unit-I

Introduction to Operational Research:-

operations research is a systematic and analytical approach to decision making and problem solving.

How to define OR:-

→ It is an Act of winning wars without actualy fighting. - Aurther Cla

→ It is a scientific approach to problem solving for executive managem
- H.M. Wagner

→ It is Art of giving bad answers to problem which otherwise have worse answer.
- T. L. Saat

Features of operations Research:-

→ Decision Making
→ Scientific Approach
→ Iter-Disciplinary Team approac
→ System Approach
→ Use of Computers
→ Objectives
→ Human factors

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Decision Making ⇒ Every industrial organisation faces multiface problems to identify the best possible solution to their problems.

To obtain optimal solution with the use of OR-techniques.

Scientific Approach:- OR is the application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide those in control of operations with optimum solutions to the problems.

Interdisciplinary Approach:- For solving a problem interdisciplinary team work is essential. This is because while attempting to solve a complex management problem, one person may not have the complete knowledge of all its aspects. (Such as economic, social, politic psychological, engineering etc).
It means we should not expect one person to find desirable solution to all management problems.

Therefore a term of individuals specializing in mathematics, statistics, economics, engineering, computer science etc.

System approach:- The main aim of the system approach is to trace out all significant and indirect effects for each proposal on all subsystem on a system and to evaluate each action in terms of effects for the system as a whole.

Use of computers:- The models of OR need lot of computation and therefore the use of computers becomes necessary. Use of computers it is possible to handle complex problems requiring large amount of calculations.

The objective of the OR model is to attempt and to locate best or optimal solution.

~~Obje~~ What is the objective?

Operations Research always try to find the best and optimal solution to the problem.

Human factors:- In deriving, Quantitative Solutions we do not consider human factors, which doubtlessly play a great role in the problems.

Advantages of operations Research Study:-

A decision maker in business and industry comes across the problem of making decisions in his day to day activities.

-> Structured Approach to problems:-
A substantial amount of time ~~&~~ effort can be saved in developing and solving OR models if one uses a logical and consistent approach.

-> Critical approach to problem solving:-
The decision maker will come to understand various components of the problem and accordingly select a mathematical model for solving the given problem.

Applications of Operations Research:-

Some of the industrial / government / busin problems that can be analysed by the OR approach have been arranged by functional areas as follows:-

- Finanance and Accounting
- Marketing
- Purchasing, Procurement and Exploral
- production Mangement
- Facilities planning
- Manufacturing
- Maintenance and project Scheduling
- personal mangement.
- Techniques and general managment
- Government

Operations research is the systematic application of quantitative methods, techniques and tools to the analysis of problems involving the operation of the systems.

(Daellenbach and George, 1978)

* OR is essentially a collection of mathematical techniques and tools which in conjunction with a system approach, are applied to solve practical decision problems of an economic or engineering nature.

* OR may be described as a scientific approach to decision-making that involves the operation of organizational system.

(F.S. Hiller and G.J. Lieberman, 198[illegible])

Linear programming problem:-

A model, which is used for optim allocation of scarce or limited resources to competing products or activities under such assumptions as certainty, linearity, fixed technology and constant profit as per unit, is linear programming.

Properties of Linear programming ~~pro~~ Model:-

a) The relationship b/w variables and constraints must be linear

b) The model must have objective fun

c) The model must have structural constraints

d) Non negativity constraint

maximize $z = 5x + 4y$ — Objective fun

For machine M_1

$0x + 2y \leq 50$

M_2 $1x + 2y \leq 25$ and

M_3 $1x + 1y \leq 15$ — Constraints

Both x and y are ≥ 0

Non negativity constraint

General L.P.P:-

The general mathematical way of representing a L.P.P. is as given below.

$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \rightarrow$ objective fun

Constraints

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1j}x_j + \cdots + a_{1n}x_n \; (\geq, =, \leq) \; b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2j}x_j + \cdots + a_{2n}x_n \; (\geq, =, \leq) \; b_2$

$\cdots \quad \cdots \quad \cdots$

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mj}x_j + \cdots + a_{mn}x_n \; (\geq, =, \leq) \; b_m$

and all x_j are ≥ 0 Non-negativity Constraint

where $j = 1, 2, \ldots n$.

Graphical method:- L.P.P involving two variables can be effectively solved by a graphical method.

Solve the L.P.P. of max $Z = 3x_1 + 4x_2$ —(i)

Subject to $4x_1 + 2x_2 \leq 40$ —(ii)

$2x_1 + 5x_2 \leq 180$ —(iii)

$x_1, x_2 \geq 0$ —(iv)

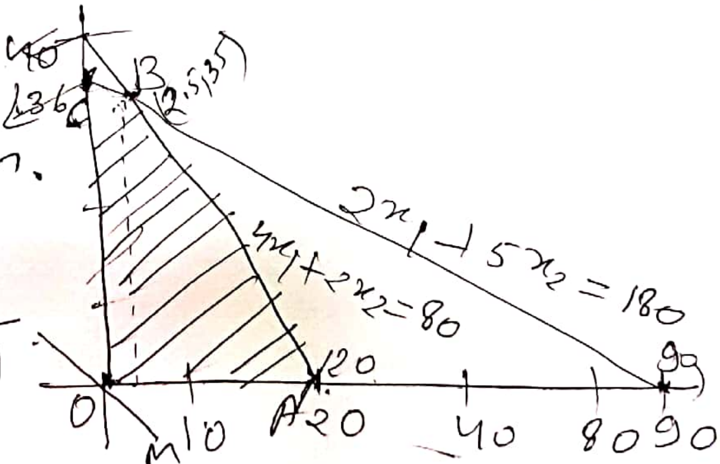
We plot the lines $4x_1 + 2x_2 = 80$

$2x_1 + 5x_2 = 180$

when we put $x_2 = 0$, $x_1 = 20, 90$

$x_1 = 0$, $x_2 = 40, 36$

This shows the desired pt (x_1, x_2) must somewhere in the shaded common region. OABC. The vertices are O(0,0), A(20,0) & B(2.5,35) & C(0,36)



Extreme point	Coordinate	$Z = 3x_1 + 4x_2$
O	(0,0)	Z = 0
A	(20,0)	Z = 60
B	(2.5,35)	$Z = 7.5 + 140$ $= \cancel{7.640}$ 147.5
C	(0,36)	Z = 144

160

$2x_1 + 5x_2 = 180 \} \times 2$

$4x_1 + 2x_2 = 80$

$8x_2 = 280$

$x_2 = 35$

$x_1 = \frac{80-70}{4}$

$= 2.5$

Maximum value $Z = 147.5$ at $x_1 = 2.5$ and $x_2 = 35$

Graphical Solⁿ method —

Extreme point Solⁿ method.—

1. Develop an L.P. model

2. Plot Constraints on graph paper and decide the feasible region

3. Examine extreme points of feasible Solⁿ space to find an optimal Solⁿ.

Examples of Maximization LPP:—

Use the graphical method to Solve the following LPP.

max $Z = 15x_1 + 10x_2$

Subject to the constraints

(i) $4x_1 + 6x_2 \le 360$, (ii) $3x_1 + 0x_2 \le 180$

(iii) $0x_1 + 5x_2 \le 200$, $x_1, x_2 \ge 0$

The given L.P problem is already mathematical form.

x_1 treats as horizontal line and x_2 treats as vertical line.

given $4x_1 + 6x_2 = 360$ (for finding the value of x,

put $x_2 = 0$,

$x_1 = 90$.

and $3x_1 = 180$, $x_1 = 60$

when put $x_1 = 0$, $x_2 = 60$

$5x_2 = 200$, $x_2 = 40$

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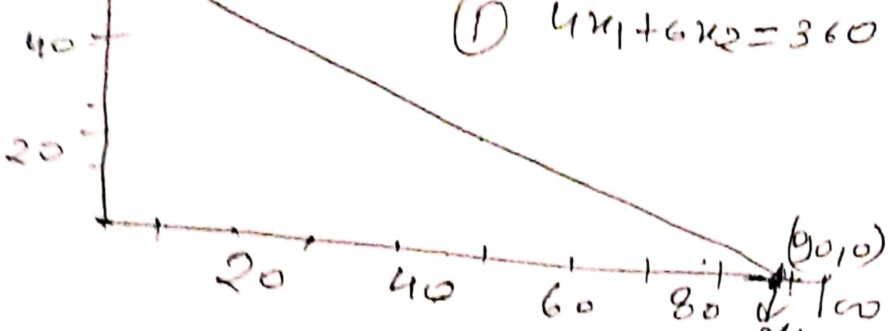
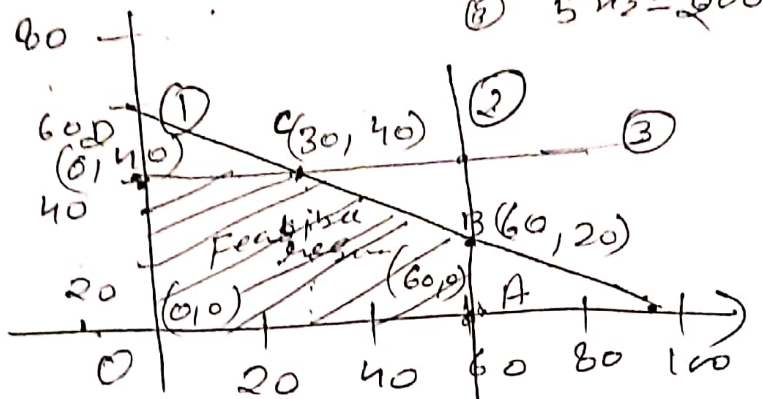


Graphical soln of LPP.

(1) $4x_1+6x_2=360$
(2) $3x_1=180$
(3) $5x_2=200$



The area which is bounded by all the constraints lines including all the boundary points is called, feasible soln ∴, Shaded area (O ABCD)

Now find optimal value of the objective fun.

Extreme point	Coordinate	$Z = 15x_1 + 10x_2$
O	(0,0)	$Z = 0$
A	(60,0)	$Z = 900$
B	(60,20)	$Z = 900+200 = 1,100$ $\boxed{1,100}$
C	(30,40)	$Z = 450+400 = 950$
D	(0,40)	$Z = 400$

Hence maximum value of $Z = 1,100$, is achieved at point B (60,20) ∴ So optimal soln of given LPP is $x_1 = 60$, $x_2 = 20$, $Z = 1,100$.

Q

Consider the Problem

max $z = 40x_1 + 80x_2$

Subject to the constraints

(i) $2x_1 + 3x_2 \leq 48$ (ii) $x_1 \leq 15$ (iii) $x_2 \leq 10$

$x_1, x_2 \geq 0$

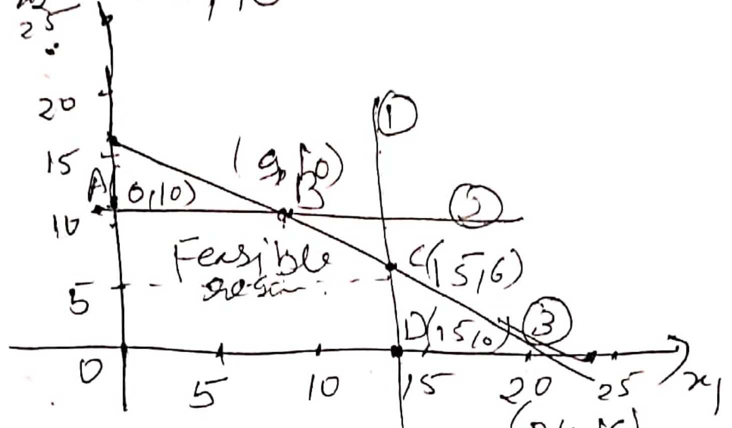
For finding extreme points the eqn

$2x_1 + 3x_2 = 48,$

$x_1 = 15,$ $x_2 = 10,$

Put $x_2 = 0,$ $x_1 = 24, 15$

$x_1 = 0,$ $x_2 = 16, 10$



Extreme point	Coordinates (x_1, x_2)	Objective fun $z = 40x_1 + 80x_2$
O	(0,0)	0
A	(0,10)	800
B	(9,10)	1,160
C	(15,6)	1080
D	(15,0)	600

maximum value of objective fun $z = 1,160,$ occur at the extreme point $(9,10),$ the optimum soln of given LPP is $x_1 = 9,$ $x_2 = 10,$ and max $z = 1,160.$

Simplex Method:- Suppose, LPP can have more than two variables, so to solve such kind of LPPs simplex method.

Step-1. Express the problem in standard form

Step-2: Introduce slack variables in objective function and in constraints (We add non-negative variable in each constraint in order to have equality of LHS and RHS.

→ Coefficient of slack variable in objective function & constraints (coefficient of the slack variable in objective fun as zero).

Step-3. Rewrite the given LPP and prepare simplex table to obtain initial basic sol^n.

C_j		c_1	c_2	c_3	0	0	0	
Basic	C_B	x_1	x_2	x_3	s_1	s_2	s_3	b_i
s_1	0	a_{11}	a_{12}	a_{13}	1	0	0	b_1
s_2	0	a_{21}	a_{22}	a_{23}	0	1	0	b_2
s_3	0	a_{31}	a_{32}	a_{33}	0	0	1	b_3
	Z_j	0	0	0	0	0	0	
	$c_j - z_j$	c_1	c_2	c_3	0	0	0	

Basic variables :- Initial basic variables are taken as slack variable to get IBFS.

→ Coefficient of basic variables
→ b_i - It represents the RHS of constraints
θ : " " the ratio b_i to the key column coefficient for each basic variables.

Step-4 Calculate Z for initial ~~var~~ basic basic variables.

Step-5 Calculate $C_j - Z_j$ for initial basic variables.

Step-6 Select the key column :- max positive value in case of maximize and max negative value in case of minimization problem.

Step-7 Select the key row :- After selecting the key column, calculate θ for each basic variable. The row which has minimum positive value is selected as key row.

Step 8. Change the basic variables. key column is considered as entering variables and key row is selected as leaving variable

~~Sele~~ Step 9: ... can have

Prepare the new simplex table.

From key row elements, divide all row elements by key no.

Fraction ratio = $\frac{\text{Corresponding key column no.}}{\text{key no.}}$

New no. of every cell = old no. − Corresponding key row no. × F.

Step-10 Repeat step 5 to 9 until the values of $C_j - Z_j$ corresponding to all variables is coming as zero or negative for maximization problem

for minimization problem then repeat steps 5 to 9 until values of $C_j - Z_j$ corresponding to all variables is coming as zero or positive.

Step-11 The soln obtained ~~as~~ is optimal.

5.

2.

key column

Solve the following L.P.P. by Simplex method!

maximize $z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \le 5$

$x_1 + x_2 \le 3$ and $x_1, x_2 \geq 0$

The given LPP is in standard form.

maximize $z = 3x_1 + 2x_2 + 0 s_1 + 0 s_2$

Subject to $2x_1 + x_2 + s_1 + 0 s_2 = 5$

$x_1 + x_2 + 0 s_1 + s_2 = 3$

where s_1 and s_2 are slack variables.

$x_1, x_2, s_1, s_2 \geq 0$

C_j		3	2	0	0		
Basic	C_B	x_1	x_2	s_1	s_2	b_i	Ratio
s_1	0	$\boxed{2}$	1	1	0	5	5/2 ← key row
s_2	0	1	1	0	1	3	3
z_j		0	0	0	0	0	
$C_j - z_j$		3	2	0	0		$z=0$

Pivot no. or key no

Selecting key column and key row

$\theta_1 = 5/2$, $\theta_2 = 3$,

least positive value is θ_1

Entering variable x_1

leaving " s_1

Pivot (key no) 2

LPP can have

Basic	C_B	x_1	x_2	S_1	S_2	b_i	Ratio
x_1	3	1	1/2	1/2	0	5/2	
S_2	0	0	1/2	-1/2	1	1/2	

fraction ratio from S_2 = corresponding key column no / key no

~~New cell value~~ = 1 =

$= \frac{1}{2}$

$2 - \frac{4}{3} = \frac{2}{3}$

$\frac{4}{3} - 2 = -\frac{2}{3}$. 0 -

new cell value = $1 - \frac{1}{2}$. corresponding key row no.

$= 1 - \frac{1}{2} \cdot 2 = 0$

$a_{22} = 1 - \frac{1}{2} = 1/2$ $\quad (1 - \frac{1}{2})$

$a_{23} = 0 - \frac{1}{2} \cdot 1 = -1/2$

$a_{24} = 1 - \frac{0}{2} \cdot 0 = 1$ $\quad \frac{5}{2} - \frac{1}{2}$

$a_{25} = 3 - \frac{5}{2} = 1/2$

C_j		3	2	0	0		
Basic	C_B	x_1	x_2	S_1	S_2	b_i	Ratio
x_1	3	1	1/2	1/2	0	5/2	5
S_2	0	0	$\boxed{1/2}$	-1/2	1	1/2	1 ⟵ key row
Z_j		3	3/2	3/2	0		
$C_j - Z_j$		0	1/2	-3/2	0		

key element ↑ key column

$2 - 3/2$

Sales

Basics of OR & LPP

C_j		3	2	0	0	
Basic	C_B	x_1	x_2	s_1	s_2	b_i
x_1	3	1	0	1	-1	2
x_2	2	0	1	-1	2	1
Z_j		3	2	1	+1	Z=8
$C_j - Z_j$		0	0	-1	-1	

Here value of $C_j - Z_j$ is either zero or negative. So this soln is optimal. here we can write the soln..

max z $10x_1 + 9x_2 + 0S_1 + S$

$3x_1 + 3x_2 \le 21$

$4x_1 + 3x_2 \le 24$ $\quad x_1, x_2 \ge 0$

$3x_1 + 3x_2 + S_1 = 21$

$4x_1 + 3x_2 + S_2 = 24$

C_j		10	9	0	0		
S_B	C_B	x_1	x_2	S_1	S_2	Soln	Ratio
S_1	0	3	3	1	0	21	$\frac{21}{3} = 7$
S_2	0	$\boxed{4}$ Pivot element	3	0	1	24	$\frac{24}{4} = 6$ → least positive
$C_j - Z_j$		10 ↑	9	0	0		
S_1	0	0	$\boxed{\frac{3}{4}}$ → Pivot element	1	$-3/4$	3	4 → least. Pivot element
x_1	10	1	$\frac{3}{4}$	0	$1/4$	6	8
$C_j - Z_j$		0	$\frac{3}{2}$ ↑	0	$-\frac{5}{2}$	60	
x_2	9	0	1	$\frac{4}{3}$	-1	4	
x_1	10	1	0	-1	1	3	
Z_j		10	9	2	1		
$C_j - Z_j$		0	0	-2	-1	66	

$x_1 = 3$

$x_2 = 4$

$Z = 66$

S_2 leaves

$x_1 \to$ enters

Pivot Row

$3 - \frac{9}{4}$

$21 - P$

$9 - \frac{30}{4}$

$\frac{3 \times 4}{3}$ $\frac{6}{3}$ 24

enters

[illegible]

leaves

$\frac{4}{3}$

$\frac{3}{4}$

$\frac{3}{4} \cdot \frac{-3}{4}$ $\quad 0 - \frac{3}{4} \cdot \frac{4}{3}$

36 30

$1 + \frac{3}{4} \cdot \frac{3}{4}$ 7

2 - 10 $\quad$ 9+10 $\quad$ 36+30 $\quad$ -36+30 $\quad$ $6 - \frac{3}{4} \cdot 4$

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(3)

$\max z = -x_1 + 3x_2 - 3x_3$

$3x_1 - x_2 + 2x_3 + s_1 = 7$

$-2x_1 + 4x_2 \qquad s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

C_j		-1	3	-3	0	0	0	
x_B	C_B	x_1	x_2	x_3	s_1	s_2	s_3	Sol
s_1	0	3	-1	2	1	0	0	7
s_2	0	-2	-4	0	0	1	0	12
s_3	0	-4	$\boxed{3}$ key element	8	0	0	1	10
z_j		0	0	0	0	0	0	
$c_j - z_j$		-1	$3 \uparrow$	-3	0	0	0	
s_1	0	key element $\boxed{5/3}$	0	$14/3$	1	0	$1/3$	$31/3$
s_2	0	$-22/3$	0	$32/3$	0	1	$4/3$	$76/3$
x_2	3	$-4/3$	1	$8/3$	0	0	$1/3$	$\frac{10}{3}$
z_j		-4	3	8	0	0	1	10
$c_j - z_j$		$3 \uparrow$	0	-11	0	0	-1	
x_1	-1	1	0	$14/5$	$3/5$	0	$1/5$	$\frac{31}{5}$
s_2	0	0	0	$468/15$	$22/5$	1	$42/15$	$\frac{1062}{15}$
x_2	3	0	1	$96/15$	$4/5$	0	$9/15$	$\frac{174}{15}$
z_j		-1	3	$82/5$	$9/5$	0	$8/15$	$143/5$
$c_j - z_j$		0	0	$-97/5$	$-9/5$	0	$-8/5$	

$x_1 = \frac{31}{5}$, $x_2 = \frac{58}{5}$, $x_3 = 0$

$z = \frac{143}{5}$

$\frac{R_3}{3}$

entering variable $= x_2$

leaving $= s_3$

key element $= 3$

$R_3/3$

$R_2 \to R_2 + 4R_3$

$R_1 \to R_1 + R_3$

$\frac{3}{5} R_1$, entering $\to x_1$

leaving $\to s_1$

key element $= \frac{5}{3}$

$R_2 \to R_2 + \frac{22}{3} R_1$

$R_3 \to R_3 + \frac{4}{3} R_1$