ज्ञानेन प्रकाशते जगत्
INDUS UNIVERSITY

VLSI Design (EC0604)
Unit-2
B.Tech (Electronics and Communication)
Semester-6

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Delay in a Logic Gate

- Express delays in process-independent unit
- Delay has two components: d = f + p
- *f*: *effort delay* = *gh* (a.k.a. stage effort)
  - Again has two components
- *g*: *logical effort*
  - Measures relative ability of gate to deliver current
  - $g \equiv 1$ for inverter
- *h*: *electrical effort* = C_{out} / C_{in}
  - Ratio of output to input capacitance
  - Sometimes called fanout
- *p*: parasitic delay
  - Represents delay of gate driving no load
  - Set by internal parasitic capacitance

$$d = \frac{d_{abs}}{\tau}$$

τ = 3RC

$\approx$ 3 ps in 65 nm process

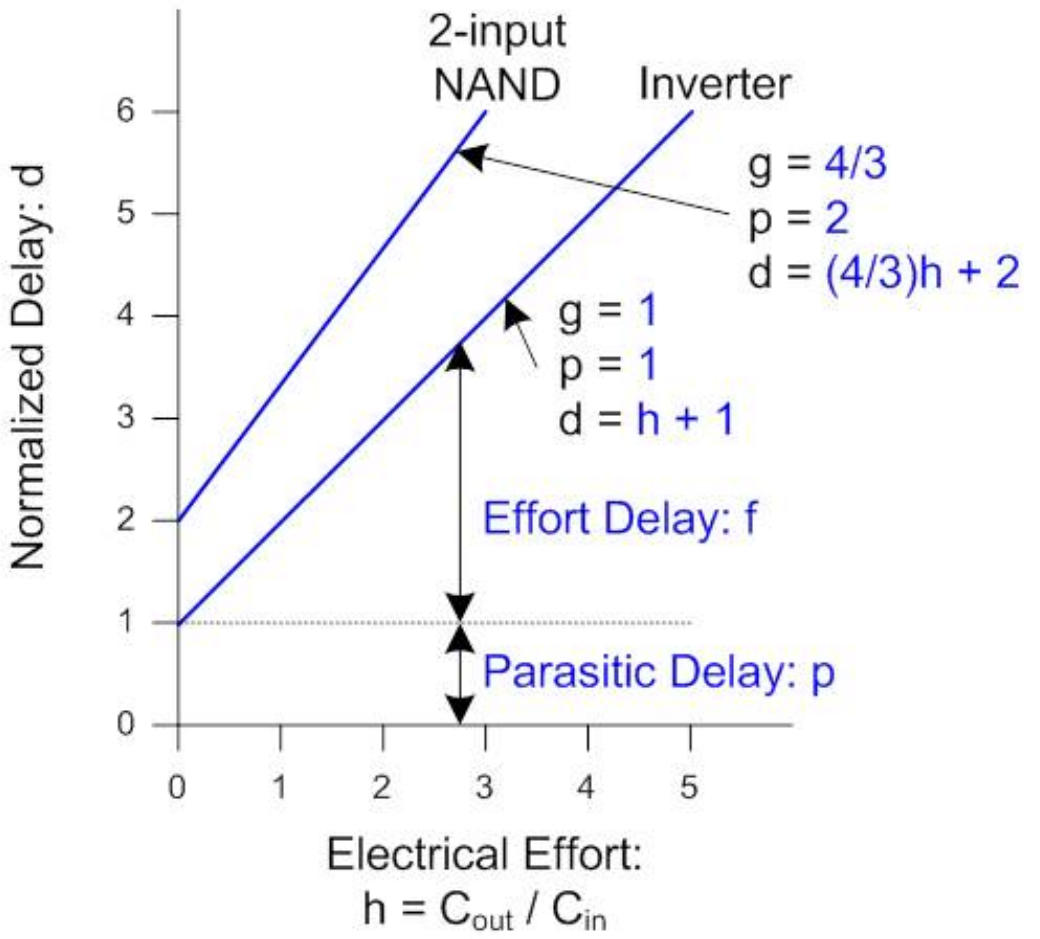
60 ps in 0.6 μm process

Delay Plots

$d = f + p$

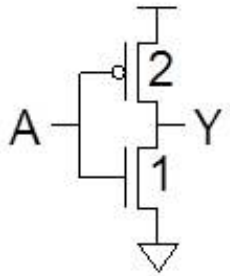
$= gh + p$

- What about NOR2?

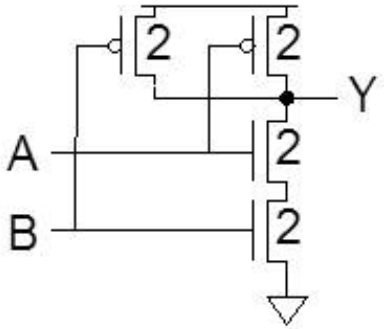


Computing Logical Effort

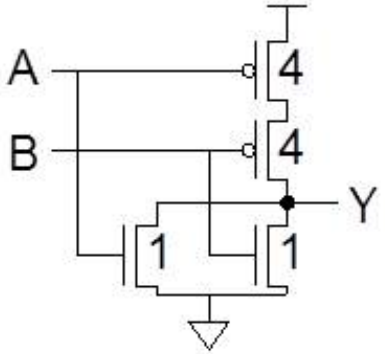
- DEF: *Logical effort is the ratio of the input capacitance of a gate to the input capacitance of an inverter delivering the same output current*.
- Measure from delay vs. fanout plots
- Or estimate by counting transistor widths



C_{in} = 3
g = 3/3



C_{in} = 4
g = 4/3



C_{in} = 5
g = 5/3

Catalog of Gates

- Logical effort of common gates

Gate type	Number of inputs				
	1	2	3	4	n
Inverter	1				
NAND		4/3	5/3	6/3	(n+2)/3
NOR		5/3	7/3	9/3	(2n+1)/3
Tristate / mux	2	2	2	2	2
XOR, XNOR		4, 4	6, 12, 6	8, 16, 16, 8	

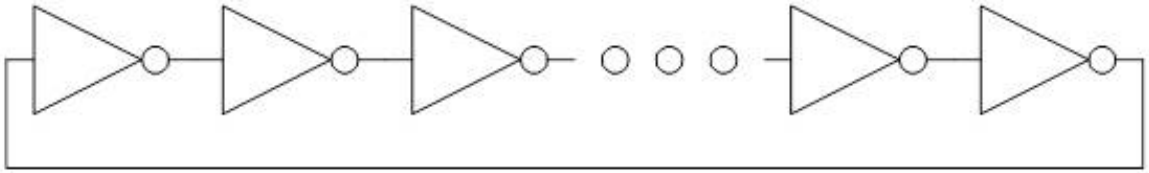
Catalog of Gates

- Parasitic delay of common gates
  - In multiples of p_{inv} (≈ 1)

Gate type	Number of inputs				
	1	2	3	4	n
Inverter	1				
NAND		2	3	4	n
NOR		2	3	4	n
Tristate / mux	2	4	6	8	2n
XOR, XNOR		4	6	8	

Example: Ring Oscillator

- Estimate the frequency of an N-stage ring oscillator



Logical Effort: g = 1

Electrical Effort: h = 1

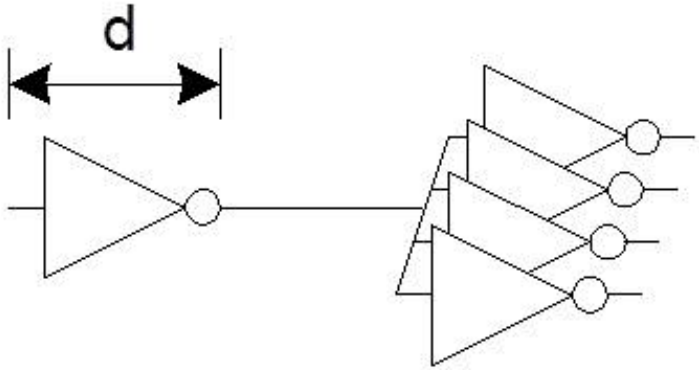
Parasitic Delay: p = 1

Stage Delay: d = 2

Frequency: $f_{osc} = 1/(2*N*d) = 1/4N$

Example: FO4 Inverter

- Estimate the delay of a fanout-of-4 (FO4) inverter



Logical Effort: g = 1

Electrical Effort: h = 4

Parasitic Delay: p = 1

Stage Delay: d = 5

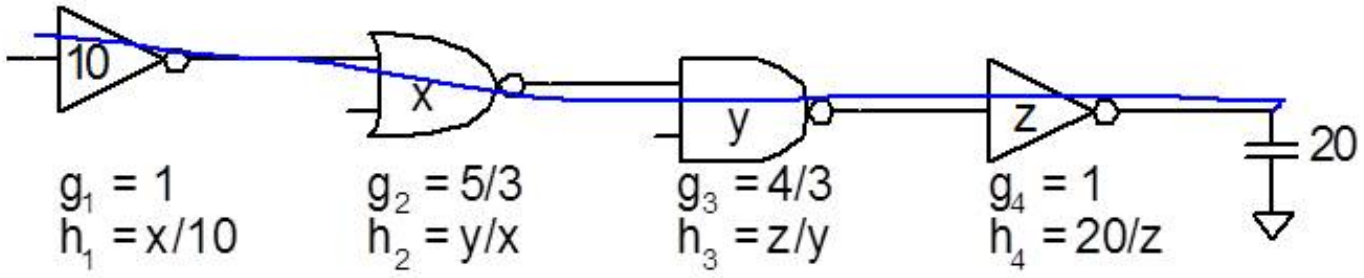
The FO4 delay is about

300 ps in 0.6 μm process

15 ps in a 65 nm process

Multistage Logic Networks

- Logical effort generalizes to multistage networks
- *Path Logical Effort* $G = \prod g_i$
- *Path Electrical Effort* $H = \frac{C_{\text{out-path}}}{C_{\text{in-path}}}$
- *Path Effort* $F = \prod f_i = \prod g_i h_i$



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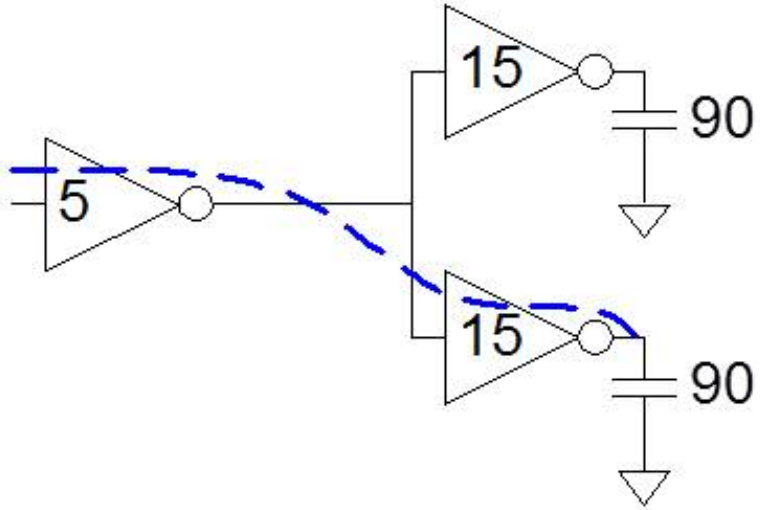
- *Path Effort* $F = \prod f_i = \prod g_i h_i$
- Can we write F = GH?

Paths that Branch

- No! Consider paths that branch:



G = 1

H = 90 / 5 = 18

GH = 18

h_1 = (15 +15) / 5 = 6

h_2 = 90 / 15 = 6

F = $g_1g_2h_1h_2$ = 36 = 2GH

Branching Effort

- Introduce *branching effort*
  - Accounts for branching between stages in path

$$b = \frac{C_{\text{on path}} + C_{\text{off path}}}{C_{\text{on path}}}$$

$$B = \prod b_i$$

Note:

$$\prod h_i = BH$$

- Now we compute the path effort
  - F = GBH

Multistage Delays

- Path Effort Delay $D_F = \sum f_i$
- Path Parasitic Delay $P = \sum p_i$
- Path Delay $D = \sum d_i = D_F + P$

Designing Fast Circuits

$$D = \sum d_i = D_F + P$$

- Delay is smallest when each stage bears same effort

$$\hat{f} = g_i h_i = F^{\frac{1}{N}}$$

- Thus minimum delay of N stage path is

$$D = NF^{\frac{1}{N}} + P$$

- This is a key result of logical effort
  - Find fastest possible delay
  - Doesn't require calculating gate sizes

Gate Sizes

- How wide should the gates be for least delay?

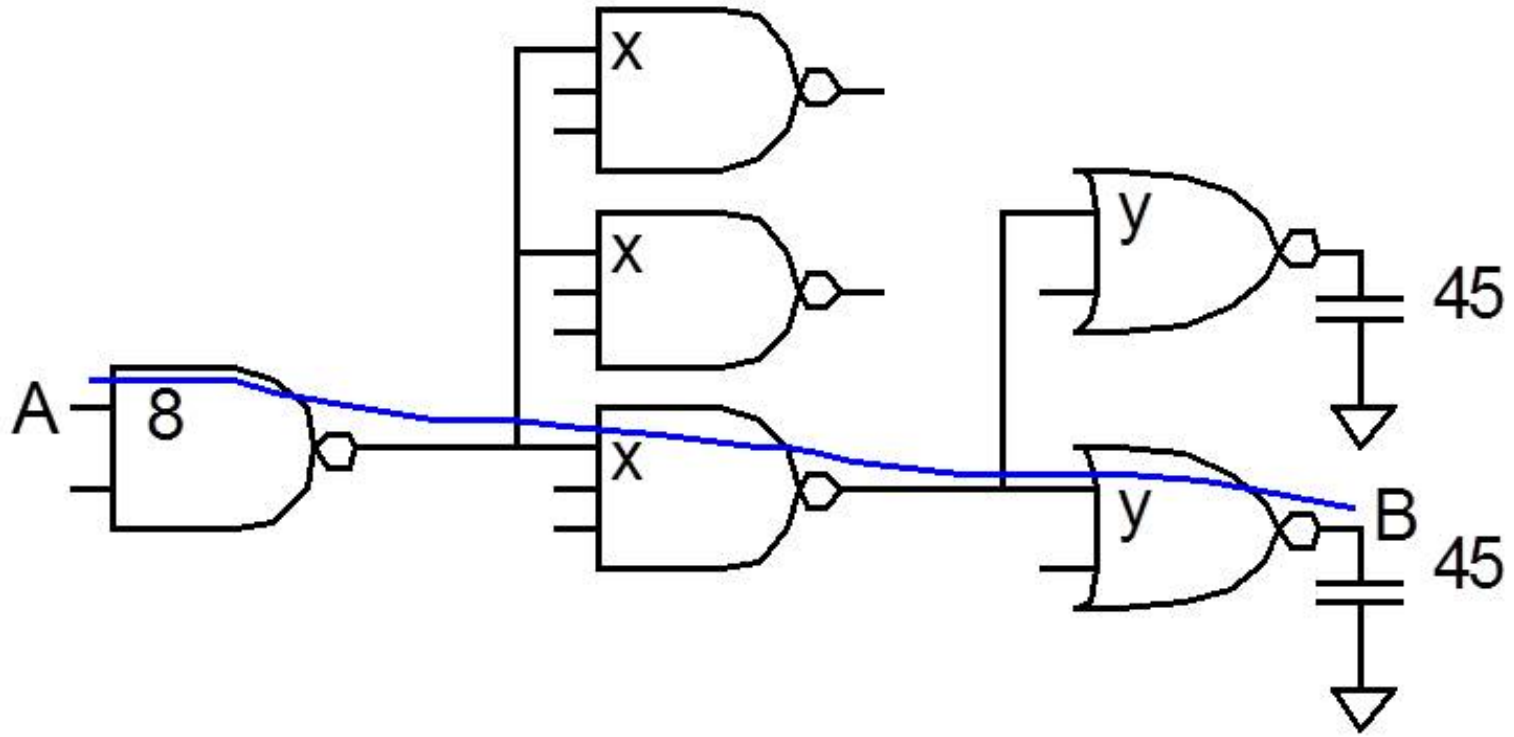
$$\hat{f} = gh = g\frac{C_{out}}{C_{in}}$$

$$\Rightarrow C_{in_i} = \frac{g_i C_{out_i}}{\hat{f}}$$

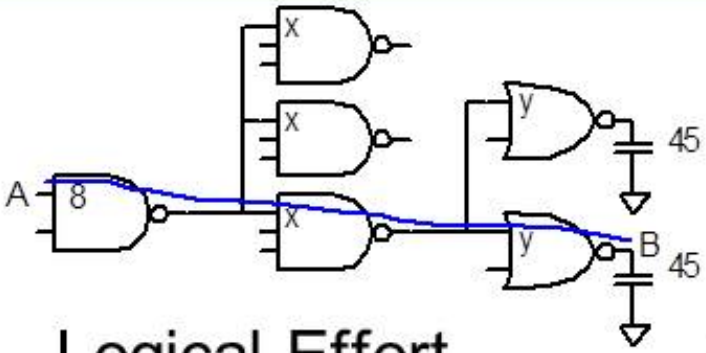
- Working backward, apply capacitance transformation to find input capacitance of each gate given load it drives.
- Check work by verifying input cap spec is met.

Example: 3-stage path

- Select gate sizes x and y for least delay from A to B



Example: 3-stage path



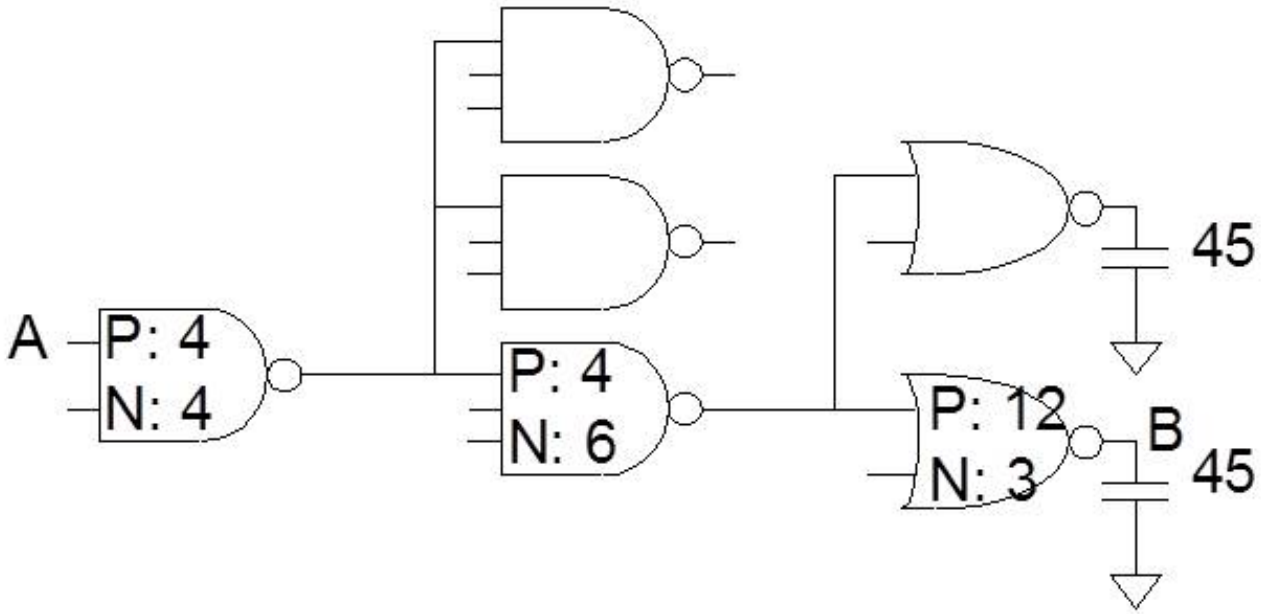
Logical Effort	G = (4/3)*(5/3)*(5/3) = 100/27
Electrical Effort	H = 45/8
Branching Effort	B = 3 * 2 = 6
Path Effort	F = GBH = 125
Best Stage Effort	$\hat{f} = \sqrt[3]{F} = 5$
Parasitic Delay	P = 2 + 3 + 2 = 7
Delay	D = 3*5 + 7 = 22 = 4.4 FO4

Example: 3-stage path

- Work backward for sizes

  y = 45 * (5/3) / 5 = 15

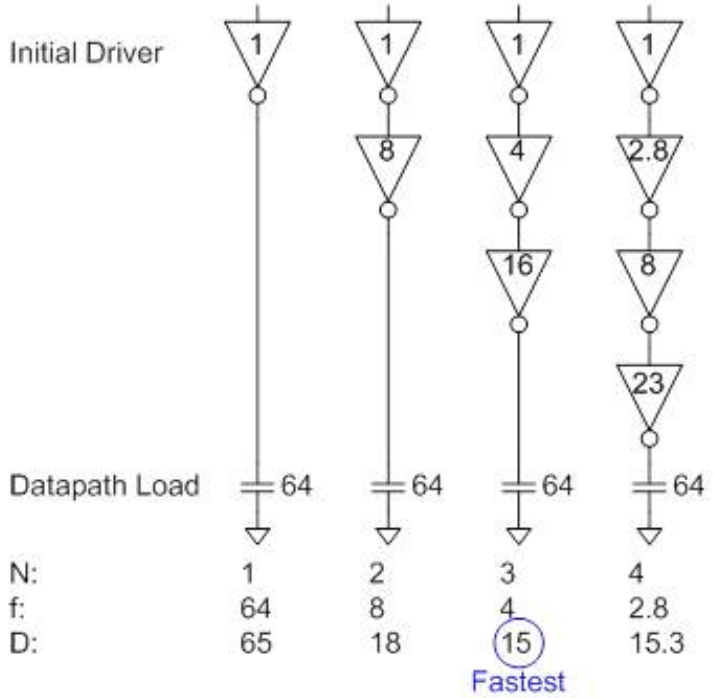
  x = (15*2) * (5/3) / 5 = 10



Best Number of Stages

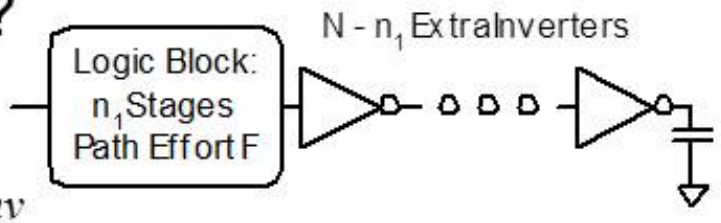
- How many stages should a path use?
  - Minimizing number of stages is not always fastest
- Example: drive 64-bit datapath with unit inverter

$$D = NF^{1/N} + P$$
$$= N(64)^{1/N} + N$$



Derivation

- Consider adding inverters to end of path
  - How many give least delay?



$$D = NF^{\frac{1}{N}} + \sum_{i=1}^{n_1} p_i + \left(N - n_1\right) p_{inv}$$

$$\frac{\partial D}{\partial N} = -F^{\frac{1}{N}} \ln F^{\frac{1}{N}} + F^{\frac{1}{N}} + p_{inv} = 0$$

- Define best stage effort $\rho = F^{\frac{1}{N}}$

$$p_{inv} + \rho\left(1 - \ln \rho\right) = 0$$

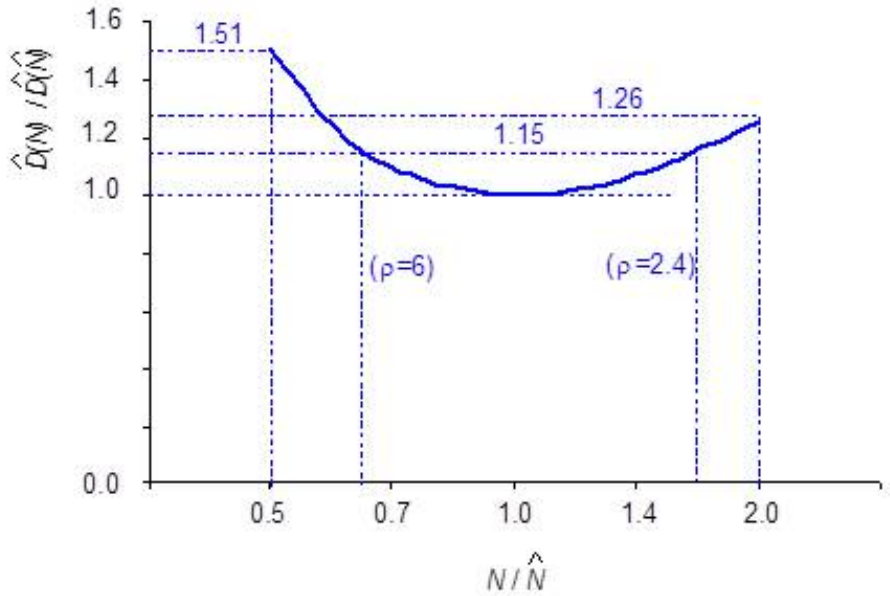
Best Stage Effort

- $p_{inv} + \rho(1 - \ln \rho) = 0$ has no closed-form solution

- Neglecting parasitics ($p_{inv} = 0$), we find $\rho = 2.718$ (e)
- For $p_{inv} = 1$, solve numerically for $\rho = 3.59$

Sensitivity Analysis

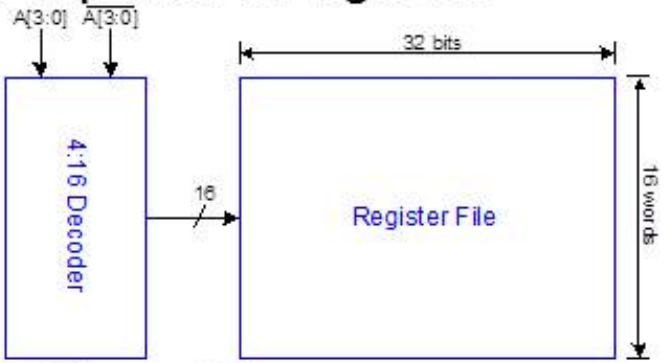
- How sensitive is delay to using exactly the best number of stages?



- $2.4 < \rho < 6$ gives delay within 15% of optimal
  - We can be sloppy!
  - I like $\rho = 4$

Example, Revisited

- Ben Bitdiddle is the memory designer for the Motoroil 68W86, an embedded automotive processor. Help Ben design the decoder for a register file.



- Decoder specifications:
  - 16 word register file
  - Each word is 32 bits wide
  - Each bit presents load of 3 unit-sized transistors
  - True and complementary address inputs A[3:0]
  - Each input may drive 10 unit-sized transistors
- Ben needs to decide:
  - How many stages to use?
  - How large should each gate be?
  - How fast can decoder operate?

Number of Stages

- Decoder effort is mainly electrical and branching

  Electrical Effort: $H = (32*3) / 10 = 9.6$

  Branching Effort: $B = 8$

- If we neglect logical effort (assume G = 1)

  Path Effort: $F = GBH = 76.8$

  Number of Stages: $N = \log_4 F = 3.1$

- Try a -stage design

Gate Sizes & Delay

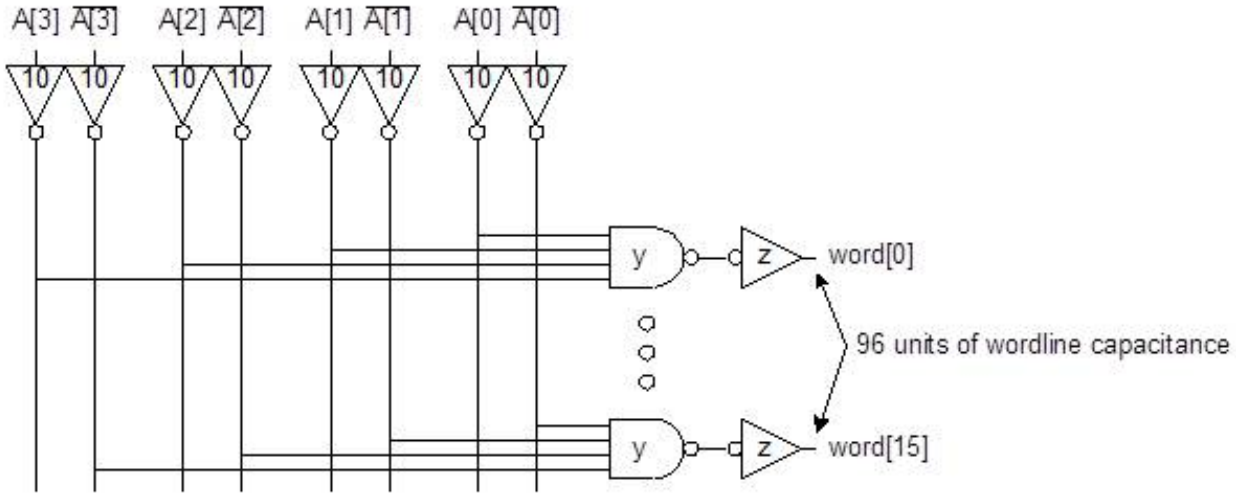
Logical Effort: G = 1 * 6/3 * 1 = 2

Path Effort: F = GBH = 154

Stage Effort: $\hat{f} = F^{1/3} = 5.36$

Path Delay: $D = 3\hat{f} + 1 + 4 + 1 = 22.1$

Gate sizes: z = 96*1/5.36 = 18 y = 18*2/5.36 = 6.7



Comparison

- Compare many alternatives with a spreadsheet
- $D = N(76.8\ G)^{1/N} + P$

Design	N	G	P	D
NOR4	1	3	4	234
NAND4-INV	2	2	5	29.8
NAND2-NOR2	2	20/9	4	30.1
INV-NAND4-INV	3	2	6	22.1
NAND4-INV-INV-INV	4	2	7	21.1
NAND2-NOR2-INV-INV	4	20/9	6	20.5
NAND2-INV-NAND2-INV	4	16/9	6	19.7
INV-NAND2-INV-NAND2-INV	5	16/9	7	20.4
NAND2-INV-NAND2-INV-INV-INV	6	16/9	8	21.6

Review of Definitions

Term	Stage	Path
number of stages	1	N
logical effort	g	$G = \prod g_i$
electrical effort	$h = \frac{C_{out}}{C_{in}}$	$H = \frac{C_{out\text{-}path}}{C_{in\text{-}path}}$
branching effort	$b = \frac{C_{on\text{-}path} + C_{off\text{-}path}}{C_{on\text{-}path}}$	$B = \prod b_i$
effort	$f = gh$	$F = GBH$
effort delay	f	$D_F = \sum f_i$
parasitic delay	p	$P = \sum p_i$
delay	$d = f + p$	$D = \sum d_i = D_F + P$

Method of Logical Effort

1) Compute path effort $F = GBH$
2) Estimate best number of stages $N = \log_4 F$
3) Sketch path with N stages
4) Estimate least delay $D = NF^{\frac{1}{N}} + P$
5) Determine best stage effort $\hat{f} = F^{\frac{1}{N}}$
6) Find gate sizes $C_{in_i} = \frac{g_i C_{out_i}}{\hat{f}}$

Limits of Logical Effort

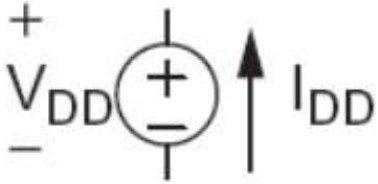
- Chicken and egg problem
  - Need path to compute G
  - But don't know number of stages without G
- Simplistic delay model
  - Neglects input rise time effects
- Interconnect
  - Iteration required in designs with wire
- Maximum speed only
  - Not minimum area/power for constrained delay

Power and Energy

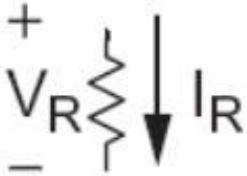
- Power is drawn from a voltage source attached to the V_{DD} pin(s) of a chip.
- Instantaneous Power: $P(t) = I(t)V(t)$
- Energy: $E = \int_0^T P(t)dt$
- Average Power: $P_{avg} = \frac{E}{T} = \frac{1}{T}\int_0^T P(t)dt$

Power in Circuit Elements

$$P_{VDD}(t) = I_{DD}(t) V_{DD}$$

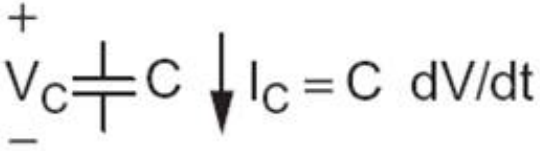


$$P_R(t) = \frac{V_R^2(t)}{R} = I_R^2(t) R$$



$$E_C = \int_0^\infty I(t) V(t)\, dt = \int_0^\infty C \frac{dV}{dt} V(t)\, dt$$

$$= C \int_0^{V_C} V(t)\, dV = \tfrac{1}{2} C V_C^2$$



Charging a Capacitor

- When the gate output rises
  - Energy stored in capacitor is

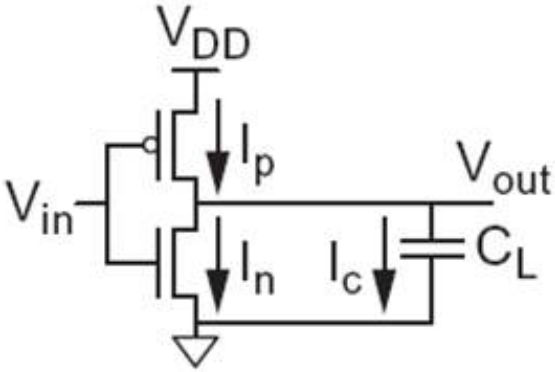
$$E_C = \frac{1}{2} C_L V_{DD}^2$$

  - But energy drawn from the supply is

$$E_{VDD} = \int_0^\infty I(t) V_{DD} dt = \int_0^\infty C_L \frac{dV}{dt} V_{DD} dt$$

$$= C_L V_{DD} \int_0^{V_{DD}} dV = C_L V_{DD}^2$$

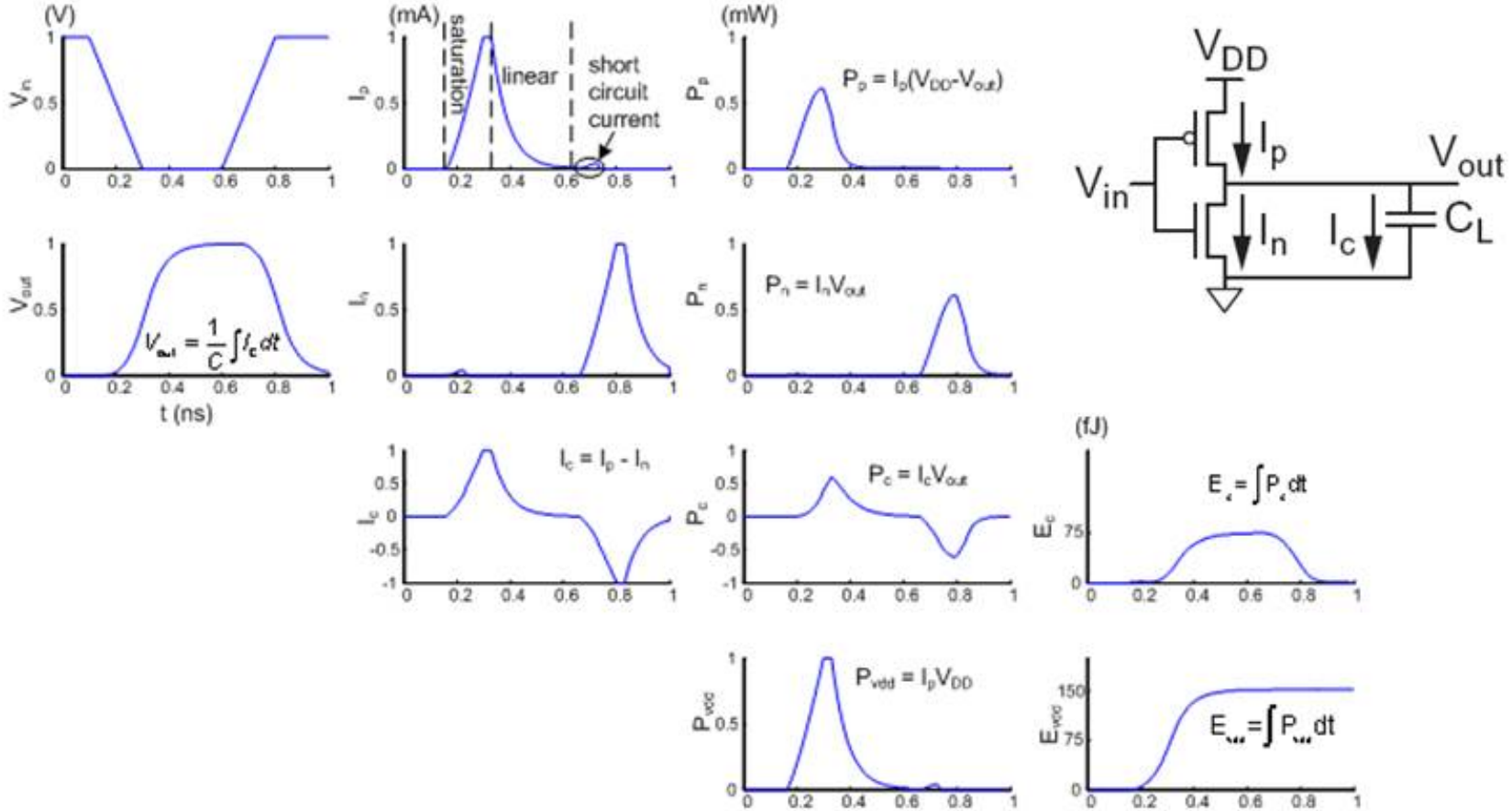
  - Half the energy from V_{DD} is dissipated in the pMOS transistor as heat, other half stored in capacitor



- When the gate output falls
  - Energy in capacitor is dumped to GND
  - Dissipated as heat in the nMOS transistor

Switching Waveforms

- Example: V_{DD} = 1.0 V, C_L = 150 fF, f = 1 GHz



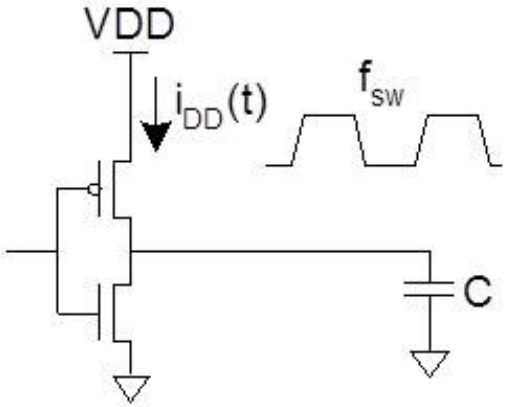
Switching Power

$$P_{\text{switching}} = \frac{1}{T}\int_0^T i_{DD}(t)V_{DD}\,dt$$

$$= \frac{V_{DD}}{T}\int_0^T i_{DD}(t)\,dt$$

$$= \frac{V_{DD}}{T}\left[Tf_{\text{sw}}CV_{DD}\right]$$

$$= CV_{DD}{}^2 f_{\text{sw}}$$



Activity Factor

- Suppose the system clock frequency = f
- Let $f_{sw} = \alpha f$, where α = activity factor
  - If the signal is a clock, $\alpha = 1$
  - If the signal switches once per cycle, $\alpha = ½$

- Dynamic power:

$$P_{\text{switching}} = \alpha C V_{DD}^{\ 2} f$$

Short Circuit Current

- When transistors switch, both nMOS and pMOS networks may be momentarily ON at once
- Leads to a blip of “short circuit” current.
- < 10% of dynamic power if rise/fall times are comparable for input and output
- We will generally ignore this component

Power Dissipation Sources

- $P_{total} = P_{dynamic} + P_{static}$
- Dynamic power: $P_{dynamic} = P_{switching} + P_{shortcircuit}$
  - Switching load capacitances
  - Short-circuit current
- Static power: $P_{static} = (I_{sub} + I_{gate} + I_{junct} + I_{contention})V_{DD}$
  - Subthreshold leakage
  - Gate leakage
  - Junction leakage
  - Contention current