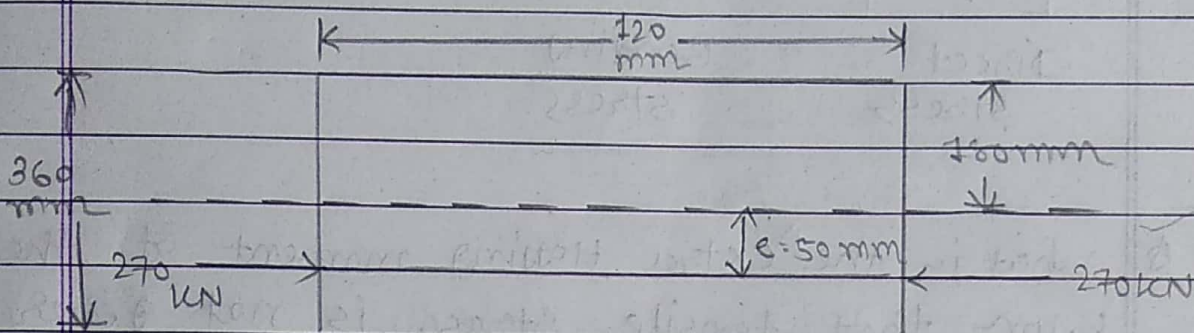
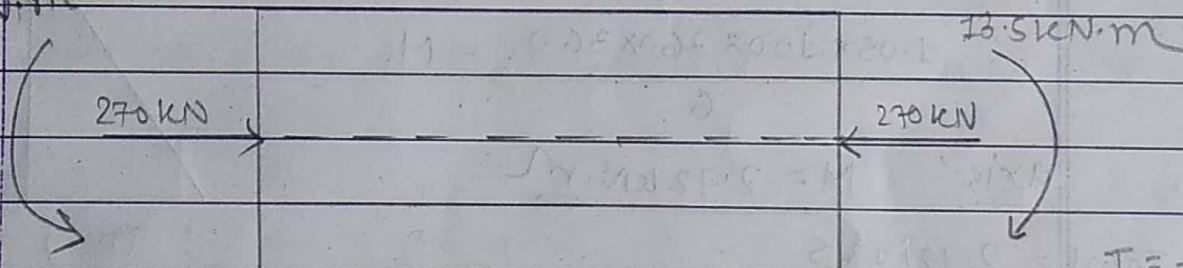


Units:  $\rightarrow 1$ \* Direct and Bending stress (combine stress)

\* calculate overall stresses at top end bottom of the beam



$$13.5 \text{ kN}\cdot\text{m} = 270 \times 0.05$$



T = +ve

C = -ve

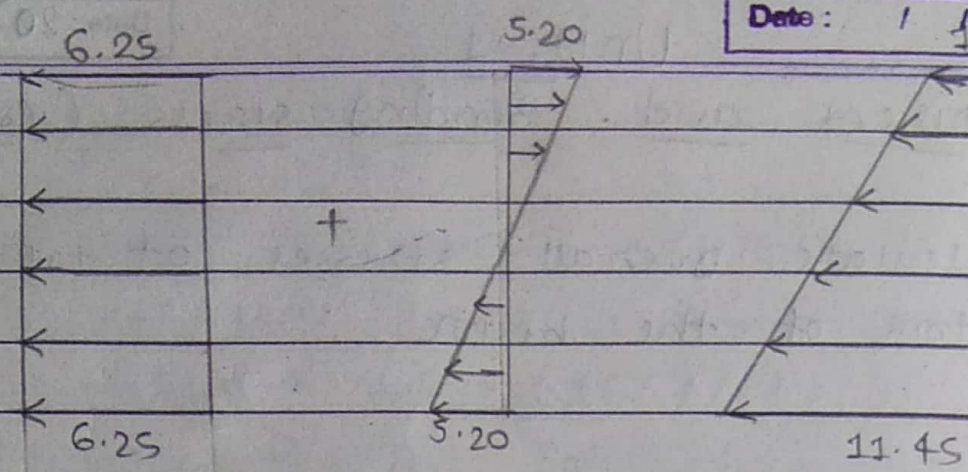
$$(1) \text{ Direct stress} = \sigma_d = \frac{P}{A} = \frac{270 \times 10^3}{120 \times 360} \\ \Rightarrow 6.25 \text{ N/mm}^2$$

$$(2) \text{ Bending stress} = \sigma_b = \frac{M}{Z} = \frac{13.5 \times 10^6 \times 6}{120 \times 360 \times 360}$$

$$Z = \frac{I}{y} = \frac{bd^3/12}{d/2} \\ = \frac{bd^2}{6}$$

$$= +5.20 \text{ N/mm}^2 \quad (\text{Top}) \\ -5.20 \text{ N/mm}^2 \quad (\text{bottom})$$

direct  $\rightarrow$  compressive



Direct stress

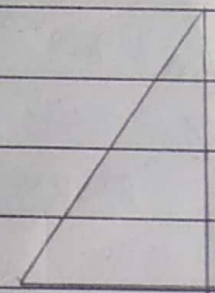
Bending stress

What is the extra Hogging moment of the beam that tensile stress is not generated

$$\sigma = \frac{M}{Z}$$

$$\therefore \frac{1.05 \times 120 \times 360 \times 360}{6} = M$$

axis:  $M = 2.72 \text{ kN.m}$



Hogging

Total =  $2.72 + 13.5$   
 $= 16.22$

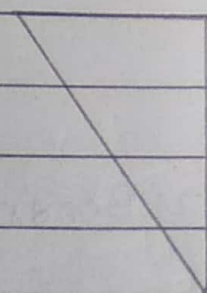
What is the extra total sagging moment (tensile stress)

$$\sigma = \frac{M}{Z}$$

$$\therefore M = \frac{11.45 \times 120 \times 360 \times 360}{6}$$

$M = 29.6 \text{ kN.m}$

Total = 29.6 kN.m



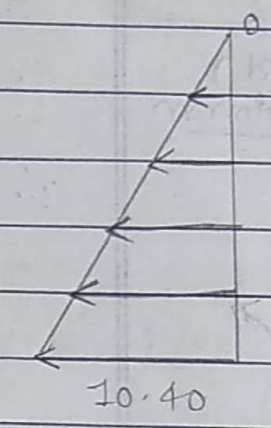
Sagging

axial force  $\rightarrow$  direct stress  
 diagram is stress vs depth

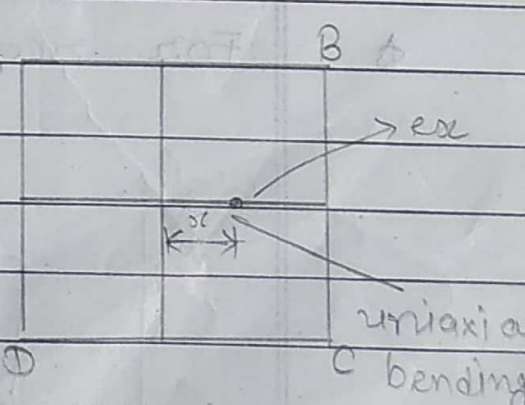
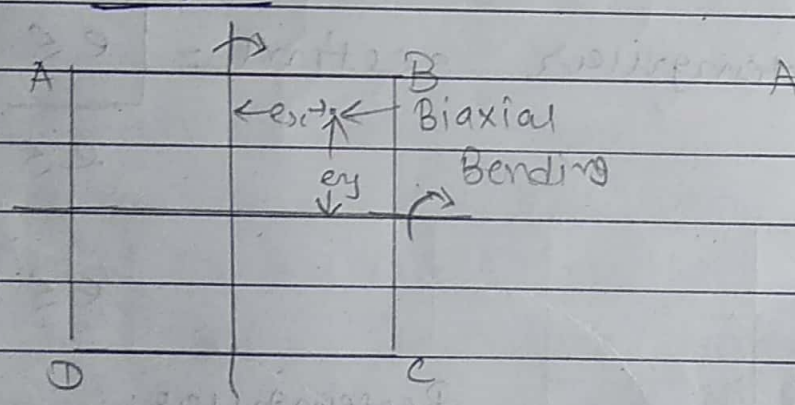
\* what is the value of external axial force (Tensile force) which is applied such that section is not subjected to tensile stress -

$\rightarrow$  direct stress  $\sigma_d = \frac{P}{A}$   
 $\therefore 1.05 = \frac{P}{120 \times 360}$

$\therefore P = 45.36 \text{ kN}$



\* Column  
 (1) Axial force  $\rightarrow$  Buckling  
 Axial force  $\rightarrow$  Bulging  
C+, T- Length of column



A =  $\sigma_d + \sigma_{bx} - \sigma_{by}$   
 B =  $\sigma_d + \sigma_{bx} + \sigma_{by}$   
 C =  $\sigma_d - \sigma_{bx} + \sigma_{by}$   
 D =  $\sigma_d - \sigma_{bx} - \sigma_{by}$

A =  $\sigma_d - \sigma_b \rightarrow$  Critical  
 B =  $\sigma_d + \sigma_b$   
 C =  $\sigma_d + \sigma_b$   
 D =  $\sigma_d - \sigma_b \rightarrow$  Critical

$\sigma_{bx} = \frac{M_x}{Z} = \frac{P y}{Z}$        $\sigma_{by} = \frac{M_y}{Z} = \frac{P x}{Z}$

unilateral  $\rightarrow$  Centroid axis

$\downarrow$  eccentrically some distance  
 Mea  $\sigma_d \geq \sigma_b$  No tension condition

$\sigma_b$   
 (maximum)

$$\sigma_d - \sigma_b \geq 0$$

( $\sigma_d$ )  
 uniform

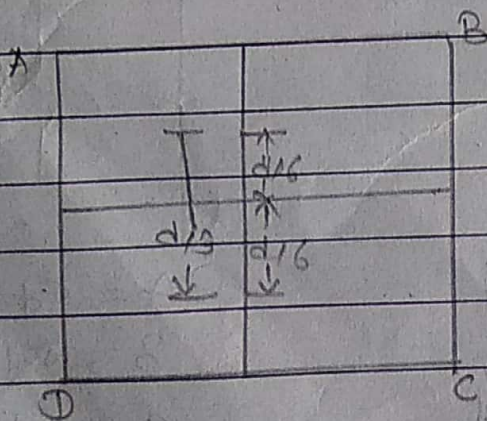
$$\therefore \frac{P}{A} - \frac{P \cdot e}{Z} \geq 0$$

$$\therefore \frac{P}{A} \geq \frac{P \cdot e}{Z}$$

$$\therefore e \leq \frac{Z}{A}$$

" Justify that eccentricity of the force acting at section should be less or equal to ratio of section Modulus to Area of the section."

For rectangular section =  $e \leq \frac{Z}{A}$   
 $e \leq \frac{bd^2/6}{bd}$   
 $e \leq b/6$



Rectangular

$$\frac{b + b}{6} = \frac{b}{3}$$


$$\frac{d}{6} + \frac{d}{6} = \frac{d}{3}$$

force should be acting at middle their part that so member is in compression.

Note  $\rightarrow$   $r_b$  is depended about  $y$ , so  $\sigma_b$  is not uniform.

$\rightarrow$   $\sigma_d$  is uniform

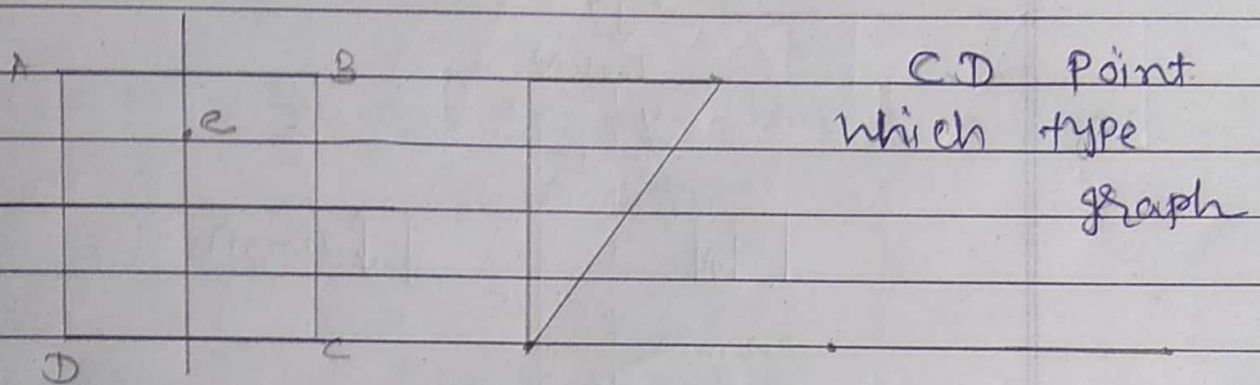
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$\frac{d}{3} f$  (Maximum)

$\frac{d}{6}$  farthest fiber of the centroidal axis.

When value of eccentricity within  $\frac{d}{6}$  value then not induce the tension but induced the compression.



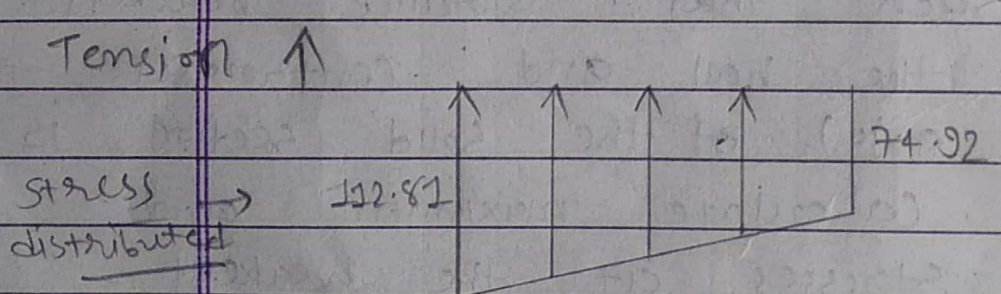
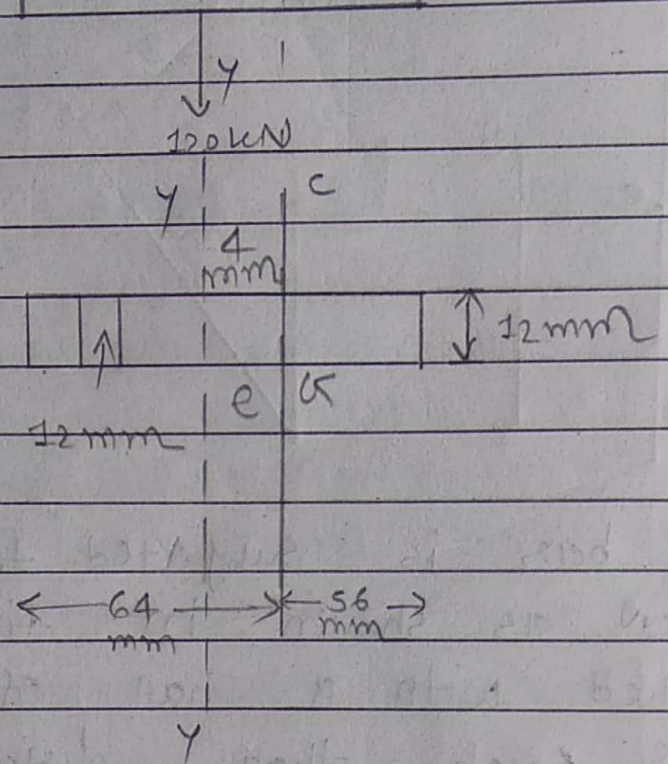
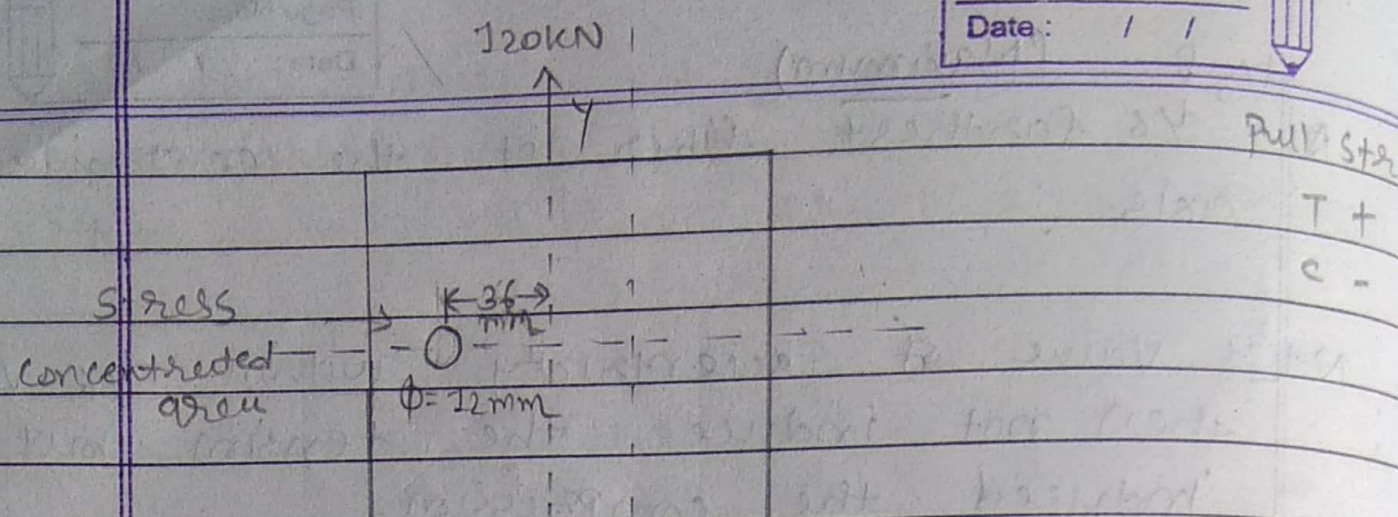
$\star$  A flat bar is subjected to an axial pull of 120 kN as shown in figure. The bar is punched with a hole of 12 mm diameter such that distance between center of the hole and centroidal axis ( $y-y$  axis) of the solid section is 36 mm. Calculate maximum and minimum stresses at the weakest section. or plot stress distribution diagram or tentative

$\rightarrow$   $b = 120$

Pull stress

T +

C -



$$\sigma_d = \frac{P}{A} = \frac{120 \times 10^3}{(120 \times 12) - 144} = \frac{120 \times 10^3}{7296} = 92.59 \text{ N/mm}^2$$

$$\sigma_{bc} = \frac{M}{I} = \frac{P_e \cdot y_c}{I} = \frac{120 \times 10^3 \times \dots}{\dots}$$



$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$= \frac{1440(0) - (12 \times 12)(36)}{1440 - 144} = 4 \text{ mm}$$

Ans.

$$\sigma_{bc} = \frac{M}{I} = \frac{P_e \cdot y_c}{I} = \frac{120 \times 10^3 \times 4 \times 56}{1.52 \times 10^6} = 17.68 \text{ N/mm}^2$$

$$\sigma_{bd} = \frac{M}{I} = \frac{P_e \cdot y_d}{I} = \frac{120 \times 10^3 \times 4 \times 64}{1.52 \times 10^6} = 20.21 \text{ N/mm}^2$$

$$\begin{aligned} \rightarrow I_{yy} &= \frac{bd^3}{12} - \left[ \left( \frac{bd^3}{12} \right)_n + Ah^2 \right] \\ &= \frac{12 \times (120)^3}{12} - \left[ \frac{12 \times (12)^3}{12} + (12 \times 12)(36)^2 \right] \\ &= 1728000 - 1728 - 186624 \end{aligned}$$

$$I_{yy} = 1.54 \times 10^6 \text{ mm}^4$$

$$\begin{aligned} I_{CG} &= I_{yy} - Ah^2 \\ &= 1.54 \times 10^6 - (12 \times 12)(36)^2 \end{aligned}$$

$$I_{CG} = 1.52 \times 10^6 \text{ mm}^4$$

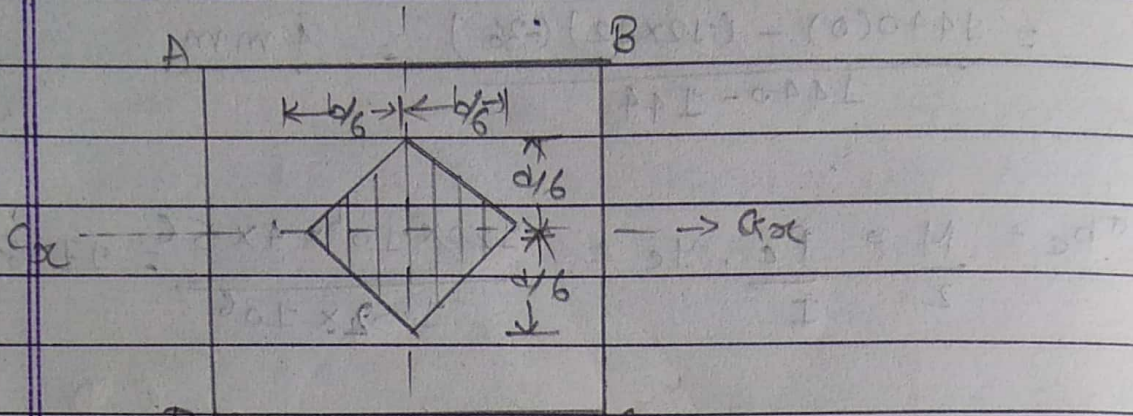
$$\begin{aligned} \rightarrow \sigma_{max} &= \sigma_d(\tau) + \sigma_b(\tau) \\ &= 92.6 + 20.21 \end{aligned}$$

$$\sigma_{max} = 112.81$$

$$\begin{aligned} \sigma_{min} &= \sigma_d(\tau) - \sigma_b(\tau) \\ &= 92.6 - 17.68 \end{aligned}$$

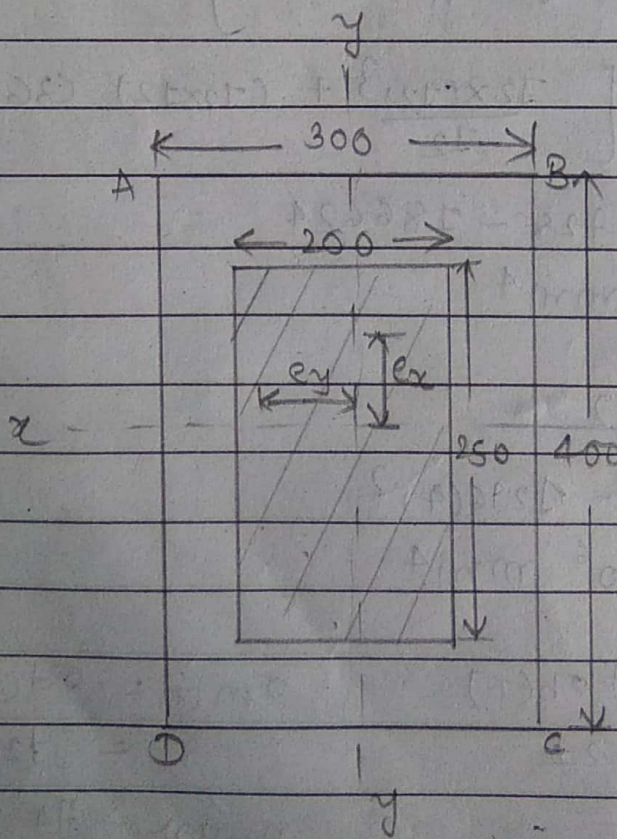
$$\sigma_{min} = 74.92$$

★ Biaxial Bending ↓  
 $\uparrow c_y$



★ core kernel ↓ of Section ↓

The Region within the section where if force is applied the section would remain under the compressive stress only.





$$z_{xc} = \frac{I_{xxc}}{y} = \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{\frac{D}{2}}$$

$$= \frac{BD^3 - bd^3}{6D}$$

$$z_y = \frac{I_{yy}}{y} = \frac{DB^3 - db^3}{6B}$$

$$z_x = \frac{300 \times (400)^3 - (200)(250)^3}{6 \times 400} = \frac{BD^3 - bd^3}{6D}$$

$$= \frac{1.92 \times 10^{10} - 3.125 \times 10^9}{2400}$$

$$= 6.697 \times 10^6 \text{ mm}^3$$

$$\rightarrow z_y = \frac{DB^3 - db^3}{6B}$$

$$= \frac{400 \times (300)^3 - (250)(200)^3}{6 \times 300}$$

$$= 4.888 \times 10^6 \text{ mm}^3$$

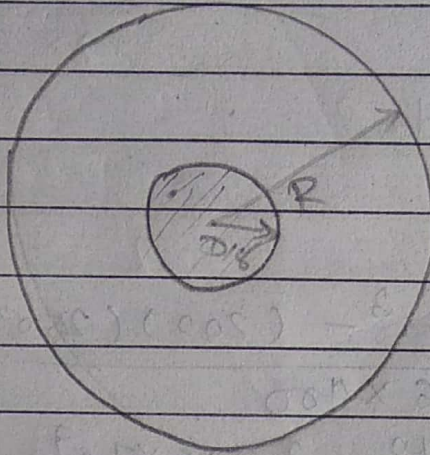
$$e_{xc} = \frac{z_y}{A} = \frac{6.697 \times 10^6}{70000}$$

$$= 95.24 \text{ mm}$$

$$\rightarrow e_y = \frac{z_x}{A} = \frac{4.88 \times 10^6}{70000}$$

$$= 69.71 \text{ mm}$$

ex: → (2)



uniform type  
of the stress

$$I_{xx} = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

$$= \frac{\pi D^4}{64} \left( 1 - \left(\frac{d}{D}\right)^4 \right)$$

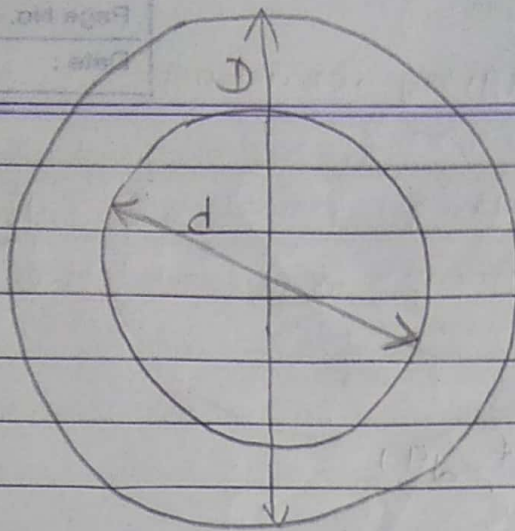
$$A = \pi R^2$$

$$e_y = \frac{z_x}{A}$$

$$= \frac{\pi/32 D^3}{\pi D^2}$$

$$= \frac{D}{8}$$

Ex-13



$$I_{xx} = \frac{\pi}{64} D^4 - \frac{\pi}{64} d^4 = \frac{\pi}{64} (D^4 - d^4)$$

$$y = \frac{D}{2}$$

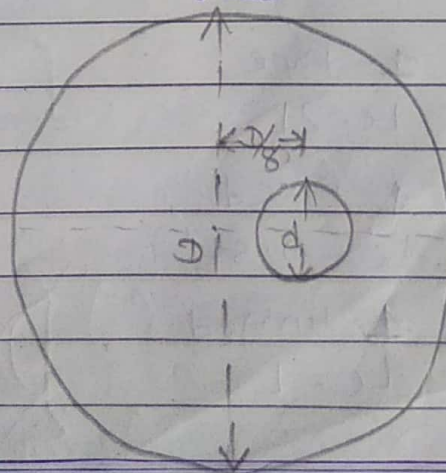
$$Z = \frac{\frac{\pi}{64} (D^4 - d^4)}{\frac{\pi}{32} D}$$

$$A = \frac{\pi}{4} (D^2 - d^2)$$

$$e = \frac{Z}{A} = \frac{\frac{\pi}{64} (D^4 - d^4)}{\frac{\pi}{32} D (D^2 - d^2)}$$

$$= \frac{(D^2 - d^2)(D^2 + d^2)}{8D(D^2 - d^2)}$$

$$= \frac{D^2 + d^2}{8D}$$

Ex-14  
H.W.

$$D = 100$$

$$d = 25$$

Structure  $\rightarrow$  To carry the compressive stress  
 Long column  $\rightarrow$  Buckling  
 Short column  $\rightarrow$  Bulging  $\rightarrow$  crushing

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Long column  $\rightarrow$  Force is acting in core then compressive  
 column  $\rightarrow$  Force is acting out of the core then  
 Buckling.

$$I_{max} = \frac{\pi}{64} (D^4 - d^4)$$

$$D = 100$$

$$d = 25$$

$$z = \frac{\pi (D^4 - d^4)}{8D}$$

$$A = \frac{\pi}{4} (D^2 - d^2)$$

$$= \frac{\pi}{4} ((100)^2 - (25)^2)$$

$$z = 390.96 \times 10^3$$

$$= 7.35 \times 10^3$$

$$e = \frac{z}{A} = \frac{390.96 \times 10^3}{7.35 \times 10^3}$$

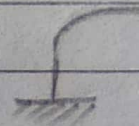
$$= 53.19$$

### ★ Euler's Theory of

Buckling failure  $\rightarrow$   $P_{cr} \rightarrow$  Cripping force

$$= \frac{\pi^2 EI}{(Le)^2}$$

(1) 1 - Fixed 2 - Free  
 $Le = 2L$



(2) 1 - fixed 2 - Fixed  
 $Le = 0.5L$



(3) 1 - Hinged 2 - Hinged  
 $Le = L$

$P_{cr} > P \rightarrow$  Relation for short columns

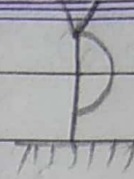
$P_{cr} < P \rightarrow$  Relation for long column

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(4) 1 Hinged 1-Fixed  
 $L_e = 0.707L$



\* Euler's Assumptions ↓

→ Material follows hook's law

(Hook's law valid upto proportional limit)

→ failure → Buckling

→ Material used to be uniform.  
Member

\* Limitations ↓

Ex: Mild steel as Material ↓

$$F_y = 250 \text{ MPa}$$

$$I = Ad^2$$

$$F_p = 200 \text{ MPa}$$

$$I = Ak^2$$

$k =$  list radius of gyration

$$P_{cr} = \frac{\pi^2 EAK^2}{(L_e)^2}$$

$$\therefore \frac{P_{cr}}{A} = \frac{\pi^2 EK^2}{(L_e)^2}$$

$$\therefore \sigma = \frac{\pi^2 EK^2}{(L_e)^2}$$

$$\therefore \sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L_e}{k}\right)^2}$$

$\left(\frac{L_e}{k}\right) \rightarrow$  Slenderness ratio ( $\lambda$ )

$$\therefore \sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$$

$$\lambda = \frac{L_e}{k}$$

$k =$  constant

$$\lambda \propto L_e$$

$$(200) = \frac{\pi^2 \times (2 \times 10^5)}{\lambda^2}$$

$$\therefore \lambda^2 = \frac{(3.141)^2 \times 2 \times 10^5}{200}$$

$$\therefore \lambda = 99.29$$

$$\lambda > 99.3$$

Ex. 1) (1) A Mild Steel cube of 25mm internal diameter and 32 mm external diameter has one end fixed and other end hinged. Calculate the crippling load assuming modulus of elasticity  $E = 2 \times 10^5 \text{ N/mm}^2$  take length of column = 3m

$$\rightarrow P_{cr} = \frac{\pi^2 EI}{(Le)^2} \quad e = \text{effective}$$

$$Le = 0.707 L$$

$$L = 3 \text{ m}$$

$$L = 3000 \text{ mm}$$

$$Le = 0.707 \times 3000$$

$$= 2121 \text{ mm}$$

$$I = \frac{\pi}{64} (D^4 - d^4)$$

$$= \frac{\pi}{64} ((32)^4 - (25)^4)$$

$$= 32.28 \times 10^3 \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(Le)^2}$$

$$= \frac{(3.14)^2 \times 2 \times 10^5 \times 32.28 \times 10^3}{(2121)^2}$$

$$= 14.14 \times 10^3 \text{ N}$$

Ex: 10) Compare the crippling load of a solid circular section of 250 mm diameter with a hollow circular of same area and 40 mm thickness. The other conditions are same for the columns.

$$(P_{cr})_s = \frac{\pi}{64} D^4 = 1.91 \times 10^8$$

$$(P_{cr})_H = \frac{\pi}{64} (D^4 - d^4)$$

$$D - d = 80 \rightarrow (i)$$

area same

$$\frac{\pi}{64} D^4 = \frac{\pi}{64} (D^4 - d^4)$$

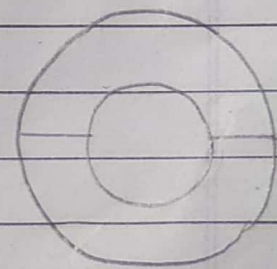
$$\therefore D^4 = D^4 - d^4$$

$$\therefore D^4 = (D+d)(D-d)(D^2+d^2)$$

$$\therefore D^4 = D+d$$

$$D-d$$

$$\therefore \frac{(250)^4}{80} = 250 + d$$



$$\therefore D+d = 781.25$$

$$D + d = 781.25$$

$$D - d = 80$$

$$2D = 861.25$$

$$\therefore D = 430.62 \text{ mm}$$

$$\therefore d = 350.62 \text{ mm}$$

$$\frac{(P_{cr})_S}{(P_{cr})_H} = \frac{I_S}{I_H} = \frac{1.91 \times 10^8}{\frac{\pi}{64} [(430.62)^4 - (350.62)^4]}$$

$$(P_{cr})_S = 0.2019 (P_{cr})_H$$

$$(P_{cr})_S = 0.2019 (P_{cr})_H$$