Unit II

Transportation and Assignment Problems

Transportation model is a special case of LPP. In this kind of model, the objective function is to transport a homogeneous commodity from various origin to different destinations at a total minimum cost. Here the available units of products from sources is known and the demand of units of products at each destination is also known. It is defined to find how the arrangement should be made so, the transportation cost is minimum.

→ Transportation model is basically a minimization model;

There are methods for solution of transportation problem

→ North West corner method

→ Least cost method

→ Vogel's approximation method

8/70

North west corner method:-

1. Balance the problem, If not add a dummy column or dummy row as the case may be and balance the problem.

2. Start allocation from left hand side top most corner cell and make allocation depending on the availability and demand. If the availability is less than the requirement then for that cell make allocation in units which is equal to the availability.
which is the smallest among the availability and requirement and allocate the smallest one to the cell.

3. When the availability or demand is fulfill for that row or column respectively, remove that row or column from the matrix, prepare a new matrix.

Then proceed allocating either sidewise or downward to satisfy the rim requirement. Continue this until all the allocations is over.

Once all the allocates are over availability and demand are satisfied, write allocations and calculate the cost of transportation.

Markets (destinations)

	D	E	F	Supply
A	4	5	1	40
B	3	4	3	60
C	6	2	8	70
Demand	70	40	60	170

The given matrix is balanced, demand and supply is same, so next step.

2.

	D	E	F	Supply
A	4(40)	5	1	~~40~~ 0
B	3	4	3	60
C	6	2	8	70
Demand	~~70~~ 30	40	60	

Allocating min value of AD cell and eliminating the source A in the next step.

	D	E	F	Supply
B	3(30)	4	3	~~60~~ 30
C	6	2	8	70
Demand	~~30~~ 0	40	60	

The demand of market D is fullfilled.

	E	F	Supply
B	4(30)	3	~~30~~ 0
C	2	8	70
Demand	~~40~~ 10	60	

Paper units.

Sources

market (destinations)

	D	E	F	Supply
A	4(40)	5	1	
B	3(30)	4(30)	3	
C	6	2(10)	8(60)	
Demand				

total Cost

$4 \cdot 40 + 3 \cdot 30 + 4 \cdot 30 + 2 \cdot 10 + 8 \cdot 60$

$= 160 + 90 + 120 + 20 + 480 = 870$

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Least Cost Method: -

The lowest cost cell in the given matrix is AF having cost of 1. Allocating min of either demand or supply in the cell.

Markets (Destination)

	D	E	F	Supply
A	4	5	1(40)	~~40~~ 0
B	3	4	3	60
C	6	2	8	70
Demand	70	40	~~60~~ 20	

	D	E	F	Supply
B	3	4	3	60
C	6	2(40)	8	~~70~~ 30
Demand	70	~~40~~ 0	20	

	D	F	Supply
B	3 (40)	3(20)	~~60~~ 40
C	6 (30)	8	~~30~~
	~~70~~	~~20~~	

	D	E		Supply
A	4	5	1(40)	40
B	3(40)	4	3(20)	60
C	6(30)	2(40)	8	70
Dem	70	40	60	170

Calculating Total Cost

= 3×40 + 6×30 + 2×40 + 1×40 + 3×20

= 120 + 180 + 80 + 40 + 60

= 480/-

VAM :-

	D	E	F	Supply	Row
A	4	5	1	40	[3]
B	3	4	3	60	[1]
C	6	2(40)	8	~~70~~ 30	[4]
Dem	70	~~40~~ 0	60		
Column Diff	[1]	[2]	[3]		

	D	F	Supply	Row Diff
A	4	1(40)	~~40~~ 0	3
B	3	3	~~60~~ 60	0
C	6	8	~~30~~ 30	2
	70	~~60~~ 20		

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	D	F	Supply	Row
B	3	3(20)	~~60~~ 40	0
C	6	8( )	30	2
Demand	70	~~20~~ 0		
Colm	3	5 ↑		

	D	E	F	Supply
A	4	5	1(40)	40
B	3(40)	4	3(20)	60
C	6(30)	2(40)	8( )	70
Demand	70	40	60	

Cost = 1×40 + 3×40 + 3×20 + 6×30 + 40×2

= 40 + 120 + 60 + 180 + 80

= 480/-

M T W T F S S M T W T F S S M T W T F S S M T W T F S S M T W T
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29

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Unit 2

Assignment Problems:-

Assignment model is another type of special application of linear programming (LP).

Basically assignment model a minimization model of L.P. The assignment problem is a special case of the transportation problem, with specific values for the decision variables and constraint constants. The number of tasks is equal to number of individuals. The problem is to determine the assignment of individuals to tasks to minimize total cost (or time), given the costs of each individual for each task.

In assignment problem, neighter resources are divisible among jobs, nor are the jobs divisible among resources.

The assignment problem is defined as

		Tasks						
		1	2	3	-	-	-	n
Resources	1	c_{11}	c_{12}	c_{13}	-	-	-	c_{1n}
	2	c_{21}	c_{22}	-	-	-		c_{2n}
	3	c_{31}	c_{32}	-	-	-	-	c_{3n}
	n	c_{n1}	c_{n2}	-	-	-	-	c_{nn}

Mathematically, the assignment model can be expressed as

$$\text{Minimize } Z = \sum_{j=1}^{n} \sum_{i=1}^{n} c_{ij} x_{ij}$$

where x_{ij} denote the assignment of resource i to task j such that

$x_{ij} = 1$, if the i^{th} resource is assigned to j^{th} task.

$= 0$, if the i^{th} resource is not assigned to j^{th} task.

Sub to $\sum_{j=1}^{n} x_{ij} = 1 \; (i = 1, 2, \ldots, n)$

$\sum_{i=1}^{n} x_{ij}$ ... $x_{ij} \geq 0$

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2 Thousand

Hungarian method:- There are following steps are given by;

Step-1. Subtract the row minimum from each row

Step-2 Subtract the column minimum from each column from the reduced matrix.

Step-3. Assign one '0' to each row & column.

Step-4. Tick all unassigned rows.

Step-5 If a row is ticked and has an assignment then tick the corresponding row.

Step-6 If a column is ticked and has an assignment then tick the corresponding

Step-7 Repeat step 5 to 6 till no more ticks is possible

M T W T F S S M T W T F S S M T W T F S S M T W T F S S M T W T F
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Step 8 Draw lines through unticked rows and ticked columns. The number of lines represents the maximum number of assignments possible.

Step-9 find out the smallest number which does not have any line passing through it.

Assignment problem by Hungarian method.

Jobs

	A	B	C	D
1	45	40	51	67
2	55	40	61	53
Workers 3	49	52	48	64
4	41	45	60	55

The given matrix is square form. It is not required to perform step-1.

In each row, for finding minimum value, subtracting it from other elements in that row. We

Jobs

Workers

	A	B	C	D
1	5	0	11	27
2	15	0	21	13
3	1	4	0	16
4	0	4	19	14

3 Check, whether at least on zero in each column or no. If it is not then, accord subtracting the min value in that column from other ele of that column.

	A	B	C	D
1	5	0	11	14
2	15	0	21	0
3	1	4	0	3
4	0	4	19	1

Step Identifying rows one by one from top to bottom until one zero is found co marking that single zero by a square in that row. Simultaneously, other zero in that crossing column we get following mat

	A	B	C	D
1	5	$\boxed{0}$	11	14
2	15	~~0~~	21	$\boxed{0}$
3	1	4	$\boxed{0}$	3
4	$\boxed{0}$	4	19	1

Step-4 No need to further check in column, there is one assignment in each row and in each column. The soln is optimal.

min cost = 41 + 40 + 48 + 53
= 182.

Jobs

		A	B	C	D
Workers	1	50	70	110	60
	2	80	50	90	60
	3	40	70	100	70
	4	100	40	60	30

The given matrix is square, no further do sum

		A	B	C	D
Workers	1	0	30	60	70
	2	30	0	40	10
	3	0	30	60	30
	4	70	10	50	10

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Checking atleast one zero in each column if there [illegible] the min value on there [illegible] other elements of the [illegible]

	A	B	C	D	
1	$\boxed{0}$	20	20	10	✓(3)
2	30	$\boxed{0}$	$\boxed{\cancel{0}}$	10	
3	$\cancel{0}$	30	20	30	✓(1)
4	70	10	10	$\boxed{0}$	

Assignment with each row and each column is not completed. So soln is not optimal

Jobs

Workers

	A	B	C	D
1	50	70	110	60
2	80	50	90	60
3	40	70	100	70
4	100	40	80	30

Row min

	A	B	C	D
1	0	20	60	10
2	30	0	40	10
3	0	30	60	30
4	70	10	50	0

Column min

	A	B	C	D
1	0	20	20	10
2	30	0	0	10

	A	B	C	D	
1	$\boxed{0}$	20	20	10	✓
2	30	$\boxed{0}$	$\cancel{0}$	10	
3	$\cancel{0}$	30	20	30	✓
4	70	10	10	$\boxed{0}$	
	✓				

1. Tick unassigned rows
2. If a ticked row has a zero, tick the corresponding column.
3. If a ticked column has an assignment, tick the corresponding row.
4. Repeat steps 2 and 3 till no more ticks are possible
5. Draw lines through unticked rows and ticked columns

$\cancel{0}$	10	10	$\cancel{0}$	✓
40	$\boxed{0}$	$\cancel{0}$	10	
$\boxed{0}$	20	10	20	✓
80	10	10	$\boxed{0}$	✓
✓				

$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$
50	$\cancel{0}$	$\boxed{0}$	20
$\boxed{0}$	[illegible]	[illegible]	[illegible]
80	$\boxed{0}$	0	$\cancel{0}$

230

Jobs

Workers

	A	B	C	D
1	24	10	21	11
2	14	22	10	15
3	15	17	20	19
4	11	19	14	13

Row min

	A	B	C	D
1	14	0	11	1
2	4	12	0	5
3	0	2	5	4
4	0	8	3	2

Column min

	A	B	C	D	
1	14	$\boxed{0}$	11	$\cancel{0}$	
2	4	12	$\boxed{0}$	4	
3	$\boxed{0}$	2	5	3	✓
4	$\cancel{0}$	8	3	1	✓
	✓				

adding the min value of uncoved row at intersection point and subtract by other uncovered element.

	A	B	C	D
1	15	$\boxed{0}$	11	$\cancel{0}$
2	5	12	$\boxed{0}$	4
3	$\boxed{0}$	1	4	2
4	$\cancel{0}$	7	2	$\boxed{0}$

$15 + 10 + 10 + 13 = \underline{48}$