इानेन प्रकाशते जगत्
INDUS UNIVERSITY

Microwave Engineering

(EC0505)

Unit-3

B.Tech. (Electronics and Communication)

Semester-V

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S-Parameters

$$Y \text{ parameters: } \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_1 \end{aligned}$$

$$Z \text{ parameters: } \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}$$

$$ABCD \text{ parameters: } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$

- All these network parameters relate total voltages and total currents at each of the two ports.
- But at microwave frequency, these parameters can not be used.

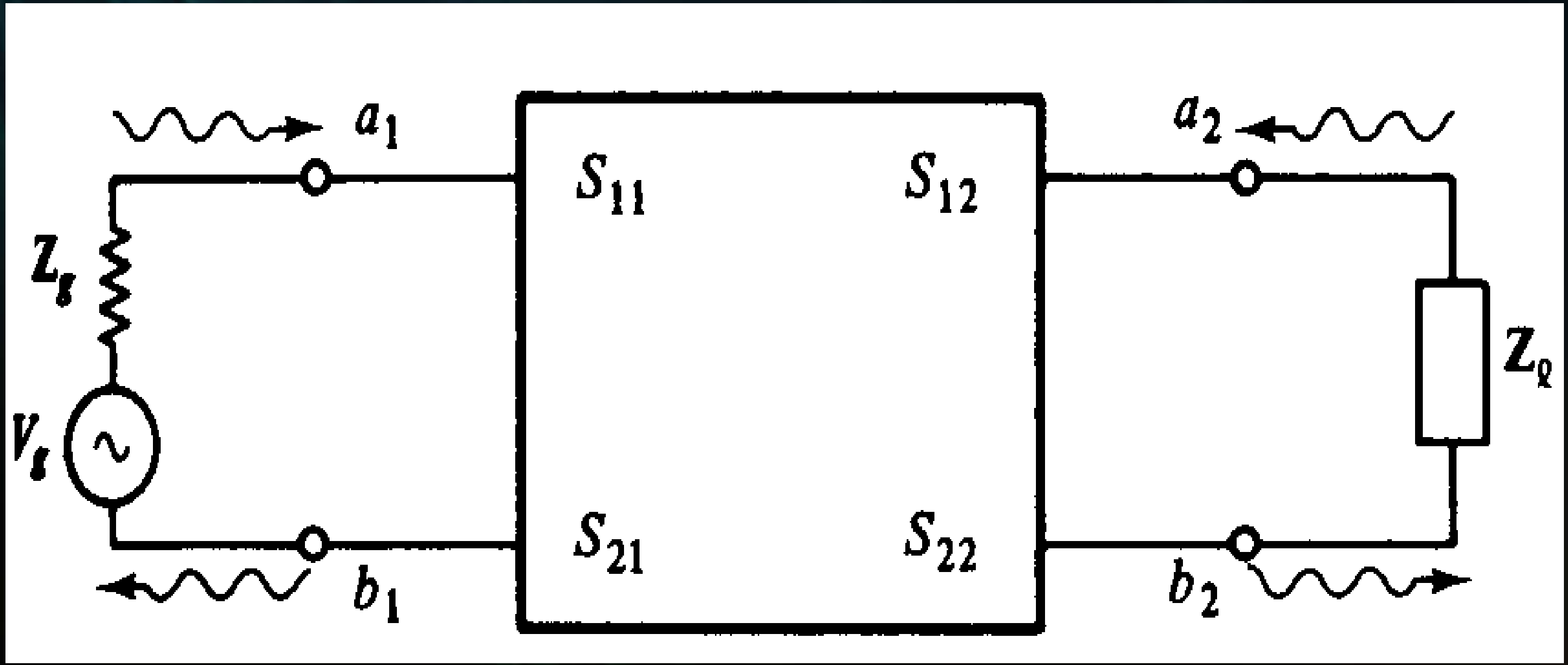
- At microwave frequency, two independent quantities required for each waveguide terminal are an incident and a reflected wave replacing voltage and current.
- Suppose that incident and reflected voltages waves on the input guide are given in magnitude and phase at the chosen reference plane by V_{1+} and V_{1-}. Similarly incident and reflected voltages waves on the reference plane two is given by V_{2+} and V_{2-}.
- It is common to normalize incident and reflected waves as follows:

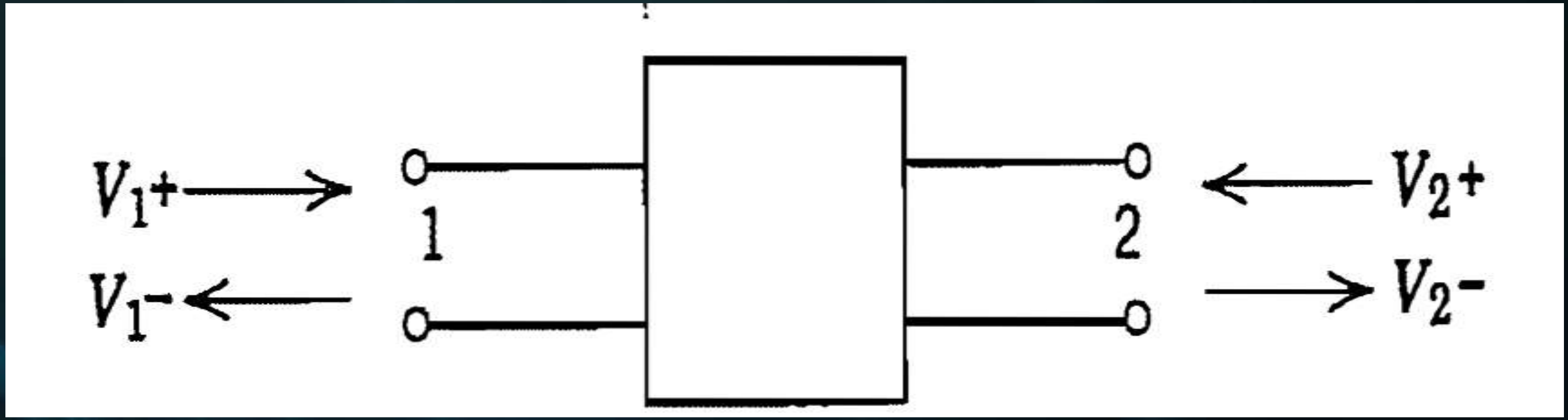
$$a_n = \frac{V_{n+}}{\sqrt{Z_{0n}}}, \qquad b_n = \frac{V_{n-}}{\sqrt{Z_{0n}}}$$

- The logical variables to use at the microwave frequencies are travelling wave rather than total voltages and total currents.
- These are S-parameters which are expressed as

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$





- In general, the voltages at the terminals of the device can be written as, $V_n = a_n + b_n$.
- Scattering coefficients are then defined, such that the voltage wave b_i leaving the 'i' port owing to a voltage wave a_j incident upon the 'j' port when no waves enter any of the other ports of the device is given by, $b_i = S_{ij} a_j$.
- In general, when a voltage is incident upon all of the 'n' ports of the device, each incident voltage will make a contribution to the total resultant voltage wave reflecting from the 'i' port.

$$b_i = S_{i1} a_1 + S_{i2} a_2 + \dots\dots\dots + S_{in} a_n$$

- Now there are reflecting waves from 'n' ports, the set of scattering equations

$$b_1 = S_{11} a_1 + S_{12} a_2 + ... + S_{1n} a_n$$

$$b_2 = S_{21} a_1 + S_{22} a_2 + ... + S_{2n} a_n$$

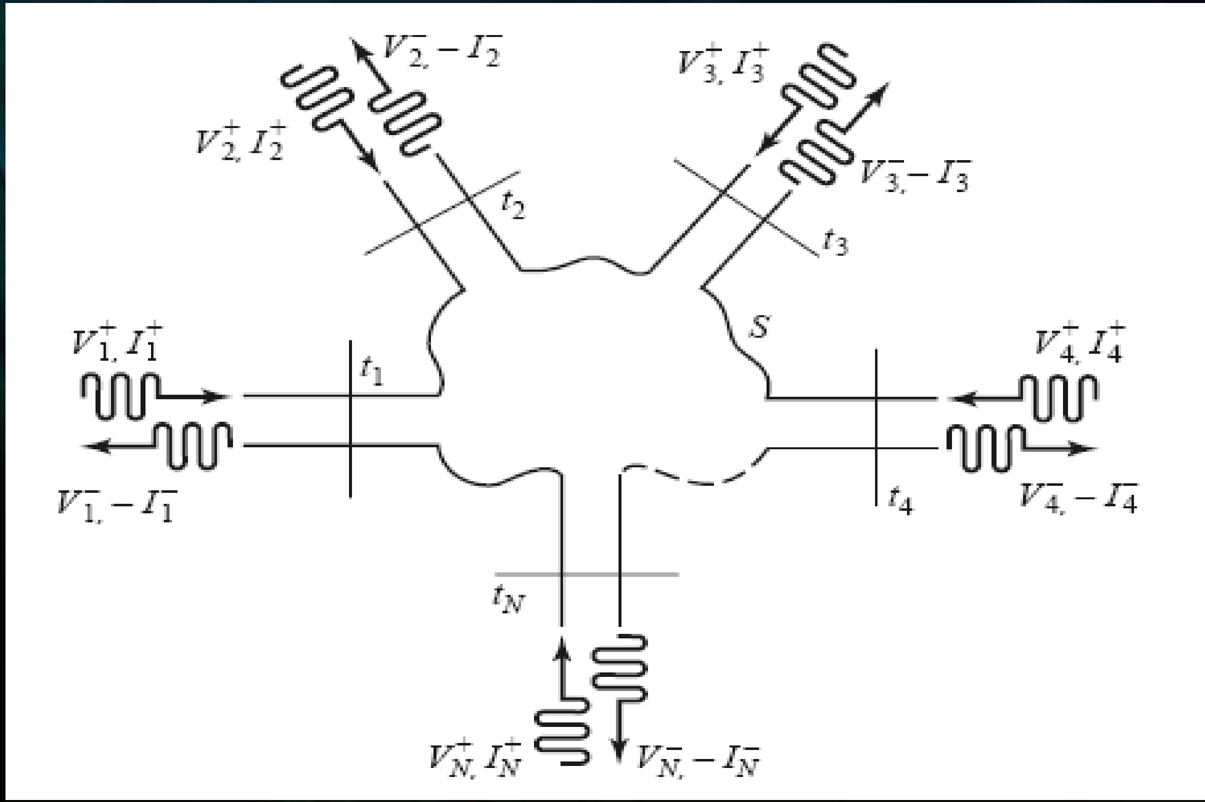
$$..$$

$$b_n = S_{n1} a_1 + S_{n2} a_2 + ... + S_{nn} a_n$$

- The diagonal element S_{ij} of the scattering matrix is the reflection coefficient at the 'j' port and represents the reflected voltage wave which would be observed at this port with an incident voltage wave of unit magnitude and zero phase, when all the other ports are terminated in matched impedances and hence no waves are reflected back into the other ports.

Properties of S parameters

Consider the N-port network shown in Figure, where V+ is the amplitude of the voltage wave incident on port n and V− is the amplitude of the voltage wave reflected from port n.



The scattering matrix is defined as

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & & & \vdots \\ S_{N1} & \cdots & & S_{NN} \\ \vdots & & & \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

$$[V^-] = [S][V^+].$$

A specific element of the scattering matrix can be determined as

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \text{ for } k \neq j}$$

Reciprocal Networks and Lossless Networks

It can be shown that the scattering matrix for a reciprocal network is symmetric, and that the scattering matrix for a lossless network is unitary.

$$V_n^+ = \frac{1}{2}(V_n + I_n),$$

$$[V^+] = \frac{1}{2}([Z] + [U])[I].$$

$$V_n^- = \frac{1}{2}(V_n - I_n),$$

$$[V^-] = \frac{1}{2}([Z] - [U])[I].$$

$$[V^-] = ([Z] - [U])([Z] + [U])^{-1}[V^+],$$

$$[S] = ([Z] - [U])([Z] + [U])^{-1}.$$

$$[S]^t = \{([Z]+[U])^{-1}\}^t([Z]-[U])^t.$$

$$[S]^t = ([Z]+[U])^{-1}([Z]-[U]),$$

$$[S] = [S]^t,$$

so the scattering matrix is symmetric for reciprocal networks.

If the network is lossless, no real power can be delivered to the network. Thus, if the characteristic impedances of all the ports are identical and assumed to be unity, the average power delivered to the network is

$$\begin{aligned} P_{avg} &= \frac{1}{2}\mathrm{Re}\{[V]^t[I]^*\} = \frac{1}{2}\mathrm{Re}\{([V^+]^t + [V^-]^t)([V^+]^* - [V^-]^*)\} \\ &= \frac{1}{2}\mathrm{Re}\{[V^+]^t[V^+]^* - [V^+]^t[V^-]^* + [V^-]^t[V^+]^* - [V^-]^t[V^-]^*\} \\ &= \frac{1}{2}[V^+]^t[V^+]^* - \frac{1}{2}[V^-]^t[V^-]^* = 0, \end{aligned}$$

For a lossless junction, the incident and reflected powers are equal

$$[V^+]^t[V^+]^* = [V^-]^t[V^-]^*.$$

$$[V^+]^t[V^+]^* = [V^+]^t[S]^t[S]^*[V^+]^*,$$

so that, for nonzero $[V^+]$

$$[S]^t[S]^* = [U],$$

$$\text{or} \quad [S]^* = \{[S]^t\}^{-1}.$$

A matrix that satisfies the above condition is called a unitary matrix.

The matrix equation can be written in summation form as

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = \delta_{ij}, \text{ for all } i, j,$$

If i = j

$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1,$$

while if $i \neq j$

$$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = 0, \text{ for } i \neq j.$$

The above equations state that the dot product of any column of [S] with the conjugate of that same column gives unity, while the dot product of any column with the conjugate of a different column gives zero (the columns are orthonormal).

Waveguide Tees

MULTIPORT JUNCTION USED AS POWER COMBINER, POWER DIVIDER AND POWER MONITORS

E Plane tees:

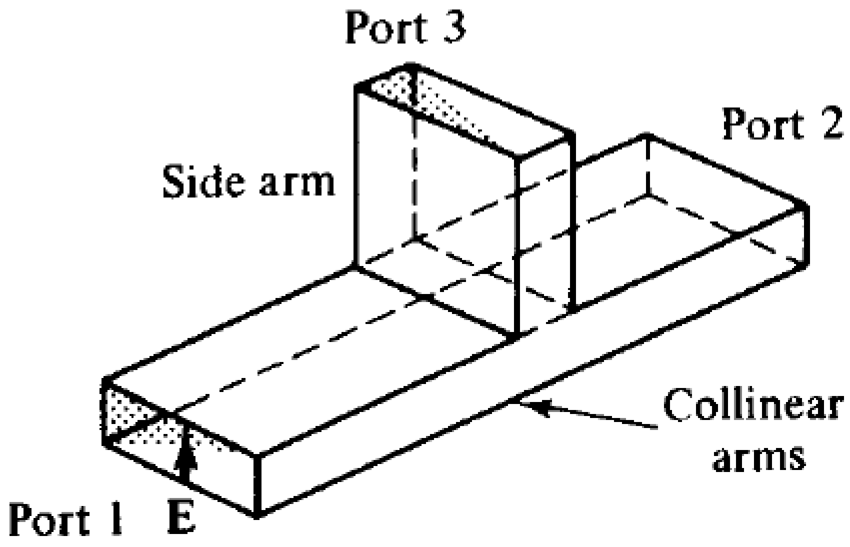
❖Axis of its side arm is parallel to the E field of the main guide

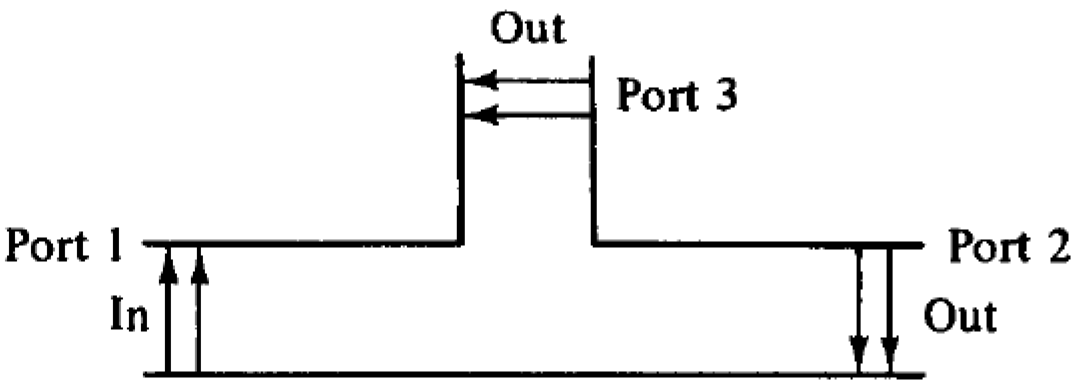
❖Collinear arms are symmetric about the side arm

Input Output

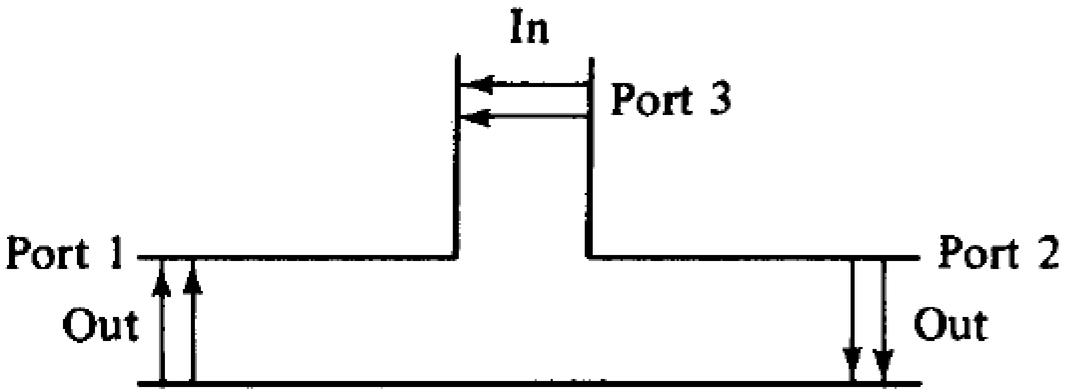
❖Port 3- Port 1 and port 2 – opposite phase & same magnitude (subtraction)

S13 = -S23 (both have opposite signs)





(a)

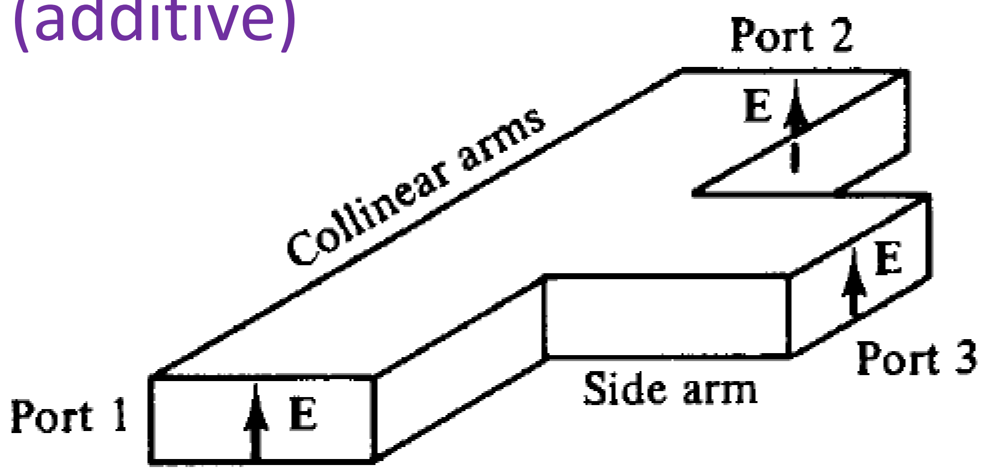


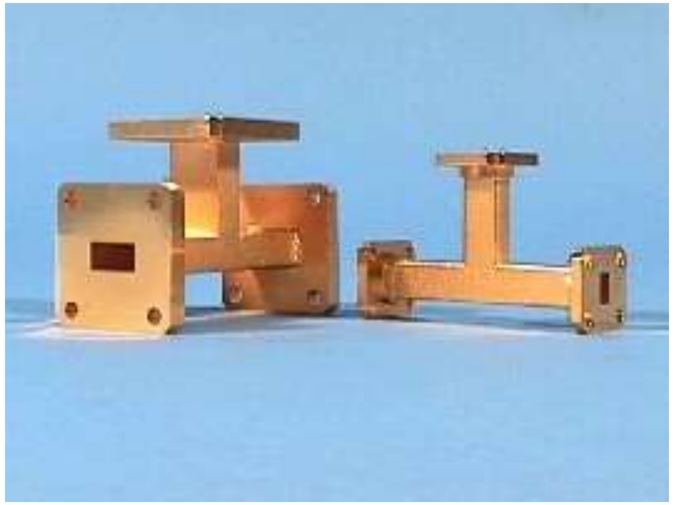
(b)

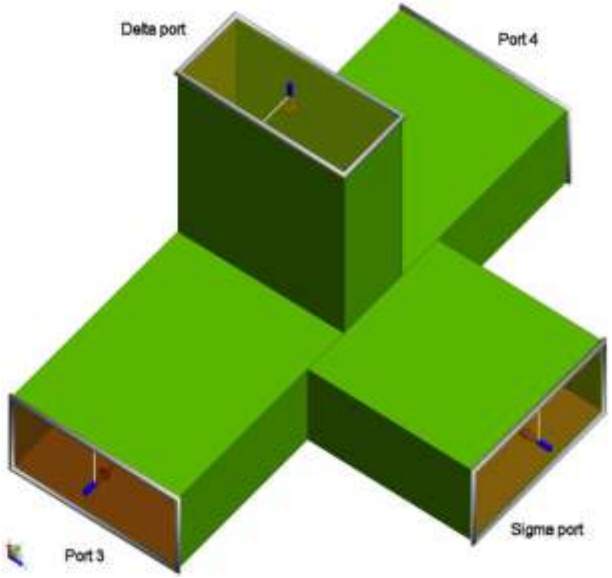
H Plane tees:

- ❖A waveguide tee in which the axis of its side arm is perpendicular to the E-field or parallel to the H-field of the main guide.
- ❖Input Output
- ❖Port 3- Port 1 and port 2 – same phase & same magnitude (additive)

S13 = S23

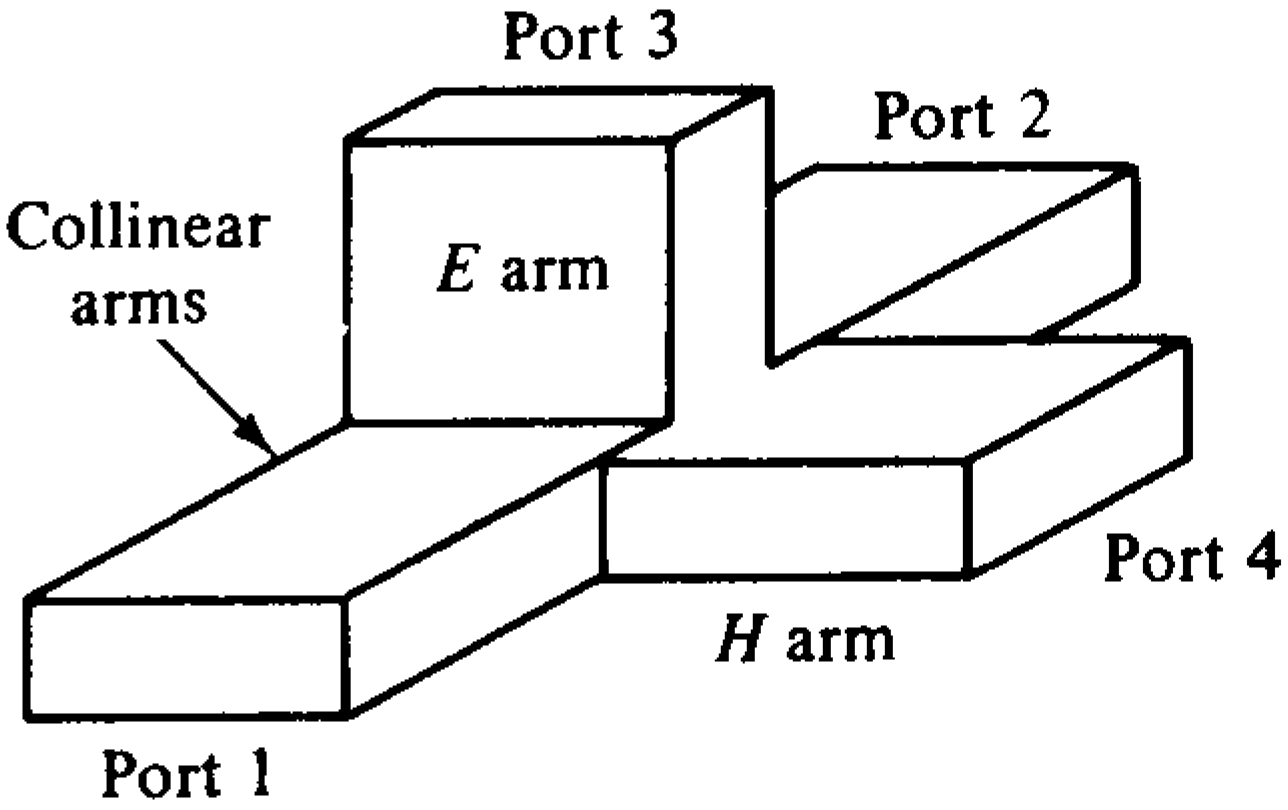





Delta port
Port 4
Sigma port
Port 3

Magic Tees (Hybrid Tees)

Combination of E-plane tee and H-plane tee.



Characteristics

1. If two waves of equal magnitude and the same phase are fed into port 1 and port 2, the output will be zero at port 3 and additive at port 4
2. If a wave is fed into port 4 (H arm), it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3 (E arm).
3. If a wave is fed into port 3 (E arm), it will produce an output of equal magnitude and opposite phase at port 1 and port 2. Output at port 4 is zero i.e $S_{43} = S_{34} = 0$.

4. If a wave is fed into one of the collinear arms at port 1 or port 2, it will not appear in the other collinear arm at port 2 or port 1 because the E arm causes a phase delay while the H arm causes the phase advance. i.e S12 = S21 = 0.

S matrix of magic tee is

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

Applications

Mixing

Duplexing

Impedance measurements.

Radar transmitters

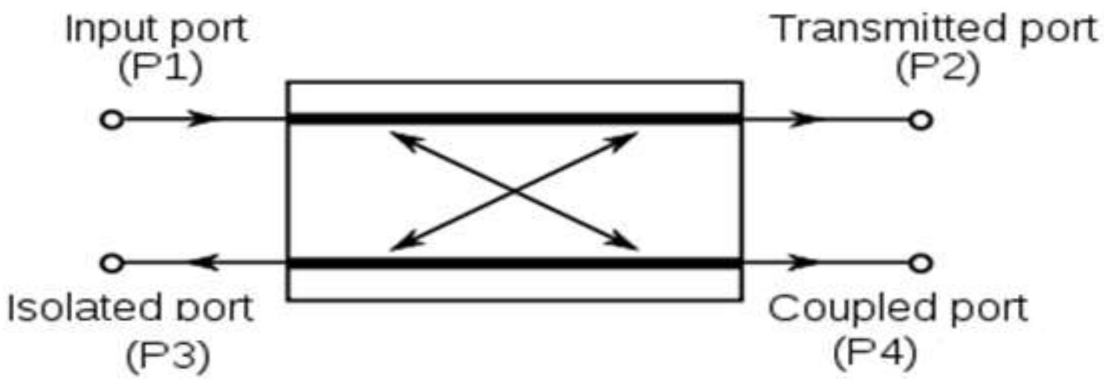
NOTE: The derivation of S-matrix for E-plane Tee, H-plane Tee & Magic Tee can be read from the Text book. "Microwave & radar Engineering by M Kulkarni, Umesh Publications" (chapter 6)

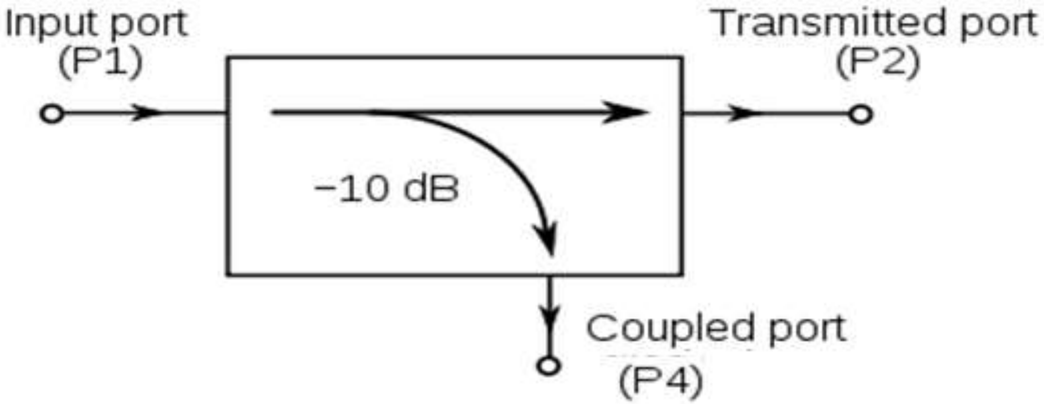
Directional coupler

- A directional coupler is a passive device which couples part of the transmission power by a known amount out through another port, often by using two transmission lines set close enough together such that energy passing through one is coupled to the other.

- The device has four ports: input, transmitted, coupled, and isolated. The term "main line" refers to the section between ports 1 and 2.

- Common properties desired for all directional couplers are wide operational bandwidth, high directivity, and a good impedance match at all ports when the other ports are terminated in matched loads.


Input port
(P1)
Transmitted port
(P2)
Isolated port
(P3)
Coupled port
(P4)


Input port
(P1)
Transmitted port
(P2)
−10 dB
Coupled port
(P4)

Bethe-hole directional coupler

- One of the most common, and simplest, waveguide directional couplers.
- This consists of two parallel waveguides, one stacked on top of the other, with a hole between them.
- Some of the power from one guide is launched through the hole into the other.

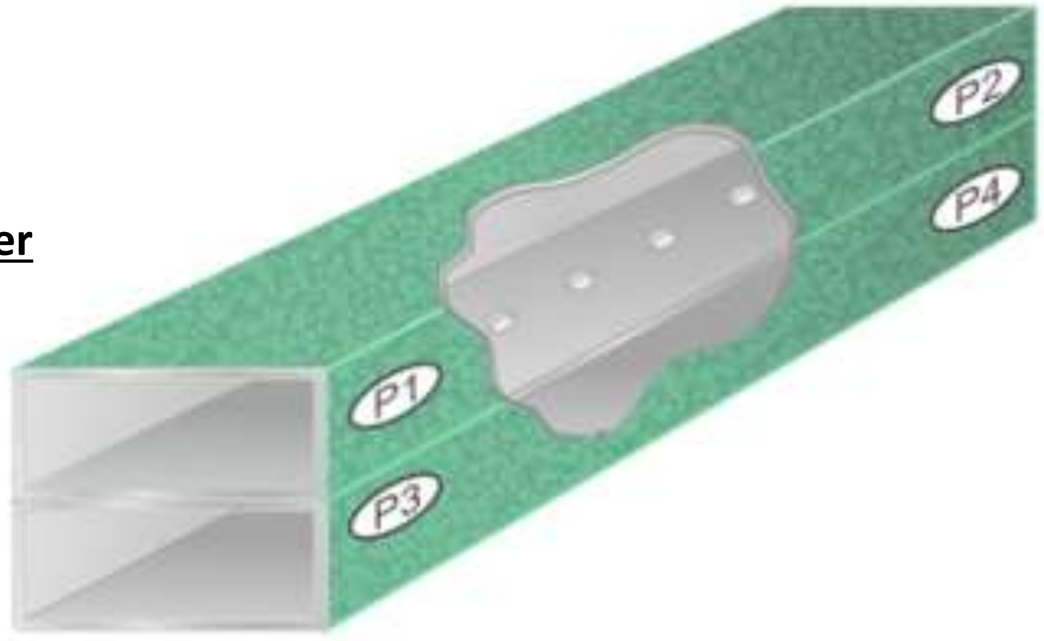


Figure: Multi Hole Directional Coupler

Cont…

- The concept of the Bethe-hole coupler can be extended by providing multiple holes. The holes are spaced $\lambda/4$ apart.

- The hole size is chosen to give the desired coupling between two waveguides.

- Design criteria are to achieve a substantially flat coupling together with high directivity over the desired band.

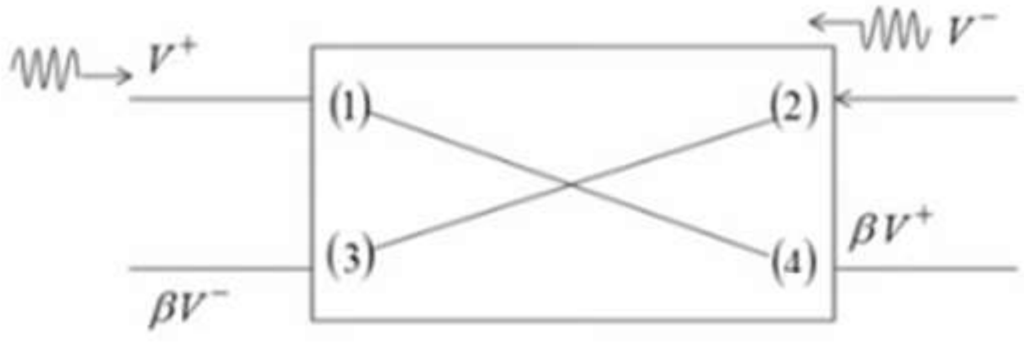
Properties of Directional Coupler

- A portion of power travelling from port 1 to port 2 is coupled to port 4 but does not go to port 3.

- A portion of power travelling from port 2 to port 1 is coupled to port 3 but does not go to port 4.

- Ports 1 and 3 are isolated and Ports 2 and 4 are isolated from each other.

How does it separate the incident wave and reflected wave for power measurements?



- Directivity is coupler's ability to separate the forward and backward waves. That is separation of incident and reflected waves.
- Some part of Incident wave at port 1 goes to the port 4. because port 1 and 3 are isolated.
- Now if there are reflections from port 2 , that reflected wave will go to port 3 only, because port 2 and 4 are isolated.
- So port 3 will have only part of reflected voltage and port 4 will have only part of incident voltage.

Performance Parameters

- Coupling Factor: How much of incident power is being coupled for the measurement purposes.

$$C = 10\log_{10}\left(\frac{P_1}{P_4}\right)$$

- Directivity: It is measure of how well the directional coupler distinguishes between the forward and reverse travelling waves.

$$D = 10\log_{10}\left(\frac{P_4}{P_3}\right)$$

- Isolation: It is the sum of Coupling factor and Directivity in dB.

$$I = 10\log_{10}\left(\frac{P_1}{P_3}\right)$$

S-matrix of directional coupler

- In a directional coupler all four ports are completely matched. Thus the diagonal elements of the S matrix are zero.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

- There is no coupling between port 1 and port 3 and between port 2 and port 4, thus

$$S_{13} = S_{31} = S_{24} = S_{42} = 0$$

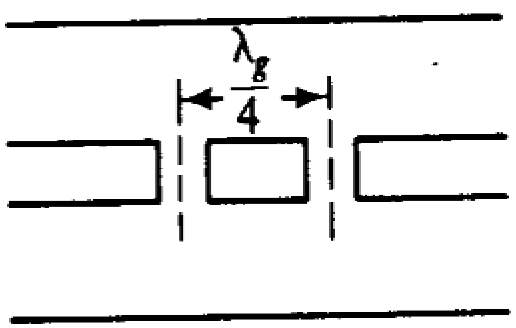
- Thus the S matrix of the directional coupler becomes,

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

- Using the symmetry and unitary properties of S parameter, it becomes

$$\mathbf{S} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$

- Several types of directional coupler are exit. Such as a two hole directional coupler, four hole directional coupler etc.



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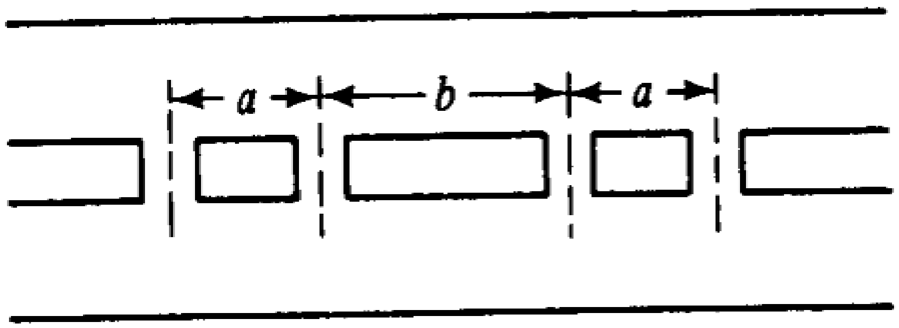

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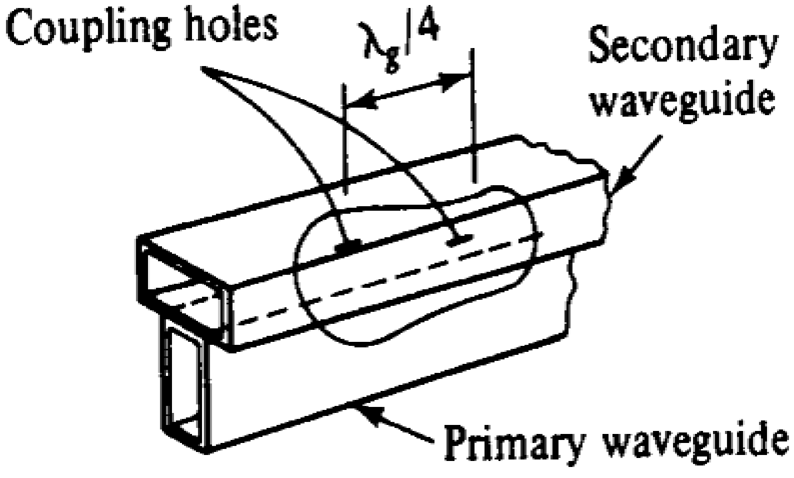
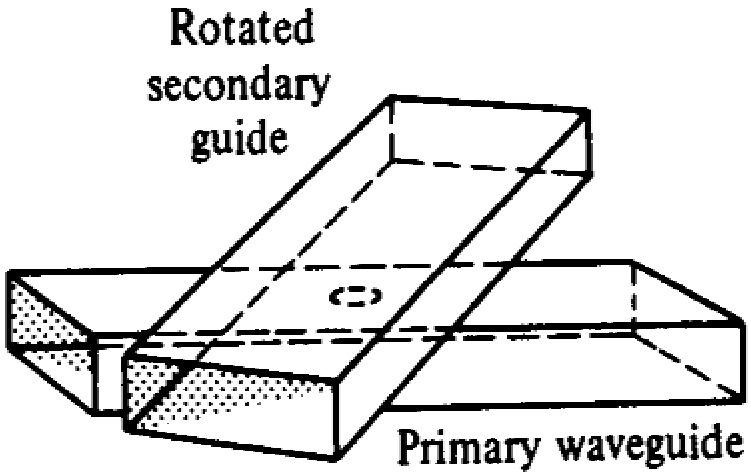
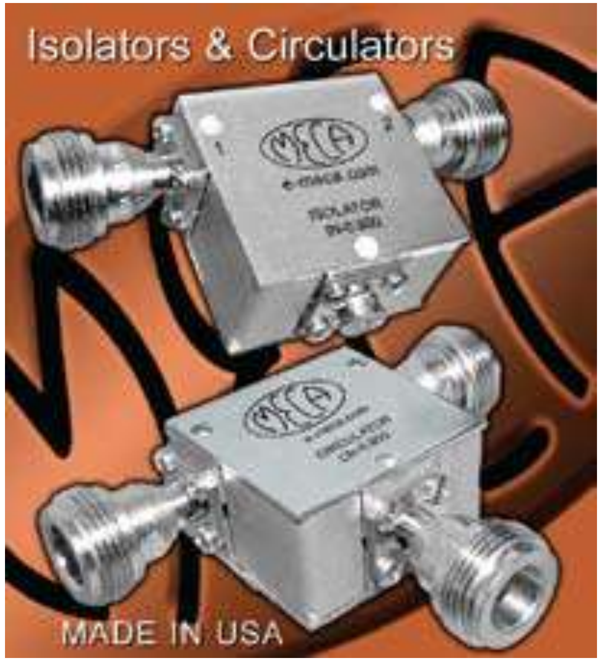




Isolator



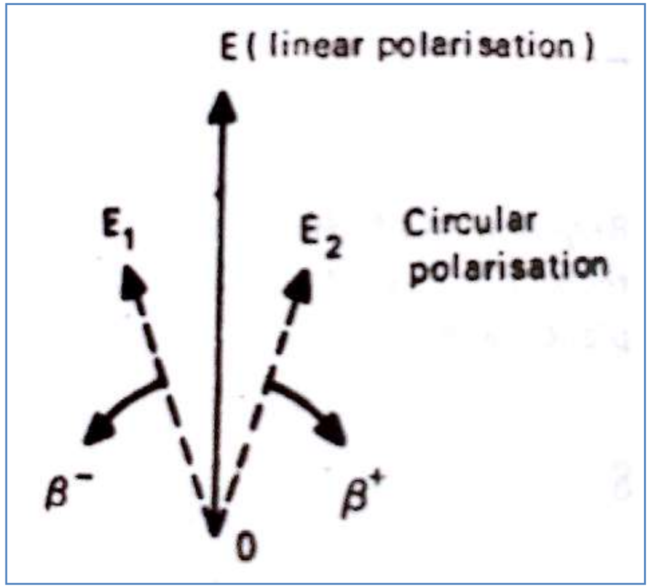
- RF isolator is a two port ferromagnetic passive device which is used to protect other RF components from excessive signal reflection.

- The interaction of the magnetic field to the ferrite material inside isolators and circulators.

- The rotary field is very strong and will cause any RF/microwave signals in the frequency band of interest at one port to follow the magnetic flow to the adjacent port and not in the opposite direction.

Farady's Rotation Principle

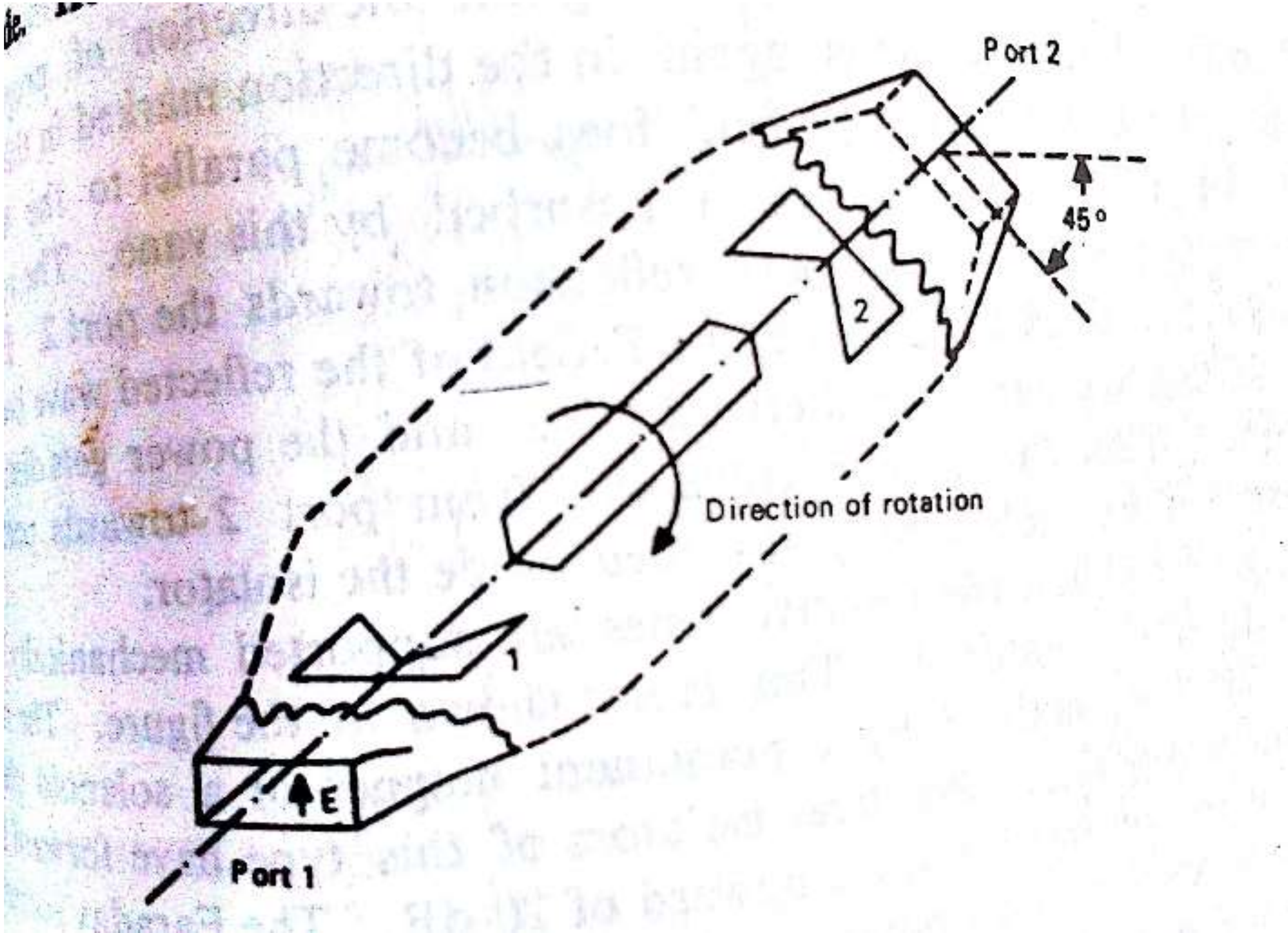
- "The rotation of direction of E-field of a linearly polarized wave occurs when passing through a ferrite medium."



•When the wave travels through one wavelength in ferrite material ,E field vector of LP wave rotates through an angle given by,

$$\Delta\Phi = \frac{\beta^{+}\lambda - \beta^{-}\lambda}{2}$$

ISOLATOR



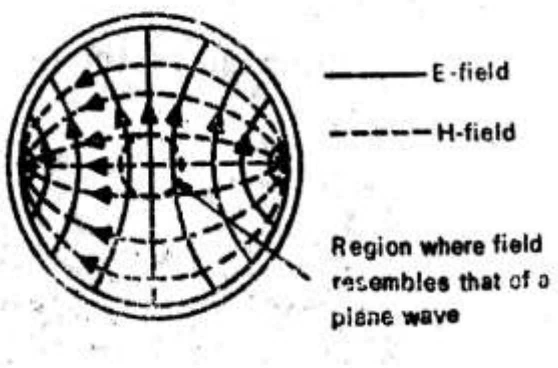
Construction of Isolator

- It uses a Faraday rotation of 45°. For this circular waveguide is used and a cylindrical ferrite rod is placed along the axis of circular waveguide.

- The ferrite rod is magnetized with stati magnetic field along the axis of waveguide.

- The circular waveguide is excited in dominant mode TE11. Thus across the c/ of the ferrite rod, field is linearly polarized.

- The length of ferrite rod and static magnetic field is chosen such that E field vector gets rotated by 45°.



Construction of Isolator

- At both the input and output ends, the circular waveguide is tapered to the rectangular waveguide sections.

- The orientation of input and output rectangular waveguides are at 45° to each other.

- The resistive cards are placed near the input and output ports and they are parallel to the broad walls of the rectangular sections.

- Resistive card at port 2 is also displaced 45° with respect to the input card.

Operation of Isolator

- A wave incident at port 1 has fixed E field direction. This excites TE11 mode in circular waveguide.

- The E field of TE11 mode is perpendicular to the resistive card numbered 1. hence power incident at port 1 is not affected by resistive card.

- Then wave encounters the magnetized ferrite rod and the direction of E field gets rotated by 45° clockwise.

- Since the orientation of port 2 and resistive card 2 is fixed at angle of 45 clockwise to port 1, the E field remains perpendicular to the resistive card 2.

- Power transmitted from 1 to 2 remains unattenuated.

Cont.

- In 2nd case, the reflected signal will have E field direction such that it is unaffected by resistive card 2.

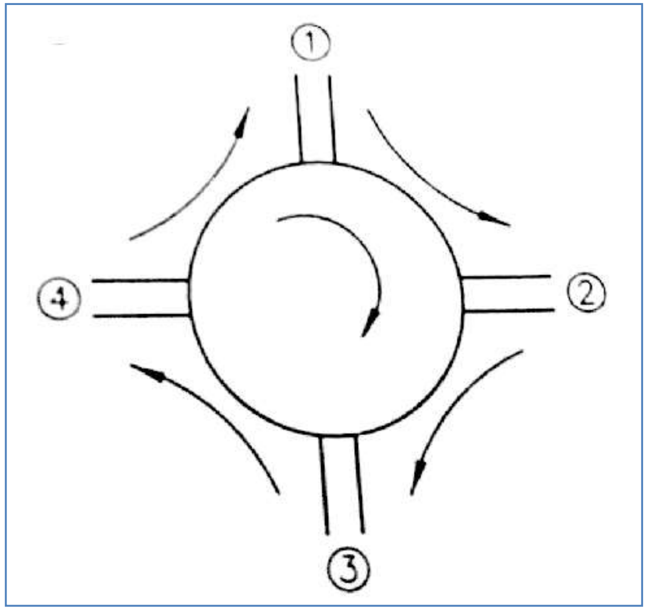
- While travelling along ferrite rod , it will have rotation of 45 clockwise.

- After this rotation , E field lines become parallel to the resistive card 1 and power is absorbed in this card.

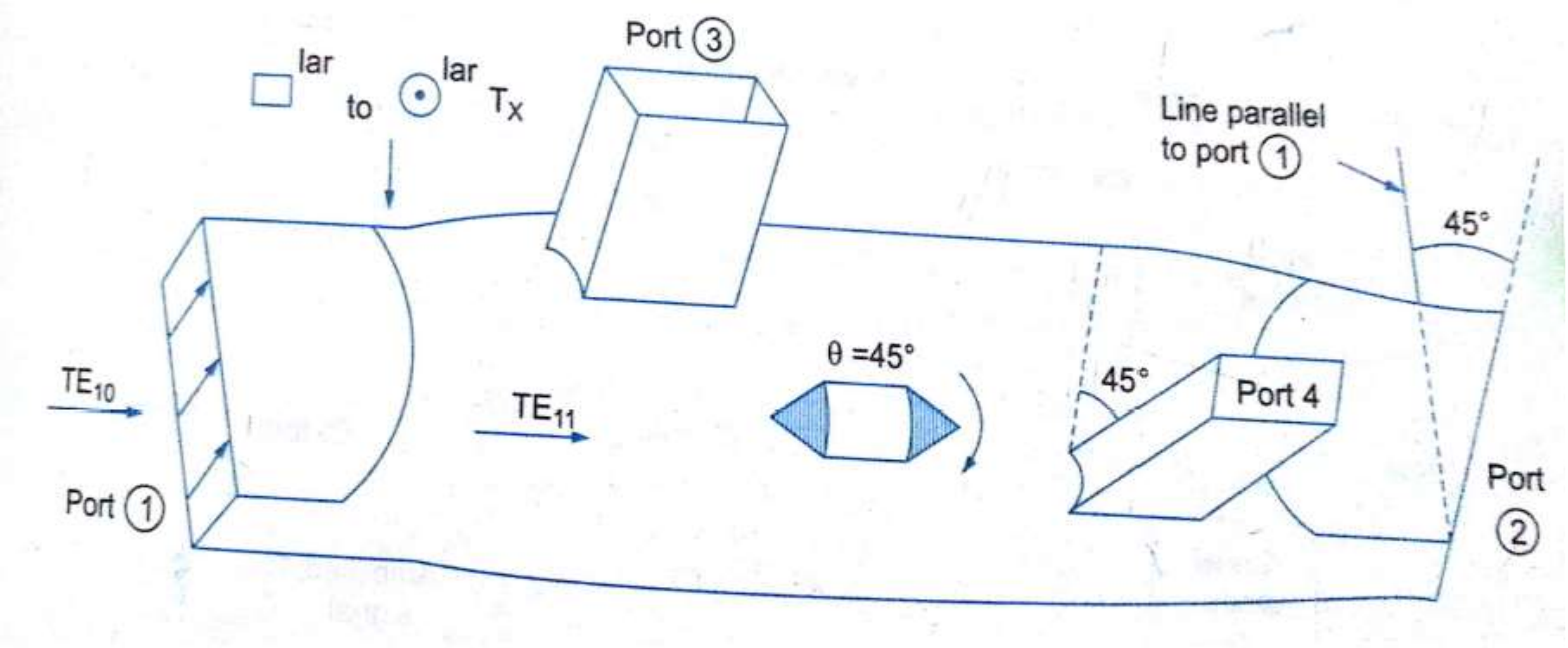
- Therefore, power travelling from 2 to 1 gets completely attenuated inside the isolator.

Circulator

- It is a 4-port microwave low power device which allows the power flow only from port 1 to 2, or port 2 to 3, or port 3 to 4, or port 4 to1 in clockwise direction.



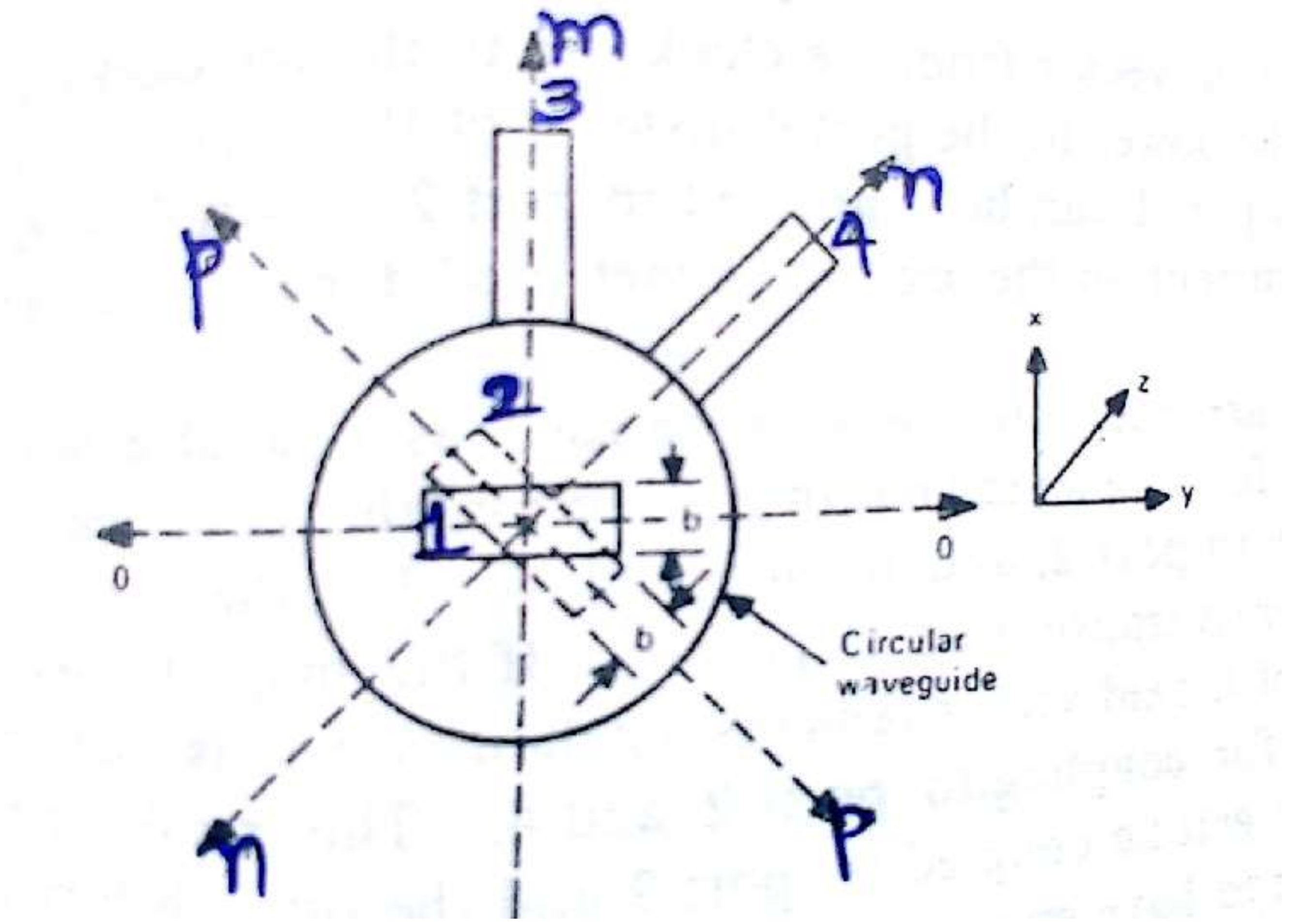
Construction of 4-port Circulator



Construction of 4-port Circulator

- It uses a magnetized ferrite rod to provide a 45° rotation of E field with the same arrangement as that of isolator.
- No resistive cards are required, instead power is coupled out to the required port.
- Ports 3 and 4 are oriented radially outward and are attached to circular section.
- Ports 1, 2, 3 and 4 are oriented such that E field can couple to these ports in numerical order after going through a rotation of 45° in clockwise direction in each step.

Operation of 4-port Circulator



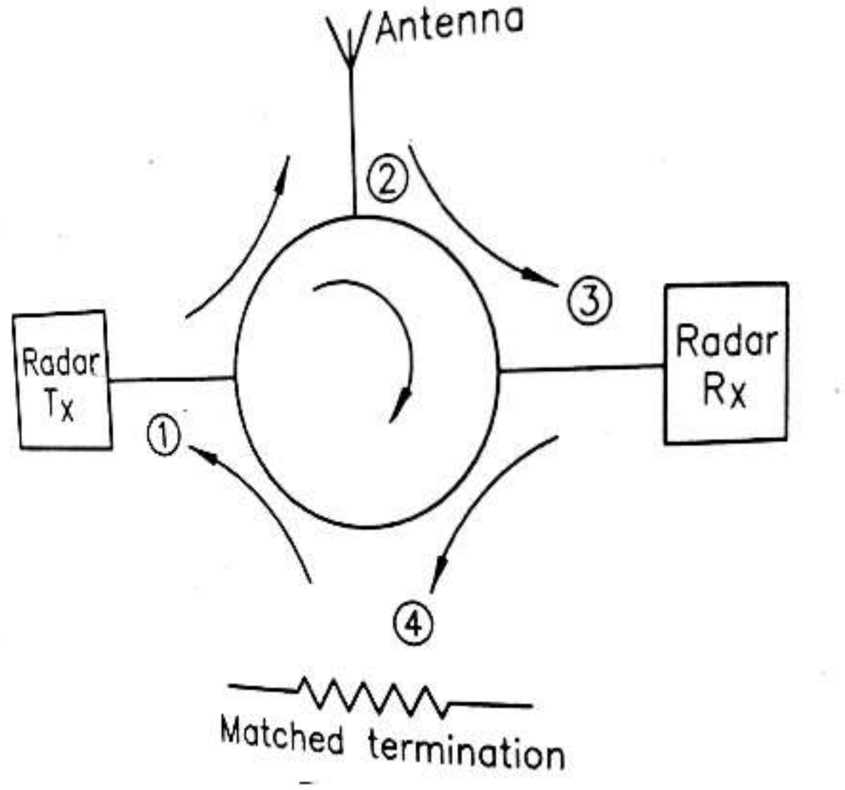
Operation of 4-port Circulator

- For a signal TE_{10} incident at port 1, E field is in the direction of “m”.
- When the wave travels along a magnetized ferrite, the direction of E field gets rotated by 45° and it coincides with the direction of “n”.
- So power incident at port 1 appears at port 2. It dose not coupled to port 3 and port 4, because E field is not significantly cut by these ports.
- Signal incident at port 2, gets rotated by 45° due to ferrite rode in clockwise direction, and it coincides with the direction of “o”. So power incident at port2 comes out of port 3 only.

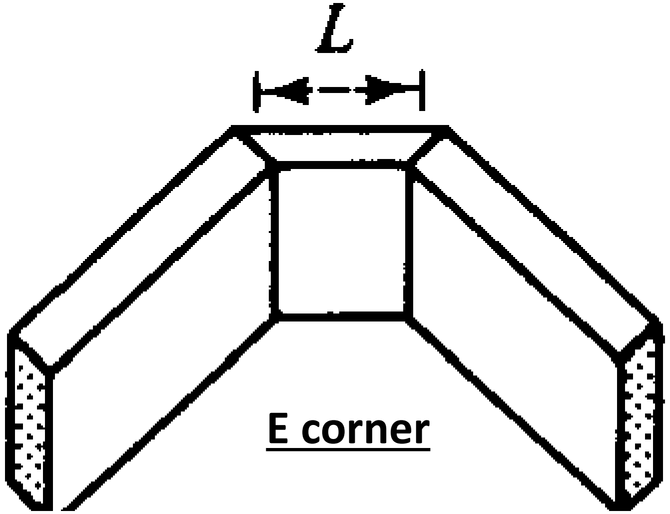
Applications of Circulator

- **Duplexer for Radar Antenna System**

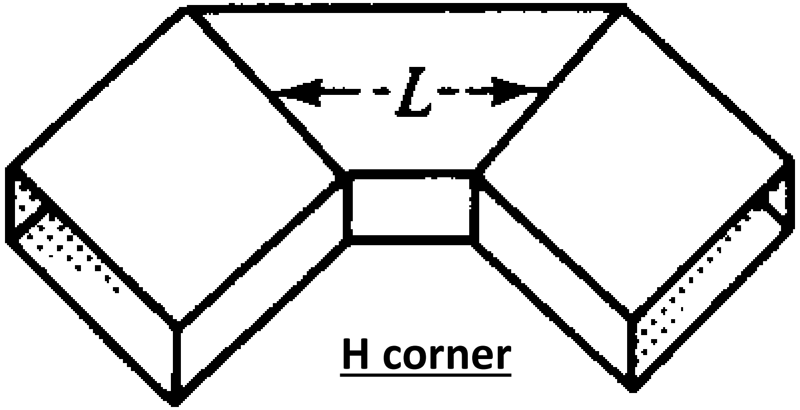
- Transmitter feeds the Antenna while the received energy is directed to the Rx.
- The high power transmitter can be isolated from the sensitive receiver.
- The same antenna can be used for transmission & reception. (Duplexer action)



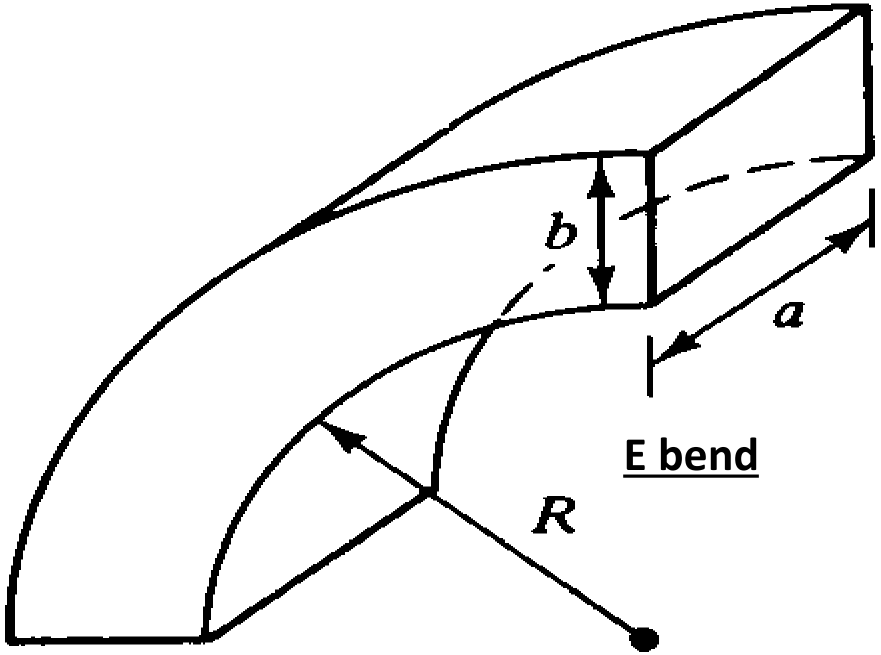
Waveguide Corner, Bends and Twists



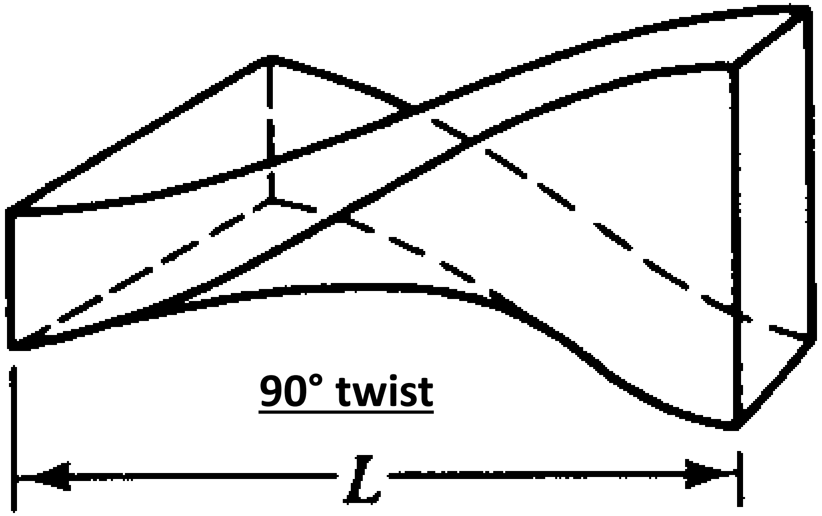
E corner



H corner



E bend



90° twist

- These waveguide components are normally used to **<u>change the direction of the guide through an arbitrary angle.</u>**

- **E bend**: the bend is in the direction of narrow wall dimension such that lines of E field are affected.
- Bends have to be gradual to minimize the reflections.
- **E-plane corner:** At low frequencies, 90 bending are preferred.
- In order to minimize reflections from the discontinuities, it is desirable to have the mean length "L", between continuities equal to an odd number of quarter-wave lengths, that is,

$$L = (2n + 1)\frac{\lambda_g}{4}$$

- If the mean length "L" is an odd number of quarter wavelengths, the reflected waves from both ends of the waveguide section are completely canceled.
- **Waveguide twists** are used to convert the vertical to horizontal polarization and vice versa.