Slope & deflection of beams.

* A beam when loaded gets deflected. The as of beam displace ⊥ to the initially straights axis of the beam. & at the same time rotate.

→ The axis of the loaded beam bends in curve is known as elastic curve or deflection curve.

→ deflection of point on the axis of the beam is the vertical distn betn its position before loading & after loading.

→ In design criteria for the deflection of a cantilever or a beam:-

(1) strength
(2) stiffness

→ As per strength criterion of the beam design, it should be strong enough to resist bending moment & shear force. or in other words, beam should be strong enough to resist the bending stresses & shear stresses.

→ As per stiffness criterion of the beam design, it should be stiff enough to resist the deflection of the beam or in other words it should be stiff enough not to deflect more than the permissible limit under the action of the loading.

* Slope and deflections of beams.

↳ Relationship betⁿ slop deflection & moment of curvature..

* methods for calculating slope & displacements of determinate structures.

(1) Double integration method.

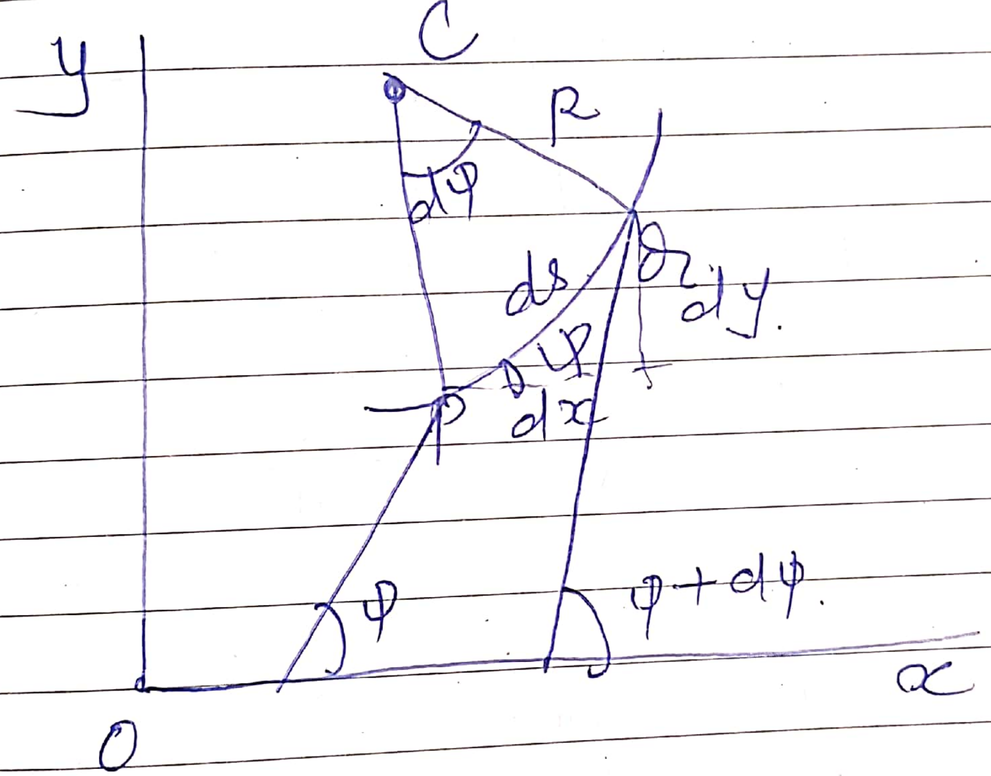
(2) Macaulay's method

(3) Moment Area method.

(4) Conjugate beam method

* Analysis of propped cantilever beams by consistent deformation method
- rigid & elastic supports.
- beams of varying MoI.

* Relation betn slope, deflection & radius of



→ Consider a small portion PQ of beam, bent into an arc as shown in fig.

ds = length of the beam PQ.
R = Radius of arc, into which the beam has been bent.
C = Centre of arc.
φ = angle which the tangent at P makes with x-x axis.

$\phi + d\phi$ = Angle which the tangent at Q makes with x-axis,

From, the geometry of Fig,

$$\angle PCQ = d\phi$$
$$ds = R \cdot d\phi$$

$$\therefore R = \frac{ds}{d\phi} = \frac{dx}{d\phi}$$
(Considering $ds = dx$)

$$\therefore \frac{1}{R} = \frac{d\phi}{dx}.$$

Or we know that if x & y be the co-ordinates of point p, then.

$$\tan\phi = \frac{dy}{dx}.$$

since ϕ is very small angle therefore taking $\tan\phi \simeq \phi$,

$\therefore \frac{1}{R}$

$$\therefore \frac{d\phi}{dx} = \frac{d^2y}{dx^2} \qquad \left(\frac{1}{R} = \frac{d\phi}{dx}\right)$$

We also know that,

$$\frac{M}{I} = \frac{E}{R}.$$

$$\therefore M = EI \times \frac{1}{R}$$

$$\boxed{M = EI \times \frac{d^2y}{dx^2}.}$$

$\therefore M = EI \frac{d^2y}{dx^2}$

Note: above eq^n is ~~not~~ based on the bending moment. effect of shear force, being very small as compared to the bending moment, is neglected.

Where, EI : flexural rigidity of the beam.

* methods for slope & deflection at a section

(1) Double Integration method
(2) Macaulay's method.

→ suitable for single load only

→ suitable for several load.

(1) Double Integration methods

$$M = EI \frac{d^2y}{dx^2}$$

Integrating the above eq^n,

$$EI \frac{dy}{dx} = \int M \quad \text{—— (1)}$$

& integrating the above eq^n once again,

If Cantilever loaded then,
Double Integration & moment
Area method.

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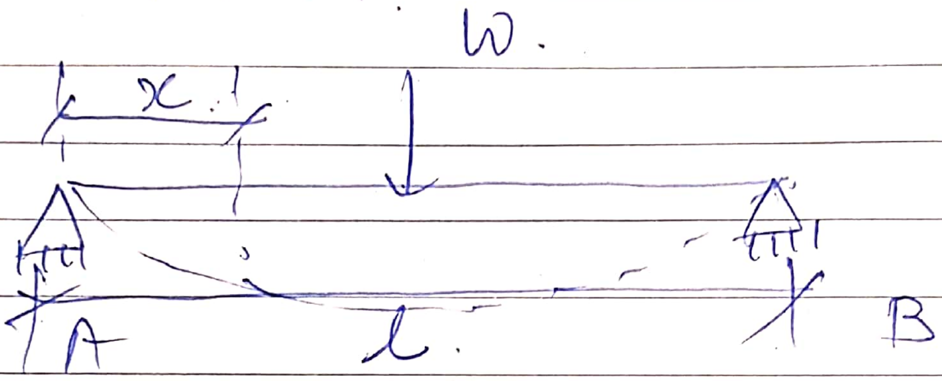
$$EIy = \iint M \quad \text{--------} (2)$$

→ It is thus obvious that after first integration the original differential eqn, we get the value of slope at any point. On further integrating, we get the value of deflection at any point.

Note:- While integrating twice the original diffn eqn, we will get two constants C_1 and C_2. The values of these constants may be found out by using the end condn.

* Simply supported beam with central point load



$\therefore \quad R_A = R_B = W/2.$

Consider a section x at a dist^n x from A.

So, $M_x = W/2 \times x$

$$\therefore \quad EI\frac{d^2y}{dx^2} = -M_x$$

$$= -W/2\, x.$$

Now, Integrating it,

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$$EI\frac{dy}{dx} = \frac{-w}{2}\frac{x^2}{2} + C_1$$

$$= -\frac{wx^2}{4} + C_1 \rightarrow 1^{st} \text{ constant of integration}$$

Now, we know that,

At, $x = l/2$, $\frac{dy}{dx} = 0$.

$$\therefore 0 = -\frac{w}{4}\left(\frac{l}{2}\right)^2 + C_1$$

$$\therefore C_1 = \frac{wl^2}{16}.$$

$$\therefore EI\frac{dy}{dx} = -\frac{wx^2}{4} + \frac{wl^2}{16} \quad (1)$$

slope.

Now, Integrating once again,

$$EIy = -\frac{wx^3}{12} + \frac{wl^2x}{16} + C_2$$

2^{nd} constant integration

At $x=0$, $y=0$,

$\therefore C_2 = 0$.

$$\therefore EIy = -\frac{wx^3}{12} + \frac{wl^2x}{16}$$

——— (2) deflection.

max^m slope will occurs at A & B.

substitute $x=0$ in eq^n — (1)

$$EI \cdot \theta_A = \frac{wl^2}{16}.$$

$$\therefore \boxed{\theta_A = \frac{wl^2}{16EI}}$$ Radians

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By symmetry, $\theta_B = -\frac{wl^2}{16EI}$.

Now, max^{m} deflection will occur at mid span, $x = l/2$ in eq^{n} —— (2).

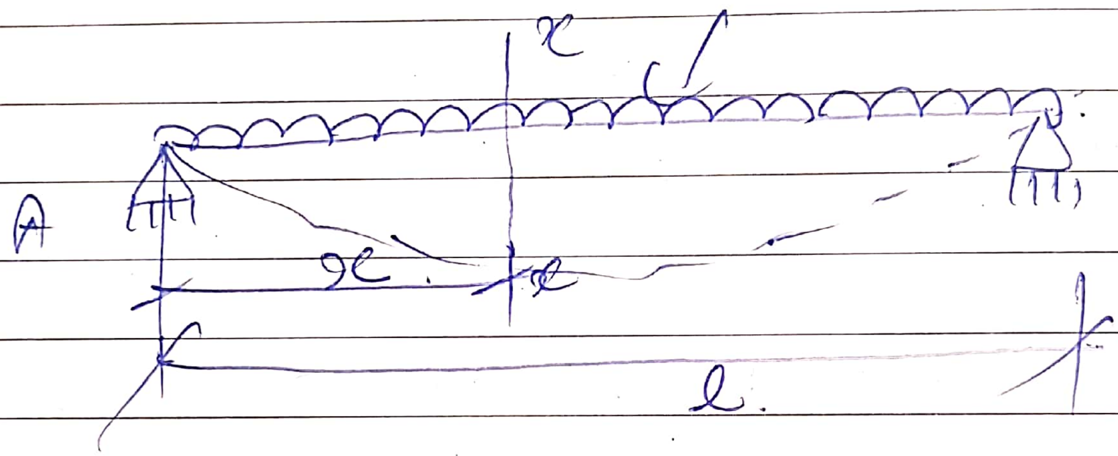
$$\therefore EIy_c = -\frac{w}{12}\left(\frac{l}{2}\right)^3 + \frac{wl^2}{16}\,\frac{l}{2}$$

$$= -\frac{wl^3}{96} + \frac{wl^3}{32}$$

$$= \frac{wl^3}{48}$$

$$\boxed{y_c = \frac{wl^3}{48EI}}$$

*. simply supported beam with udl on entire span.



$$\therefore \quad R_A = R_B = \frac{wl}{2}.$$

$$M_x = \frac{wl}{2} x - \frac{wx^2}{2}.$$

$$EI \frac{d^2y}{dx^2} = -M_x$$

$$= -\frac{wlx}{2} + \frac{wx^2}{2}.$$

Integrating it,

$$EI\frac{dy}{dx} = -\frac{wlx^2}{4} + \frac{wx^3}{6} + C_1$$

At $x = l/2$, $\frac{dy}{dx} = 0$,

$$0 = -\frac{wl}{4}\left(\frac{l}{2}\right)^2 + \frac{w}{6}\left(\frac{l}{2}\right)^3 + C_1$$

$$0 = -\frac{wl^3}{16} + \frac{wl^3}{48} + C_1$$

$$\therefore C_1 = \frac{wl^3}{24}$$

$$EI\frac{dy}{dx} = -\frac{wlx^2}{4} + \frac{wx^3}{6} + \frac{wl^3}{24}$$

(1) slope.

Integrating once again,

$$EIy = -\frac{wlx^3}{12} + \frac{wx^4}{24} + \frac{wl^3}{24}x + C_2$$

At A, $x=0$, $y=0$,

$C_2 = 0$,

$$\therefore EIy = -\frac{wlx^3}{12} + \frac{wx^4}{24} + \frac{wl^3x}{24}$$

(2) deflection

$\therefore$ max^m slope is at the supports,

$x=0$, in eq^n (1),

$$EI\theta_A = +\frac{wl^3}{24}$$

$$\therefore \theta_A = \frac{wl^3}{24EI} \text{ radians}$$

(3)

max^m deflection is at mid span,

$x = l/2$ in eq^n (2)

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$$E7 \quad y_c = -\frac{wl}{12}\left(\frac{l}{2}\right)^3 + \frac{w}{24}\left(\frac{l}{2}\right)^4 + \frac{wl^3}{24}\left(\frac{l}{2}\right).$$

$$= \frac{5}{384} wl^4.$$

$$\boxed{y_c = \frac{5}{384} wl^4}$$

Or example. eg.

2) Macaulay's Method for slope and deflection:

→ The problems of deflections in beams are bit tedious & laborious, specially when a beam is sub. to point loads or in general discontinuous loads.

→ Mr. W.H. Macaulay devised a method, a continuous expression, for bending moment and integrating it such a way that the constants of integration are valid for all sections of the beams; even though law of bending moment varies from section to section.

* The following rules are observed while using macaulay's method.

1) Always take origin on the extreme left of the beam.

2) Take left clockwise moment ~~positive~~ as ~~negative~~ positive and left anticlockwise moment as ~~positive~~ Negative.

3) While calculating the slopes & deflections it is convenient to use the values first in terms of kN and meters.

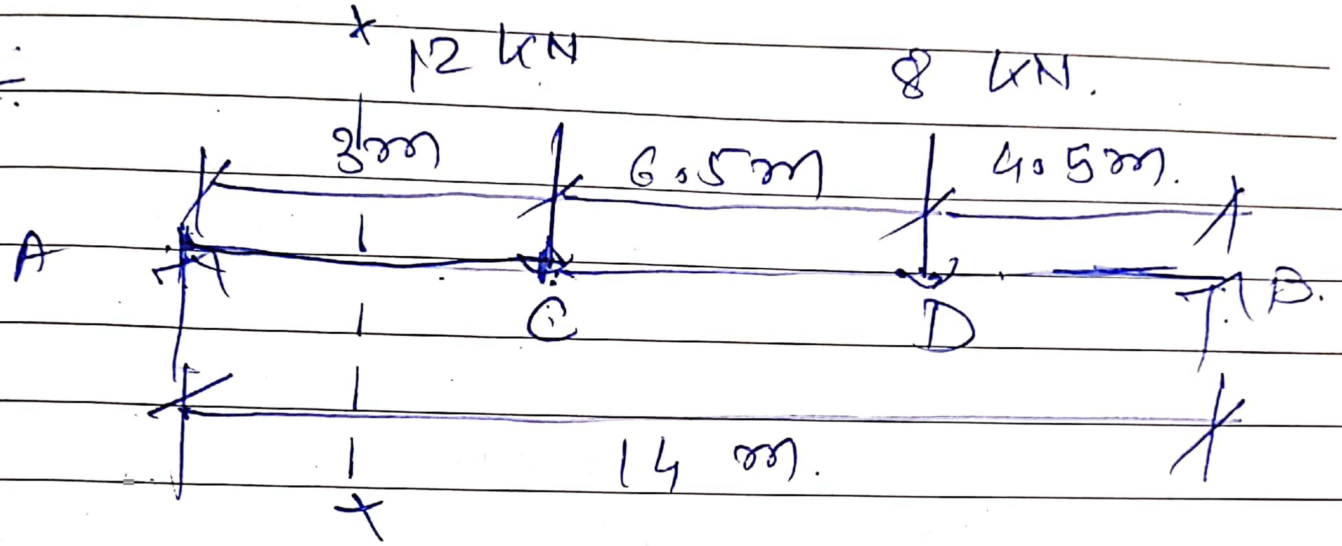
4) The constant of integration C_1 and C_2 are written in the first part of the continuous expression.

(5) The quantity $(x-a)$ should be integrated as $\frac{(x-a)^2}{2}$ & not as $\frac{x^2}{2} - ax$.

(6) quantity $(x-a)$ should be written as $(x-a)^0$, when it is a dist^n to be multiplied with moment.

(7) For any term when $(x-a) < 0$, i.e negative, the term is neglected.

Ex:



Take $E = 200$ GPa,
$I = 160 \times 10^6$ mm^4.

Calculate the deflections of the beam under the loads C & D.

Soln :- Taking moment about A,

$$R_B \times 14 = (12 \times 3) + (8 \times 9.5)$$

$$R_B = 8 \text{ kN } (\uparrow)$$

$$R_A = (12+8) - 8 = 12 \text{ kN } (\uparrow)$$

Now taking A as the origin & using macaulay's method, the bending moment at any section x at a dist^n x from A.

$$\cancel{EI \frac{d^2y}{dx^2} = -(12 \times 10^3)x}$$

$$M_x = 12x \; | \; -12(x-3) \; | \; -8(x-9.5)$$

$$EI \frac{d^2y}{dx^2} = -12x \; | \; +12(x-3) \; | \; +8(x-9.5)$$

Integrate it,

$$EI \frac{dy}{dx} = -\frac{12x^2}{2} + C_1 \; | \; +12\frac{(x-3)^2}{2}$$

$$| \; +8\frac{(x-9.5)^2}{2}.$$

$$\therefore EI \frac{dy}{dx} = -6x^2 + C_1 \; | \; +6(x-3)^2 \; | \; +4(x-9.5)^2$$

— (1) Slope.

Integrate once again,

$$EI\, y = -\frac{6x^3}{3} + C_1 x + C_2 \; \Big| + \frac{6(x-3)^3}{3} + \frac{4(x-9.5)^3}{3} \Big|$$

$$EI\, y = -2x^3 + C_1 x + C_2 \; \Big| + 2(x-3)^3 + \frac{4}{3}(x-9.5)^3 \Big| \quad \text{——— (2) deflection.}$$

Applying boundary cond^t, | neglect negative terms.

when $x = 0$, $y = 0$,

from eq^n (2) $C_2 = 0$,

when $x = 14$, $y = 0$ ∴ from eq^n (2),

$$0 = -2(14)^3 + C_1(14) + 0 \; \Big| + 2(14-3)^3 + \frac{4}{3}(14-9.5)^3 \Big|$$

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$\therefore C_1 = \frac{2704.5}{14}$

$$\boxed{C_1 = 193.17}$$

$$EI\frac{dy}{dx} = -6x^2 + 193.17 \; | + 6(x-3)^2 \; | + 4(x-9.5)^2$$

—— (3) slope.

$$EIy = -2x^3 + 193.17x \; | + 2(x-3)^3 \; | + \frac{4}{3}(x-9.5)^3$$

—— (4) deflecti

for deflection at C, substitute $x = 3$ in eq^n (4),

$$EIy_C = -2(3)^3 + 193.17 \times 3 + 0.$$

$$EIy_C = 525.51$$

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$$y_C = \frac{525.51}{EI} \; kN.m^3.$$

$$y_C = \frac{525.51 \times 10^{12}}{\cancel{EI}\; 200\times10^3 \times 160 \times 10^6}$$

$$y_C = \boxed{16.4\ mm} \downarrow$$

for deflection at D, $x = 9.5$ in eqn (4),

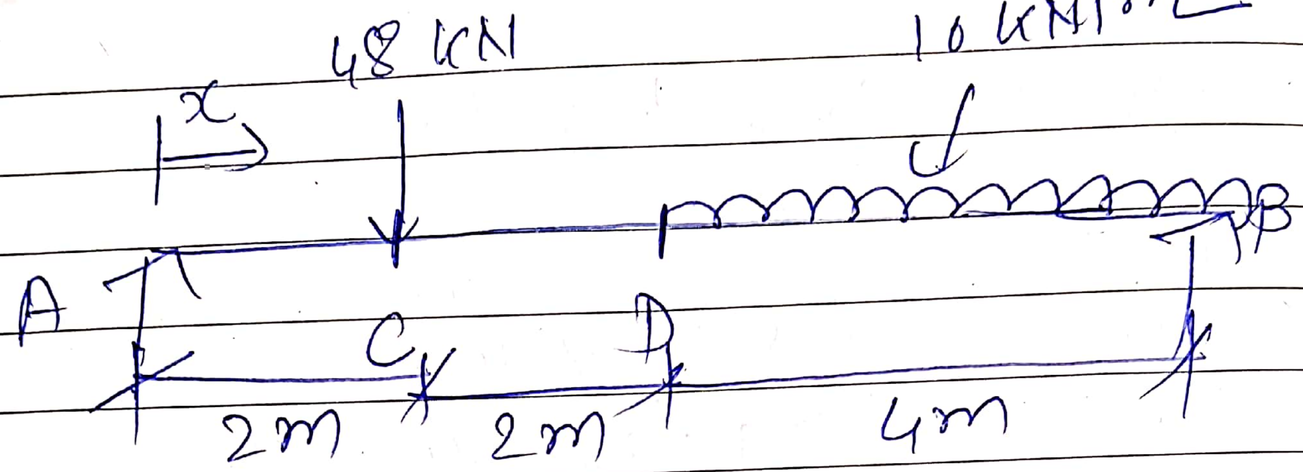
$$EI\,y_D = -2(9.5)^3 + 193.17(9.5) + 2(9.5-3)^3 + 0.$$

$$\therefore y_D = \frac{669.61}{EI} \; kN.m^3$$

$$= \frac{669.61 \times 10^{12}}{200\times10^3 \times 160 \times 10^6}$$

$$\boxed{y_D = 20.9\ mm} \downarrow$$

Ex



Find slope at A & B / deflection at C & D.

$E = 200\ kN/mm^2,$

$I = 6.5 \times 10^8\ mm^4.$

Hint → $R_B \times 8 = 48 \times 2 + (10 \times 4 \times 6)$

$R_B = 42\ kN\ (\uparrow)$

$R_A = (48 + 10 \times 4) - 42$

$= 46\ kN\ (\uparrow)$

$M_x = 46x \;|\; -48(x-2) \;|\; -10\dfrac{(x-4)^2}{2}$

$= 46x \;|\; -48(x-2) \;|\; -5(x-4)^2$

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$\therefore EI \frac{d^2y}{dx^2} = -M_x$.

↓ put M_x value

& 1st integrate → slope

again 2nd integrate → deflection

when $x=0$, $y=0$ → $\boxed{C_2=0}$

$x=8$, $y=0$ —— eqn (2)

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$\boxed{C_1 = 261.33 \text{ kN.m}^2}$

Now,

$$\boxed{\begin{aligned}\theta_A &= 0.00201 \text{ rad} \\ \theta_B &= -0.00185 \text{ rad}\end{aligned}}$$

Moment Area Method

→ This method was developed by Charles E. Greene.

→ It is a powerful technique of finding out deflections of beams, particularly cantilever beams.

→ It is convenient to use this method with great advantage in the foll^n types of problems:-

(1) Cantilever beams (slope at the fixed end is zero)

(2) Simply supported beams carrying symmetrical loading (slope at mid span is zero)

(3) Beams fixed at both ends. (slope at each end is zero).

* Moment - Area theorem - 1 :-

→ "The change of slope bet^n any two points on an elastic curve is equal to the net area of $\frac{M}{EI}$ diagram bet^n these points."

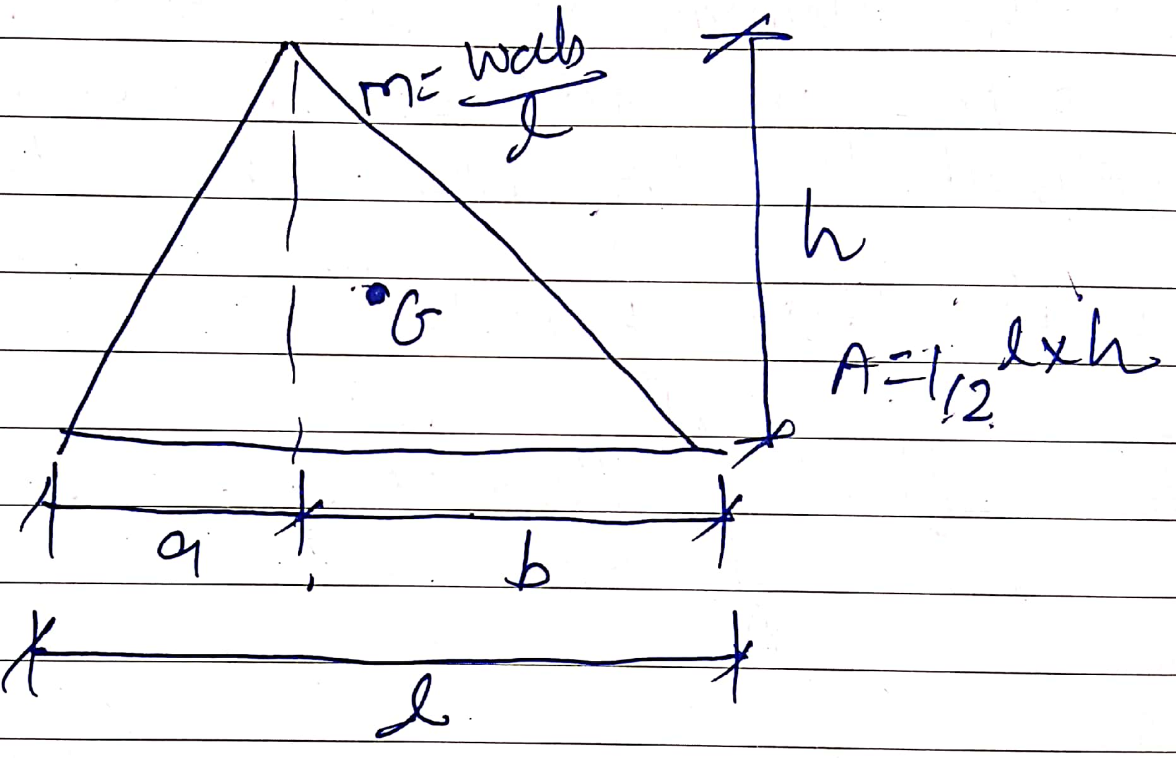
$$\theta = \int_A^B \frac{M}{EI} \cdot dx$$

* Moment - Area theorem - 2 :-

→ "The displacement of point B from the tangent at A is equal to the moment of $\frac{M}{EI}$ diagram bet^n A and B, taken about B".

* Area and ~~c.g~~ position of the c.g of parabola.

(1)



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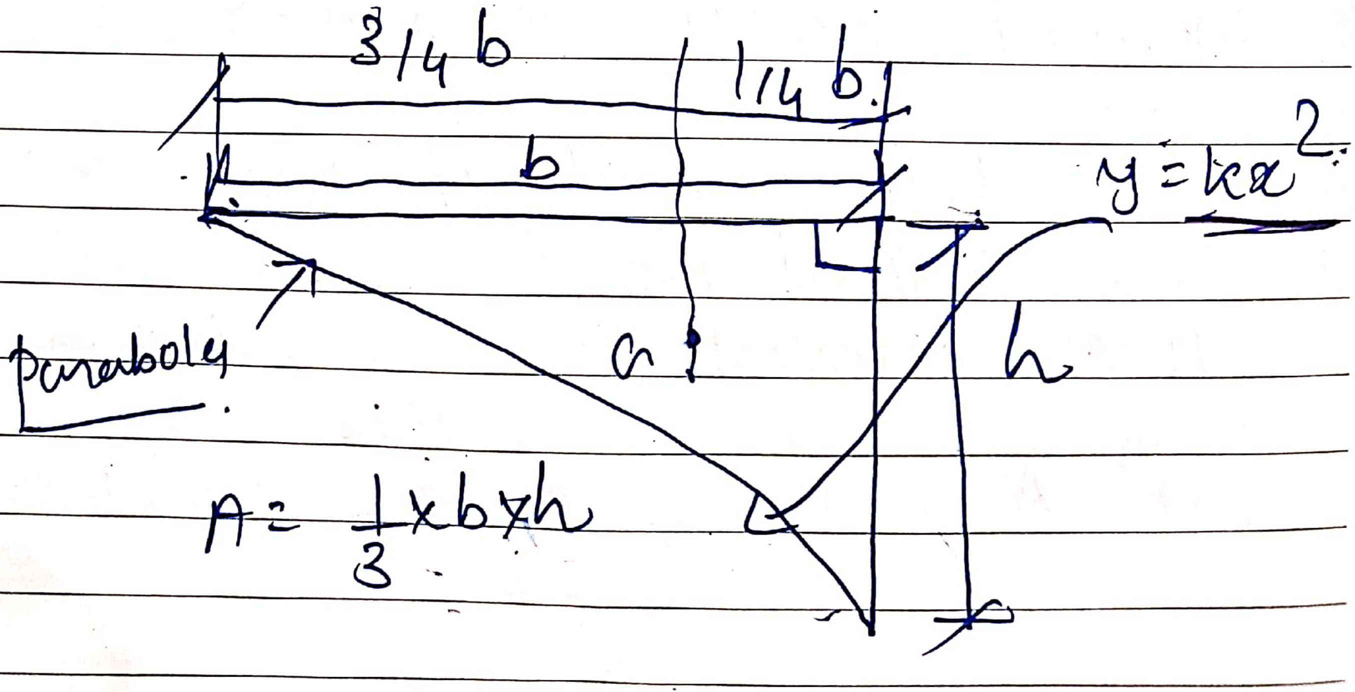

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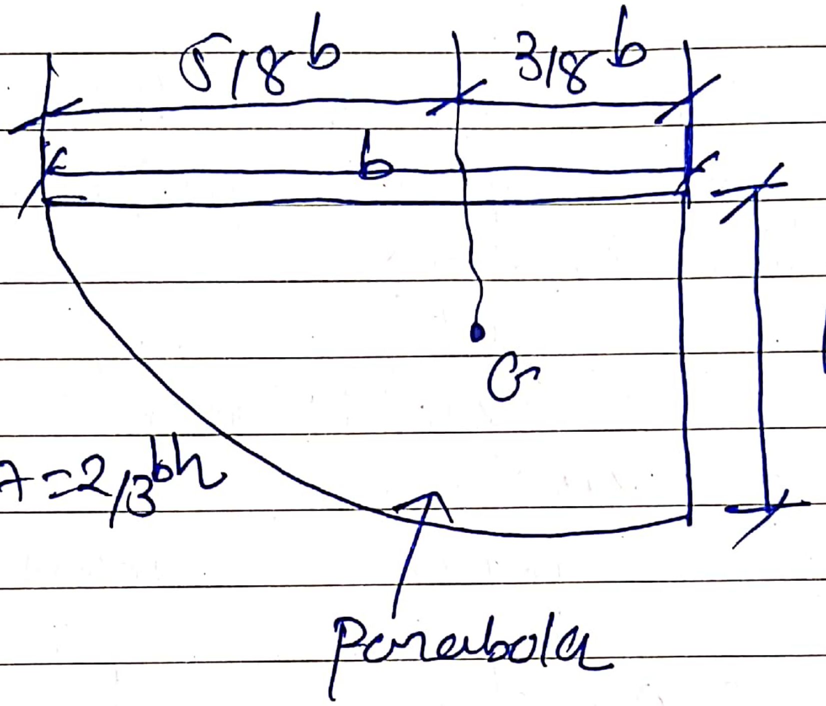
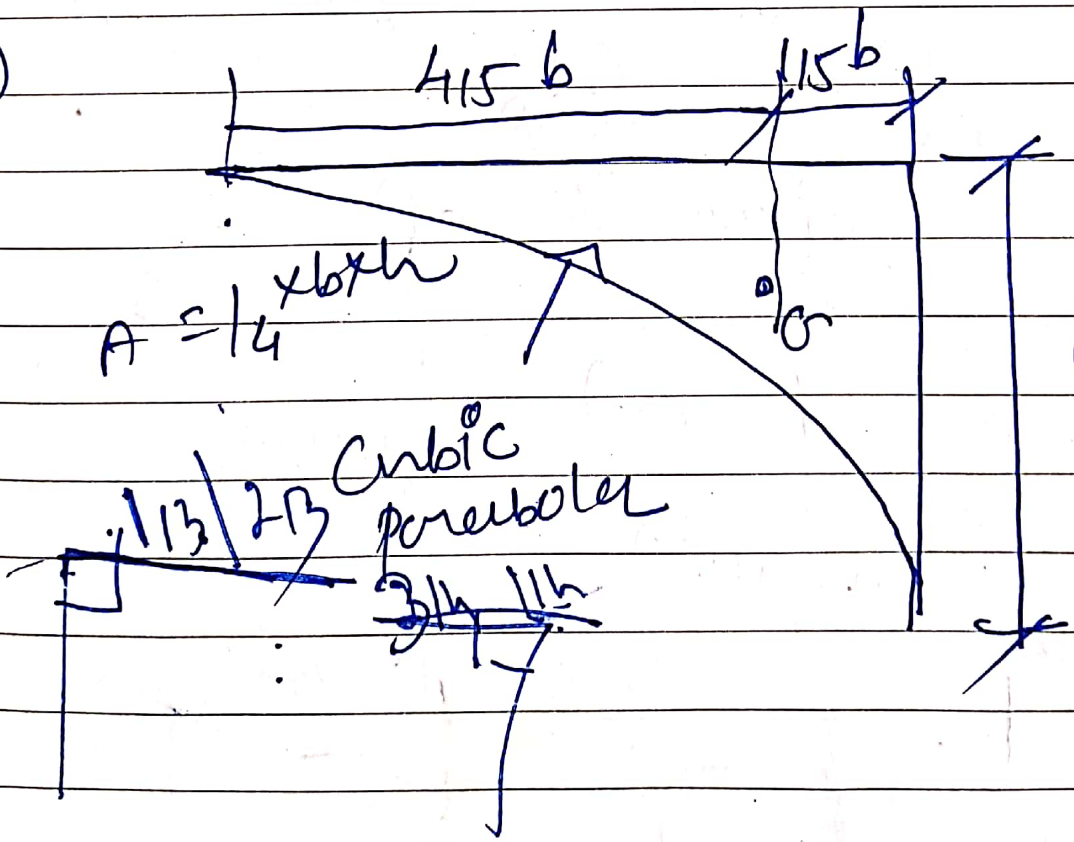
(3)

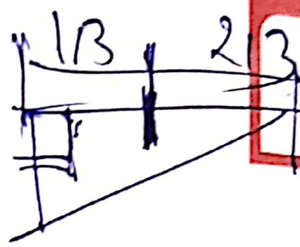


$y = kx^2$

(4)



$y = kx^3$

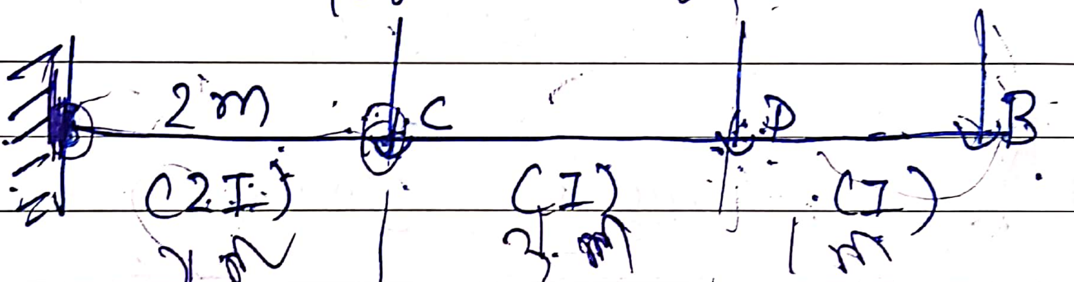


Ex: Determine slope and deflections at the free end of cantilever B as shown in fig.

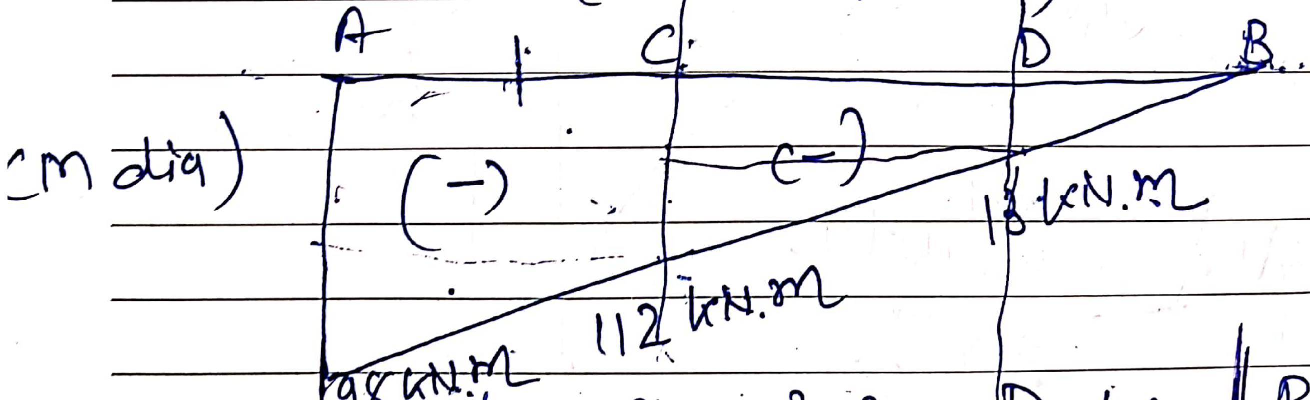
Take $E = 2 \times 10^5 \, N/mm^2$

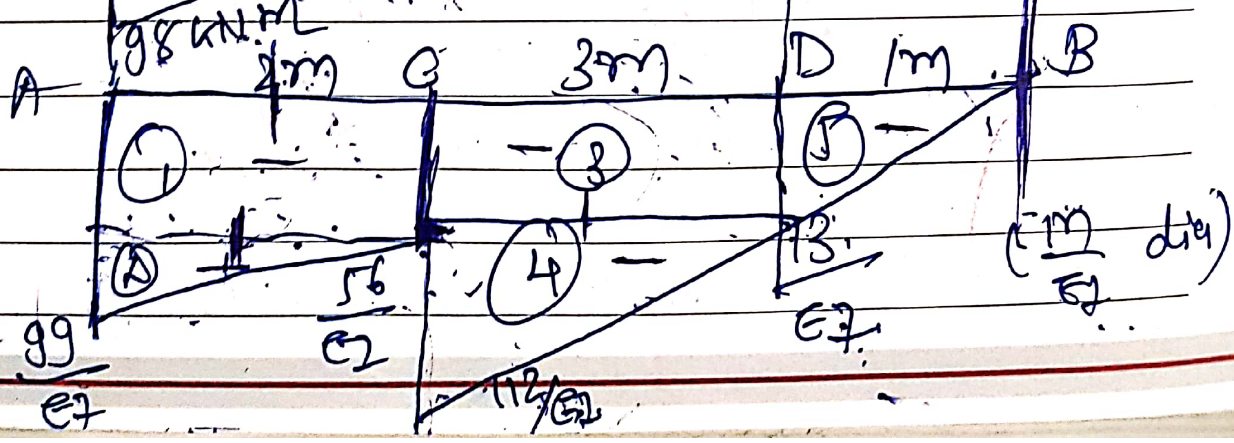
$I = 5 \times 10^8 \, mm^4$

Sol^n:



(Given beam)





θ_B = slope at B,

$= -\left[\text{Area of } \frac{M}{EI} \text{ diagram bet}^{n}\ B \& A\right]$.

$= -\left[2 \times \frac{(-56)}{EI} + -\frac{1}{2} \times 2 \times \frac{(99-56)}{EI}\right.$

$+ 3 \times \frac{(-13)}{EI} - \frac{1}{2} \times 3 \times \frac{(112-13)}{EI}$

$\left. + \frac{1}{2} \times 1 \times \frac{(-13)}{EI}\right]$

$= \frac{349}{EI}\ kN.m^2$

$= \frac{349 \times 10^9}{2 \times 10^5 \times 5 \times 10^8}$

$= 0.00349$ radian

$= 0.20$ degree $(\circlearrowright)$

y_B = deflection at B,

$= -$ [moment of $\frac{M}{EI}$ diagram bet^n B and A about B]

$$= -\left[\frac{-112}{EI}\times 5 - \frac{43}{EI}\times\left(4+\frac{2}{3}\times 2\right) - \frac{39}{EI}\times 2.5 - \frac{148.5}{EI}\times\left(1+\frac{2}{3}\times 3\right) - \frac{6.5}{EI}\times\left(\frac{2}{3}\times 1\right)\right]$$

$$= \frac{1}{EI}\left[560 + 229.33 + 97.5 + 445.5 + 4.33\right]$$

$$= \frac{1336.86}{EI} \quad \cancel{kNm^3} \quad kN.m^3$$

Conjugate beam Method

→ It is a modified form of moment area method and may be conveniently used for finding out the slope & deflection of beams.

→ A conjugate beam is a fictitious beam, which has the same length as the actual beam, but is supported in such a manner that when it is loaded with M/EI diagram of the actual beam, the SF and BM at a section in the conjugate beam give the slope & the deflection at the corresponding section of the actual beam.

* Change of support in conjugate beam :-

(1) Fixed end :-

– At the fixed end of the actual beam, slope & deflection both are zero. The shear & moment in the corresponding conjugat beam must be zero. A Free end only can comply with such a condition.

→ Thus, a fixed end of an actual beam, becomes a free end in a conjugate beam.

(2) Hinge or roller end :-

- At the hinge or roller end of the actual beam, there is a rotation but no deflection. In a corresponding conjugate beam, at this end, there should be shear but no moment. A hinge or roller end of conjugate beam comply with such a condition.
- Thus, a hinge or roller end of an actual beam, remains hinge or roller in a conjugate beam.

(3) Free end:-

→ At the free end of an actual beam, there exist slope & deflection both. In the corresponding conjugate beam, at this end, there should be shear and moment both. Fixed end only can comply with such a condition.

→ Thus, a free end of an actual beam, becomes a fixed end in a conjugate beam.

(4) Internal hinge:-

→ At an internal hinge of an actual beam, there exist slope & deflection both. Thus, a simple support would be required.

→ Thus, an internal hinge of an actual beam, becomes a simple support in a conjugate beam.

(5) Internal Support :-

→ An internal support in actual beam permit rotation but does not deflect. There is shear force on both side of internal support, which should be equal and opposite in nature. Slope is continuous over support.

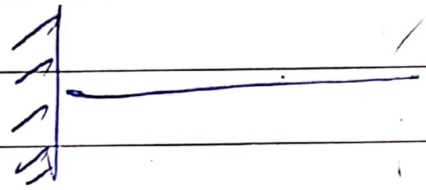
→ But at the end support, there is shear force on one side only. Hence, there is slope on one-side only.

→ Thus, an internal support of an actual beam becomes internal hinge for a conjugate beam.

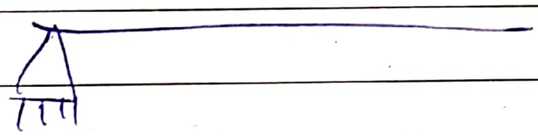
Support conditions

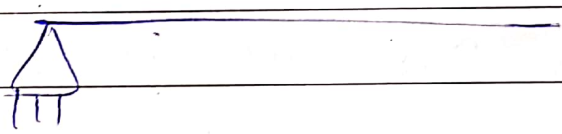
Actual beam — Conjugate beam

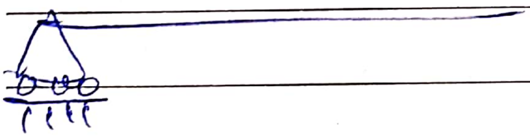
(1) Fixed end — Free end

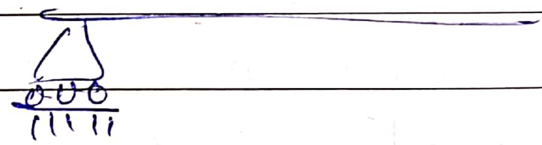


(2) Hinge or roller end

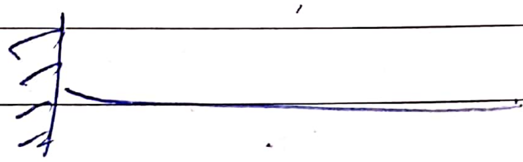






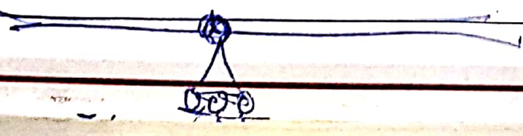


(3) Free end. — Fixed end



(4) Internal hinge — Internal support

(5) Internal support — Internal hinge



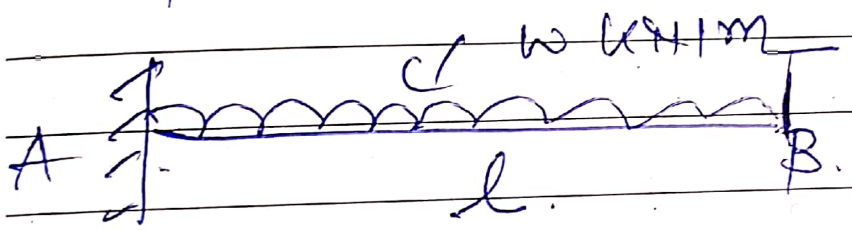
JUNE जून 2016

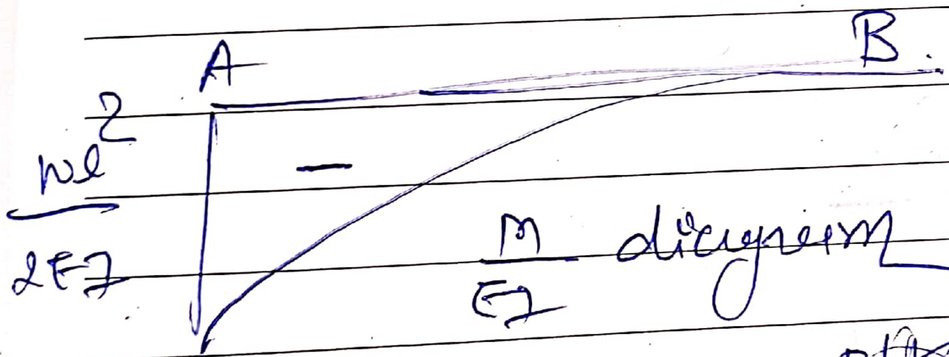
JUNE | 2016

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

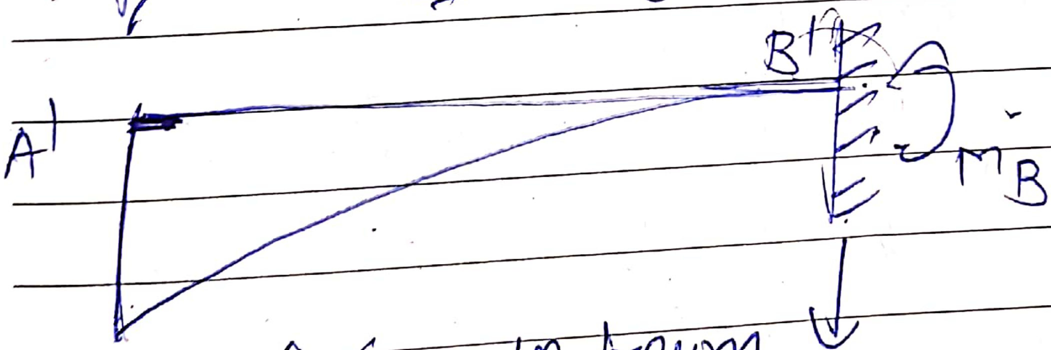
1
Wednesday
बुधवार

Ex: A Cantilever beam of span l, is sub. to load w kN/m on entire span, find slope and deflection at free end.





$\frac{M}{EI}$ diagram

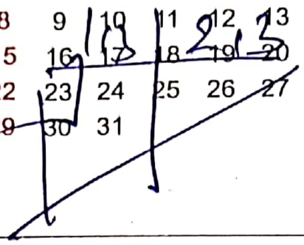


Conjugate beam

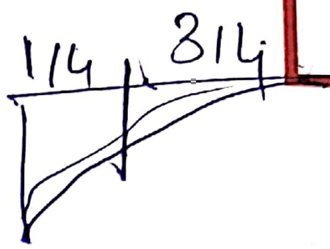
ओएनजीसी ONGC

MAY | 2016

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				



JUNE जून 2016



2
Thursday
गुरुवार

$M_B = 0,$

$M_A = -w.l.l/2 = \frac{-wl^2}{2}$

θ_B = slope at B,

= S.F at B' in conjugate beam

$= 1/3 \times l \times \frac{wl^2}{2EI}$

$= \frac{wl^3}{6EI}$ kN.m^3.

y_B = deflection at B,

= B.M at B' in conjugate beam

$= \frac{wl^3}{6EI} \times 3/4 \times l$

$= \frac{wl^4}{8EI}$ kN.m^3.

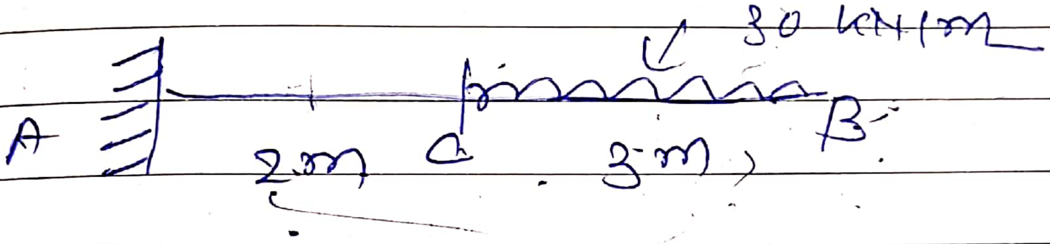
JUNE
जून
2016

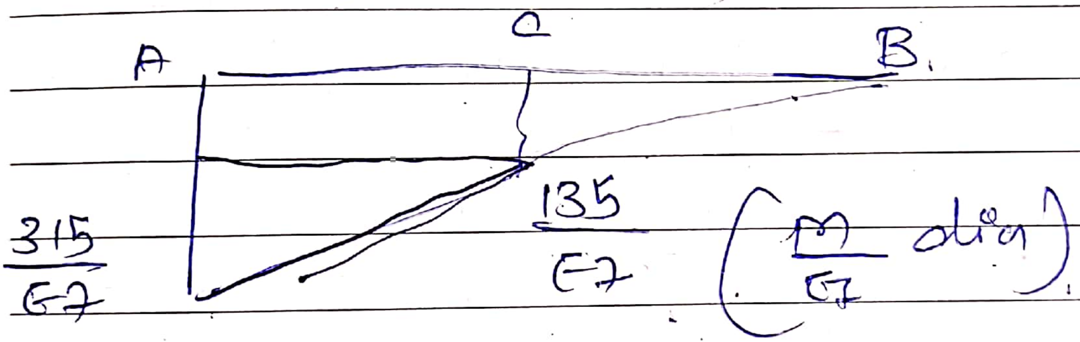
3
Friday
शुक्रवार

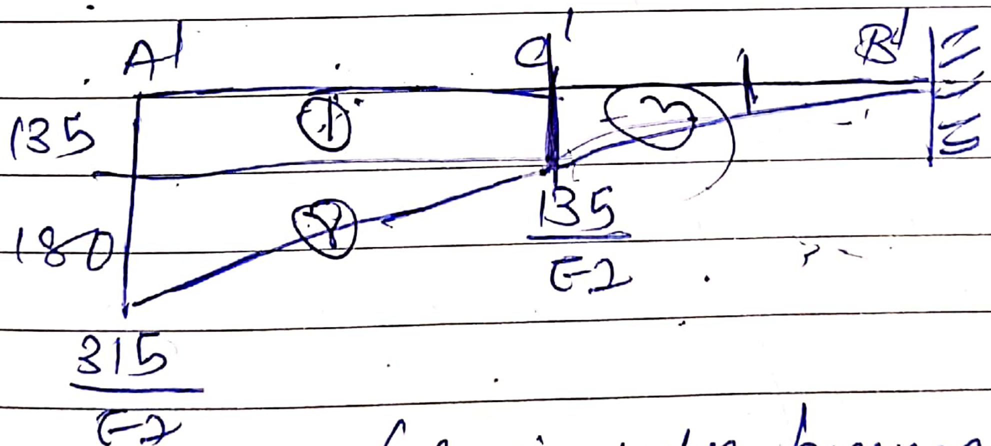
JULY | 2016

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Ex. Find $\theta_B, \theta_C, y_B, y_C$ for a beam shown below. $E = 2\times10^5\ N/mm^2$, $I = 3\times10^8\ mm^4$.







(Conjugate beam)

ओएनजीसी ONGC

MAY | 2016

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

JUNE जून 2016

4 Saturday शनिवार

$M_B = 0,$

$M_C = -30 \times 3 \times 1.5 = -135\ kN.m$

$M_A = -30 \times 3 \times 3.5 = -315\ kN.m.$

θ_B = slope at B,

= S.F at B' in conjugate beam.

$$= \left(\frac{1}{2} \times 2 \times \frac{180}{EI}\right) + \left(2 \times \frac{135}{EI}\right)$$

5 Sunday रविवार

$$+ \left(\frac{1}{3} \times 3 \times \frac{135}{EI}\right)$$

$$= \frac{585}{EI}\ kN.m^2$$

$$= \frac{585 \times 10^7}{2 \times 10^5 \times 3 \times 10^8}$$

$= 9.75 \times 10^{-3}$ radian

$= \boxed{0.56^0}$

JUNE
जून
2016

JULY | 2016

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

6
Monday
सोमवार

θ_C = slope at C,

$$= \frac{180}{EI} + \frac{270}{EI}$$

$$= \frac{450}{EI} \text{ kN.m}^2$$

$$= \frac{450 \times 10^9}{2\times10^5 \times 3\times10^8} = 7.5\times10^{-3} \text{ radian}$$

$$= \boxed{0.43^\circ}$$

y_B = deflection at B,

= B.M at B' on conjugate beam

$$= \frac{180}{EI} \times \left(3 + 2/3 \times 2\right) + \frac{270}{EI} \times (3\times1)$$

$$+ \frac{135}{EI} \times \left(\frac{3}{4}\times3\right)$$

$$= \frac{2163.75}{EI} \text{ kN.m}^3$$

$$= \frac{2163.75\times10^{12}}{2\times10^5 \times 3\times10^8} = \boxed{36.03 \text{ mm}}$$

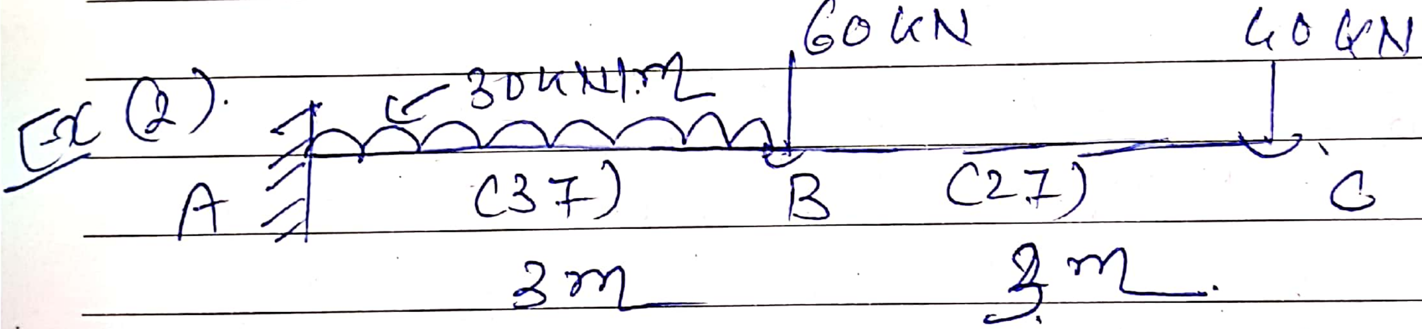
y_C = deflection at C

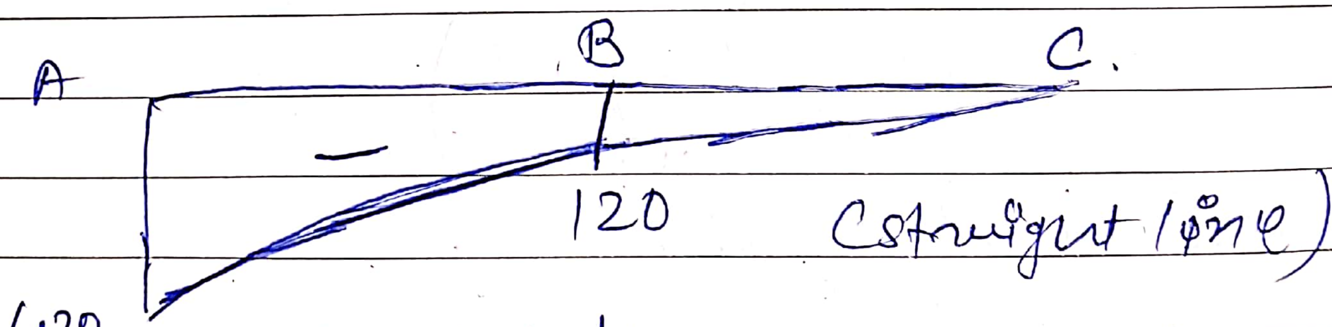
$$= \frac{180}{EI} \times \left(\frac{2}{3} \times 2\right) + \frac{270}{EI} \times 1$$

$$= \frac{510}{EI} \text{ kN.m}^3$$

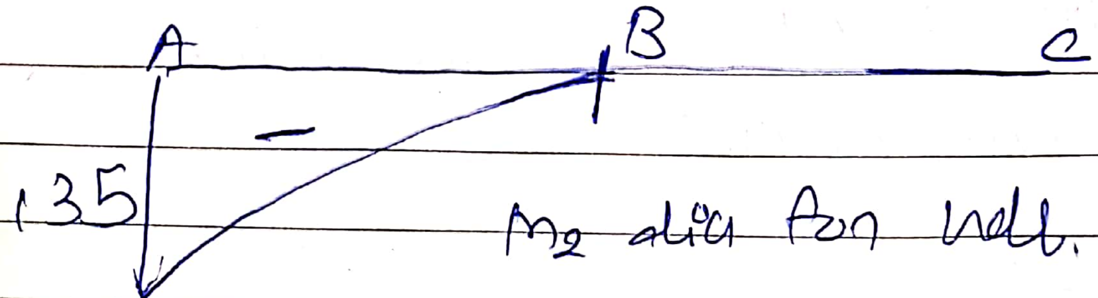
$$= \frac{510 \times 10^{12}}{2 \times 10^5 \times 3 \times 10^8} = \boxed{8.5 \text{ mm}}$$

Ex (2).





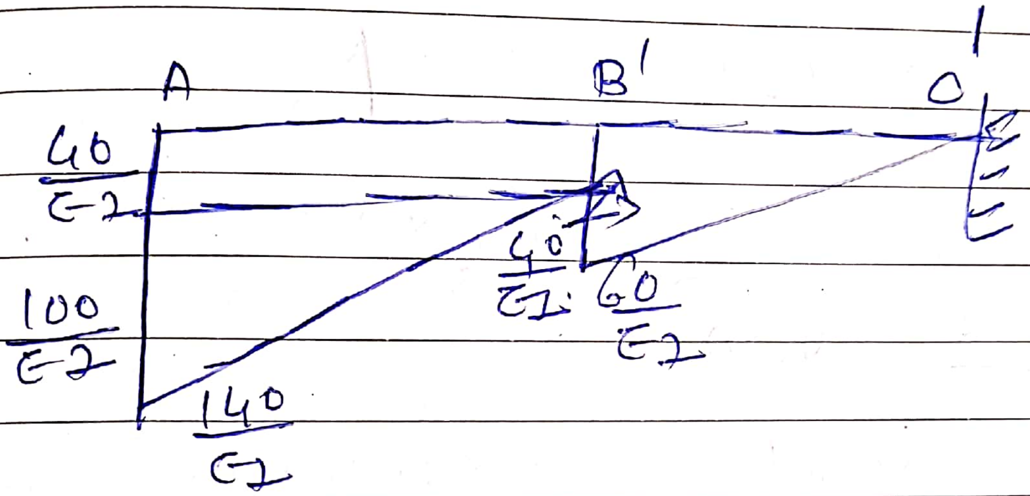
M_1 dia for point load

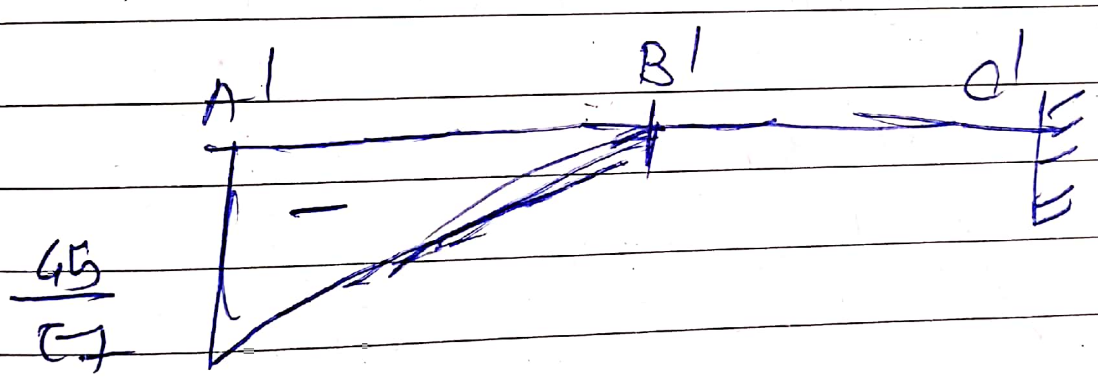


M_2 dia for udl.

JULY | 2016

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30





(Conjugate beam)

$$\theta_C = \frac{1}{2} \times 3 \times \frac{100}{EI} + 3 \times \frac{40}{EI} + \frac{1}{2} \times 3 \times \frac{60}{EI} + \frac{1}{3} \times 3 \times \frac{45}{EI}$$

$$= \frac{405}{EI} \ kN.m^2$$

$$= \frac{405 \times 10^9}{2 \times 10^5 \times 3 \times 10^8} = 6.75 \times 10^{-3} \text{ radian}$$

$$= \boxed{0.387^\circ}$$

θ_B = slope at B,

$$= \cancel{\frac{315}{E}} \ \frac{150}{EI} + \frac{120}{EI} + \frac{45}{EI}$$

$$= \frac{315}{EI} \ kN.m^2$$

$$= \frac{315 \times 10^9}{2 \times 10^5 \times 3 \times 10^8} = 5.25 \times 10^{-3} \text{ radian}$$

$$= \boxed{0.30^\circ}$$

$$y_c = \frac{150}{EI} \times \left(3 + \frac{2}{3} \times 3\right) + \frac{120}{EI} \times (3 + 1.5)$$

$$+ \frac{90}{EI} \times \left(2/3 \times 3\right) + \frac{45}{EI} \times \left(3 + \frac{3}{4} \times 3\right)$$

$$= \frac{1706.25}{EI} \; kN.m^3$$

$$= \frac{1706.25 \times 10^{12}}{2 \times 10^5 \times 3 \times 10^8}$$

$$= \boxed{28.44 \; mm}$$

$$y_B = \frac{150}{EI} \times \left(2/3 \times 3\right) + \frac{120}{EI} \times 1.5$$

$$+ \frac{45}{EI} \times \left(3/4 \times 3\right)$$

$$= \frac{581.25}{EI} \; kN.m^3$$

$$= \frac{581.25 \times 10^{12}}{2 \times 10^5 \times 3 \times 10^8} = \boxed{9.68 \; m}$$