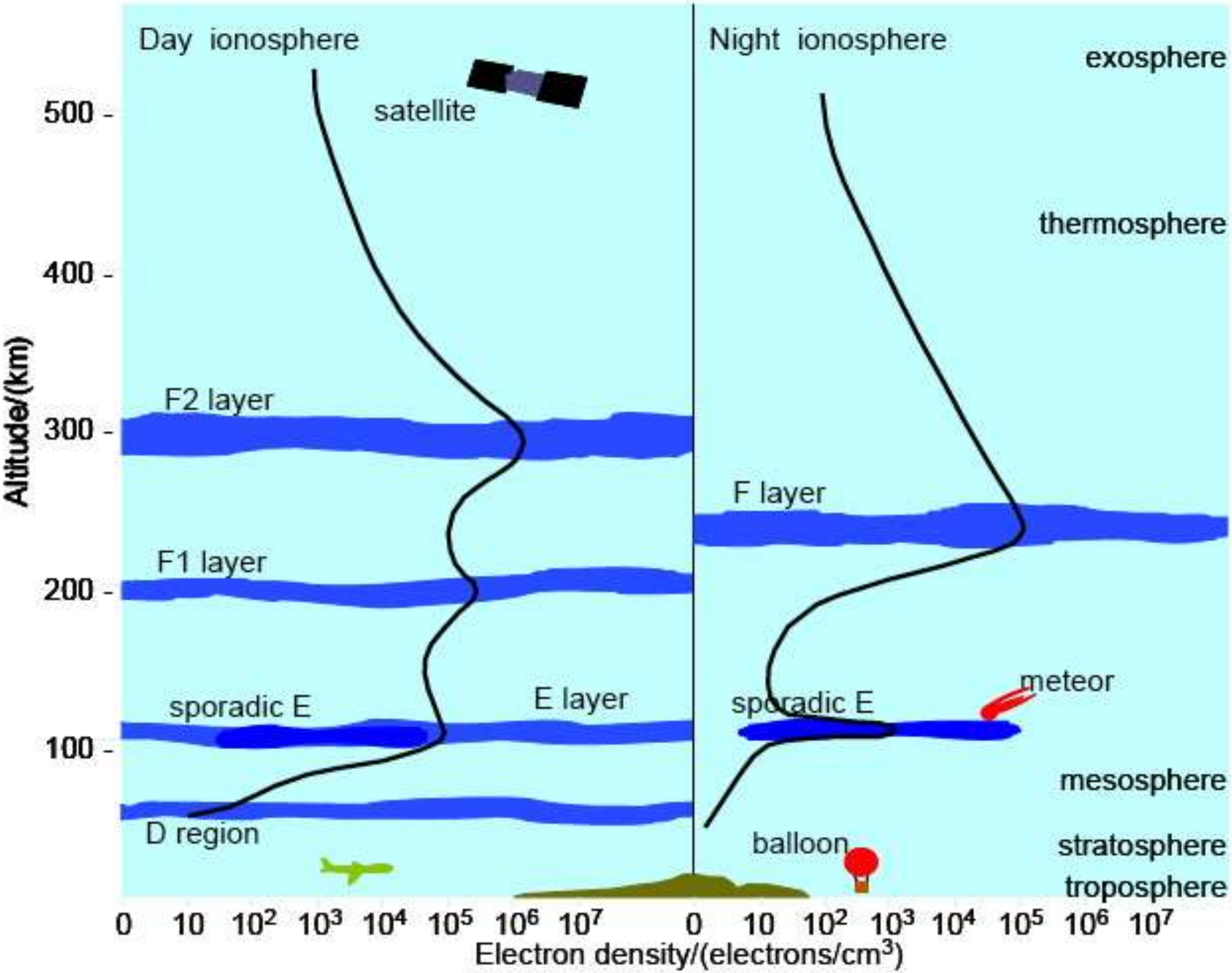
F1 LAYER

- Average **<u>height of 180km and thickness 20km</u>**.
- Remains highly ionized irrespective of time.
- Most HF waves penetrates through and are reflected by F2 region.
- Doubled absorption.
- Once while entering F2 and other after reflection by F2.
- Supports radio wave propagation during night.

F2 LAYER

- Most important layer for HF waves reflection.
- Daytime : 250km to 450km.
- Night time : 300km with F1 layer.
- Highest thickness of 200km.
- This layer appears in day as well as night.
- Though ionization density is high, actual air density is low.
- **So molecules travel more distance before collision.**

- So this distance is called **free path**.
- As rate of collision is low, ionization continues at night as well.
- So **greater HF reception** as F regions combine and other regions that causes absorption disappear .
- More electron density during day.
- It depends upon time, season, latitude, etc.
- The electron density reduces during night than day. This is shown graphically as:



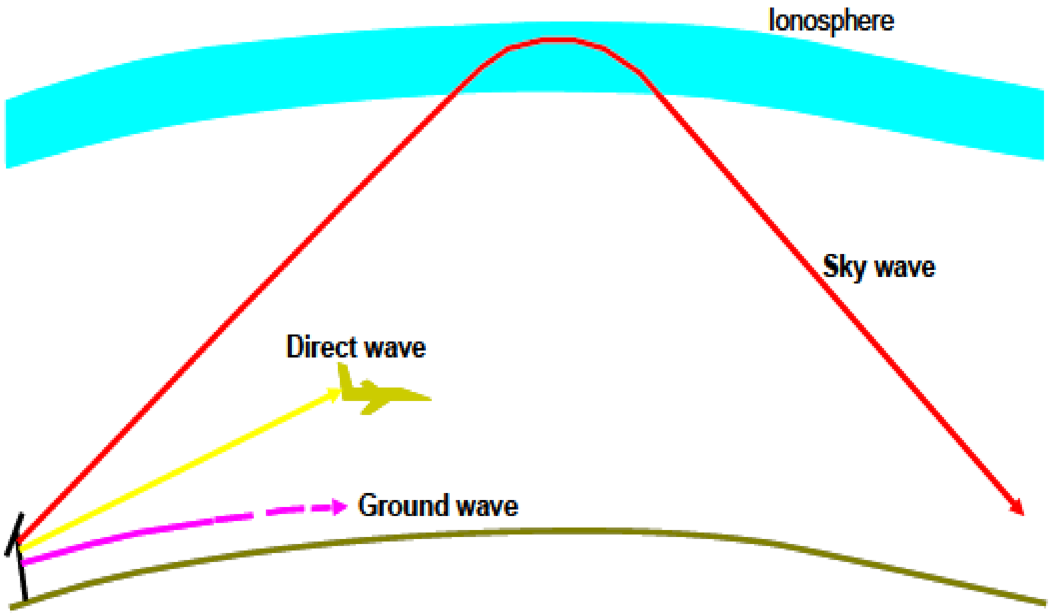
Day ionosphere
Night ionosphere
exosphere
satellite
thermosphere
F2 layer
F layer
F1 layer
sporadic E
E layer
sporadic E
meteor
mesosphere
D region
balloon
stratosphere
troposphere
Altitude/(km)
500
400
300
200
100
0 10 10^2 10^3 10^4 10^5 10^6 10^7
0 10 10^2 10^3 10^4 10^5 10^6 10^7
Electron density/(electrons/cm^3)

Propagation Modes

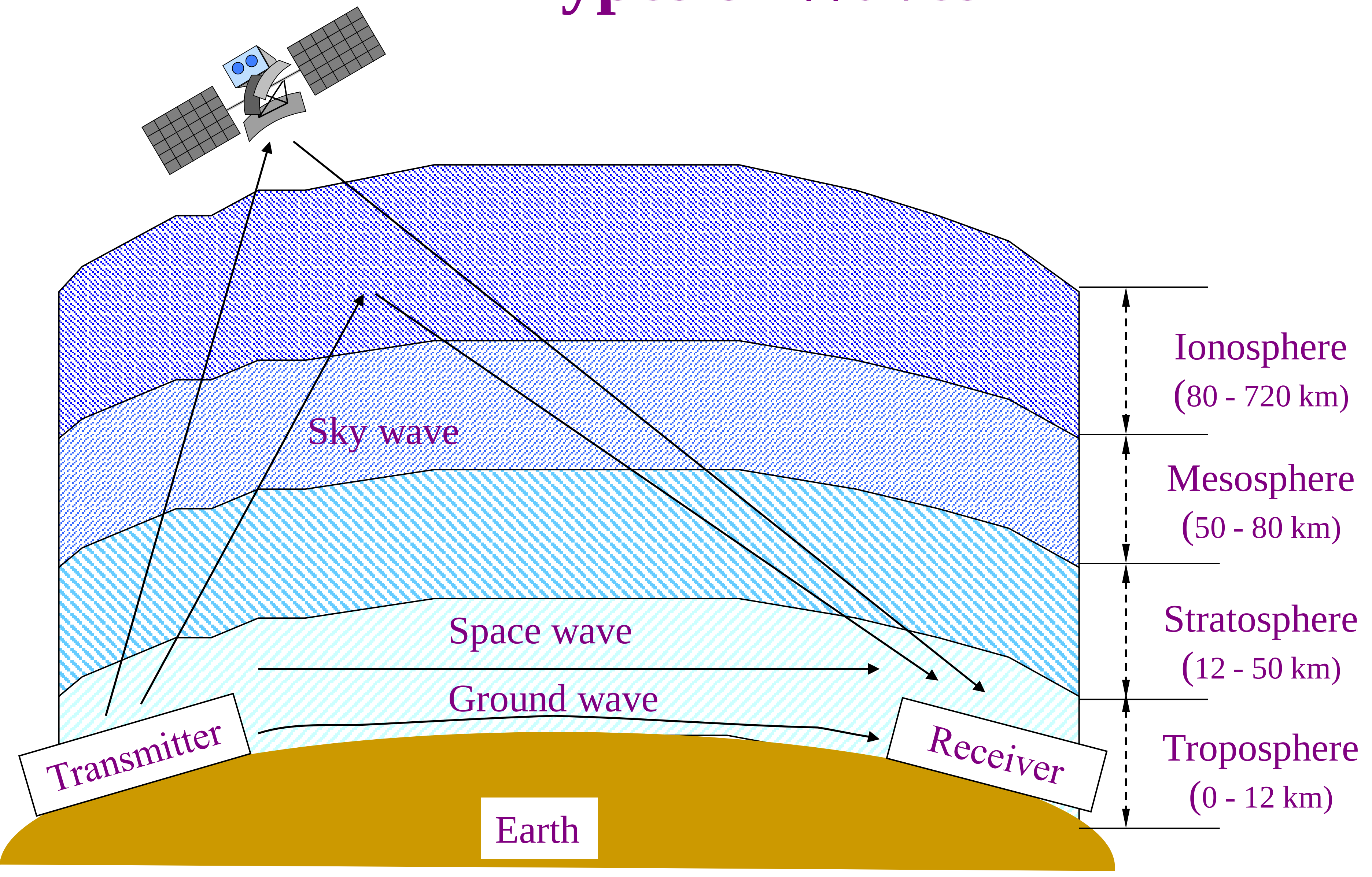
- Ground-wave (< 2MHz) propagation
- Sky-wave (2 – 30 MHz) propagation
- Line-of-sight Space-wave (> 30 MHz) propagation

Type of waves

- **ground wave:** near the ground for short distances, up to 100 km over land and 300 km over sea.
- **direct wave:** this wave may interact with the earth-reflected wave depending on terminal separation, frequency and polarisation;
- **sky wave:** reflected by the ionosphere; all distances.



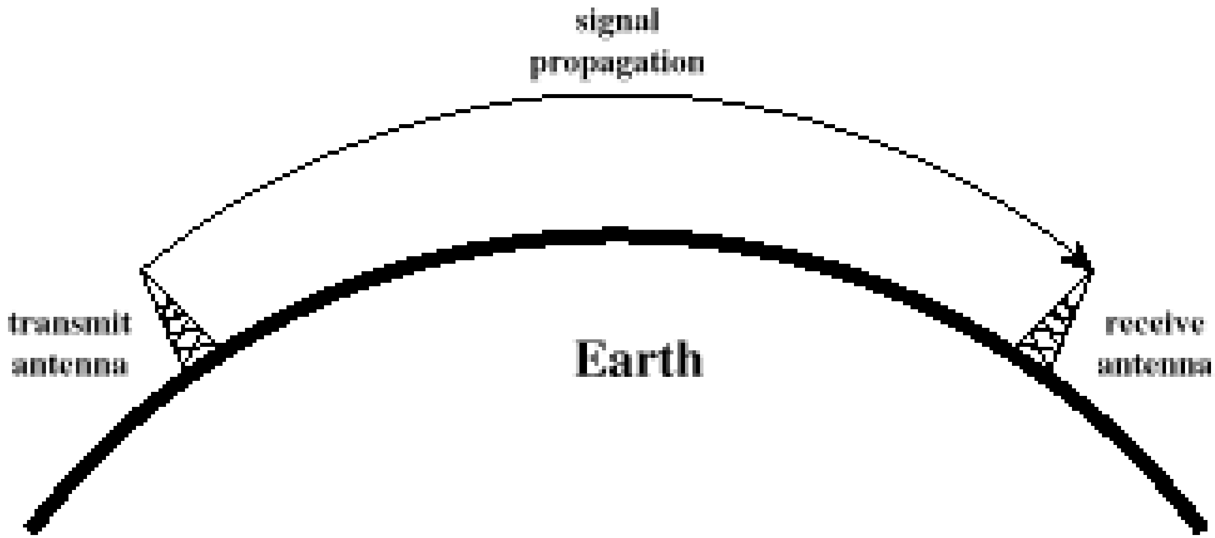
Types of Waves



Ground-Wave Propagation

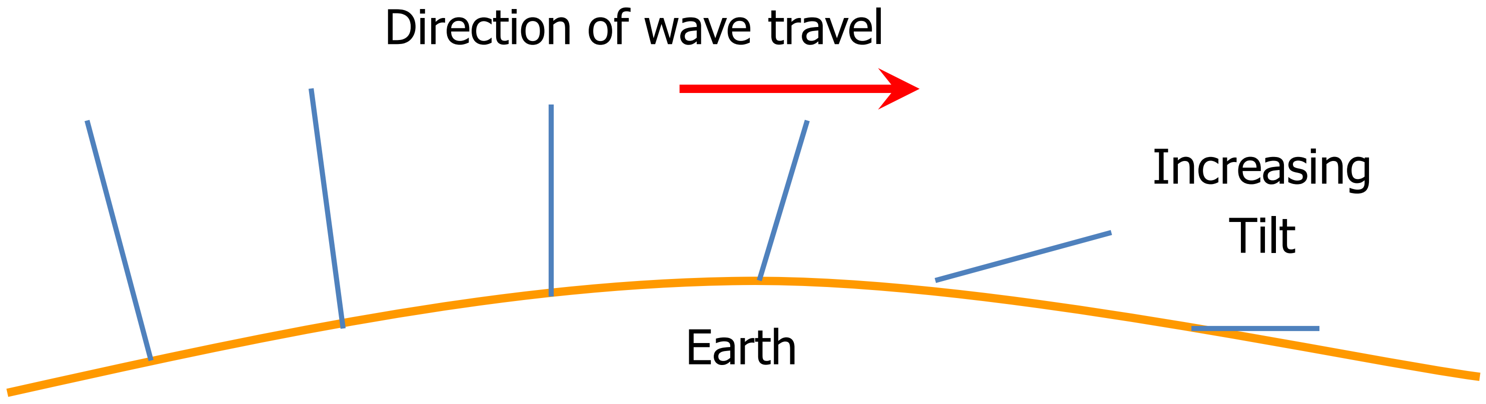
- *Ground waves* refer to the propagation of radio **waves parallel to and adjacent to the surface of the Earth**, following the curvature of the Earth.
- **Lower frequency, below 3 MHz,** travel efficiently as ground waves.
- It propagate in vertical polarization, with their magnetic field horizontal and **electric field vertical**

Ground Wave Propagation



- The radio signal spreads out from the transmitter along the surface of the Earth.
- Instead of just travelling in a straight line the radio **signals tend to follow the curvature of the Earth**.
- This is because currents are induced in the surface of the earth and this action slows down the wave-front in this region, causing the wave-front of the radio communications signal to tilt downwards towards the Earth.
- With the wave-front tilted in this direction it is able to curve around the Earth and be received well beyond the horizon

- **Ground waves are attenuated** as they follow the earth’s surface as a result wavefront "lean over" at a progressively greater angle in the direction of propagation as they travel, until they are dissipated.



Factors Affecting Ground-Wave Propagation

- As the wavefront of the ground wave travels along the Earth's surface it is attenuated. The degree of attenuation is dependent upon a variety of factors:
- Frequency
- Ground
- Antenna and its polarization

EFFECT OF FREQUENCY

- Frequency of the radio signal is one of the major determining factor as losses rise with increasing frequency.
- As a result, it makes this form of propagation impracticable above the bottom end of the HF portion of the spectrum (3 MHz).
- Typically a signal at 3.0 MHz will suffer an attenuation that may be in the region of 20 to 60 dB more than one at 0.5 MHz dependent upon a variety of factors in the signal path including the distance.

EFFECT OF GROUND

- The surface wave is also very dependent upon the nature of the ground over which the signal travels.
- Ground conductivity, terrain roughness and the dielectric constant all affect the signal attenuation.
- The ground penetration varies, becoming greater at lower frequencies, and this means that it is not just the surface conductivity that is of interest.
- At the higher frequencies this is not of great importance, but at lower frequencies penetration means that ground strata down to 100 meters may have an effect.

ANTENNA AND ITS POLARIZATION

- The type of antenna and its polarization has a major effect on ground wave propagation
- Vertical polarization is subject to considerably less attenuation than horizontally polarized signals

Application of Ground Wave

- Military communications , especially one-way transmissions to ships and submarines.
- Radio amateurs have an allocation at 137 kHz in some parts of the world.
- Radio broadcasting using surface wave propagation uses the higher portion of the LF range in Europe, Africa and the Middle East.

Disadvantages of Ground Wave

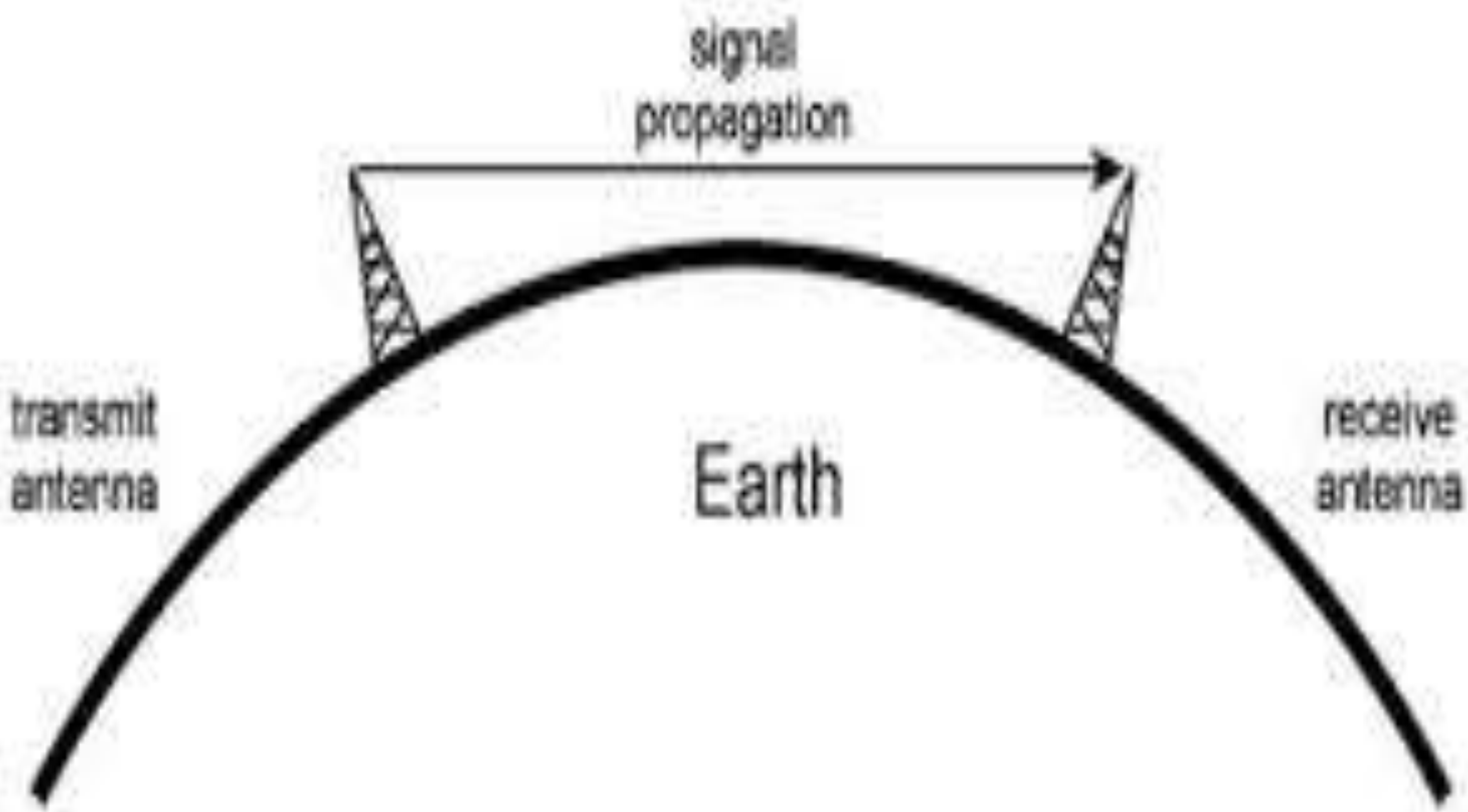
- Required relatively high transmission power.
- Limited to VLF,LF and MF, therefore required large antenna.
- Ground waves losses very considerably with surface materials.

Advantages of Ground Wave

- Given enough amount of power for transmission, ground wave can be used to communicate between any two locations in the world
- Ground waves are relatively unaffected by changing of atmospheric conditions.

Direct Mode(Line-of-sight) Space wave propagation

- Line-of-sight is the direct propagation of radio waves between antennas that are visible to each other.
- It is the most common of the radio propagation modes at higher frequencies. **Because radio signals can travel through many non-metallic objects, radio can be picked up through walls.**
- Examples : 1. propagation between a satellite and a ground antenna 2. reception of television signals from a local TV transmitter etc.



(c) Line-of-sight (LOS) propagation (above 30 MHz)

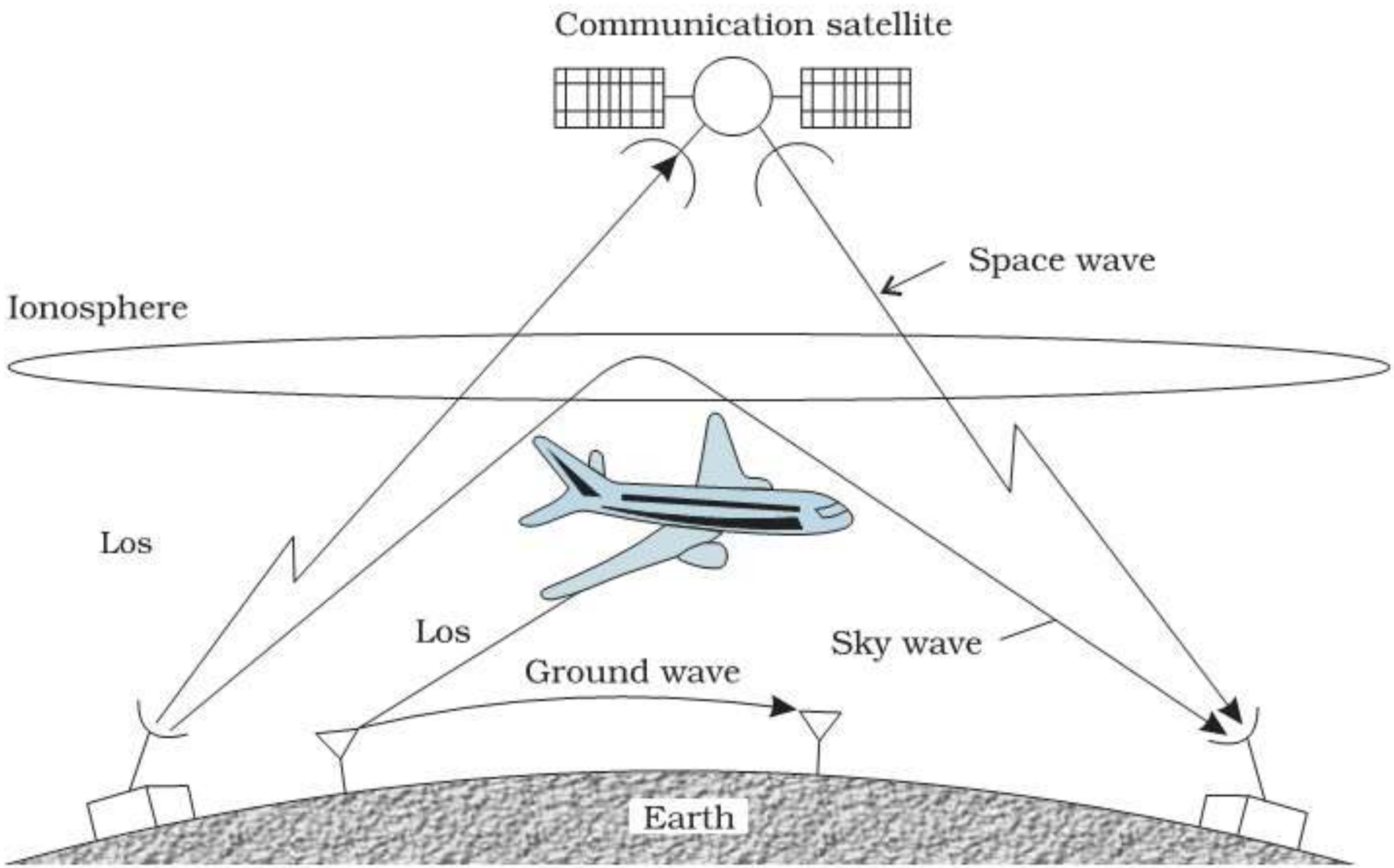
What is Space Wave Propagation?

- These waves occur within the lower 20 km of the atmosphere, and are comprised of a transmitted and reflected wave.

- The radio waves having high frequencies are basically called as space waves. These waves have the ability to propagate through atmosphere, from transmitter antenna to receiver antenna.

- These waves can travel directly or can travel after reflecting from earth’s surface to the troposphere surface of earth. So, it is also called as **Tropospherical Propagation.**

Principle used in space wave propagation



Various propagation modes for em waves.

Sky wave propagation.

- The space wave follows two distinct paths from the transmitting antenna to the receiving antenna - one through the air directly to the receiving antenna,(LOS path) the other reflected from the ground to the receiving antenna.

- The primary path of the space wave is directly from the transmitting antenna to the receiving antenna. So, the receiving antenna must be located within the radio horizon of the transmitting antenna.

- Because space waves are refracted slightly, even when propagated through the troposphere, the radio horizon is actually about one-third farther than the line-of-sight or natural horizon.

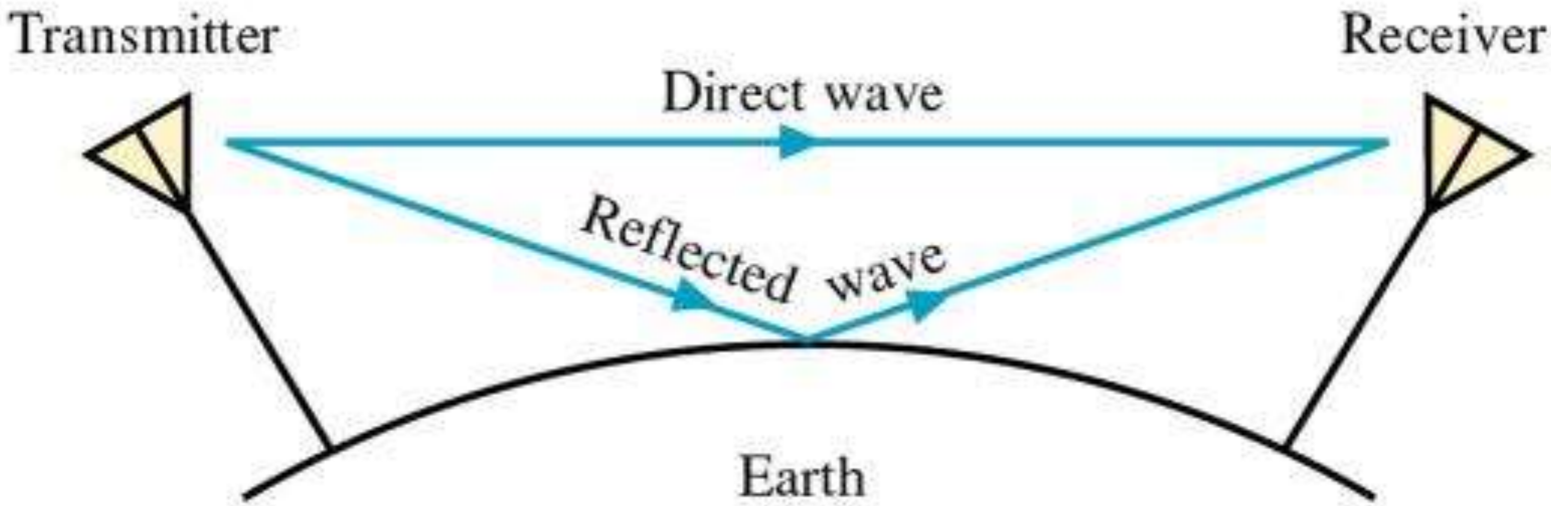


Fig 2: Two path propogation of space waves

- Although space waves suffer little ground attenuation, they nevertheless are susceptible to fading. This is because space waves actually follow two paths of different lengths (direct path and ground reflected path) to the receiving site and, therefore, may arrive in or out of phase.

- If these two component waves are received in phase, the result is a reinforced or stronger signal. Likewise, if they are received out of phase, they tend to cancel one another, which results in a weak or fading signal.

Limitations of space waves

As a form of electromagnetic radiation, like light waves, radio waves are affected by the phenomena of reflection, refraction, diffraction, absorption, polarization, and scattering.

There are some limitations of space wave propagation:

- These waves are limited to the curvature of the earth.
- These waves have line of sight propagation, means their propagation is along the line of sight distance.

Effect of curvature of earth

- Effect of curvature of earth When the distance between the transmitting and receiving antennas is large, curvature of earth has considerable effect on Surface wave plasma.
- The field strength at the receiver becomes small as the direct ray may not be able to reach the receiving antenna. The earth reflected rays diverge after their incidence on the earth. The curvature of earth creates shadow zones.

Effect of imperfection of earth

- Earth is basically imperfect and electrically rough.
- When a wave is reflected from perfect earth, its phase change is 180° .
- But actual earth makes the phase change different from 180°. The amplitude of ground reflected ray is smaller than that of direct ray.
- The field at the receiving point due to space is reduced by earth's imperfection and roughness

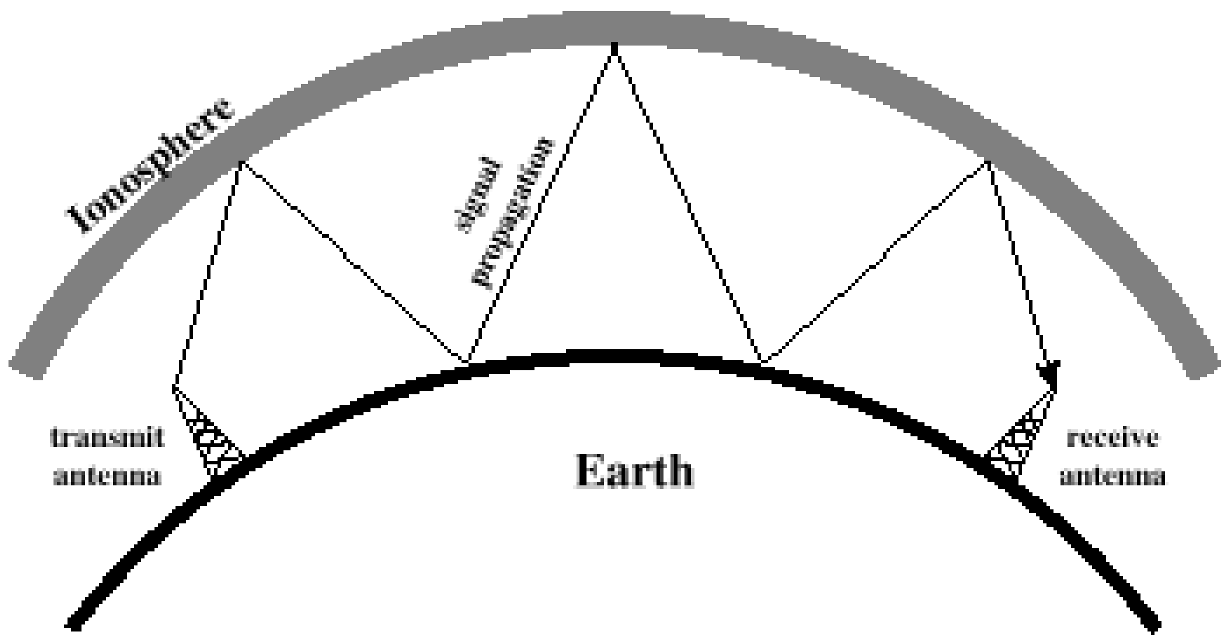
Applications of Space wave propagation

- Used in transmission of TV signals.
- Satellite Phones.
- Military Applications.
- Satellite Communication.
- Generation of Weather Maps.
- Internet.

Ionospheric Sky wave Propagation

- It refers to the **propagation of radio waves *reflected or refracted* back toward Earth from the ionosphere ,an electrically charged layer of the upper atmosphere .**
- It is not limited by the curvature of the Earth, this propagation can be used to communicate beyond the horizon, at intercontinental distances. It is mostly used in the short wave frequency bands (GHz). This is also called sky wave propagation.

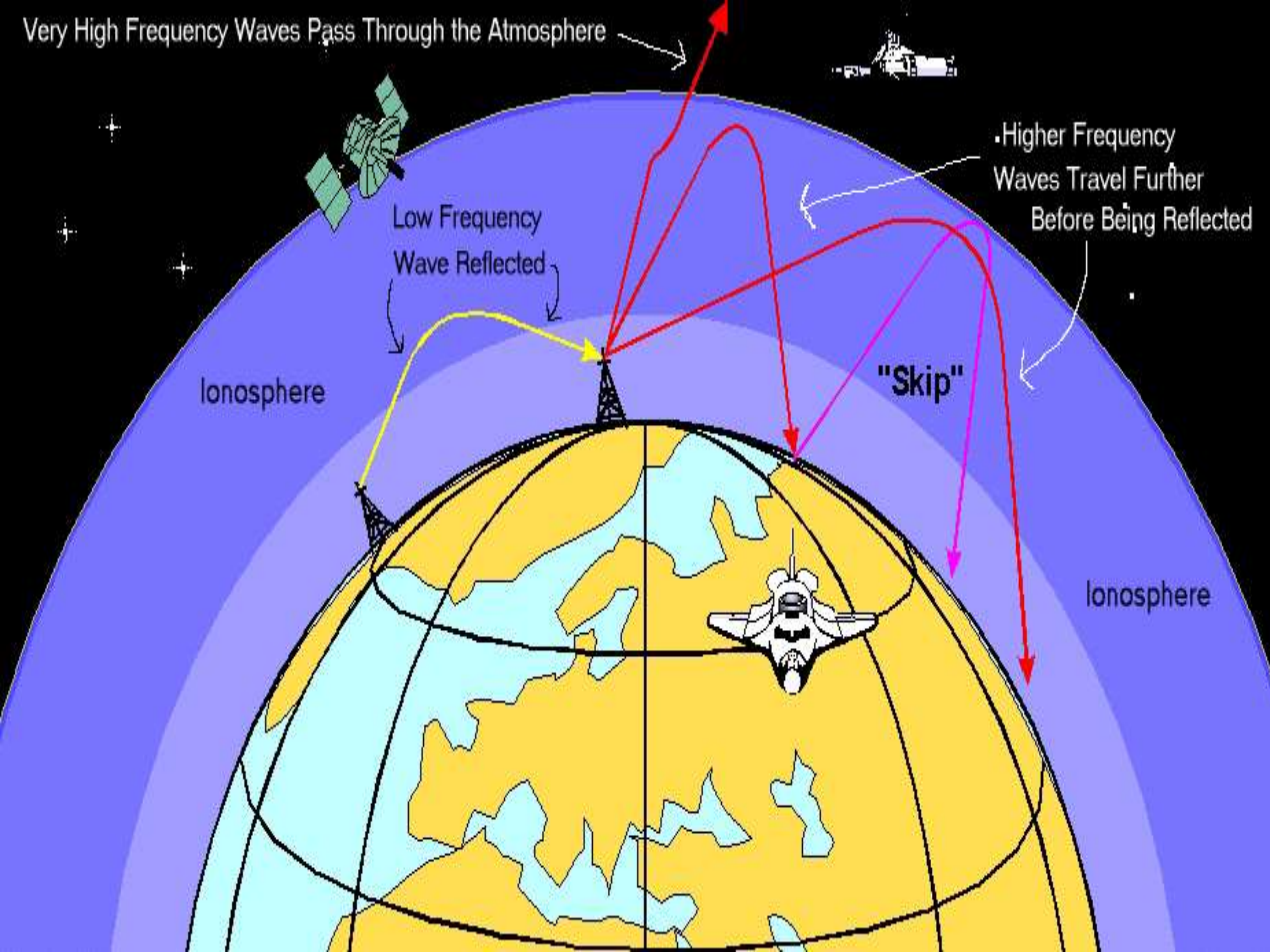
Sky Wave Propagation



i) **Low Frequency Operation**: At low frequency, below 100KHz the change in electron and ion density within a distance of wavelength is vary large. There is abrupt change in refractive index. **Here reflection of waves takes place.**

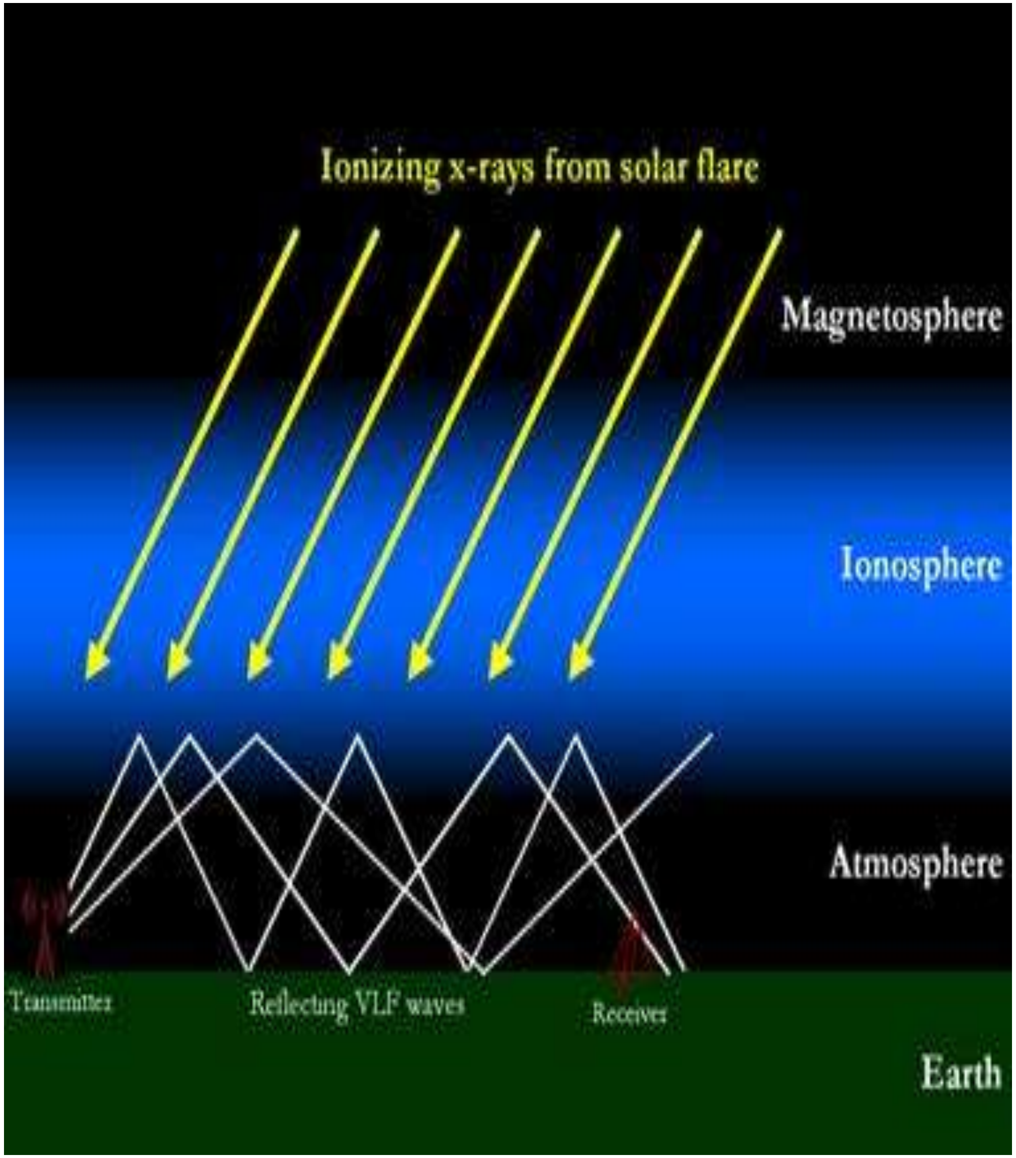
ii) **High Frequency Operation:** Here the wavelength is so small that the ionization density (and hence refractive index) changes slightly in the course of wavelength. Under such conditions the ionosphere may be treated as a dielectric with continuously changing refractive index, **causing refraction.**

iii) **Mid Frequency Operation**: for mid frequencies, the ionospheric region is considered to be consisting of several thin but discrete layers, each layer having constant ionization density. Here **there is partial refraction and reflection**.


Very High Frequency Waves Pass Through the Atmosphere
Higher Frequency
Waves Travel Further
Before Being Reflected
Low Frequency
Wave Reflected
Ionosphere
"Skip"
Ionosphere

- **Long-range communication in the high-frequency band** is possible because of refraction in a region of the upper atmosphere called the **ionosphere**
- The ionosphere can cause radio waves to bend (**refraction**).The more dense the ionization ,the higher the degree of refraction, and at higher frequencies.
- **VHF and higher radio waves usually pass through the ionosphere and escape into space.** HF radio waves are most affected by refraction.
- while the **low frequency waves reflect off the ionosphere and essentially "skip" around the earth.**

- For the **very low frequency (VLF)** waves that the space weather monitors track, the ionosphere and the ground produce a “waveguide” through which radio signals can bounce and make their way around the curved Earth.



Applications of Sky wave Propagation

- H.F. (High Frequency 3 to 30MHz)

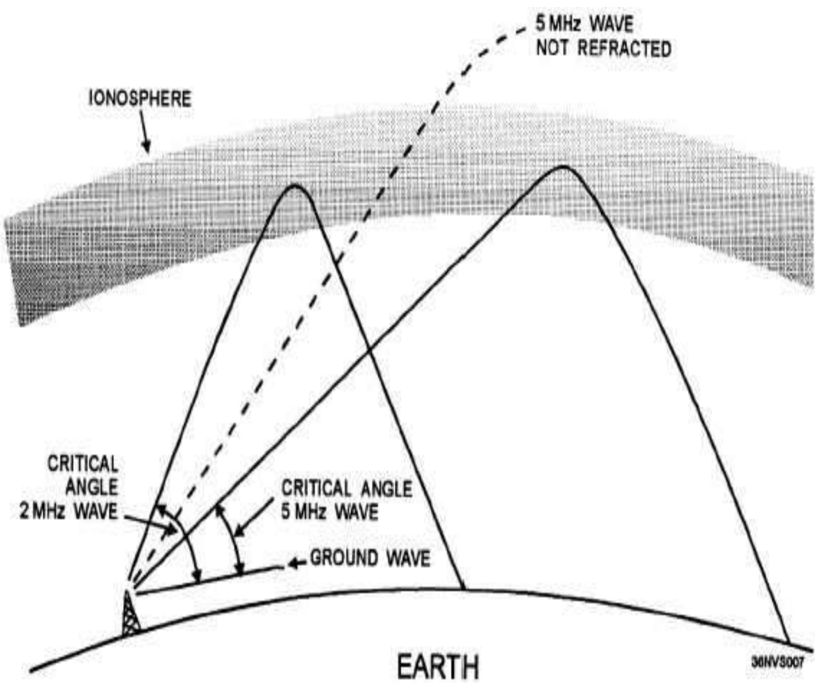
  Long Range communications. Shipping, Aircraft, World Broadcast Communications, Radio Amateurs.
- Use involves reflecting the signal off the ionosphere back down to waiting recieving stations. Prone to atmospheric changes causing fading and noise. Range from 500 to thousands of Kilometres.

Various Parameters

- Critical frequency
- Maximum Usable Frequency (MUF)
- Virtual Height
- Skip Distance
- Critical Angle

Critical frequency

- The highest frequency that returns from an ionospheric layer at a vertical incidence is called the critical frequency for the particular layer.
- It is proportional to the square root of maximum Electron density in the layer. $f_c = 9(N_m)^{1/2}$



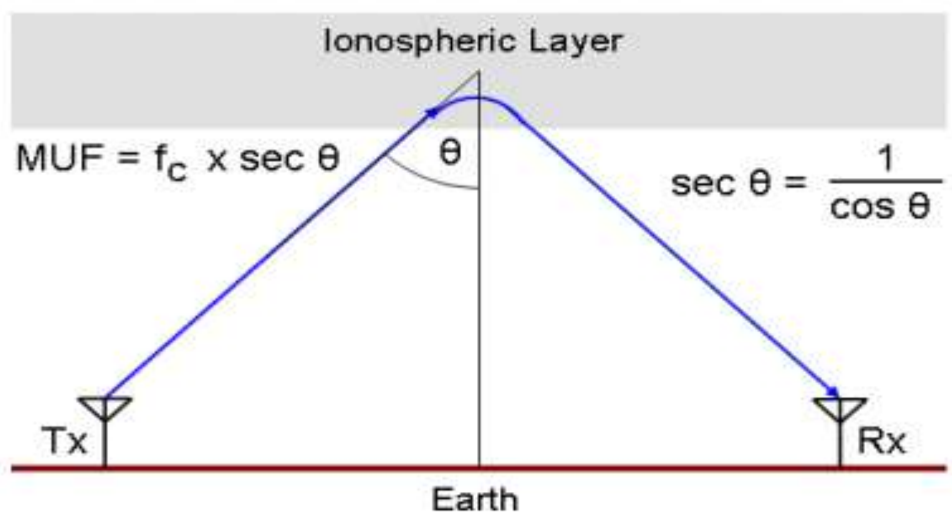
- It is the highest frequency which can be refracted by a particular layer at vertical incidence but it is not the highest frequency which will get refracted for any other angle of incidence.
- Radio waves transmitted at frequencies higher than the critical frequency of a given layer will pass through the layer and be lost in space.
- But if the same waves enter an upper layer with a higher critical frequency, they will be refracted back to Earth.

Maximum Usable Frequency(MUF)

- MUF is the highest radio frequency that can be used for transmission between two points via reflection from the ionosphere.
- When a signal is transmitted using HF propagation, over a given path there is a maximum frequency that can be used. This results from the fact that as the signal frequency increases it will pass through more layers and eventually travelling into outer space. When the signal passes through all the layers communication will be lost.

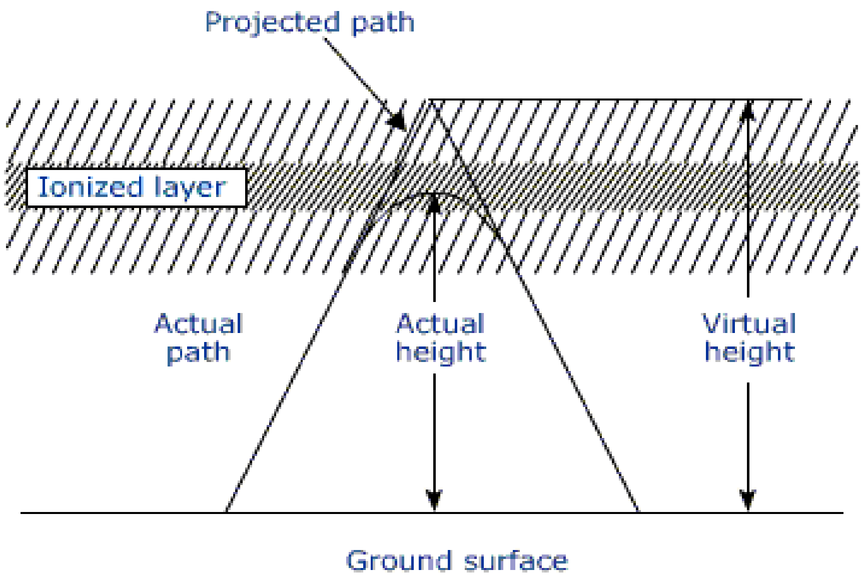
Maximum Usable Frequency(MUF)

- MUF is defined as the highest frequency at which it is reflected by the ionospheric layer at the angle of incidence other than normal incidence.
- MUF depends on time ,day ,distance direction and solar activity.
- MUF is the highest frequency that can be used by sky waves.



Virtual Height

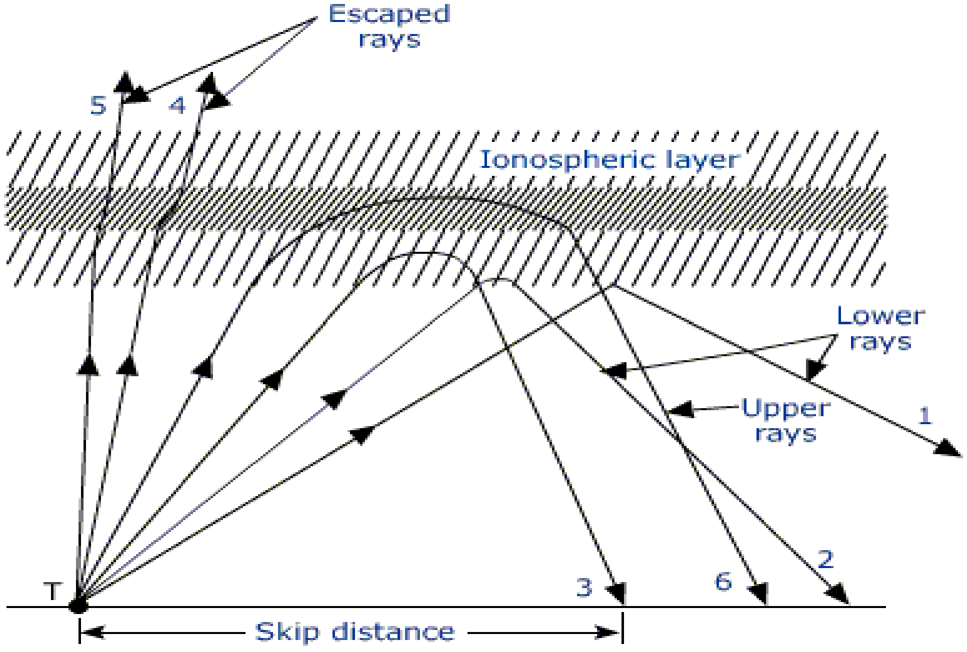
- The virtual height is that height from which a wave sent up at an angle appears to be reflected.



- As the wave is refracted, it is bent down gradually rather than sharply.
- However, below the ionized layer, the incident and refracted rays follow paths that are exactly the same as they would have been if reflection had taken place from a surface located at a greater height, called the virtual height of this layer.

Skip Distance

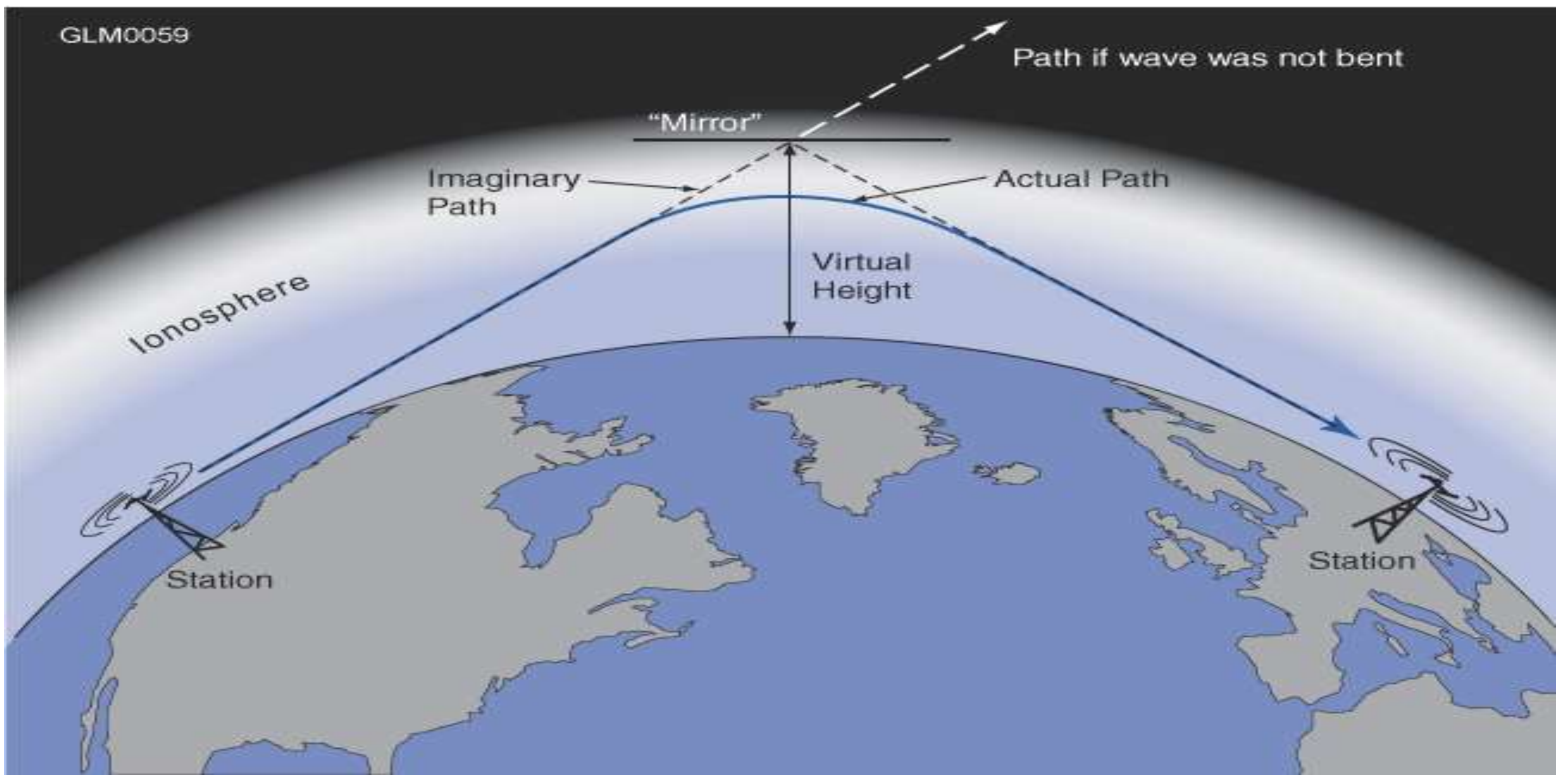
- The skip distance is the shortest distance from a transmitter, measured along earth's surface, at which a sky wave of fixed frequency will be returned to earth.



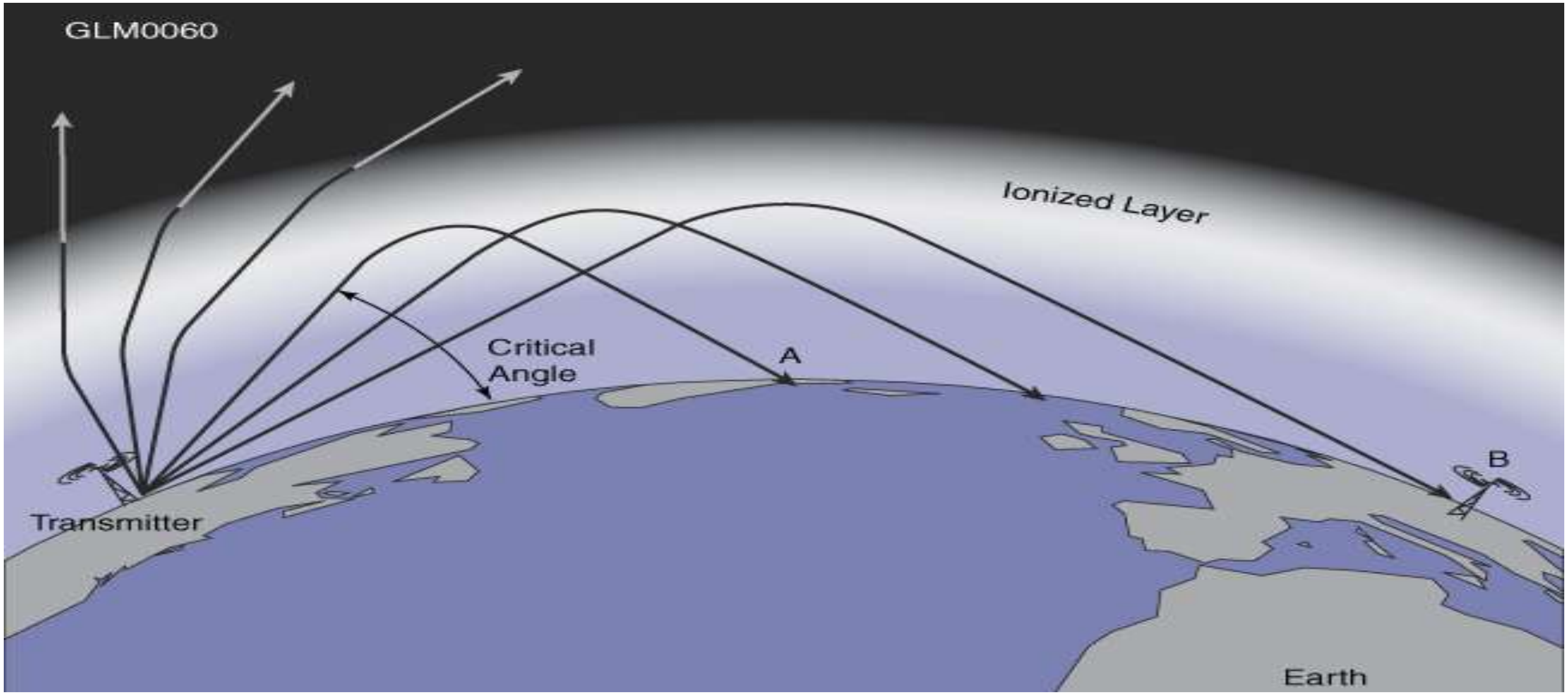
- The highest frequency that is returned to earth in the vertical direction is called the **critical frequency, f_c.**

- The highest frequency that returns to earth over a given path is called the **maximum usable frequency (MUF).** Because of the general instability of the ionosphere, the *optimum working frequency* (OWF) = 0.85 MUF, is used instead.

- The **virtual height** is the height from which the radio wave appears to be reflecting.



- **<u>The critical angle</u>** is the angle at which a radio wave must hit the ionosphere to reflect back to the Earth.



Propagation mechanisms

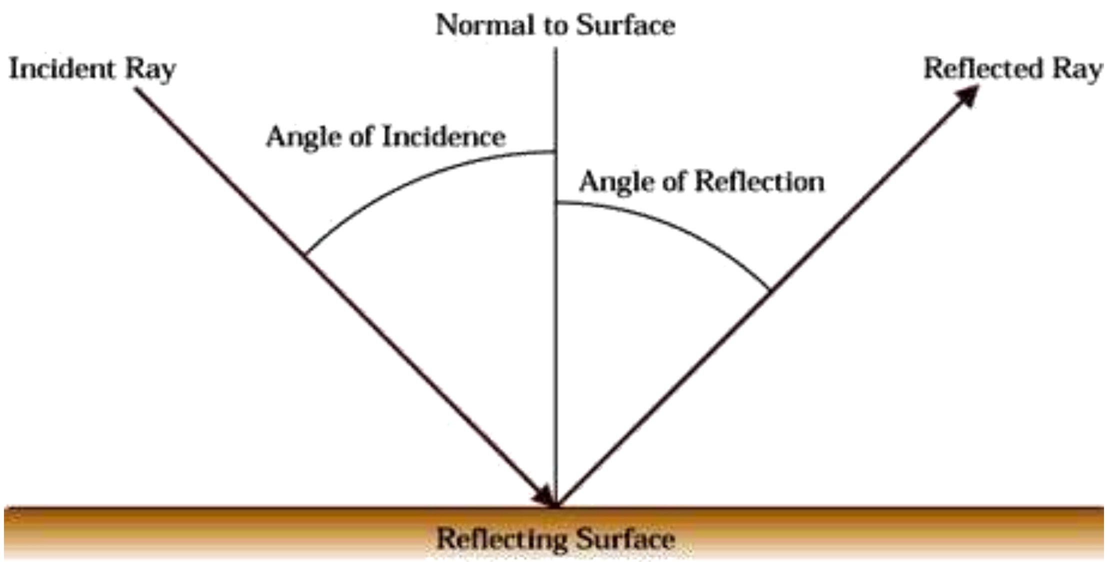
- When the dimension of the object is:
  - Very Large compared to the wavelength
    - Reflection/Refraction
  - Comparable/Larger compared to the wavelength
    - Diffraction
  - Small compared to the wavelength
    - Scattering
  - Very small compared to the wavelength
    - Unaffected

Reflection, Refraction, and Diffraction

- These three properties are shared by light and radio waves
- For both reflection and refraction, it is assumed that the **surfaces involved are much larger than the wavelength**; if not, diffraction will occur

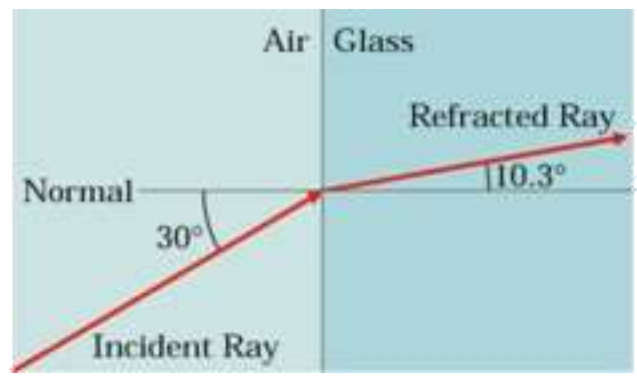
Reflection

- Reflection of waves from a smooth surface (***specular reflection***) results in the angle of reflection being equal to the angle of incidence.



Refraction

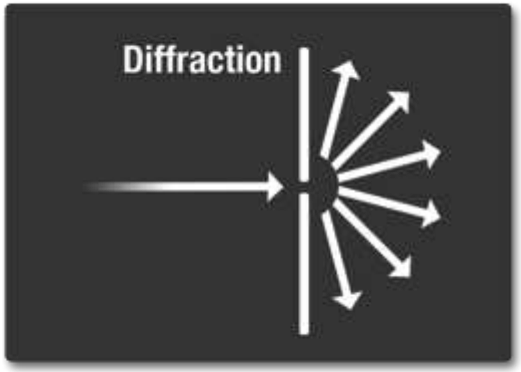
- A transition from one medium to another results in the **<u>bending of radio waves</u>**, just as it does with light
- Snell's Law governs the behavior of electromagnetic waves being refracted:

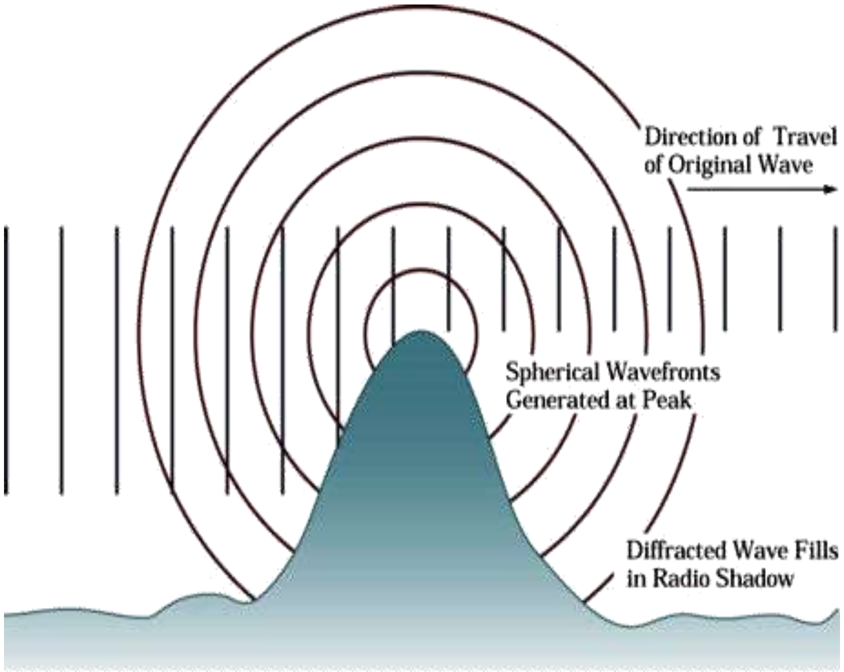


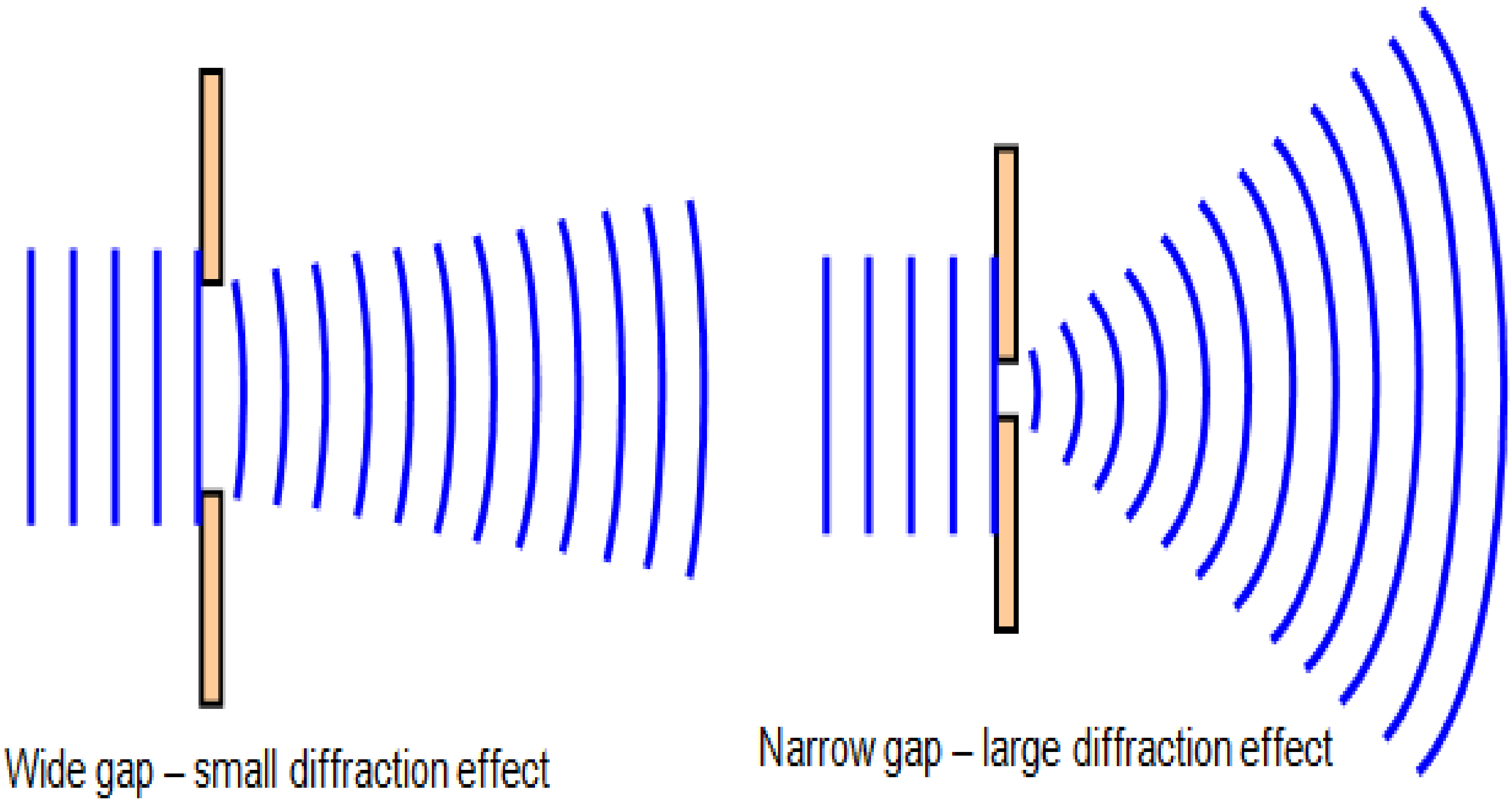
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Diffraction

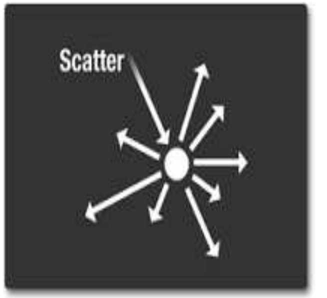
- As a result of diffraction, electromagnetic waves can appear to **"go around corners"**
- It is defined as **the bending of light around the corners of an obstacle or aperture into the region of geometrical shadow of the obstacle.**
- Diffraction is more apparent when the object has sharp edges, that is when the **dimensions are comparable to the wavelength**






Wide gap – small diffraction effect
Narrow gap – large diffraction effect

Scattering



- Scattering is a physical process that causes radiation to deviate from a straight trajectory.
- The wavelength dependence of scattering is first and foremost determined by this ratio of the size of the scattering particles to the wavelength of the light.
- Scatter diameters much less than the wavelength results in *Rayleigh scattering*.
- Larger diameters result in *Mie scattering*.

Rayleigh scattering

- Rayleigh scattering occurs when light travels in transparent solids and liquids, but is most prominently seen in gases. The amount of Rayleigh scattering that occurs to a beam of light is dependent upon the size of the particles and the wavelength of the light; in particular, the scattering coefficient, and hence the intensity of the scattered light, varies for small size particles inversely with the fourth power of the wavelength.
- This wavelength dependence (**$\sim\lambda^{(-4)}$**) means that blue light is scattered much more than red light.