ज्ञानेन प्रकाशते जगत्

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Unit-2

B.Tech. (Electronics and Communication)

Semester-VI

Asst. Prof. Zalak Patel

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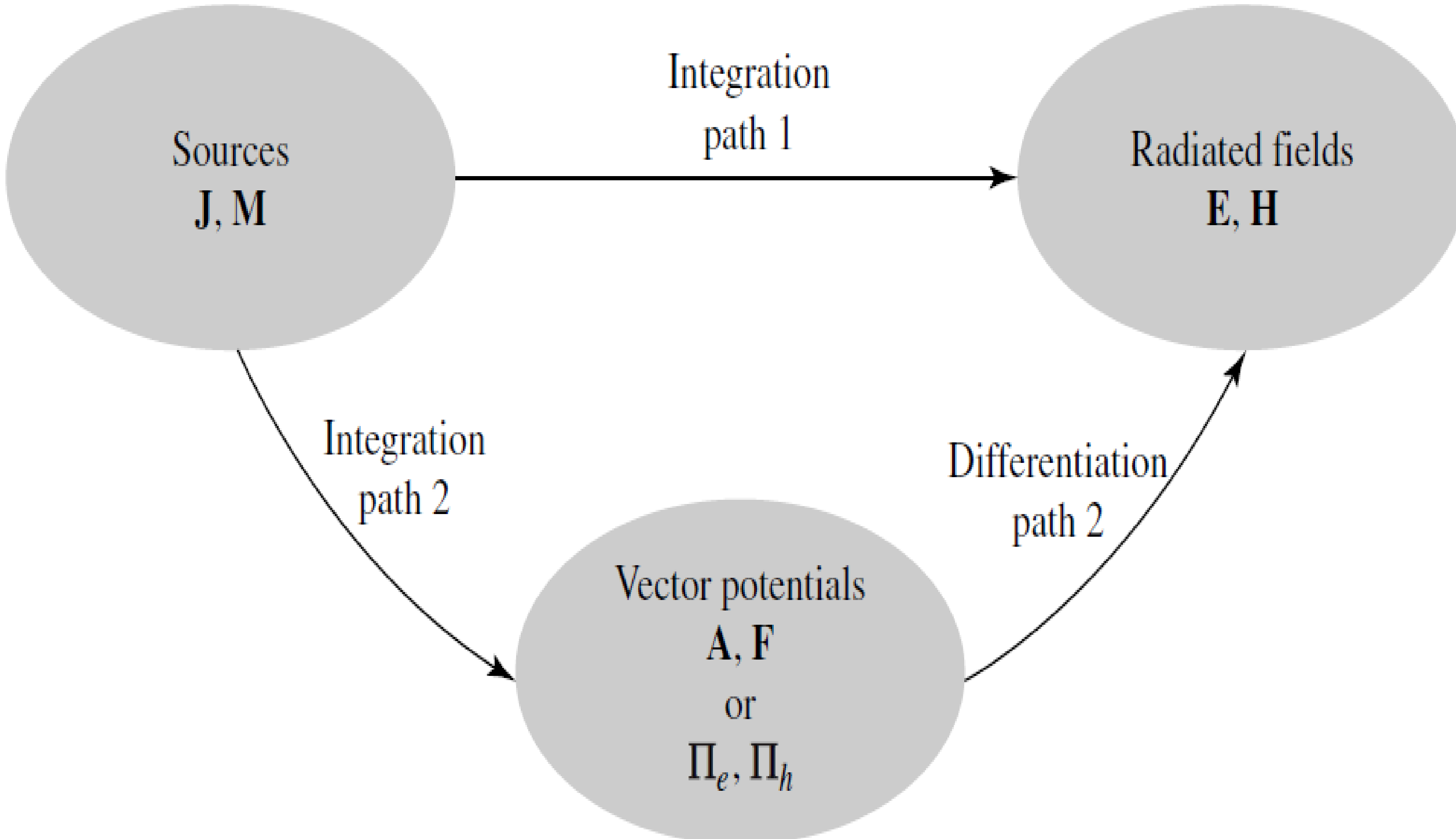
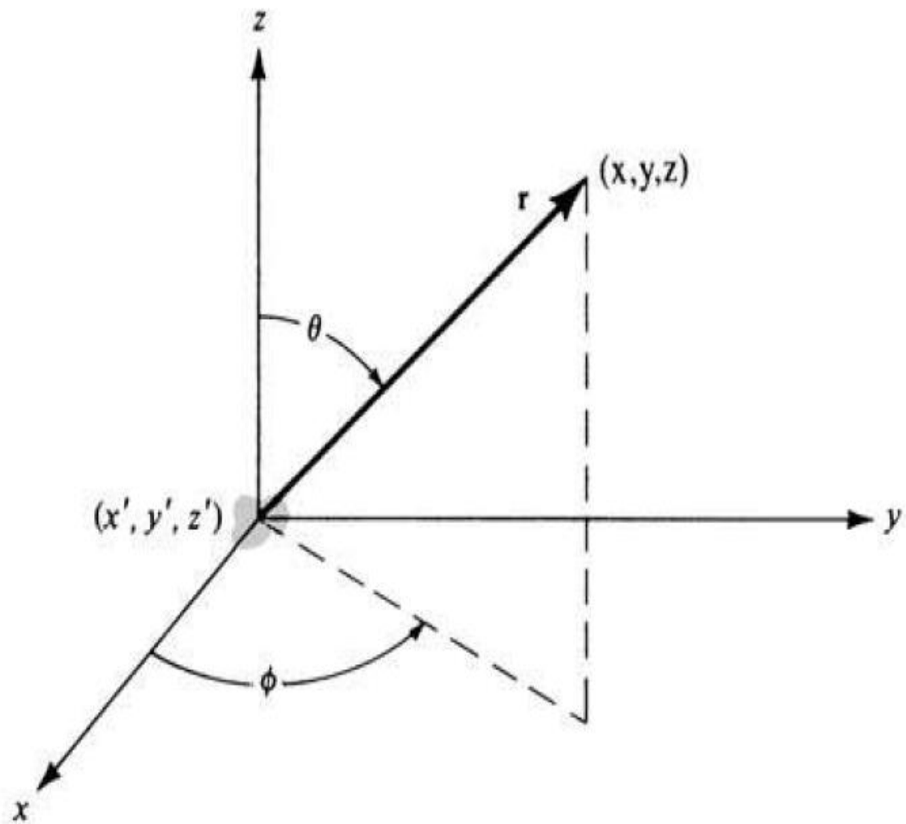
Radiation Integrals and Auxiliary Potential Functions


Integration
path 1
Sources
J, **M**
Radiated fields
E, **H**
Integration
path 2
Differentiation
path 2
Vector potentials
A, **F**
or
Π_e, Π_h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(x,y,z)
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(x′, y′, z′)
y
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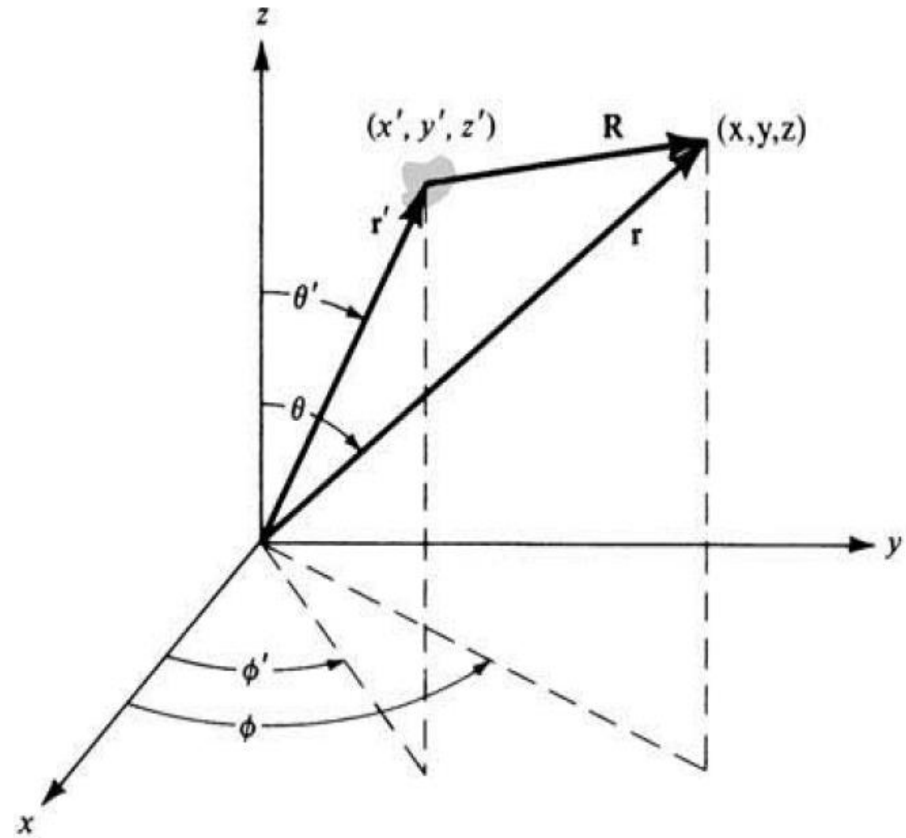
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THE VECTOR POTENTIAL A FOR AN ELECTRIC CURRENT SOURCE J

- The vector potential A is useful in solving for the EM field generated by a given harmonic electric current J. The magnetic flux B is always solenoidal; that is, $\nabla \cdot B = 0$.
- Therefore, it can be represented as the curl of another vector because it obeys the vector identity.

$$\nabla \cdot \nabla \times A = 0$$

$$B = \mu H = \nabla \times A$$

- Maxwell's curl equation

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \times E = -j\omega\mu H = -j\omega\nabla \times A$$

$$\nabla\times[E+j\omega A]=0$$

- From the vector identity

$$\nabla\times(-\nabla\phi e)=0$$

$$E+j\omega A=-\nabla\phi e$$

$$E=-\nabla\phi e-j\omega A$$

- Taking the curl of both sides

$$\nabla\times(\mu H)=\nabla(\nabla\cdot A)-\nabla^2 A$$

- For a homogeneous medium

$$\mu\nabla\times H=\nabla(\nabla\cdot A)-\nabla^2 A$$

- Maxwell's equation

$$\nabla\times H=J+j\omega\varepsilon E$$

$$\mu J + j\omega\mu\varepsilon E = \nabla(\nabla \cdot A) - \nabla^2 A$$

$$\nabla^2 A + k A^2 = -\mu J + \nabla(\nabla \cdot A) + \nabla(j\omega\mu\varepsilon\phi e) = -\mu J + \nabla(\nabla \cdot A + j\omega\mu\varepsilon\phi e)$$

- Now in order to simplify above equation
- The divergence of A

$$\nabla \cdot \mathbf{A} = -j\omega\epsilon\mu\phi_e \Rightarrow \phi_e = -\frac{1}{j\omega\mu\epsilon}\nabla \cdot \mathbf{A}$$

- Which is known as the Lorentz condition

$$\nabla^2\mathbf{A} + k^2\mathbf{A} = -\mu\mathbf{J}$$

$$\mathbf{E}_A = -\nabla\phi_e - j\omega\mathbf{A} = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla \cdot \mathbf{A})$$

THE VECTOR POTENTIAL F FOR A MAGNETIC CURRENT SOURCE M

- Although magnetic currents appear to be physically unrealizable.
- The fields generated by a harmonic magnetic current in a homogeneous region, with J = 0 but M = 0, must satisfy ∇ · D = 0.
- Therefore, E can be expressed as the curl of the vector potential F by

$$\mathbf{E}_F = -\frac{1}{\epsilon}\nabla \times \mathbf{F}$$

$$\nabla \times \mathbf{H}_F = j\omega\epsilon\mathbf{E}_F$$

$$\nabla \times (\mathbf{H}_F + j\omega\mathbf{F}) = 0$$

- From the vector identity,

$$\mathbf{H}_F = -\nabla\phi_m - j\omega\mathbf{F}$$

- Taking the curl

$$\nabla \times \mathbf{E}_F = -\frac{1}{\epsilon}\nabla \times \nabla \times \mathbf{F} = -\frac{1}{\epsilon}[\nabla\nabla\cdot\mathbf{F} - \nabla^2\mathbf{F}]$$

- Maxwell's equation

$$\nabla \times \mathbf{E}_F = -\mathbf{M} - j\omega\mu\mathbf{H}_F$$

$$\nabla^2\mathbf{F} + j\omega\mu\epsilon\mathbf{H}_F = \nabla\nabla\cdot\mathbf{F} - \epsilon\mathbf{M}$$

$$\nabla^2\mathbf{F} + k^2\mathbf{F} = -\epsilon\mathbf{M} + \nabla(\nabla\cdot\mathbf{F}) + \nabla(j\omega\mu\epsilon\phi_m)$$

- Now suppose,

$$\nabla \cdot \mathbf{F} = -j\omega\mu\epsilon\phi_m \Rightarrow \phi_m = -\frac{1}{j\omega\mu\epsilon}\nabla \cdot \mathbf{F}$$

$$\nabla^2\mathbf{F} + k^2\mathbf{F} = -\epsilon\mathbf{M}$$

$$\mathbf{H}_F = -j\omega\mathbf{F} - \frac{j}{\omega\mu\epsilon}\nabla(\nabla \cdot \mathbf{F})$$

ELECTRIC & MAGNETIC FIELDS FOR ELECTRIC (J) & MAGNETIC (M) CURRENT SOURCES

- The electric and magnetic fields generated by an electric current source J and a magnetic current source M. The procedure requires that the auxiliary potential functions A and F generated, respectively, by J and M are found first.
- The total fields are then obtained by the superposition of the individual fields due to A and F (J and M).
- Find A,

$$\mathbf{A} = \frac{\mu}{4\pi} \iiint_V \mathbf{J} \frac{e^{-jkR}}{R} \, dv'$$

- Find F,

$$\mathbf{F} = \frac{\epsilon}{4\pi} \iiint_V \mathbf{M} \frac{e^{-jkR}}{R} \, dv'$$

- The total fields are then determined by,

$$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_F = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\mathbf{A}) - \frac{1}{\epsilon}\nabla \times \mathbf{F}$$

$$\mathbf{H} = \mathbf{H}_A + \mathbf{H}_F = \frac{1}{\mu}\nabla \times \mathbf{A} - j\omega\mathbf{F} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\mathbf{F})$$

Far Field Radiation

- The fields radiated by antennas of finite dimensions are spherical waves.
- For these radiators, a general solution to the vector wave equation of in spherical components , each as a function of r, θ, ϕ, takes the general form of

$$\mathbf{A} = \hat{\mathbf{a}}_r A_r(r,\theta,\phi) + \hat{\mathbf{a}}_\theta A_\theta(r,\theta,\phi) + \hat{\mathbf{a}}_\phi A_\phi(r,\theta,\phi)$$

- The amplitude variations of r in each component of are of the form 1/rn, Neglecting higher order terms of 1/rn,

$$\mathbf{A} \simeq [\hat{\mathbf{a}}_r A'_r(\theta,\phi) + \hat{\mathbf{a}}_\theta A'_\theta \theta,\phi) + \hat{\mathbf{a}}_\phi A'_\phi(\theta,\phi)]\frac{e^{-jkr}}{r},$$

$$\mathbf{E} = \frac{1}{r}\{-j\omega e^{-jkr}[\hat{\mathbf{a}}_r(0) + \hat{\mathbf{a}}_\theta A'_\theta(\theta,\phi) + \hat{\mathbf{a}}_\phi A'_\phi(\theta,\phi)]\} + \frac{1}{r^2}\{\cdots\} + \cdots$$

$$\mathbf{H} = \frac{1}{r}\left\{j\frac{\omega}{\eta}e^{-jkr}[\hat{\mathbf{a}}_r(0) + \hat{\mathbf{a}}_\theta A'_\phi(\theta,\phi) - \hat{\mathbf{a}}_\phi A'_\theta(\theta,\phi)]\right\} + \frac{1}{r^2}\{\cdots\} + \cdots$$

- Neglecting higher order terms of 1/rn, the radiated E- and H-fields have only θ and ϕ components. They can be expressed as,

$$\left.\begin{aligned} E_r &\simeq 0 \\ E_\theta &\simeq -j\omega A_\theta \\ E_\phi &\simeq -j\omega A_\phi \end{aligned}\right\} \Rightarrow \boxed{\mathbf{E}_A \simeq -j\omega \mathbf{A}}$$

(for the θ and ϕ components only since $E_r \simeq 0$)

$$\left.\begin{aligned} H_r &\simeq 0 \\ H_\theta &\simeq +j\frac{\omega}{\eta}A_\phi = -\frac{E_\phi}{\eta} \\ H_\phi &\simeq -j\frac{\omega}{\eta}A_\theta = +\frac{E_\theta}{\eta} \end{aligned}\right\} \Rightarrow \boxed{\mathbf{H}_A \simeq \frac{\hat{\mathbf{a}}_r}{\eta} \times \mathbf{E}_A = -j\frac{\omega}{\eta}\hat{\mathbf{a}}_r \times \mathbf{A}}$$

(for the θ and ϕ components only since $H_r \simeq 0$)

Reciprocity

- The reciprocity theorem, as applied to circuits, which states that "in any network composed of linear, bilateral, lumped elements, if one places a constant current (voltage) source between two nodes (in any branch) and places a voltage (current) meter between any other two nodes (in any other branch), makes observation of the meter reading, then interchanges the locations of the source and the meter, the meter reading will be unchanged"
- Let us assume that within a linear and isotropic medium, but not necessarily homogeneous, there exist two sets of sources J1,M1, and J2,M2 which are allowed to radiate simultaneously or individually inside the same medium at the same frequency and produce fields E1,H1 and E2,H2, respectively.
- The sources and fields satisfy,

$$-\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = \mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{H}_2 \cdot \mathbf{M}_1 - \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_1 \cdot \mathbf{M}_2$$

- Which is called the Lorentz Reciprocity Theorem in differential form.
- Taking a volume integral of both sides, and using the divergence theorem on the left side.

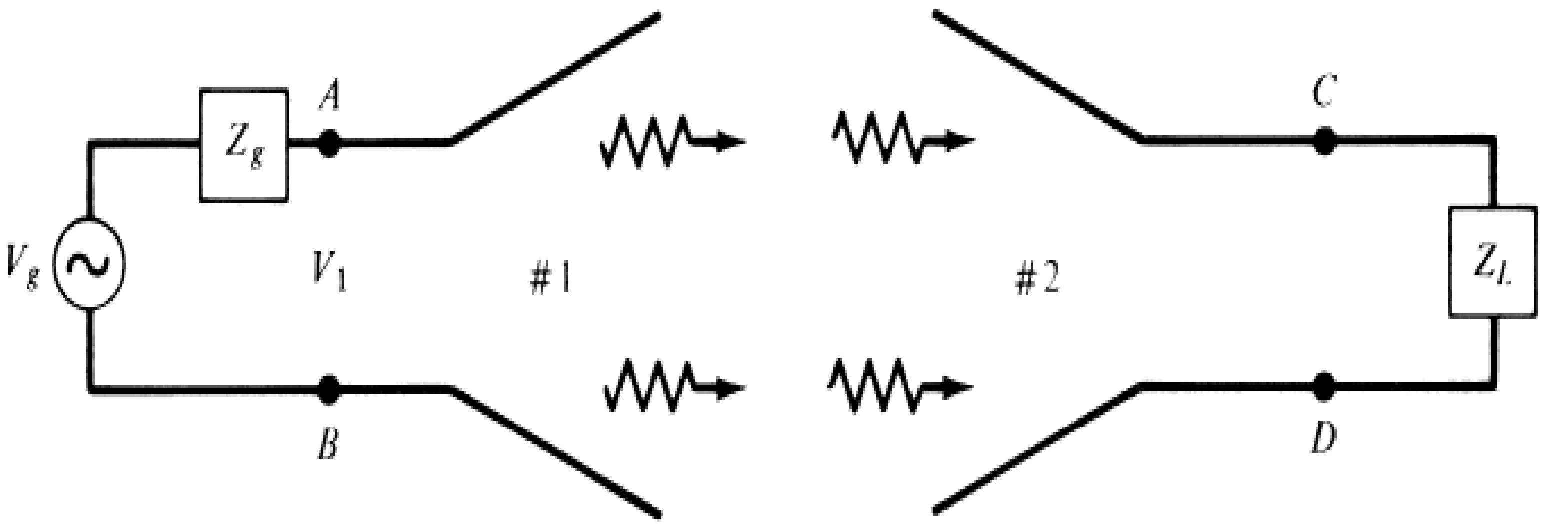
$$-\oiint_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{s}'$$

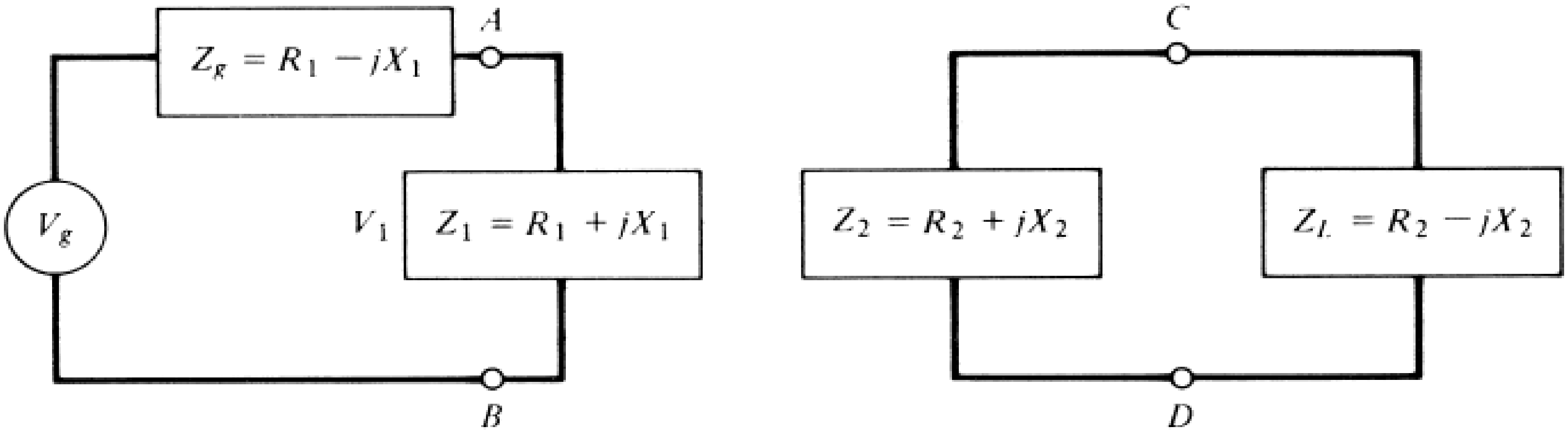
$$= \iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{H}_2 \cdot \mathbf{M}_1 - \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_1 \cdot \mathbf{M}_2)\, dv'$$

- Which is designated as the Lorentz Reciprocity Theorem in integral form.
- For a source-free (**J1 = J2 = M1 = M2 = 0) region,**

$$\nabla \cdot (\mathrm{E}_1 \times \mathrm{H}_2 - \mathrm{E}_2 \times \mathrm{H}_1) = 0$$

- **Reciprocity for Two Antennas:** Two antennas, whose input impedances are Z1 and Z2, are separated by a linear and isotropic medium.
- One antenna (#1) is used as a transmitter and the other (#2) as a receiver.
- The internal impedance of the generator Zg is assumed to be the conjugate of the impedance of antenna #1 while the load impedance ZL is equal to the conjugate of the impedance of antenna #2





- The power delivered by the generator to antenna #1 is given by

$$P_1 = \frac{1}{2}\mathrm{Re}[V_1 I_1^*] = \frac{1}{2}\mathrm{Re}\left[\left(\frac{V_g Z_1}{Z_1 + Z_g}\right)\frac{V_g^*}{(Z_1 + Z_g)^*}\right] = \frac{|V_g|^2}{8R_1}$$

- If the transfer admittance of the combined network consisting of the generator impedance, antennas, and load impedance is Y21, the current through the load is VgY21 and the power delivered to the load is,

$$P_2 = \tfrac{1}{2}\mathrm{Re}[Z_2(V_g Y_{21})(V_g Y_{21})^*] = \tfrac{1}{2}R_2|V_g|^2|Y_{21}|^2$$

- The ratio is,

$$\frac{P_2}{P_1} = 4R_1 R_2 |Y_{21}|^2$$

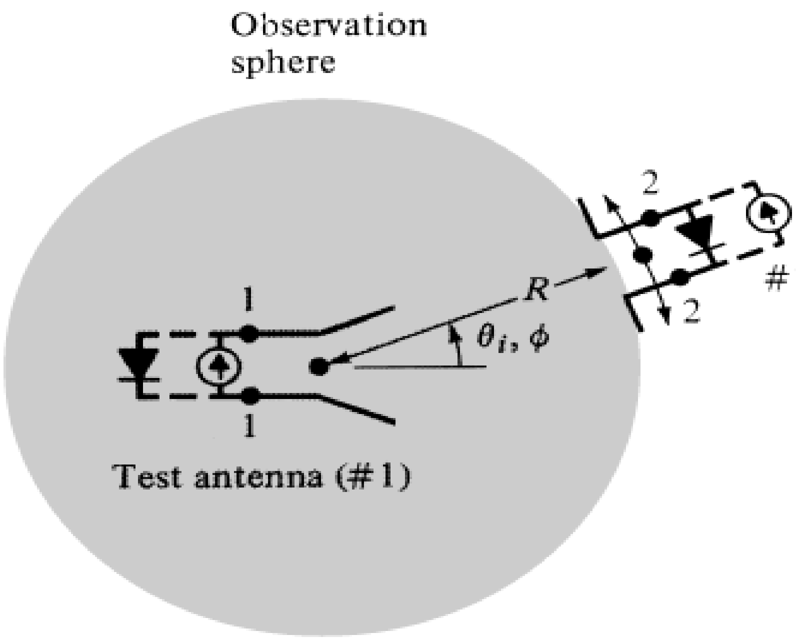
- In a similar manner, we can show that when antenna #2 is transmitting and #1 is receiving, the power ratio of P1/P2 is given by

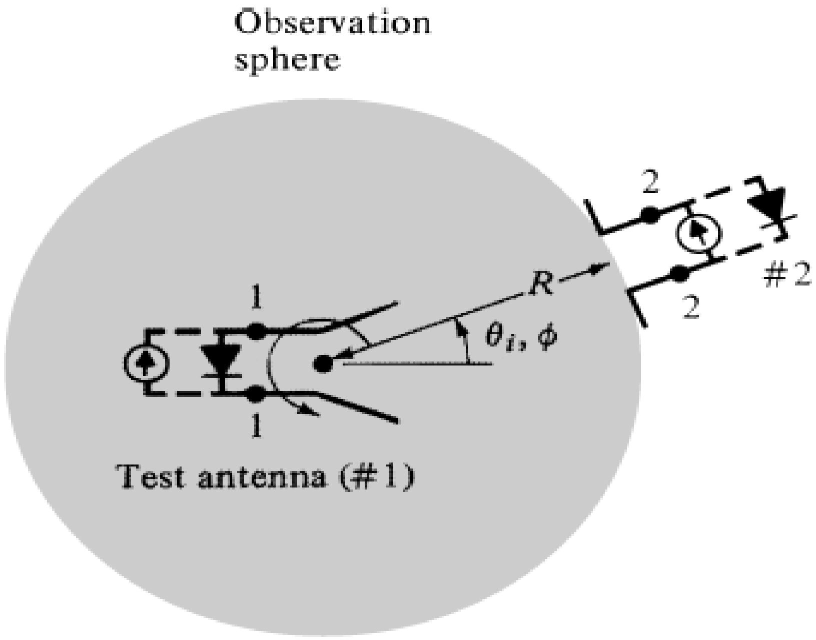
$$\frac{P_1}{P_2} = 4R_2 R_1 |Y_{12}|^2$$

- Under conditions of reciprocity (Y12 = Y21), the power delivered in either direction is the same.

Reciprocity for Antenna Radiation Patterns

- The radiation pattern is a very important antenna characteristic.
- The only other restriction for reciprocity to hold is for the antennas in the transmit and receive modes to be polarization matched, including the sense of rotation.





- The antenna under test is #1 while the probe antenna (#2) is oriented to transmit or receive maximum radiation. The voltages and currents V1, I1 at terminals 1–1 of antenna #1 and V2, I2 at terminals 2–2 of antenna #2 are related by,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \text{self-impedance of antenna \#1}$$

$$Z_{22} = \text{self-impedance of antenna \#2}$$

$$Z_{12}, Z_{21} = \text{mutual impedances between antennas \#1 and \#2}$$

- If a current I1 is applied at the terminals 1–1 and voltage V2 (designated as V2oc) is measured at the open (I2 = 0) terminals of antenna #2, then an equal voltage V1oc will be measured at the open (I1 = 0) terminals of antenna #1 provided the current I2of antenna #2 is equal to I1.

$$Z_{21} = \left.\frac{V_{2oc}}{I_1}\right|_{I_2=0}$$

$$Z_{12} = \left.\frac{V_{1oc}}{I_2}\right|_{I_1=0}$$

- If the medium between the two antennas is linear, passive, isotropic, and the waves monochromatic, then because of reciprocity,

$$Z_{21} = \left.\frac{V_{2oc}}{I_1}\right|_{I_2=0} = \left.\frac{V_{1oc}}{I_2}\right|_{I_1=0} = Z_{12}$$

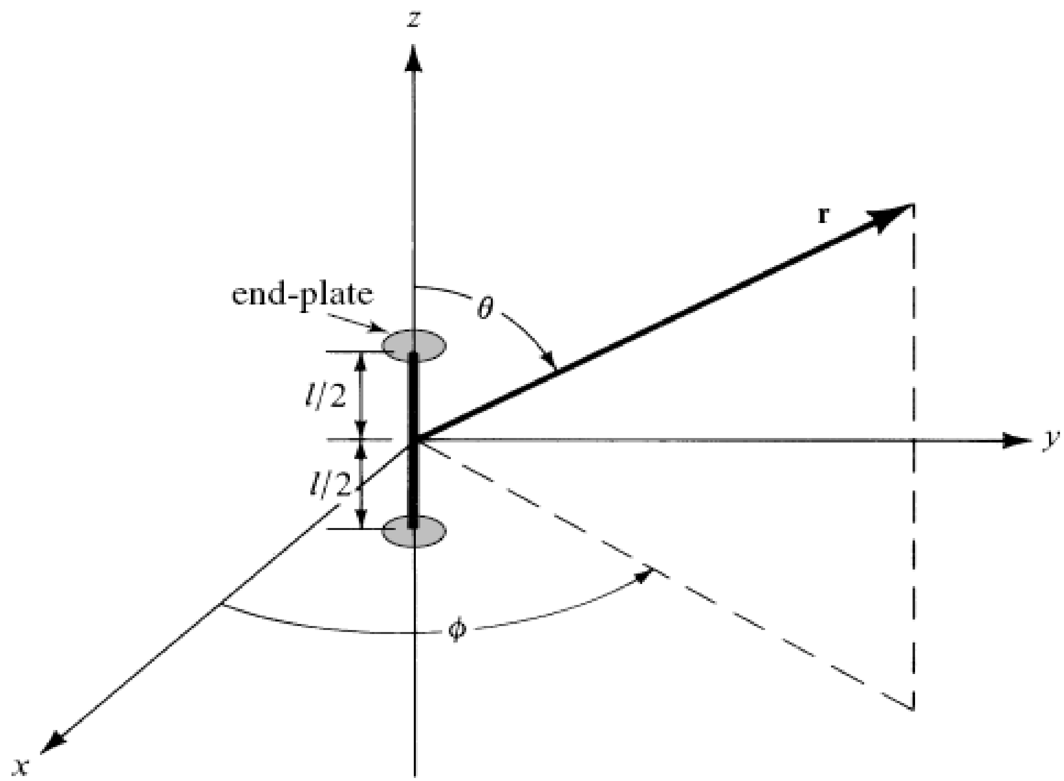
- If in addition I1 = I2, then,

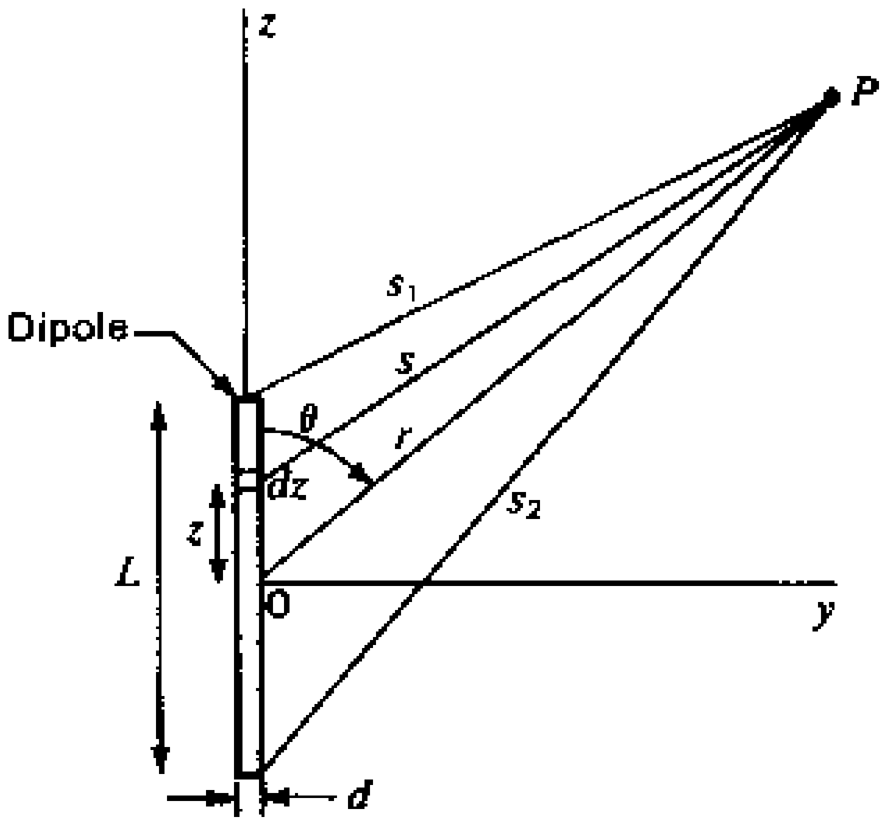
$$V_{2oc} = V_{1oc}$$

Short Dipole

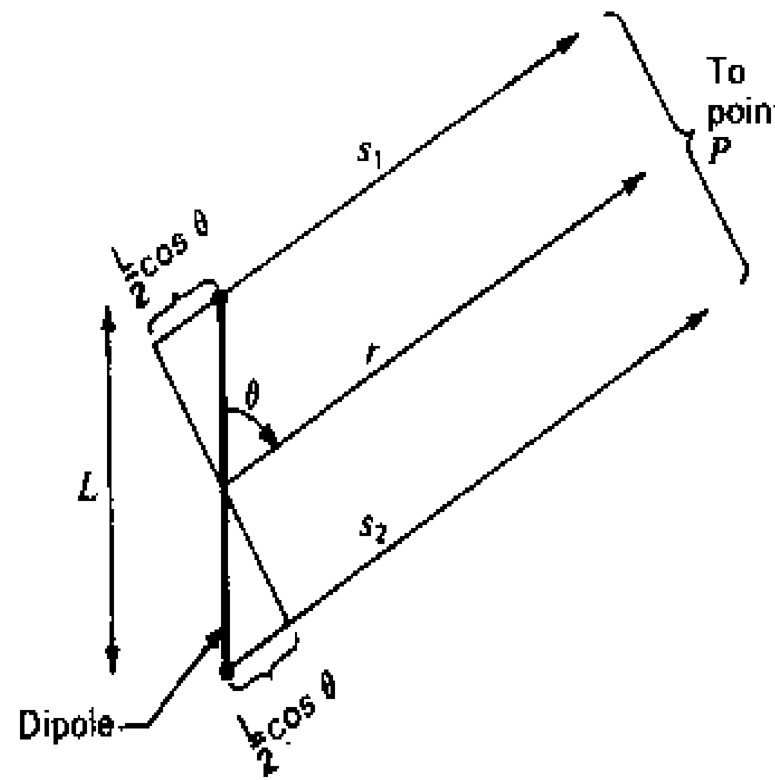
Introduction

- An infinitesimal linear wire (l λ) is positioned symmetrically at the origin of the coordinate system and oriented along the z axis, as shown in Figure





Near field region



Far field region

- Although infinitesimal dipoles are not very practical, they are used to represent capacitor-plate (also referred to as top-hat-loaded) antennas.
- The spatial variation of the current is assumed to be constant and given by,

$$\mathbf{I}(z') = \hat{\mathbf{a}}_z I_0$$

- To find the fields radiated by the current element, the two-step procedure is used. It will be required to determine first **A and F and then find the E and H.**
- Since the source only carries an electric current **Ie only;**

  Im and the potential function F are zero.
- To find **A**

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \int_C \mathbf{I}_e(x', y', z') \frac{e^{-jkR}}{R} dl'$$

- Where (x, y, z) represent the observation point coordinates, (x, y, z) represent the coordinates of the source, R is the distance from any point on the source to the observation point.

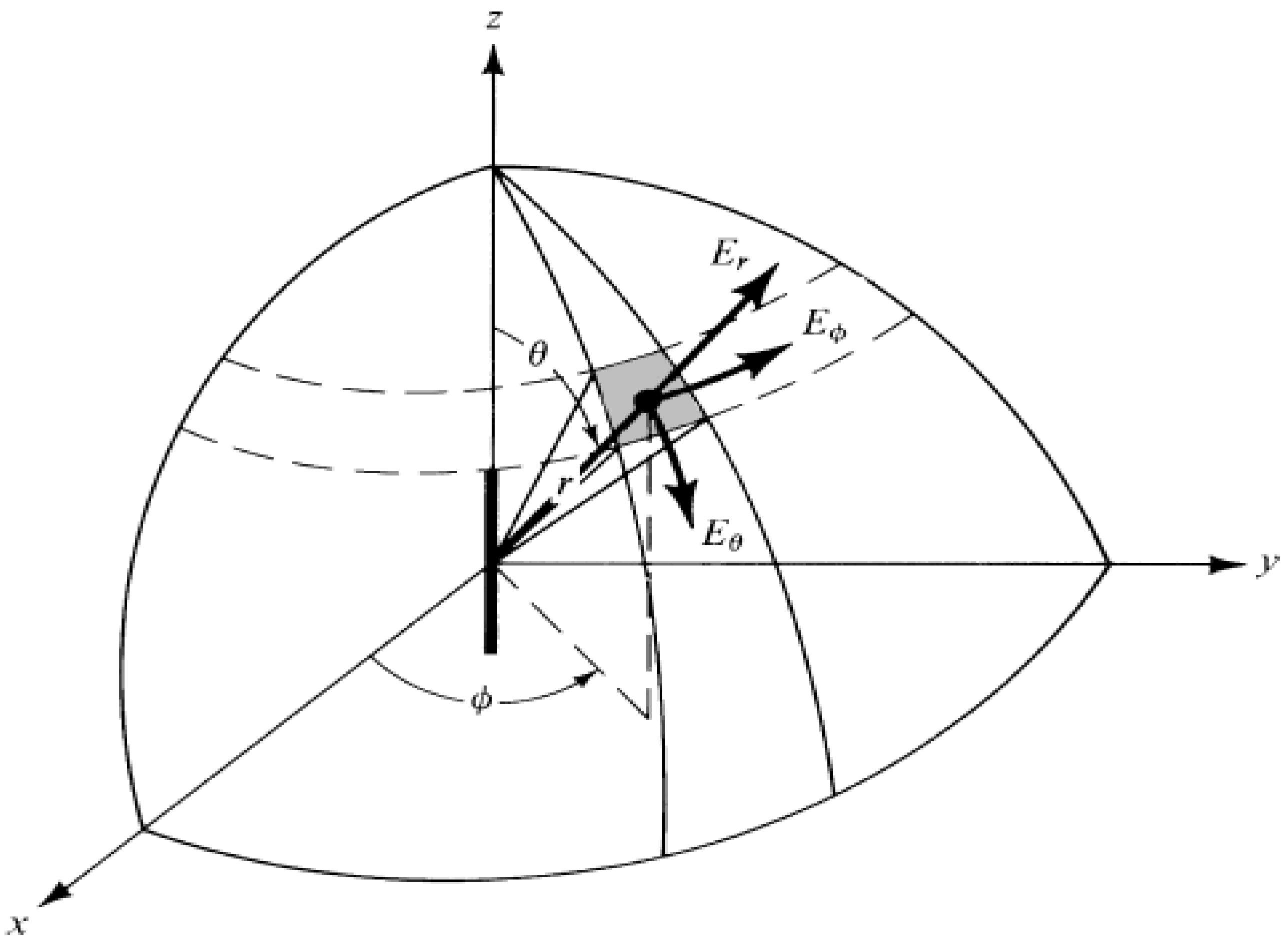
$$\mathbf{I}_e(x', y', z') = \hat{\mathbf{a}}_z I_0$$

$$x' = y' = z' = 0 \text{ (infinitesimal dipole)}$$

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} = \sqrt{x^2 + y^2 + z^2}$$

$$= r = \text{constant}$$

$$dl' = dz'$$



z
y
x
E_r
E_ϕ
E_θ
θ
r
ϕ

$$\mathbf{A}(x, y, z) = \hat{\mathbf{a}}_z \frac{\mu I_0}{4\pi r} e^{-jkr} \int_{-l/2}^{+l/2} dz' = \hat{\mathbf{a}}_z \frac{\mu I_0 l}{4\pi r} e^{-jkr}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

For this problem, $A_x = A_y = 0$

$$A_r = A_z \cos\theta = \frac{\mu I_0 l e^{-jkr}}{4\pi r} \cos\theta$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I_0 l e^{-jkr}}{4\pi r} \sin\theta$$

$$A_\phi = 0$$

$$\mathbf{H} = \hat{\mathbf{a}}_\phi \frac{1}{\mu r} \left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta} \right]$$

$$H_r = H_\theta = 0$$

$$H_\phi = j\frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$\mathbf{E} = \mathbf{E}_A = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\mathbf{A}) = \frac{1}{j\omega\epsilon}\nabla\times\mathbf{H}$$

$$E_r = \eta\frac{I_0 l \cos\theta}{2\pi r^2}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = j\eta\frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0$$

Power Density and Radiation Resistance

- For a lossless antenna, the real part of the input impedance was designated as radiation resistance. It is through the mechanism of the radiation resistance that power is transferred from the guided wave to the free-space wave.
- To find the input resistance for a lossless antenna, the Poynting vector is formed in terms of the **E- and H-fields radiated by the antenna. By integrating the Poynting vector over** a closed surface (usually a sphere of constant radius), the total power radiated by the source is found.
- The real part of it is related to the input resistance.

- For the infinitesimal dipole, the complex Poynting vector can be,

$$\mathbf{W} = \frac{1}{2}(\mathbf{E} \times \mathbf{H}^*) = \frac{1}{2}(\hat{\mathbf{a}}_r E_r + \hat{\mathbf{a}}_\theta E_\theta) \times (\hat{\mathbf{a}}_\phi H_\phi^*)$$

$$= \frac{1}{2}(\hat{\mathbf{a}}_r E_\theta H_\phi{}^* - \hat{\mathbf{a}}_\theta E_r H_\phi{}^*)$$

- whose radial Wr and transverse Wθ components are given, respectively, by

$$W_r = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}\left[1 - j\frac{1}{(kr)^3}\right]$$

$$W_\theta = j\eta\frac{k|I_0 l|^2 \cos\theta \sin\theta}{16\pi^2 r^3}\left[1 + \frac{1}{(kr)^2}\right]$$

- The complex power moving in the radial direction is obtained by integrating over a closed sphere of radius r.

$$P = \oiint_S \mathbf{W} \cdot d\mathbf{s} = \int_0^{2\pi} \int_0^{\pi} (\hat{\mathbf{a}}_r W_r + \hat{\mathbf{a}}_\theta W_\theta) \cdot \hat{\mathbf{a}}_r r^2 \sin\theta \, d\theta \, d\phi$$

$$P = \int_0^{2\pi} \int_0^{\pi} W_r r^2 \sin\theta \, d\theta \, d\phi = \eta \frac{\pi}{3} \left| \frac{I_0 l}{\lambda} \right|^2 \left[1 - j \frac{1}{(kr)^3} \right]$$

$$P_{\text{rad}} = \eta \left(\frac{\pi}{3} \right) \left| \frac{I_0 l}{\lambda} \right|^2$$

- Since the antenna radiates its real power through the radiation resistance, for the infinitesimal dipole,

$$P_{\text{rad}} = \eta \left(\frac{\pi}{3} \right) \left| \frac{I_0 l}{\lambda} \right|^2 = \frac{1}{2} |I_0|^2 R_r$$

$$R_r = \eta \left(\frac{2\pi}{3}\right) \left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \left(\frac{l}{\lambda}\right)^2$$

- The reactance of an infinitesimal dipole is capacitive. This can be illustrated by considering the dipole as a flared open-circuited transmission line, Since the input impedance of an open-circuited transmission line a distance l/2 from its openend is given by Zin = −jZc cot (βl/2), where Zc is its characteristic impedance, it will always be negative (capacitive) for l << λ.

Near-Field (kr << 1) Region

- The equation can be reduced in much simpler form and canbe approximated by,

$$\left.\begin{aligned}
E_r &\simeq -j\eta \frac{I_0 l e^{-jkr}}{2\pi k r^3}\cos\theta \\
E_\theta &\simeq -j\eta \frac{I_0 l e^{-jkr}}{4\pi k r^3}\sin\theta \\
E_\phi &= H_r = H_\theta = 0 \\
H_\phi &\simeq \frac{I_0 l e^{-jkr}}{4\pi r^2}\sin\theta
\end{aligned}\right\} \quad kr \ll 1$$

- The E-field components, Er and Eθ **, are in time-phase but they are in time-phase quadrature** with the **H-field component Hϕ; therefore there is no time-average power** flow associated with them.
- This is demonstrated by forming the time-average power density as,

$$\mathbf{W}_{\mathrm{av}} = \tfrac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*] = \tfrac{1}{2}\mathrm{Re}[\hat{\mathbf{a}}_r E_\theta H^*{}_\phi - \hat{\mathbf{a}}_\theta E_r H^*{}_\phi]$$

$$\mathbf{W}_{\mathrm{av}} = \frac{1}{2}\mathrm{Re}\left[-\hat{\mathbf{a}}_r\, j\frac{\eta}{k}\left|\frac{I_0 l}{4\pi}\right|^2 \frac{\sin^2\theta}{r^5} + \hat{\mathbf{a}}_\theta\, j\frac{\eta}{k}\frac{|I_0 l|^2}{8\pi^2}\frac{\sin\theta\cos\theta}{r^5}\right] = 0$$

Far-Field (kr >> 1) Region

- Er will be smaller than Eθ because Er is inversely proportional to r2 where Eθ is inversely proportional to r.

$$\left.\begin{aligned} E_\theta &\simeq j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta \\ E_r &\simeq E_\phi = H_r = H_\theta = 0 \\ H_\phi &\simeq j\frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta \end{aligned}\right\} \quad kr \gg 1$$

- The **E- and H-field components are perpendicular to each other, transverse to the** radial direction of propagation, and the r variations are separable from those of θ and ϕ. The shape of the pattern is not a function of the radial distance r, and the fields form <u>**a Transverse ElectroMagnetic (TEM) wave whose wave impedance is equal to the intrinsic impedance of the medium.**</u>

Directivity

- The real power P_{rad} radiated by the dipole,

$$\mathbf{W}_{\text{av}} = \frac{1}{2}\text{Re}(\mathbf{E} \times \mathbf{H}^*) = \hat{\mathbf{a}}_r \frac{1}{2\eta}|E_\theta|^2 = \hat{\mathbf{a}}_r \frac{\eta}{2}\left|\frac{kI_0 l}{4\pi}\right|^2 \frac{\sin^2\theta}{r^2}$$

- Radiation intensity U,

$$U = r^2\ W_{\text{av}} = \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2 \sin^2\theta = \frac{r^2}{2\eta}|E_\theta(r,\theta,\phi)|^2$$

- The maximum value occurs at $\theta = \pi/2$ and it is equal to,

$$U_{\text{max}} = \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2$$

- The directivity reduces to,

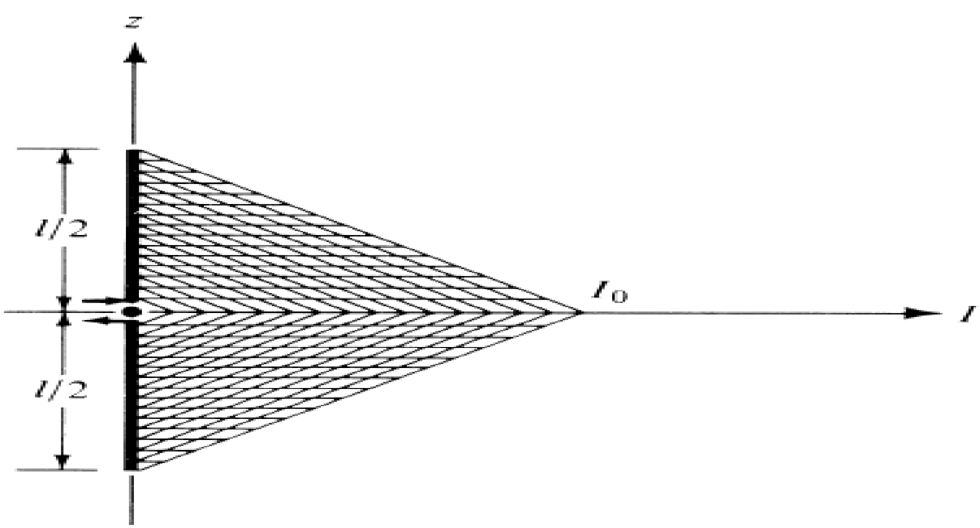
$$D_0 = 4\pi \frac{U_{\max}}{P_{\text{rad}}} = \frac{3}{2}$$

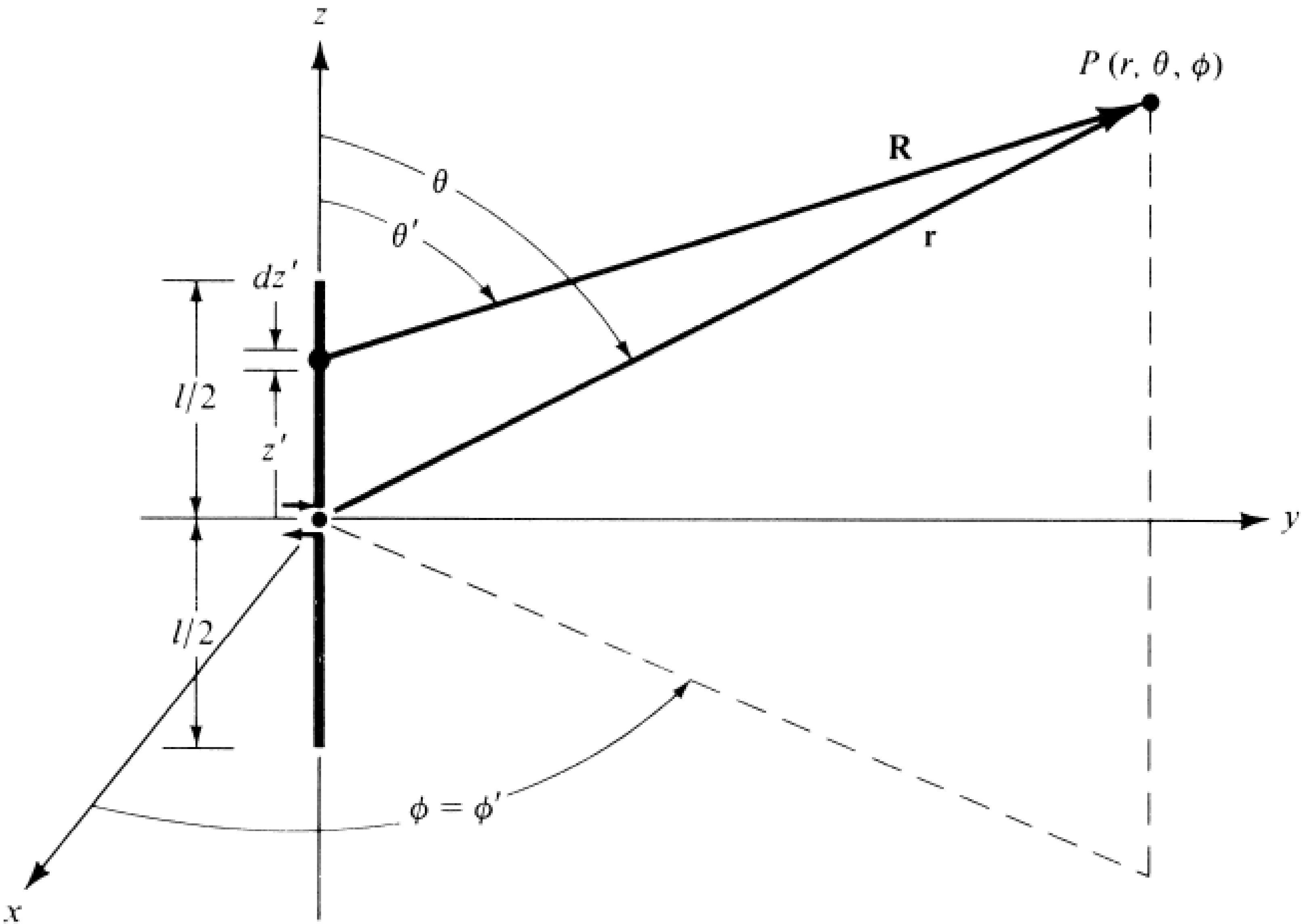
- The maximum effective aperture,

$$A_{em} = \left(\frac{\lambda^2}{4\pi}\right) D_0 = \frac{3\lambda^2}{8\pi}$$

Small Dipole

- A better approximation of the current distribution of wire antennas, **<u>whose lengths are usually λ/50 < l ≤ λ/10, is the triangular variation.</u>**
- The most convenient geometrical arrangement for the analysis of a dipole is usually to have it positioned **<u>symmetrically about the origin with its length directed along the *z-axis,*</u>**





z
P (r, θ, φ)
R
θ
θ′
r
dz′
l/2
z′
y
l/2
φ = φ′
x

$$\mathbf{I}_e(x', y', z') = \begin{cases} \hat{\mathbf{a}}_z I_0 \left(1 - \frac{2}{l} z'\right), & 0 \leq z' \leq l/2 \\ \hat{\mathbf{a}}_z I_0 \left(1 + \frac{2}{l} z'\right), & -l/2 \leq z' \leq 0 \end{cases}$$

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \left[\hat{\mathbf{a}}_z \int_{-l/2}^{0} I_0 \left(1 + \frac{2}{l} z'\right) \frac{e^{-jkR}}{R} dz' + \hat{\mathbf{a}}_z \int_{0}^{l/2} I_0 \left(1 - \frac{2}{l} z'\right) \frac{e^{-jkR}}{R} dz' \right]$$

- Because the overall length of the dipole is very small (usually l ≤ λ/10), the values of R for different values of z along the length of the wire (−l/2 ≤ z ≤ l/2) are not much different from r. Thus R can be approximated **by R = r** throughout the integration path. The maximum phase error in by allowing R = r for λ/50 < l ≤ λ/10**, will be kl/2 = π/10 rad = 18◦ for l = λ/10.**

$$\mathbf{A} = \hat{\mathbf{a}}_z A_z = \hat{\mathbf{a}}_z \frac{1}{2}\left[\frac{\mu I_0 l e^{-jkr}}{4\pi r}\right]$$

- The potential function becomes a more accurate approximation as $kr \rightarrow \infty$.
- *Since the potential function for the triangular distribution is* one-half of the corresponding one for the constant (uniform) current distribution, the corresponding **fields of the former are one-half of the latter**. Thus we can write the **E and H-fields radiated by a small dipole as,**

$$\left.\begin{aligned} E_\theta &\simeq j\eta \frac{k I_0 l e^{-jkr}}{8\pi r}\sin\theta \\ E_r &\simeq E_\phi = H_r = H_\theta = 0 \\ H_\phi &\simeq j\frac{k I_0 l e^{-jkr}}{8\pi r}\sin\theta \end{aligned}\right\} \quad kr \gg 1$$

- The radiation resistance of the antenna is strongly dependent upon the current distribution.
- As compare infinitesimal dipole, it can be shown that for the **small dipole its radiated power is one-fourth (1/4) of previous case.**
- Thus the radiati

$$R_r = \frac{2P_{\text{rad}}}{|I_0|^2} = 20\pi^2 \left(\frac{l}{\lambda}\right)^2$$

Finite Length Dipole

- **Current Distribution:** For a very thin dipole (ideally zero diameter), the current distribution can be written, to a good approximation, as,

$$\mathbf{I}_e(x'=0, y'=0, z') = \begin{cases} \hat{\mathbf{a}}_z I_0 \sin\left[k\left(\frac{l}{2} - z'\right)\right], & 0 \le z' \le l/2 \\ \hat{\mathbf{a}}_z I_0 \sin\left[k\left(\frac{l}{2} + z'\right)\right], & -l/2 \le z' \le 0 \end{cases}$$

- Experimentally it has been verified that the current in a center-fed wire antenna has sinusoidal form with nulls at the end points.

- **<u>The finite dipole antenna is subdivided into a number of infinitesimal dipoles of length *Δz'*.</u>** As the number of subdivisions is increased, each infinitesimal dipole approaches a length dz.
- For an infinitesimal dipole of length dz positioned along the z-axis at z, the electric and magnetic field components in the far field are,

$$dE_\theta \simeq j\eta \frac{kI_e(x', y', z')e^{-jkR}}{4\pi R} \sin\theta \, dz'$$

$$dE_r \simeq dE_\phi = dH_r = dH_\theta = 0$$

$$dH_\phi \simeq j \frac{kI_e(x', y', z')e^{-jkR}}{4\pi R} \sin\theta \, dz'$$

- A very thin dipole of finite length l is symmetrically positioned about the origin with its length directed along the z-axis.
- Because the wire is assumed to be very thin *(x′ = y′ = 0).*

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} = \sqrt{x^2 + y^2 + (z - z')^2}$$

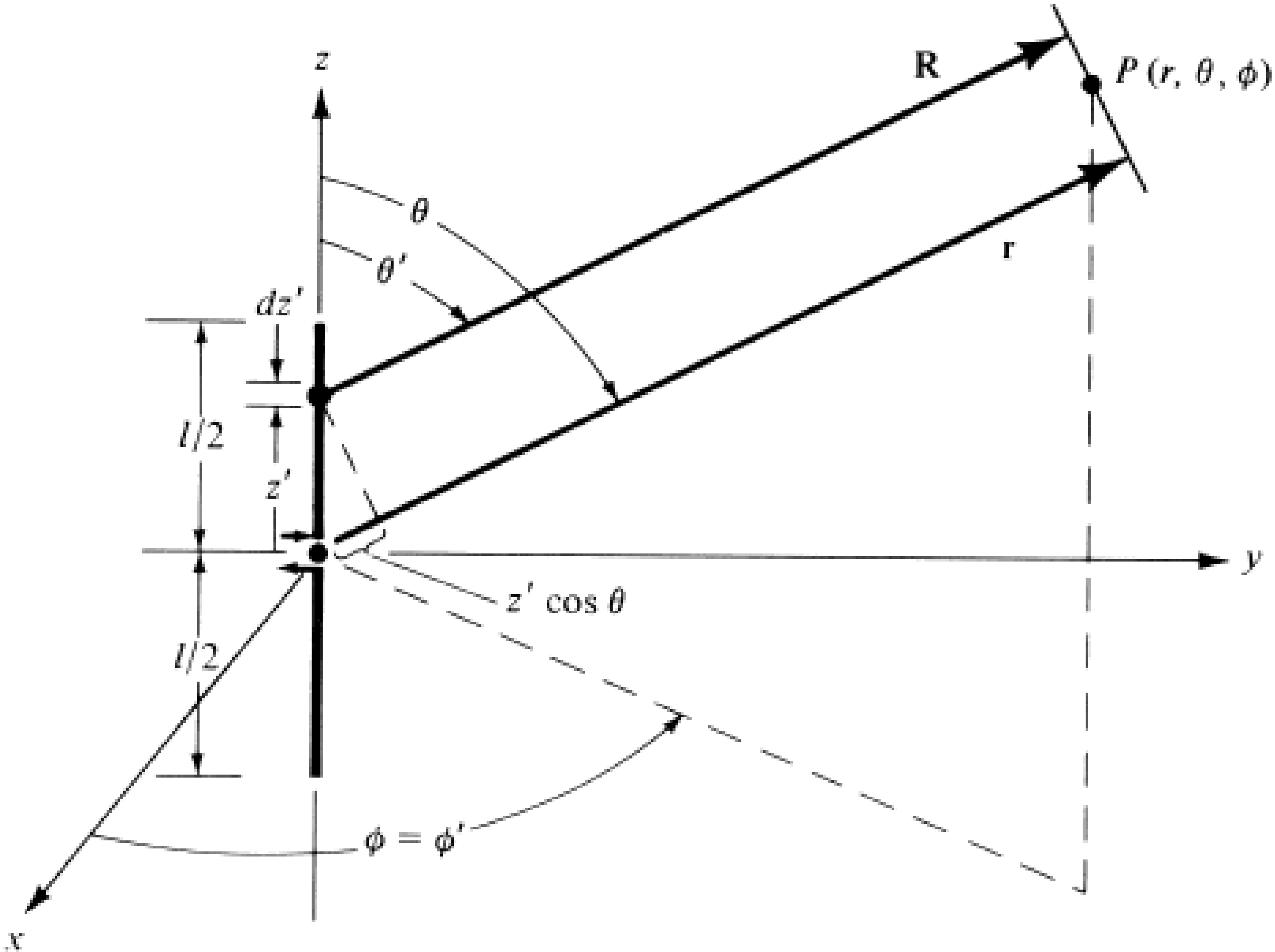
$$R = \sqrt{(x^2 + y^2 + z^2) + (-2zz' + z'^2)} = \sqrt{r^2 + (-2rz'\cos\theta + z'^2)}$$

- Using the binomial expansion,

$$R = r - z'\cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) + \frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right) +$$

- For far field region,

$$R \simeq r - z'\cos\theta$$



(b) Geometrical arrangement for far-field approximations

Far-field Approximations

$$R \simeq r - z' \cos\theta \qquad \text{for phase terms}$$

$$R \simeq r \qquad \text{for amplitude terms}$$

- Using the far-field approximations

$$dE_\theta \simeq j\eta \frac{kI_e(x', y', z')e^{-jkr}}{4\pi r} \sin\theta e^{+jkz'\cos\theta}\, dz'$$

- Summing the contributions from all the infinitesimal elements, the summation reduces, in the limit, to an integration.

$$E_\theta = \int_{-l/2}^{+l/2} dE_\theta = j\eta \frac{ke^{-jkr}}{4\pi r} \sin\theta \left[\int_{-l/2}^{+l/2} I_e(x', y', z')e^{jkz'\cos\theta}\, dz' \right]$$

- The factor outside the brackets is designated as the **element factor** and that within the brackets as the **space factor**. For this antenna, the element factor is equal to the field of a unit length infinitesimal dipole located at a reference point (the origin). In general, the element factor depends on the type of current and its direction of flow while the space factor is a function of the current distribution along the source.
- For the current distribution of above equations,

$$E_\theta \simeq j\eta \frac{kI_0e^{-jkr}}{4\pi r}\sin\theta \left\{\int_{-l/2}^{0} \sin\left[k\left(\frac{l}{2}+z'\right)\right] e^{+jkz'\cos\theta}\,dz' + \int_{0}^{+l/2} \sin\left[k\left(\frac{l}{2}-z'\right)\right] e^{+jkz'\cos\theta}\,dz'\right\}$$

- The above integral can be solved by,

$$\int e^{\alpha x} \sin(\beta x + \gamma)\, dx = \frac{e^{\alpha x}}{\alpha^2 + \beta^2}[\alpha \sin(\beta x + \gamma) - \beta \cos(\beta x + \gamma)]$$

$$E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]$$

$$H_\phi \simeq \frac{E_\theta}{\eta} \simeq j \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]$$