Microwave Engineering

(EC0505)

Unit-2

B.Tech. (Electronics and Communication)

Semester-V

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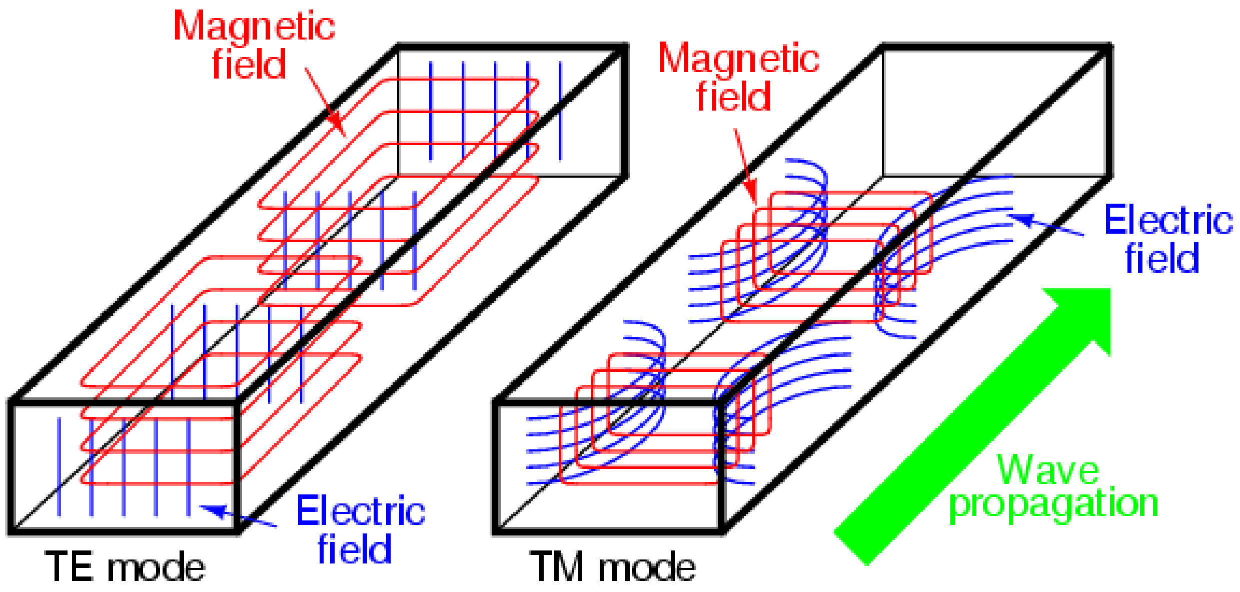
Waveguides



Magnetic flux lines appear as continuous loops
Electric flux lines appear with beginning and end points

Transmission Line	Waveguide
Two or more conductors separated by some insulating medium (two-wire, coaxial, microstrip, etc.).	Metal waveguides are typically one enclosed conductor filled with an insulating medium (rectangular, circular)
Normal operating mode is the TEM	Operating modes are TE or TM modes (cannot support a TEM mode).
No cutoff frequency for the TEM mode. Transmission lines can transmit signals from DC up to high frequency.	Waveguide operates at a frequency above the respective TE or TM mode cutoff frequency for that mode to propagate.
Signal attenuation at high frequencies	Lower signal attenuation at high frequencies than transmission lines
Small cross-section transmission lines (like coaxial cables) can only transmit low power levels	Metal waveguides can transmit high power levels
Large cross-section transmission lines (like power transmission lines) can transmit high power levels.	Large cross-section (low frequency) waveguides are impractical due to large size and high cost.

Waveguide

Waveguide:

The efficient transfer of information or EM energy from one point to another in a chosen direction is performed by specially designed electromagnetic structure or media called, electromagnetic **waveguide.**

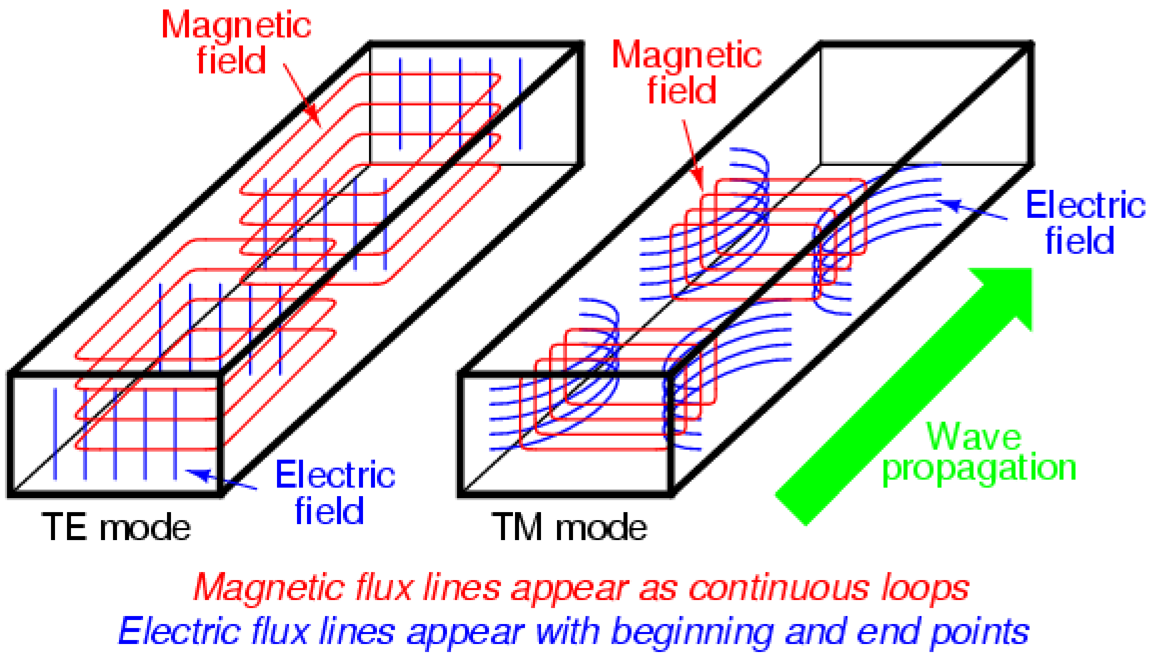
A *waveguide* is a special form of transmission line consisting of a hollow-metallic tube of uniform c/s for transmitting EM waves by successive reflections from inner walls of the tube.

The tube wall provides distributed inductance, while the empty space between the tube walls provide distributed capacitance.

- When an electromagnetic wave propagates down a hollow tube, only one of the fields -- either electric or magnetic -- will actually be transverse to the wave's direction of travel.

- The other field will “loop” longitudinally to the direction of travel, but still be perpendicular to the other field.

Whichever field remains transverse to the direction of travel determines whether the wave propagates in *TE* mode (**T**ransverse **E**lectric) or *TM*(**T**ransverse **M**agnetic) mode.



Mode means Field pattern

The transverse modes are classified into different types:

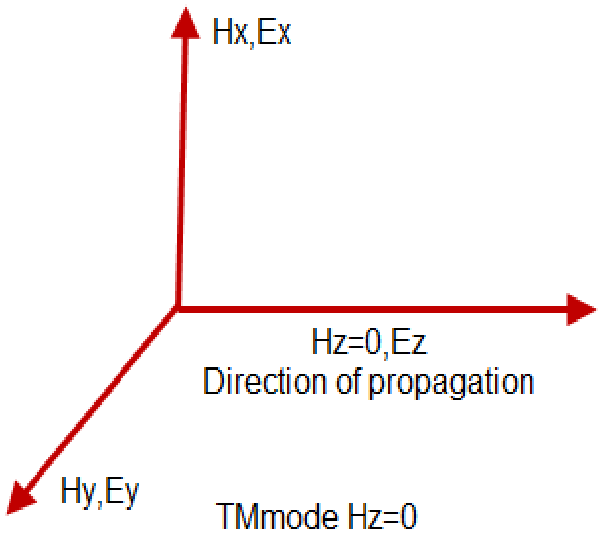
• TE modes (Transverse Electric) have no electric field in the direction of propagation. **$E_z=0$**

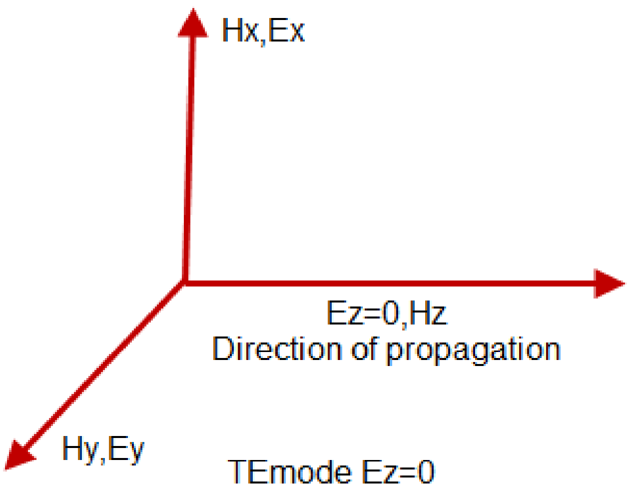
•TM modes (Transverse Magnetic) have no magnetic field in the direction of propagation. **$H_z=0$**

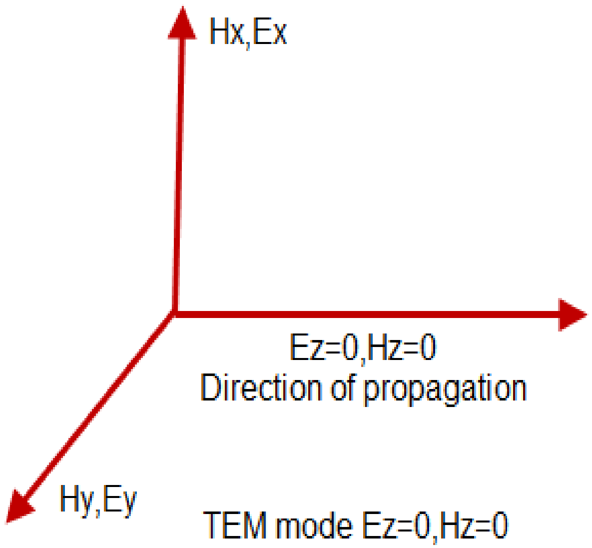
•TEM modes (Transverse Electromagnetic) have no electric nor magnetic field in the direction of propagation. **$E_z=0$, $H_z=0$**

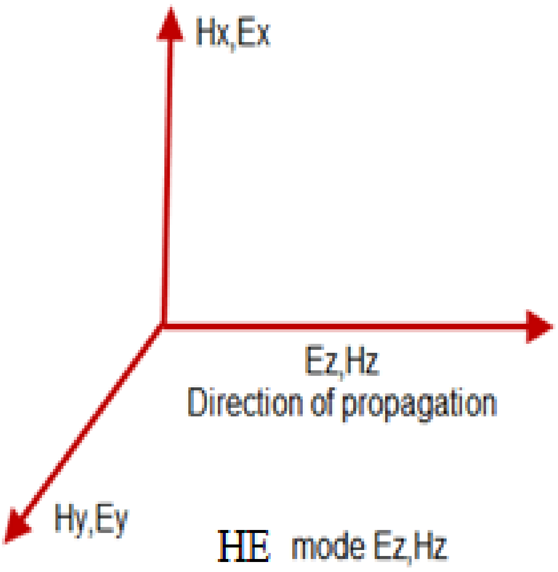
•Hybrid modes have both electric and magnetic field components in the direction of propagation. **E_z,H_z nonzero**

TEM mode is only possible with two conductors and cannot exist in a waveguide.


Hx,Ex
Hz=0,Ez
Direction of propagation
Hy,Ey
TMmode Hz=0


Hx,Ex
Ez=0,Hz
Direction of propagation
Hy,Ey
TEmode Ez=0


Hx,Ex
Ez=0,Hz=0
Direction of propagation
Hy,Ey
TEM mode Ez=0,Hz=0


Hx,Ex
Ez,Hz
Direction of propagation
Hy,Ey
HE mode Ez,Hz

❑Electromagnetic waveguides are analyzed by solving_Maxwell's equations, or their reduced form, the electromagnetic wave equation, with boundary conditions determined by the properties of the materials and their interfaces.

❑**These equations have multiple solutions, or modes**, which are eigenfunctions of the equation system.

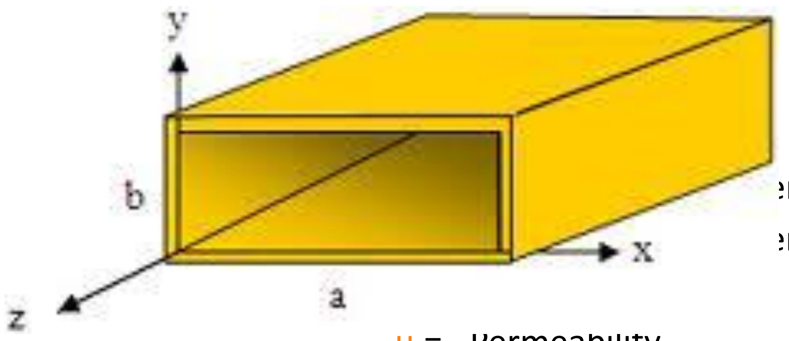
❑**Each mode has its cutoff frequency below which the mode cannot exist in the guide. i.e. if the cut off frequency of the mode is lower than the operating frequency then only that mode can exist in a waveguide.**

❑Waveguide propagation modes depend on the operating signal frequency and the shape and size of the guide.

❑**The mode with the lowest cutoff frequency is termed the dominant mode of the guide.** It is usual to choose the size of the guide such that only this one mode can exist in the frequency band of operation. And other higher modes can not exist as they are more lossy in nature.

❑**Design criterion: The cut off frequency of dominant mode is chosen 10% below the operating signal frequency & accordingly waveguide size is decided.**

Rectangular Waveguide



ngth variations of fields in the "a" direction

ngth variations of fields in the "b" direction

μ = Permeability

The lower cutoff frequency (or wavelength) for a particular mode in rectangular waveguide is determined by the following equations:

$$(f_c)_{mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Rectangular Waveguide Theory:

Proceeding from the Maxwell curl equations:

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \quad \Rightarrow \quad \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\bar{H}$$

or

$$\hat{x}: \quad \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\hat{y}: \quad -\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) = -j\omega\mu H_y$$

$$\hat{z}: \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

However, the spatial variation in z is known so that

$$\frac{\partial\left(e^{-j\beta z}\right)}{\partial z}=-j\beta\left(e^{-j\beta z}\right)$$

Consequently, these curl equations simplify to

$$\frac{\partial E_z}{\partial y}+j\beta E_y=-j\omega\mu H_x \qquad (1)$$

$$-\frac{\partial E_z}{\partial x}-j\beta E_x=-j\omega\mu H_y \qquad (2)$$

$$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=-j\omega\mu H_z \qquad (3)$$

We can perform a similar expansion of Ampère's equation $\nabla\times\overline{H}=j\omega\varepsilon\overline{E}$ to obtain

$$\frac{\partial H_z}{\partial y}+j\beta H_y=j\omega\varepsilon E_x \qquad (4)$$

$$-j\beta H_x-\frac{\partial H_z}{\partial x}=j\omega\varepsilon E_y \qquad (5)$$

$$\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}=j\omega\varepsilon E_z \qquad (6)$$

From equation (1),

$$H_x = \frac{j}{\omega\mu}\left(\frac{\partial E_z}{\partial y} + j\beta E_y\right)$$

Substituting for E_y from (5) we find

$$H_x = \frac{j}{\omega\mu}\left[\frac{\partial E_z}{\partial y} + j\beta\frac{1}{j\omega\varepsilon}\left(-j\beta H_x - \frac{\partial H_z}{\partial x}\right)\right]$$

$$= \frac{j}{\omega\mu}\frac{\partial E_z}{\partial y} + \frac{\beta^2}{\omega^2\mu\varepsilon}H_x - \frac{j\beta}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

or,

$$\boxed{H_x = \frac{j}{k_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right)} \qquad (7)$$

where $\boxed{k_c^2 \equiv k^2 - \beta^2}$ and $k^2 = \omega^2\mu\varepsilon$.

$$\boxed{H_y = -\frac{j}{k_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right)} \quad (8)$$

$$\boxed{E_x = \frac{-j}{k_c^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right)} \quad (9)$$

$$\boxed{E_y = \frac{j}{k_c^2}\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right)} \quad (10)$$

Most important point: From (7)-(10), we can see that **all transverse** components of $\bar{E}$ and $\bar{H}$ can be determined from **only the axial** components E_z and H_z. It is this fact that allows the mode designations TEM, TE, and TM.

- K_c is the cut-off wave-number, k is the wave number of free space frequency and β is the phase constant of the waveguide.

- **TEM WAVE:** transverse electromagnetic waves are characterized by $E_z=H_z=0$.

- The cut-off wave number $k_c=0$ for TEM waves.

$$Z_{\text{TEM}} = \frac{E_x}{H_y} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}} = \eta,$$

- **TE WAVE:** transverse electric waves (H-wave) are characterized by $E_z = 0$ and H_z is non zero.
- So the transverse field can be written as,

$$H_x = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial x},$$

$$H_y = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial y},$$

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y},$$

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}.$$

$$Z_{\mathrm{TE}} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta},$$

- **TM WAVE:** transverse magnetic waves (E-waves) are characterized by $H_z = 0$ and E_z is non zero.
- So the transverse field can be written as,

$$H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y},$$

$$H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x},$$

$$E_x = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial x},$$

$$E_y = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial y}.$$

$$Z_{\mathrm{TM}} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \frac{\beta\eta}{k},$$

Rectangular Waveguides: *Fields inside*

- assuming waveguide is filled with
- lossless dielectric material and
- walls of perfect conductor,

the wave inside should obey…

$$\nabla^2 E + k^2 E = 0$$

$$\nabla^2 H + k^2 H = 0$$

$$\text{where} \quad k^2 = \omega^2 \mu \varepsilon$$

Then applying on the z–component…

$$\nabla^2 E_z + k^2 E_z = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + k^2 E_z = 0$$

Solving by method of Separation of Variables :

$$E_z(x, y, z) = X(x)Y(y)Z(z)$$

from where we obtain :

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2$$

Fields inside the waveguide

$$\frac{X''}{X}+\frac{Y''}{Y}+\frac{Z''}{Z}=-k^2 \qquad k_c^{\,2}=k^2-\beta^2=k_x^2+k_y^2$$

$$-k_x^2-k_y^2-\beta^2=-k^2$$

which results in the expressions:

$$X''+k_x^2X=0 \qquad X(x)=c_1\cos k_x x+c_2\sin k_x x$$

$$Y''+k_y^2Y=0 \qquad Y(y)=c_3\cos k_y y+c_4\sin k_y y$$

$$Z''+\beta^2Z=0 \qquad Z(z)=c_5e^{+j\beta z}+c_6e^{-j\beta z}$$

Substituting

$$X(x) = c_1 \cos k_x x + c_2 \sin k_x x$$

$$Y(y) = c_3 \cos k_y y + c_4 \sin k_y y$$

$$E_z(x, y, z) = X(x)Y(y)Z(z)$$

$$Z(z) = c_5 e^{+j\beta z} + c_6 e^{-j\beta z}$$

$$E_z = (c_1 \cos k_x x + c_2 \sin k_x x)(c_3 \cos k_y y + c_4 \sin k_y y)(c_5 e^{+j\beta z} + c_6 e^{-j\beta z})$$

If only looking at the wave traveling in $+z$-direction :

$$E_z = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)e^{-j\beta z}$$

Similarly for the magnetic field ,

$$H_z = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y)e^{-j\beta z}$$

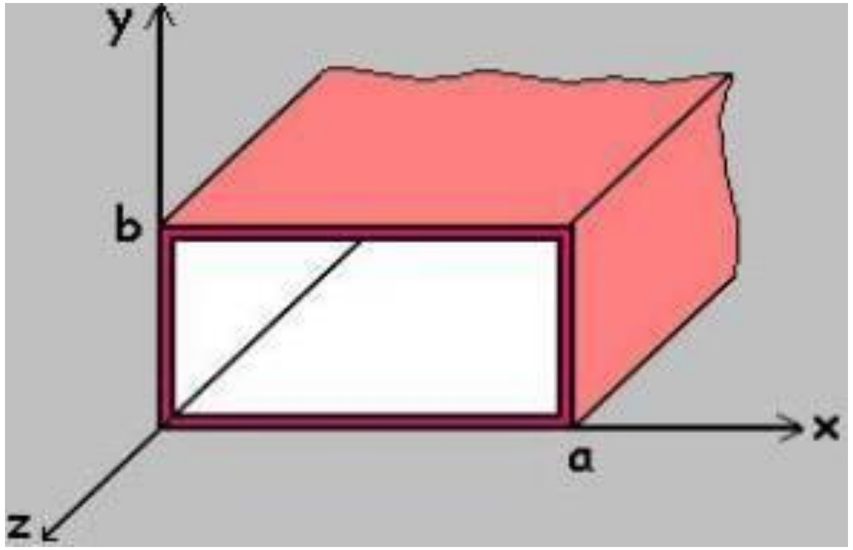
TE Mode

Find Ex & Ey

$$H_z = \left(B_1 \cos k_x x + B_2 \sin k_x x\right)\left(B_3 \cos k_y y + B_4 \sin k_y y\right) e^{-j\beta z}$$

- Boundary conditions: $E_x = 0 \text{ at } y = 0, b \;\; \& \,\forall x \to 0 \text{ to } a \;\; \text{(top \& bottom wall)}$
  $E_y = 0 \text{ at } x = 0, a \;\; \& \,\forall y \to 0 \text{ to } b \;\; \text{(LH \& RH side wall)}$

From these, we conclude:
X(x) is in the form of cos $k_x x$,
where $k_x = m\pi/a$, $m = 0,1,2,3,\ldots$
Y(y) is in the form of cos $k_y y$,
where $k_y = n\pi/b$, $n = 0,1,2,3,\ldots$
So the solution for $E_z(x,y,z)$ is



$$H_z = B_1 B_3 \left(\cos k_x x\right)\left(\cos k_y y\right) e^{-j\beta z}$$

Figure from: www.e[illegible]ilkent.edu.tr/~microwave/programs/magnetic/rect/info.htm

TE Mode

- Substituting

$$H_z = H_o \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

where again

$$K_C{}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

- Note that <u>*n* and *m* cannot be both zero</u> because the fields will all be zero.

TM Mode (H_z=0) transverse magnetic, E_z exists

$$E_z = \left(A_1 \cos k_x x + A_2 \sin k_x x\right)\left(A_3 \cos k_y y + A_4 \sin k_y y\right) e^{-j\beta z}$$

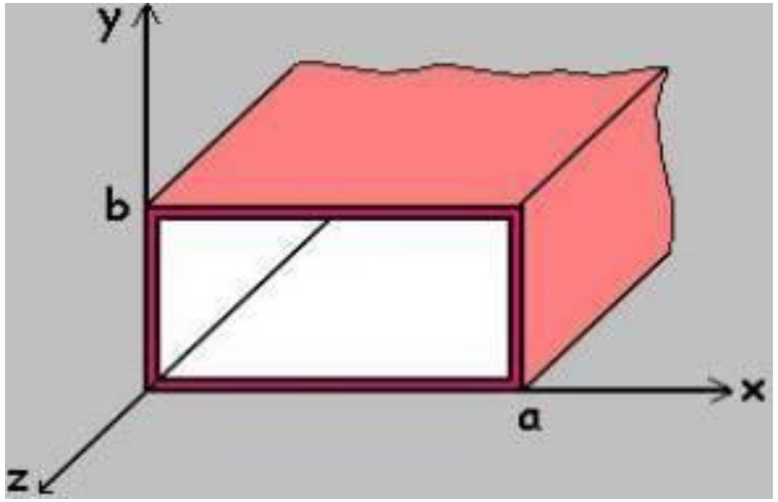
- Boundary conditions: $E_z = 0 \ \text{at}\ y = 0, b \ \ \& \ \forall x \rightarrow 0 \ to \ a$ (top & bottom walls)
  $E_z = 0 \ \text{at}\ x = 0, a \ \ \& \ \forall y \rightarrow 0 \ to \ \mathrm{b}$ (LH & RH side walls)

From these, we conclude:

X(x) is in the form of sin $k_x x$, where $k_x = m\pi/a$, m=1,2,3,…

Y(y) is in the form of sin $k_y y$, where $k_y = n\pi/b$, n=1,2,3,…

So the solution for $E_z(x,y,z)$ is



$$E_z = A_2 A_4 \left(\sin k_x x\right)\left(\sin k_y y\right) e^{-j\beta z}$$

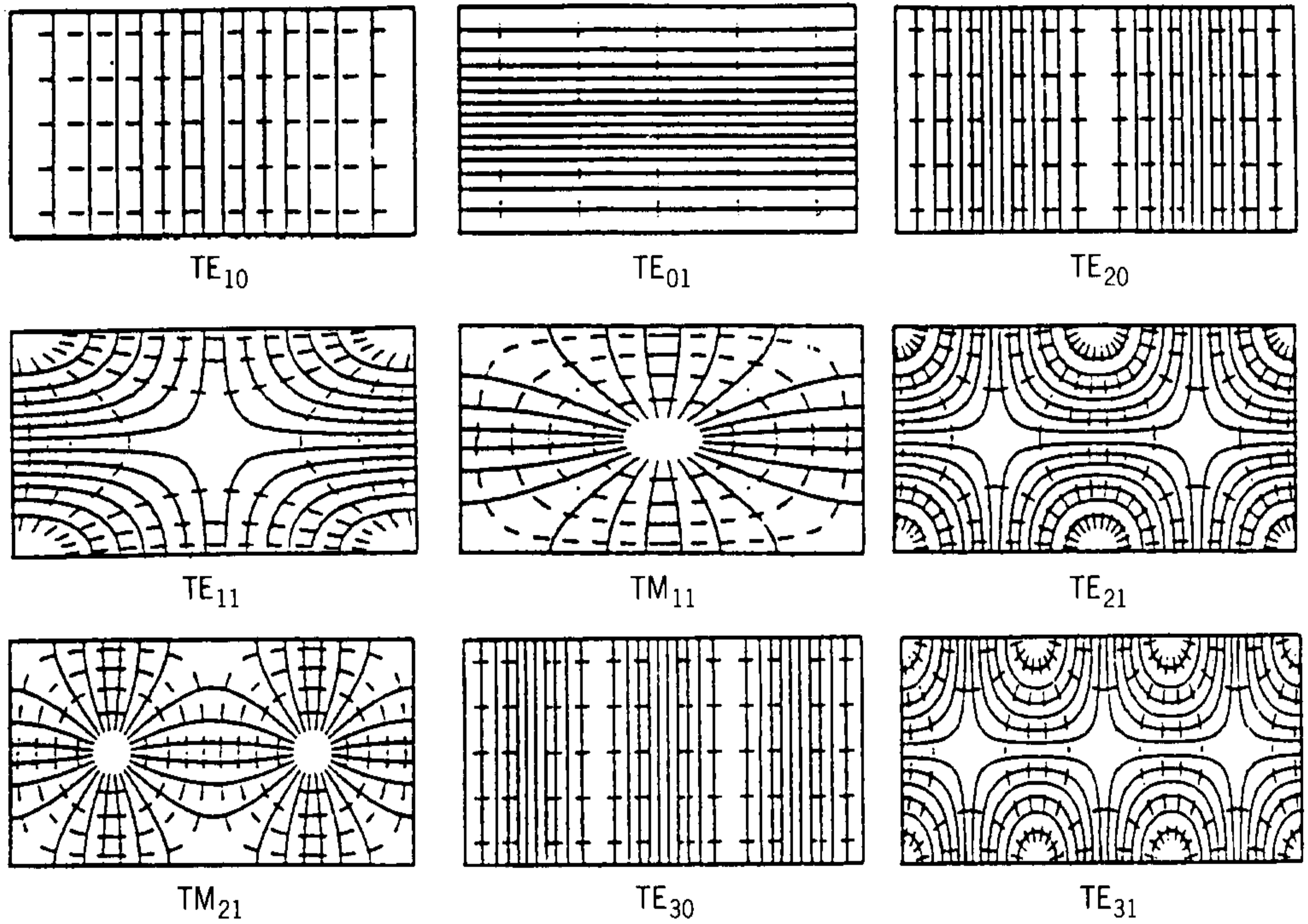
TM Mode

- Substituting

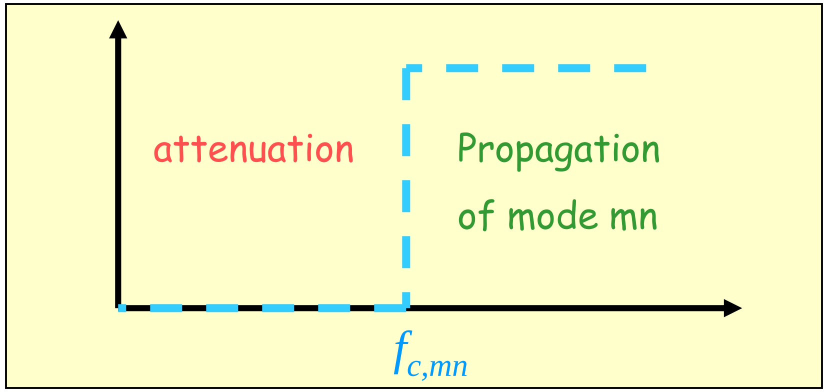
$$E_z = E_o \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

where

$$K_C{}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \qquad = k^2 - \beta^2$$


TE_{10}
TE_{01}
TE_{20}
TE_{11}
TM_{11}
TE_{21}
TM_{21}
TE_{30}
TE_{31}

Cutoff



- The cutoff frequency is the frequency below which attenuation occurs and above which propagation takes place. (High Pass)

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \omega_c\sqrt{\mu\epsilon}$$

- The phase constant becomes

$$\beta_g = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Phase velocity and impedance

- The phase velocity is defined as

$$v_g = \frac{\omega}{\beta_g} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}} \qquad v_p = 1\Big/\sqrt{\mu\varepsilon} = C$$

$$\boxed{v_p = \frac{\omega}{\beta} = \frac{C}{\sqrt{1-\left(\dfrac{\lambda_0}{\lambda c}\right)^2}}}$$

- The **phase velocity** of a wave is the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels.

- And the intrinsic impedance of the mode is

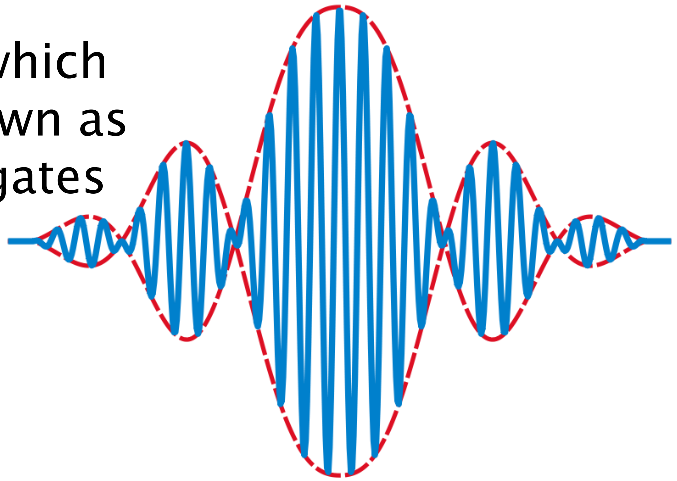
$$Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \eta\sqrt{1-\left[\frac{f_c}{f}\right]^2} \qquad \eta = \sqrt{\mu\Big/\varepsilon}$$

Group velocity:

$$v_g = \frac{d\omega}{d\beta} = C\sqrt{1-\left(\frac{\lambda_0}{\lambda c}\right)^2}$$

$$v_g v_p = C^2$$

The **group velocity** of a wave is the velocity with which the overall shape of the waves' amplitudes — known as the *modulation* or *envelope* of the wave — propagates through space.



Example:

1. The dimension of a waveguide is 2.5X1cms.The frequency is 8.6GHz Find the possible modes that can propagate through the waveguide also find the cutoff frequencies for the same.

2. A rectangular waveguide is filled by dielectric material of r =9, with inside dimension of 7X3.5cm. It operates in the dominant TE10 mode. Determine (i) cut off frequency (ii) phase velocity at a frequency of 2 GHz (iii) guided wavelength at the same frequency.

Method of excitation of modes