Strain Energy

(1)

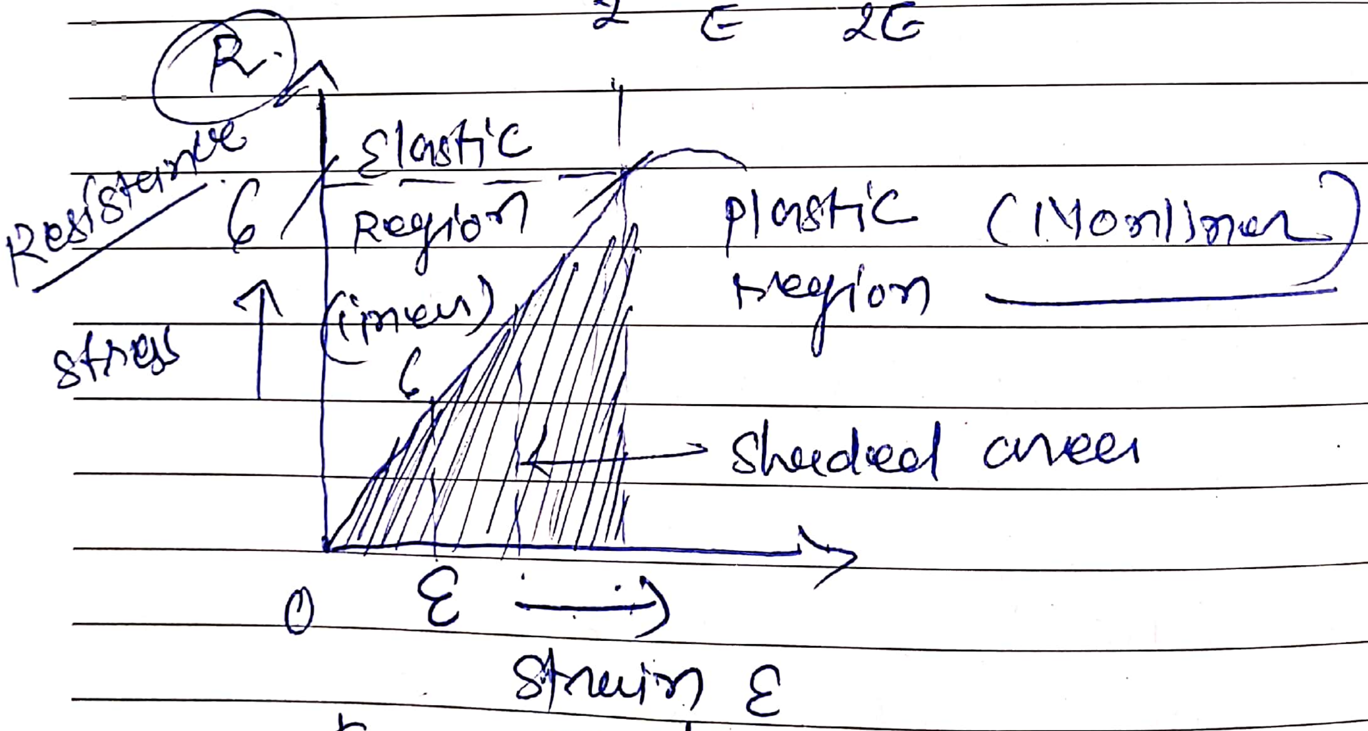
⇒ Whenever some load is attached to a hanging wire, it extends & load moves downwards by an amount equal to extension of the wire. A little consideration will show that when load is moves downwards, it loses its energy. This energy absorbed or stored in the stretched wire, which may be released by removing the load. This energy, which is absorbed in body, when strained within its elastic limit is .... strain energy = work done.

(2) Resilience: Capacity of strained body for doing ~~so~~ work (when it springs back) on the removal of straining force.

It is said to be resilence of the body.

Strain energy per unit volume $= \frac{1}{2}\sigma\varepsilon$

$= \frac{1}{2}\sigma.\frac{\sigma}{E} = \frac{\sigma^2}{2E}$



δl

deformation

$R = \sigma.A$

Energy stored $U = \frac{1}{2}$ OB BC

$= \frac{1}{2}R.\delta l$

$\sigma.\varepsilon\delta l$

but $\varepsilon = \frac{\delta l}{l}$

$\sigma . A$

So, $U = \frac{1}{2} R . \varepsilon l$

$= \frac{1}{2} \sigma . A . \varepsilon l$

$= \frac{1}{2} \sigma . \varepsilon A l$

$= \frac{1}{2} \times$ stress × strain × vol^m of the member.

It is desired to have the expression in terms of the max. stress induced in terms of the dimensions of the bar,

$\varepsilon = \frac{\sigma}{E}$, So, strain energy stored becomes,

$U = \frac{1}{2} \times \sigma \times \frac{\sigma}{E} \times A l$ or $\frac{\sigma^2}{2E} \times$ vol^m of longitudinal

(3) Proof Resilience :- It is used for max^m strain energy, which can be stored in body up to the elastic limit. called as Proof Resilience.

(4) Modulus of Resilience :-

Proof Resilience per unit vol^m of material is called ... & is an imp. property of the material.

* Types of loading

(1) Gradually (2) suddenly (3) Impact.

(1): Strain energy stored in body, when the load is gradually Applied.

→ loading starts from zero & increase gradually till the body is fully loaded.

eg. when we lower a body with the help of crane, body first touches the platform on which it is to be placed on further releasing the chain, the platform goes on loading till it is fully loaded by the body.

Work done = force × distⁿ

$= \text{Avg. load} \times \text{deformation}$

$= P/2 \times \delta l$

$= P/2\,(\varepsilon . l) = 1/2\, \sigma \varepsilon A l$

$= 1/2\, \sigma^2/E \times A l = \dfrac{\sigma^2}{2E} \times V = U$

∴ ~~modulus of resilience~~

$= \sigma^2/2E \times V / V = \boxed{\sigma^2/2E}$

(2) Strained energy stored in body when load is suddenly applied.

→ In factories & workshop's, load is suddenly applied on body. e.g. when lower body with the help of crane, body is first of just above the platform on which it is to be placed. If the chain breakes at once at this moment the whole load of body begin to act on platform.

work done = $P \times \delta l$

$\therefore U = \frac{\sigma^2}{2E} \times Al$

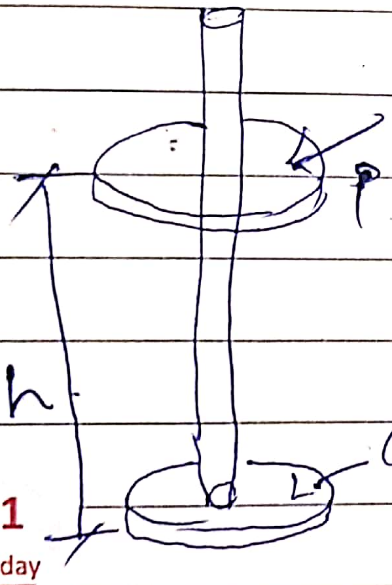
$\frac{\sigma^2}{2E} \times Al = P \times \delta l = P \times \frac{\sigma}{E} l$

$$\boxed{\sigma = \frac{2P}{A}}$$

stress induced in this case is twice the stress induced when same load is applied grad.

(3)

*. Strain energy stored in body, when load is applied with impact



In factories & workshops, Impact load is applied on body, eg. when we lower body with the help of crane, & chain breaks while load is being lowered the loads falls through a distⁿ, before it touches a platform.

Work done = load × dist^n moved

$$= P(h + \delta l)$$

$$U = \frac{\sigma^2}{2E} \times Al.$$

$$\therefore \frac{\sigma^2}{2E} \times Al = P\left(h + \frac{\sigma}{E} l\right) = Ph + \frac{P\sigma l}{E}$$

$$\therefore \sigma^2\left(\frac{Al}{2E}\right) - \sigma\left(\frac{Pl}{E}\right) - Ph = 0.$$

multiplying both sides by $\left(\frac{E}{Al}\right)$.

$$\left(\frac{\sigma^2}{2}\right) - \sigma\left(\frac{P}{A}\right) - \left(\frac{PEh}{Al}\right) = 0$$

quadratic eqn,

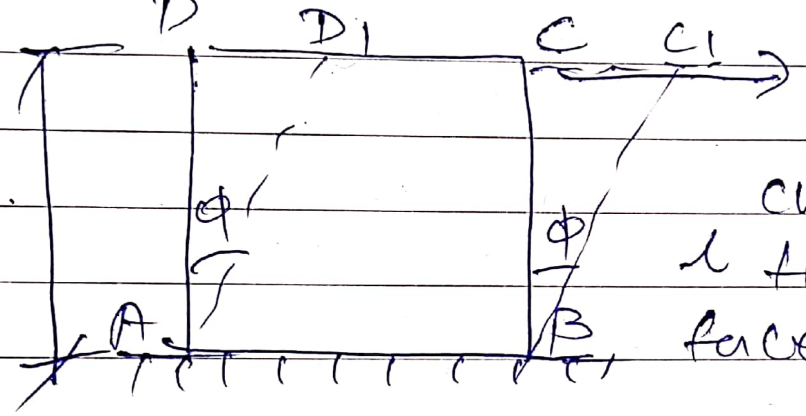
$$\sigma = \frac{P}{A} \pm \sqrt{\left(\frac{P}{A}\right)^2 + \left(\frac{4\times 1}{2}\right)\left(\frac{PEh}{Al}\right)}$$

$$\boxed{\sigma = \frac{P}{A}\left[1 \pm \sqrt{1 + \frac{2AEh}{Pl}}\right]}$$

If δ is very small compared to h,
then, work done $= Ph$

$$\therefore \sigma = \sqrt{\frac{2EPh}{Al}}$$

* Strain energy stored in body due to shear stress.



Cube ABCD of length l fixed at bottom face AB...

Force P is applied gradually then the avg. force is equal to P/2.

Work done $= P/2 \times DD_1$

$(DD_1 = AD \times \phi)$

$U = \frac{1}{2} \times P \times AD \times \phi$

$= \frac{1}{2} \times \tau \times DC \times l \times AD \times \phi$ $\qquad P = \tau \times DC \times l$

$= \frac{1}{2} \times \tau \times \phi \times DC \times AD \times l$.

$= \frac{1}{2} \times \tau \times \frac{\tau}{N} \times V$.

$\boxed{U = \frac{\tau^2}{2N} \times V}$.

$\therefore$ modulus of resilience $= \frac{\tau^2}{2N} \times V / V = \frac{\tau^2}{2N}$

"श्रीगणेशाय" [redacted] Monika

Statically Determinate Structure:

→ The structure for which the reactions at the supports and the internal forces in the members can be found out by the conditions of static equillibrium, is called a statically determinate structure.

– Three basic conditions of static equillibrium

i.e. $\Sigma H = 0$ Δshape

$\Sigma V = 0$

$\Sigma M = 0$.

Statically Indeterminate Structure:

→ The structure for which the reactions at the supports and the internal forces in the members cannot be found out by the condition of static equillibrium it is called statically indeterminate structure.

(i) Externally Indeterminate Structure:

- If the equations of static equillibrium are not sufficient to determine all the unknown reactions ($\Sigma H, \Sigma V, \Sigma M$) acting on a structure, it is called externally indeterminate structure, or externally redundent structure.

(ii) Internally Indeterminate Structure:

- If the equations of static equillibrium are not sufficient to determine the internal forces and moments in the member of the structure, even though all the external forces (Including applied forces & reactions) acting on the structure are known as internally indeterminate structure or internally redundent structure.

Degree of static Indeterminacy: (Degree of Redundancy) D_S

→ It may be defined as the number of unknown forces in excess of equations of statics.

∴ Degree of static indeterminacy

D_S = total number of unknown forces − no. of eq^n available in static condition

∴ D_S = extra unknowns
= redundants.

The indeterminacy of a structure may be either external or internal or both.

∴ Static indeterminacy = external indeterminacy + internal "

∴ $\underbrace{D.S = D_{se} + D_{si}}$

Total S.I = Ext. S.I + Int S.I.

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ree of Redundancy of Beam :

Beam : → only externally indeterminate

→ shear & moment at any point in the beam are readily known once the reaction components are determined. so, beam is statically determinate internally.

∴ for beam

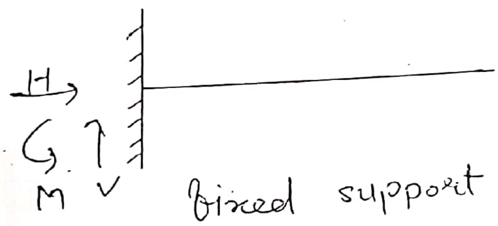
$$S.I. = \text{external } S.I$$

$$D_s = D_{se}.$$

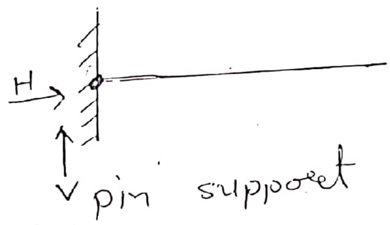
$$S.I. = \text{total no. of reaction} - \text{total no. of available condition}$$

$$= R - r$$

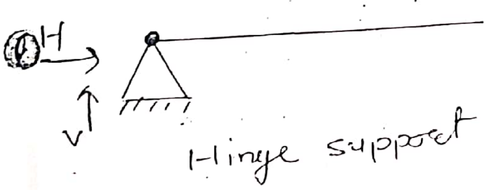
Reaction Components at Supports :



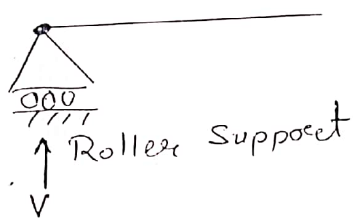
fixed support



pin support



Hinge support



Roller Support

→ Internal Hinge (pin) will provide one extra condition of equilibrium.

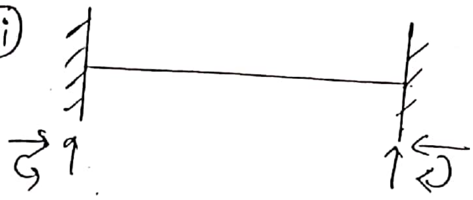
→ Link will provide two extra conditions.

(i)

$S.I = R - r$
$= 3 - 3$
$= 0$

Beam is Determinate.

(ii)



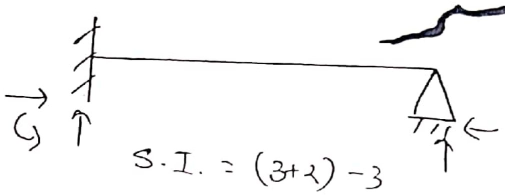
$S.I. = R - r$
$= (3+3) - 3$
$= 3.$

Beam is Indeterminate to three degree

(3)

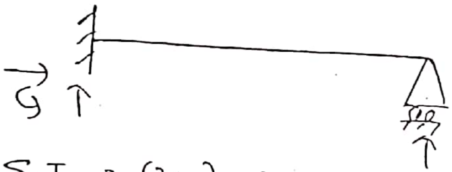
$S.I. = (2+2) - 3$
$= 1$

(4)



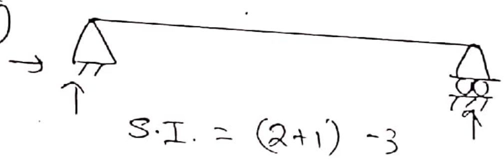
$S.I. = (3+2) - 3$
$= 2.$

(5)



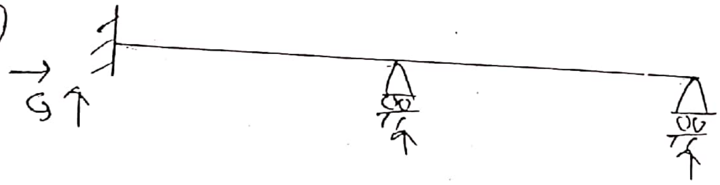
$S.I = (3+1) - 3$
$= 1$

(6)



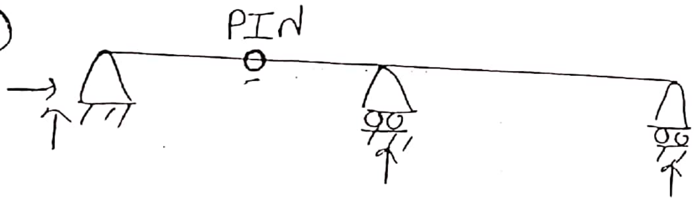
$S.I. = (2+1) - 3$
$= 0.$

(7)

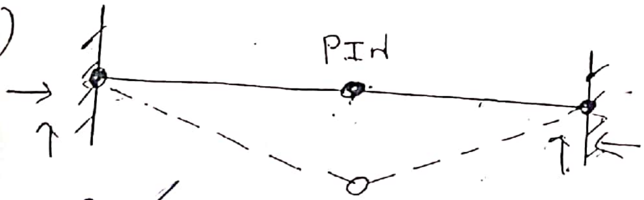


$\therefore S.I = (3+1+1) - 3$
$= 2.$

(8)



$S.I. = (2+1+1) - (3+1)$
$= 0.$



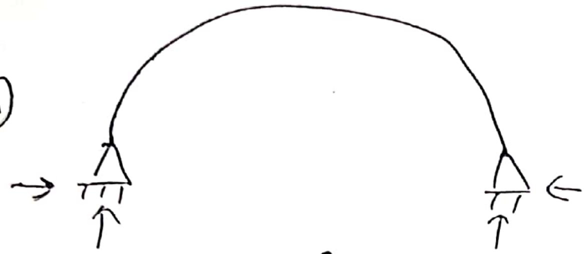
$SI = (2+2) - (3+1)$
$= 0.$

Beam is determinate but unstable.

9

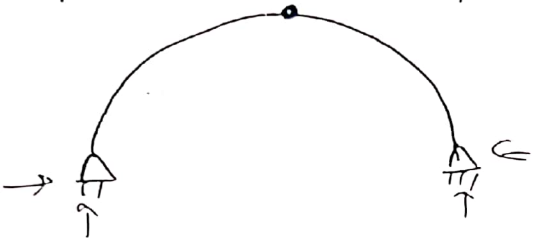
$$S.I. = (3+3) - 3 = 3.$$

(11)



$$S.I. = (2+2) - 3 = 1$$

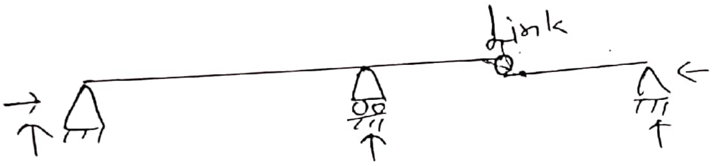
(12)



$$S.I. = (2+2) - (3+1) = 0.$$

stable, determinate.

(10)



$$S.I. = (2+1+2) - (3+2) = 0$$

Link → 2 extra condition.

* Plane Truss :-

A truss is composed of links or bars, assumed to be connected by frictionless pins at the joints, and arranged so that the area enclosed within the boundaries of the structure is subdivided by the bars into geometrical figures which are usually triangles.

→ If all the members of a truss lie in one plane, it is called a plane truss.

→ If members of a truss lie in three dimensions, it is called a space truss.

→ For truss, the loads applied at joint only.

Plane truss: two equillibrium equations are available.

$\Sigma H = 0$ & $\Sigma V = 0$.

Types of Truss

Perfect Truss

$m = 2j - r$

or $m + r = 2j$

Stable & Determinate

Imperfect Truss

$m \neq 2j - r$

Deficient Truss

$m < 2j - r$

not enough unknowns so, it is unstable

Redundent Truss

$m > 2j - r$.

stable & statically indeterminate

where, m = no. of members or no. of unknown forces in members

J = no. of joints ⇒ $2j$. eqn available

r = no. of condition eqn available for evaluation of reaction components.

- $m + r$ → total no. of unknowns

for plane truss $r = 3$ (normally)

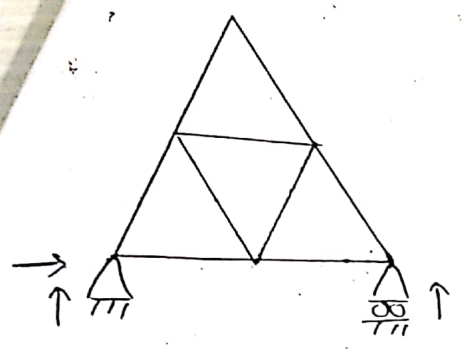
for plane truss S.I. = internal SI + external S.I.

external SI = $R - r$

internal S.I $= (m + r) - 2j$

$= m - (2j - r)$

ple:→



$m = 9 \quad j = 6$

$R = 3 \quad r = 3$ (conditions)

ext. SI. $= R - r = 3 - 3 = 0$. stable determinate

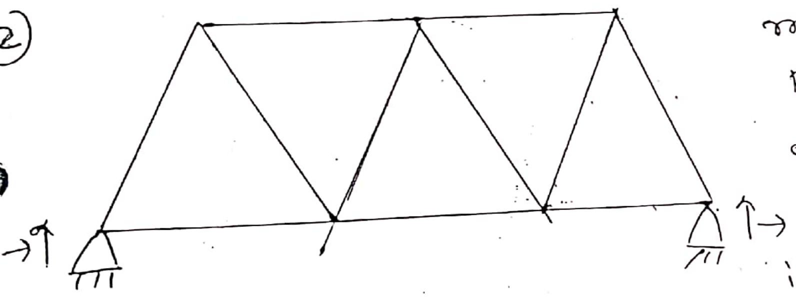
int SI $= (m + r) - 2j$

$= (9+3) - (2 \times 6) = 0$. " "

total S.I = ext. SI + int SI

$= 0$. stable & determinate

(2)



$m = 11 \quad j = 7$

$R = 4 \quad r = 3$

ext SI $= R - r = 4 - 3 = 1$.

stable, indeterminate

int SI $= (m + r) - 2j$

$= (11 + 3) - 2 \times 7$

$= 14 - 14 = 0$.

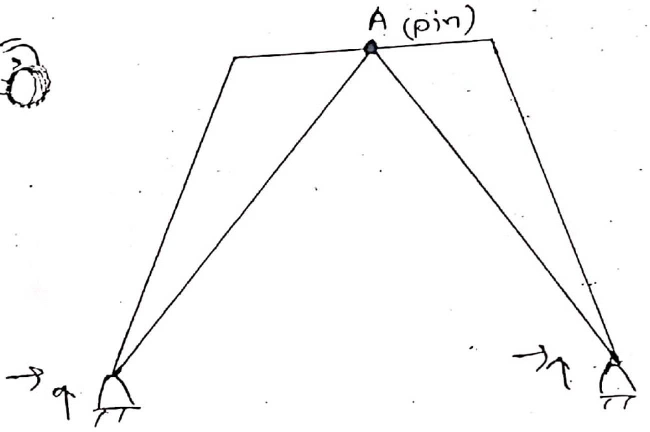
stable, determinate

total SI = ext. SI + int SI

$= 1 + 0 = 1$

stable & indeterminate.

(3)



$m = 6 \quad j = 5 \quad R = 4$

$r = 3 + 1 = 4$.

ext SI $= R - r = 4 - 4 = 0$

int SI $= (m + r) - 2j$

$= (6 + 4) - 2 \times 5$

$= 0$.

stable & determinate.

frame.

- In case of pin jointed plane frames or trusses the members carry only axial forces & hence two equations are available at each joint, viz $\boxed{\Sigma H = 0 \quad \Sigma V = 0}$
- in case of rigid jointed plane frame, each member carry three unknown internal forces ($\Sigma H, \Sigma V, \Sigma M$).

Internal Indeterminacy :

$D_{si} = 3C$ — C = no. of areas (cells) completely enclosed by members of the frame

(i) rigid jointed frame

$D_{si} = 3C - r'$ — r' = no. of members connected to pin - 1

External Indeterminacy :

$D_{se} = R - r$ → Equillibrium condition

reaction

rigid jointed frame — $r = 3$ w/o hinge or link

total SI — $D_s = D_{se} + D_{si}$

OR

$D_s = (3m + R) - 3j$

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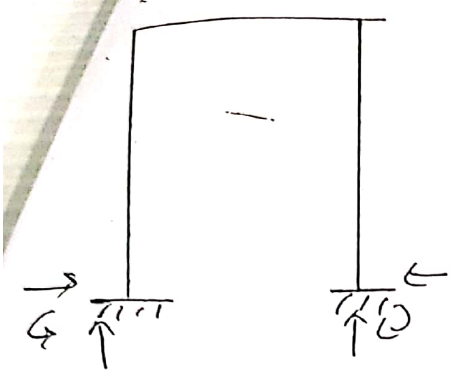
ple's



$m = 3 \quad j = 4. \quad R = 6 \quad r = 3 \quad c = 0$

$D_{se} = R - r = 6 - 3 = 3$
stable, indeterminate

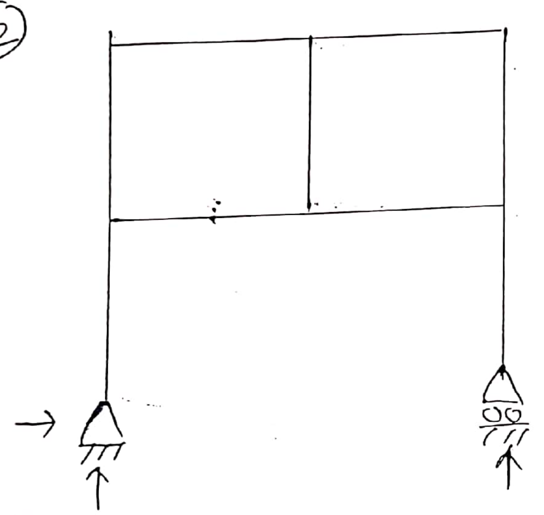
$D_{si} = 3c = 3 \times 0 = 0.$
stable, determinate.

$\therefore D_S = 3 + 0 = 3$ stable, indeterminate

OR

$D_s = 3m + R - 3j$
$= 3(3) + 6 - (3 \times 4) = 9 + 6 - 12 = 3.$
stable, indeterminate

(2)



$D_S = 3m + R - 3j$

$m = 9 \quad R = 3 \quad J = 8 \quad r = 3 \quad C = 2$

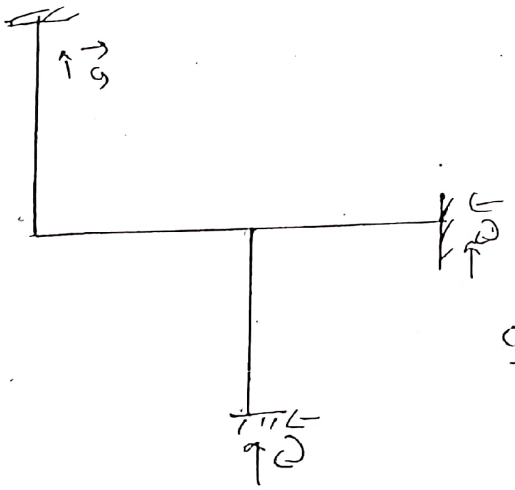
$= (3 \times 9) + 3 - (3 \times 8)$
$= 27 + 3 - 24$
$= 6$ stable, indeterminate.

OR

$D_{si} = 3 \times C = 3 \times 2 = 6.$
$D_{se} = R - r = 3 - 3 = 0.$
$D_s = 6 + 0 = 6.$

$R = 9, \; m = 4, \; j = 5$

(3)



$D_s = 3m + R - 3j$

$m = 4 \quad R = 9 \quad J = 5$

$= 3 \times 4 + 9 - 3 \times 5$
$= 6.$

OR $D_{se} = R - r = 9 - 3 = 6$

$D_{si} = 3 \times 0 = 0.$

$C = 0 \qquad D_s = 6.$

GRID :

→ A grid is a plane structure composed of continuous members that either intersect or cross each other.

→ All the members of grid, are generally lie in one plane.

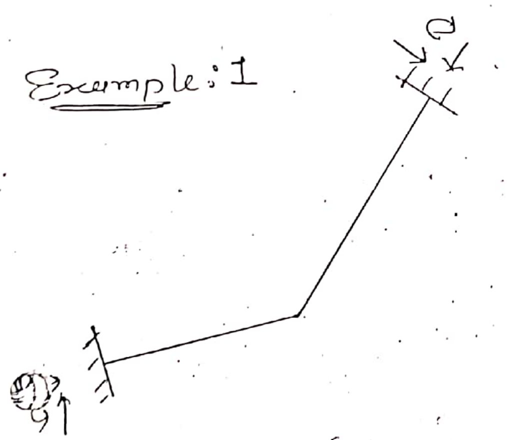
→ External loads act normal to the plane of the grid, & couples have their vectors in the plane of the grid.

→ The members of a grid are subjected to vertical shear force, bending moment & twisting moment at any cross section.

→ As the external loads act normal to the plane of the grid ; there is no axial force in the grid members.

$$S.I = 3m + R - 3j$$

Example: 1



$m = 2 \quad j = 3$

$R = 6$

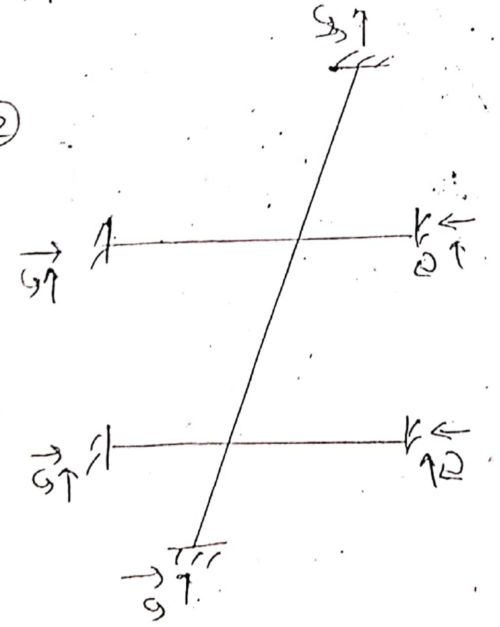
$D_s = 3m + R - 3j$

$= (3\times2) + 6 - (3\times3)$

$= 3.$

stable, indeterminate.

(2)



$m = 7, \quad J = 8 \quad R = 18$

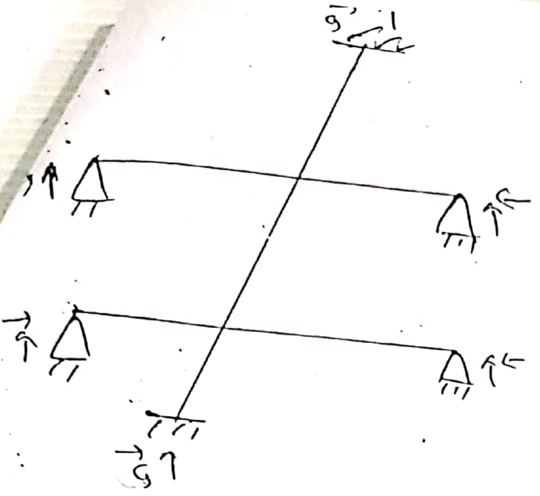
$D_s = 3m + R - 3j$

$= (3\times7) + 18 - (3\times8)$

$= 21 + 18 - 24$

$= 15$

stable, indeterminate.



$m = 7 \quad j = 8 \quad R = 14$

$D_s = 3m + R - 3j$

$= (3\times7) + 14 - 3\times8$

$= 11$

stable, indeterminate.

✗ Space Truss :-

① In the case of space truss, all the members of the truss do not lie in one plane. Very often, space truss is formed by combining a series of plane trusses.

- The members of a space truss are subjected to axial forces only.
- The equilibrium of an entire space truss or sections of a space truss is described by the six scalar eqn.

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$

$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0.$

for space truss

(i) $m + r = 3j$ → stable, statically determinate

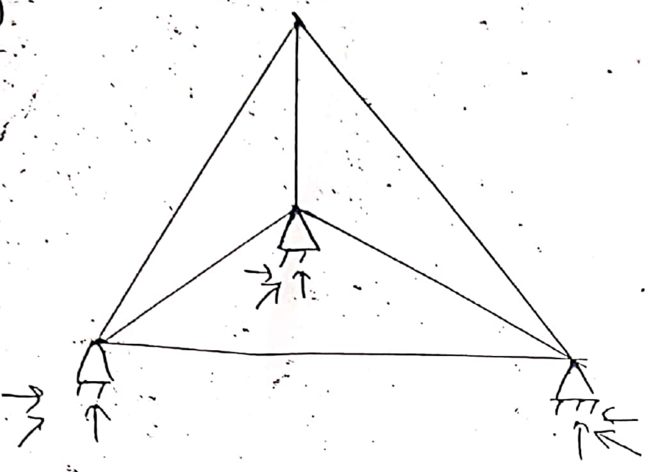
(ii) $m + r < 3j$ → unstable (internally)

(iii) $m + r > 3j$ → stable, statically indeterminate.

$r = 6$ – for space truss

Example:

1.



$m = 6 \quad j = 4$

$R = 3 \times 3 = 9$

$r = 6$.

$D_{se} = R - r$

$= 9 - 6 = 3$.

∴ stable, indeterminate

$D_{si} = (m + r) - 3j$

$= (6 + 6) - 3 \times 4$

$= 0$

stable, determinate.

(2) ∴ $D_s = D_{se} + D_{si}$

$= 3 + 0$

$= 3$

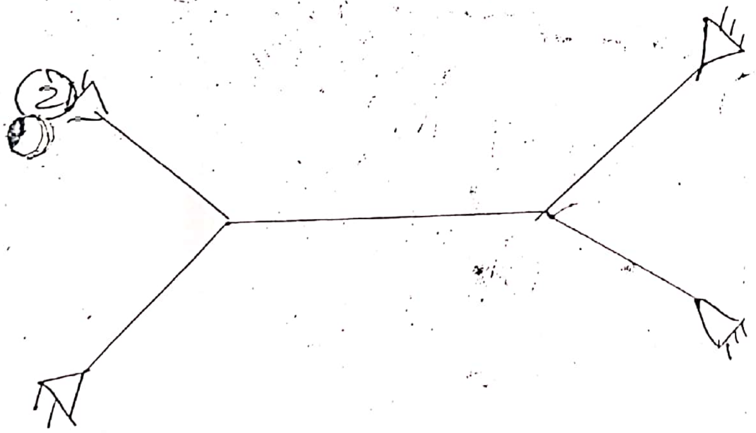
so, stable & indeterminate.

OR $D_s = m + R - 3j$

$= 6 + 9 - (3 \times 4)$

$= 3$. stable, indeterminate.

(2)



$m = 5 \quad j = 6$

$R = 4 \times 3 = 12 \quad r = 6$

$D_{se} = R - r$

$= 12 - 6$

$= 6$ stable, indetermi

$D_{si} = (m + r) - 3j$

$= (5 + 6) - 3 \times 6$

$= -7$ unstable.

$D_s = D_{se} + D_{si}$

$= 6 - 7$

$= -1$

unstable

OR $D_s = m + R - 3j$

$= 5 + 12 - 3 \times 6$

$= -1$

unstable

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Space Frame (Rigid Joint)

All the members of space frame do not lie in one plane.

→ In the case of a rigid-jointed space frame, every members carries six unknown internal forces i.e. ~~six~~ three forces & three moments.

Internal Indeterminacy:

$D_{si} = 6 \cdot C$ $\quad$ C = no. of cuts required to make open configuration

External Indeterminacy:

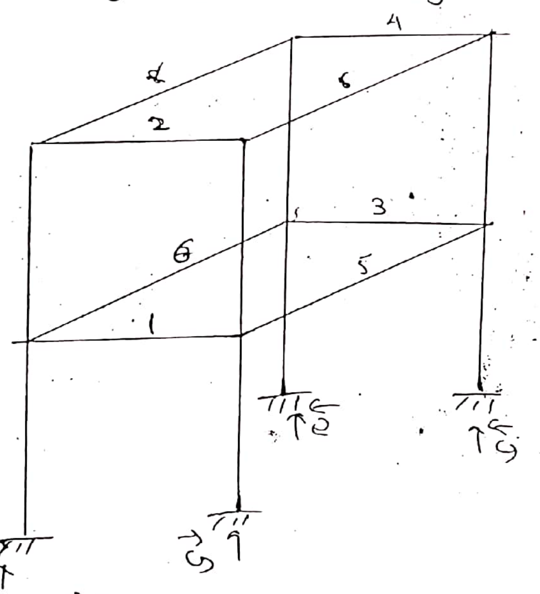
$D_{se} = R - r$

$\therefore D_s = D_{se} + D_{si}$

OR

$D_s = 6m + R - 6j$

Ex: I



six reaction at each support.

$m = 16 \quad j = 12$

$R = 4 \times 6 = 24$

$r = 6$

$C = 6$

$D_s = 6m + R - 6j$

$= (6 \times 16) + 24 - (6 \times 12)$

$= 96 + 24 - 72$

$= 48$

$D_{se} = R - r$

$= 24 - 6 = 18$

$D_{si} = 6 \cdot C$

$= 6 \times 5$

$= 30$

$D_s = 48$

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Kinematic Indeterminacy :
(Degree of Freedom)

If the displacement components of the joints of a structure cannot be determined by compatibility equations alone, the structure is said to be kinematically indeterminate structure.

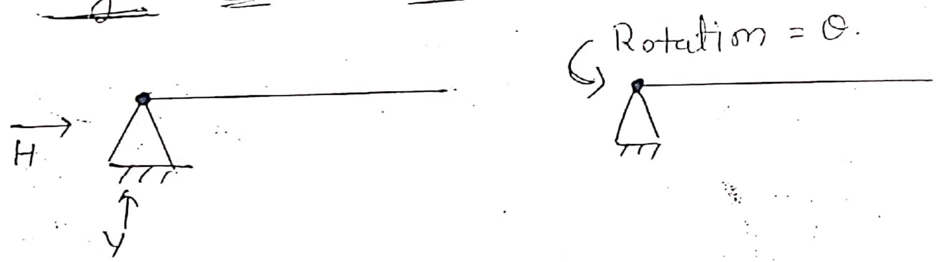
→ The number of additional equations necessary for the determination of all the independent displacements components is known as the degree of kinematic indeterminacy or the degree of redundancy of the structure.

→ Kinematic Indeterminacy : total no. of possible displacements in a structure.

Possible Displacements at Various Supports :

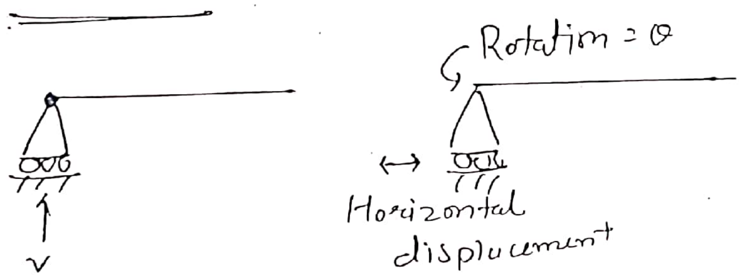
(i) Hinged or Pinned Support : Reaction H & V prevent H & V displacement

∴ so, only rotation is permitted at hinge support.

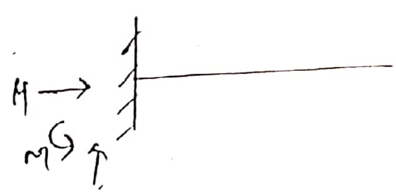


(ii) Roller Support :

V → ✗
H & θ → ✓

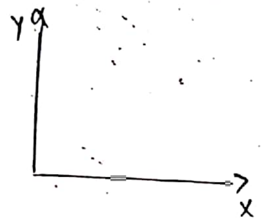


(iii) Fixed Support : no displacement are possible at fixed support.



Degree of Freedom per joint:

Plane Truss: pin jointed plane truss



at each joint two displacements ($\Delta x, \Delta y$) are possible.

$\therefore D_k = 2j - e$

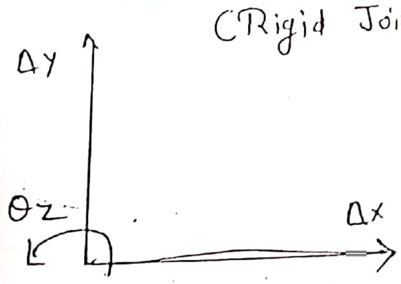
where = $2j$ = total displacements of all joints
e = no. of eqn of compatibility
= no. of constrains impost by support condition
= no. of external reaction components
= R

$SI = (m + R) - 2j$

$\therefore \boxed{D_k = 2j - R}$

2. **Plane Frame:** (Rigid Jointed)
three joint displacements are possible $\Delta x, \Delta y, \theta_z$

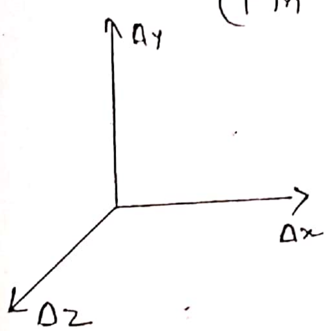
$D_k = 3j - R$



$(3m + R) - 3j$

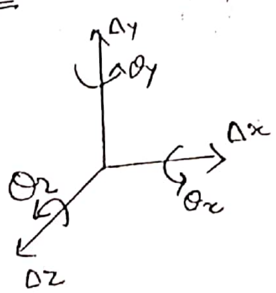
3. **Space Truss:** (Pin Truss)
$\Delta x, \Delta y, \Delta z$ - three displacement are possible

$\boxed{D_k = 3j - R}$



$SI = (m + R) - 3j$

4. **Space Frame:** Rigid Jointed



$D_k = 6j - R$

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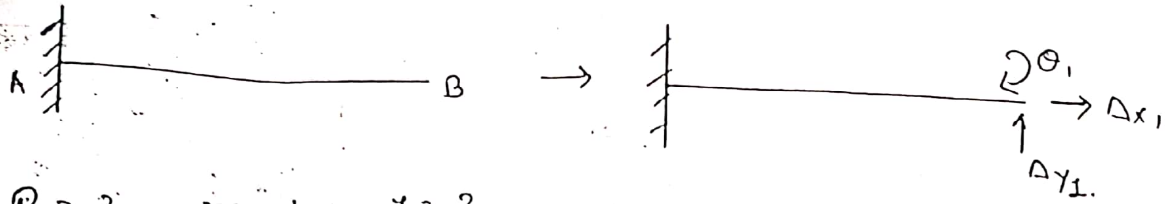
Example:

1. 

$R = 3$ $m = 1$ $j = 2$

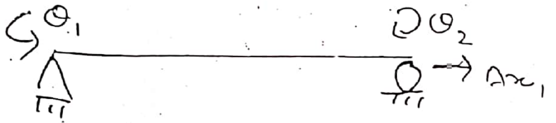
$D_k = 3j - R = 3 \times 2 - 3$

$= 3.$ $(\Delta x_1, \Delta y_1, \theta_1)$

if axial deformation is neglected

$\therefore D_k = (3j - R) - m = 3 - 1$

$= 2.$ $(\theta_1, \Delta y_1)$

2. 

$R = (2+1) = 3$

$m = 1$ $J = 2$

$D_k = 3J - R = (3 \times 2) - 3$

$= 3.$ $(\theta_1, \theta_2, \Delta x_1)$

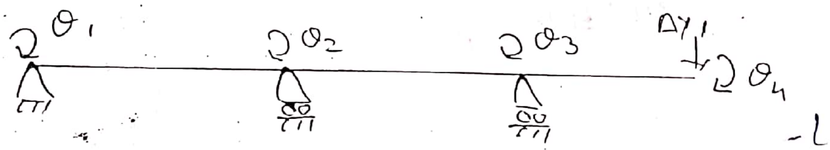
3. 

$R = 4$ $J = 2$

$m = 1$

$D_k = 3j - R = (3 \times 2) - 4$

$= 2.$ (θ_1, θ_2)

4. 

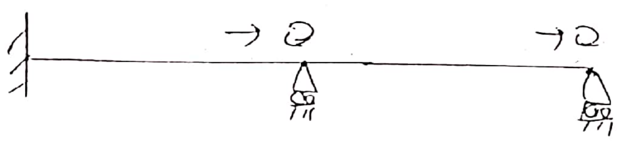
$-12 - 4$

$= 8$

$R = 4$ $m = 3$ $J = 4$

$D_{k\,neol} = 3j - (R + m)$

$= (3 \times 4) - (4 + 3) = 5$ $(\theta_1, \theta_2, \theta_3, \theta_4, \Delta y_1)$

5. 

$m = 2$ $J = 3$

$R = 5$

$D_k = 3j - R$

$= 3 \times 3 - 5$ θ_1, θ_2 Δy_1, Δy_2

$= 4$

$D_{k\,neol} = 3j - R - m$

$= 9 - 5 - 2 = 2$ θ_1 θ_2

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