turning force is applied to transmit the energy by rotation.

Shaft is used to transmit the power. Wheel or axle or shaft that is designed to transmit movement or power from machine support & change direction of belt from 1 part to another.

Torsion

⇒ bar or member is sub. to equal and opposite couples at the ends acting on two parallel planes at right angles to the longitudinal axis of the bar the c/s twist relative to each other is called torsion or twisting moment.

★ Assumptions :-

(1) The material of the shaft is uniform throughout.

(2) The twist along the shaft is uniform.

(3) Normal c/s of the shaft, which were plane and circular before the twist, remain plane and circular even after the twist.

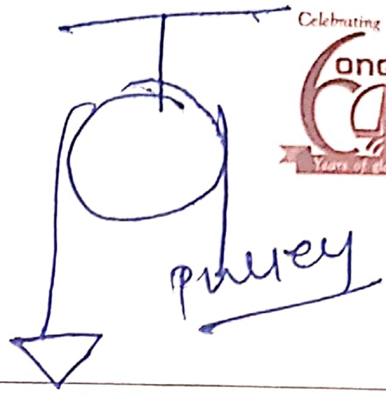
(4) All dia of the normal c/s, which were straight before the twist, remain straight with their magnitude unchanged, after the twist.

MARCH
मार्च
2016

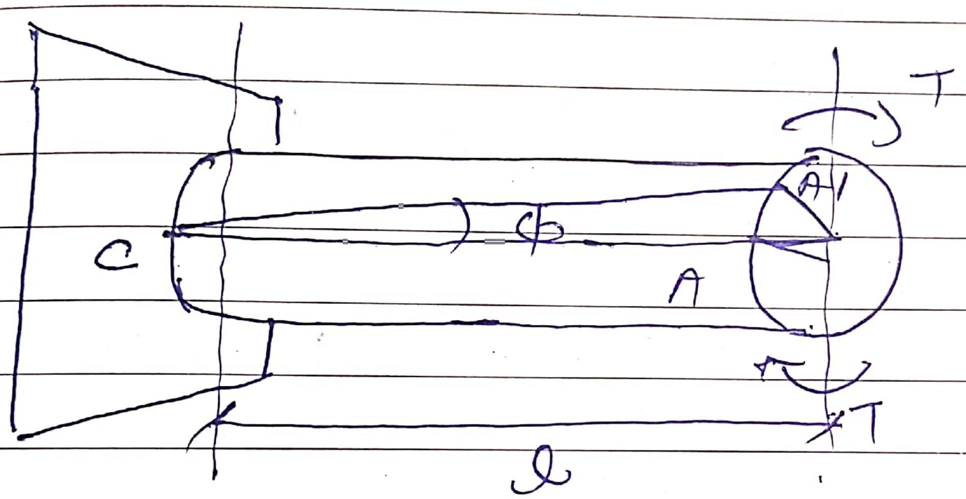
3
Thursday
गुरुवार

APRIL | 2016

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30



* Pure torsion formula :-



Consider a circular shaft fixed at one end & sub. to torque at the other end as shown in fig.

let, T = Torque in N.mm
l = length of the shaft in mm
R = Radius of the circular shaft in mm

Now, As result of this torque, every c/s of the shaft will be sub. to shear stresses. Let the line CA on the surface of the shaft be deformed to CA' and OA to OA'.

$\angle ACA' = \phi$ in degrees.
$\angle AOA' = \theta$ in radians.
τ = shear stress induced at the surface
C = modulus of rigidity / torsional rigidity.

Shear strain = Deformation per unit length

$$= \frac{AA'}{l} = \tan\theta = \phi$$

$$AA' = R\theta$$

$$\phi = \frac{AA'}{l} = \frac{R\theta}{l} \quad \text{——— (1)}$$

$$\phi = \frac{\tau}{C} \quad \text{——— (2)}$$

equating, (1) & (2)

$$\frac{\tau}{C} = \frac{R\theta}{l} \quad \therefore \frac{\tau}{R} = \frac{C\theta}{l}$$

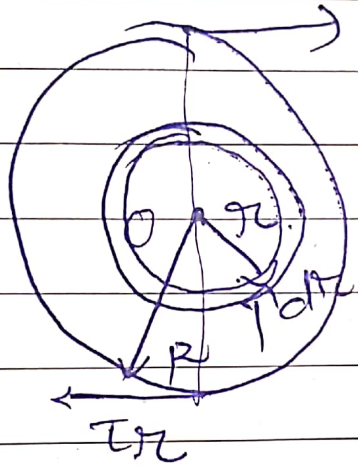
$$\therefore \tau = \frac{C\theta . R}{l}$$

$\sigma = \frac{P}{A}.$

5
Saturday
शनिवार

*. Strength of solid shaft.

Shaft sub. to torque T as shown in fig.



here, consider an elementary area at radius r and thickness dr.

6
Sunday
रविवार

$$\text{Area} = 2\pi r\,dr.$$

$$\text{Shear Stress} = \tau_r$$

τ is the max. shear stress at radius R,

$$\tau_r = \frac{r}{R} \times \tau$$

Shear force in small area,

$$\cancel{\delta Q} = \frac{2\pi r\,dr\,\tau_r}{\cancel{A}\ \ \cancel{R}}$$

force

$$= 2\pi r\,dr \times \frac{r}{R} \times \tau$$

$$\sigma = \frac{F}{A}$$

$$= \frac{\tau}{R} \times 2\pi r^2\,dr.$$

Scanned with CamScanner

Now, turning moment of this element,

$$\delta T = \delta Q . r$$
$$= \frac{\tau}{R} \times 2\pi r^3 dr$$

Total twisting moment,

$$T = \int_0^R \frac{\tau}{R} \times 2\pi r^3 dr$$

$$= \frac{2\pi\tau}{R}\left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{\cancel{2}\pi\tau}{R}\frac{R^4}{\cancel{4}\,2}$$

D^2 ~~$2R$~~

$$= \frac{\tau}{R}\left(\frac{\pi}{2} R^4\right) = \frac{\pi}{2}\tau R^3$$

where ~~$\frac{\pi}{2} R^4 = J$, polar moment of inertia.~~

where $2R = D$

$$T = \frac{\pi}{2} \tau \times \left(\frac{D}{2}\right)^3 = \boxed{\frac{\pi}{16} \tau D^3 \text{ N.mm}}$$

$$\therefore \tau = \frac{16T}{\pi D^3}$$ $\therefore$ The MOI of the plane area, with respect to an axis $\perp$ to the plane of the fig. is called polar moment of inertia.

$$\therefore \frac{16T}{\pi D^3} = \frac{C\theta R}{l}$$

$R = D/2$ $$\therefore \frac{16T}{\pi D^3 \times R} = \frac{C\theta}{l}$$

$$\therefore \frac{16T \times 2}{\pi D^3 \times D} = \frac{C\theta}{l}$$

$$= \frac{T}{\frac{\pi}{32} D^4} = \frac{C\theta}{l}$$

$J = \rightarrow \frac{\pi}{32} D^4$

$$\therefore \frac{T}{J} = \frac{C\theta}{l}$$

where J is the polar moment of inertia.

$$\boxed{\frac{\tau}{R} = \frac{T}{J} = \frac{C\theta}{l}}$$

Resistance against twisting moment.

If hollow circular shaft the polar moment of inertia,

$$J = \frac{\pi}{32}(D^4 - d^4)$$

Where, τ = shear stress.

T = Torque in N.mm

C = modulus of rigidity

mm⁴ ⟸ J = polar moment of inertia

l = length of shaft in mm

R = Radius of circular shaft in mm

θ = angle.

Power transmitted by shaft

No. of revolution

work done per min = f × dist^n = $T \cdot 2\pi NT$

sec = $\frac{2\pi NT}{60}$ KN.m.

MARCH मार्च 2016

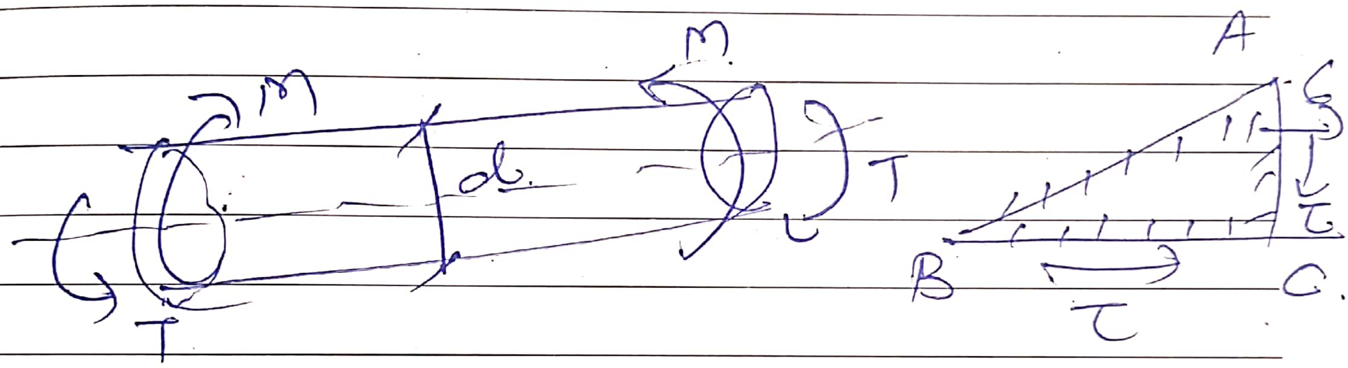
APRIL | 2016

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Celebrating 60 ONGC Years of glory

10 Thursday गुरुवार 22/07/2019

* Shafts subjected to bending (T) and Torsion (M).



Twisting and bending moments on shafts.

let, M and T subjected to be the BM & Twisting moment on the c/s of dia d.

Numerical

σ = Normal stress due to B.M

$$\sigma = \frac{32M}{\pi d^3}$$

τ = shear stress due to twisting moment

$$\tau = \frac{16T}{\pi d^3}$$

power transmitted $= \frac{2\pi NT}{60}$ kW.

max. principal stress at element,

$$P_1 = \frac{6}{2} + \sqrt{\left(\frac{6}{2}\right)^2 + \tau^2}$$

$$= \frac{16M}{\pi d^3} + \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16M}{\pi d^3} + \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

$$\therefore P_1 \times \frac{\pi d^3}{16} = M + \sqrt{M^2+T^2}$$

$$\therefore P_1 \times \frac{\pi d^3}{32} = \frac{M+\sqrt{M^2+T^2}}{2}$$

$$M_e = \frac{M+\sqrt{M^2+T^2}}{2}.$$

B.M due to max. principal stress in shaft is known as equivalent B.M (M_e).

similarly, max. shear stress developed on the surface of the shaft.

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3} \times \sqrt{M^2 + T^2}$$

$$\tau_{max} \times \frac{\pi d^3}{16} = \sqrt{M^2 + T^2} = T_e$$

equivalent twisting moment.

The twisting moment corresponding to max. shear stress on the surface of the shaft is known as equivalent twisting moment, T_e.

$$T_e = \sqrt{M^2 + T^2}.$$

Ex. A solid circular shaft of 100mm dia is transmitting 120 KW at 180 RPM. Find the intensity of shear stress in the shaft.

(P) → 150.

$$120 = \frac{2\pi N T}{60}$$

$$120 = 15.7T$$

$$T = 7.64 \text{ kN.m}$$
$$= 7.64 \times 10^6 \text{ N.mm}$$

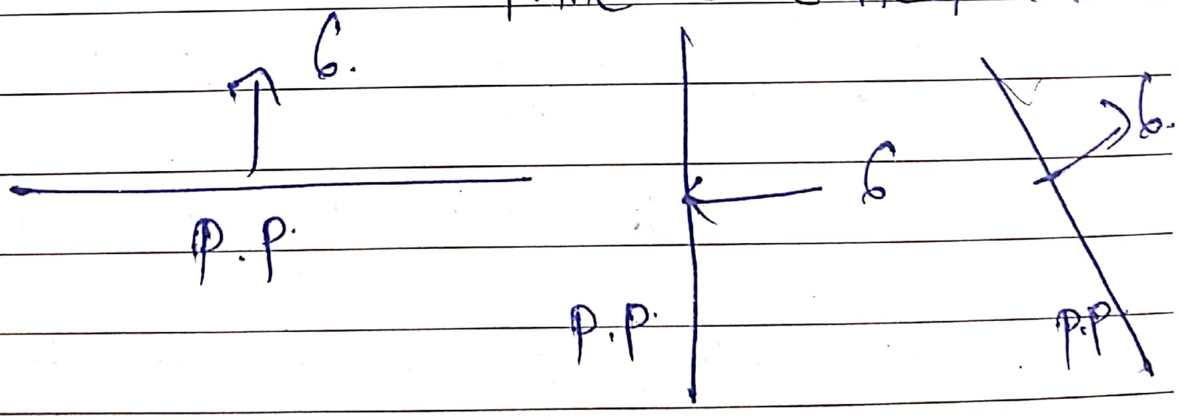
$$7.64 \times 10^6 = \frac{\pi}{16} \times \tau \times D^3$$

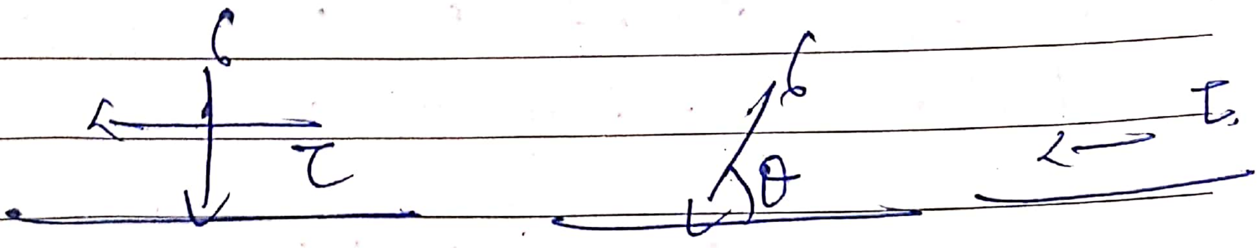
$$\boxed{\therefore \tau = 39 \text{ MPa.}}$$

Chap: Principal Stress & principal Planes.

Principal planes :- At any point in a strained material, there are three planes, mutually perpendicular to each other, which carry direct stresses only, and no shear stress.

Principal Stress :- Magnitude of direct stress, across a principal plane is called....





*. methods for the stresses on an oblique section of a body

→ determination of stresses.

(1) Analytical method

(2) Graphical method

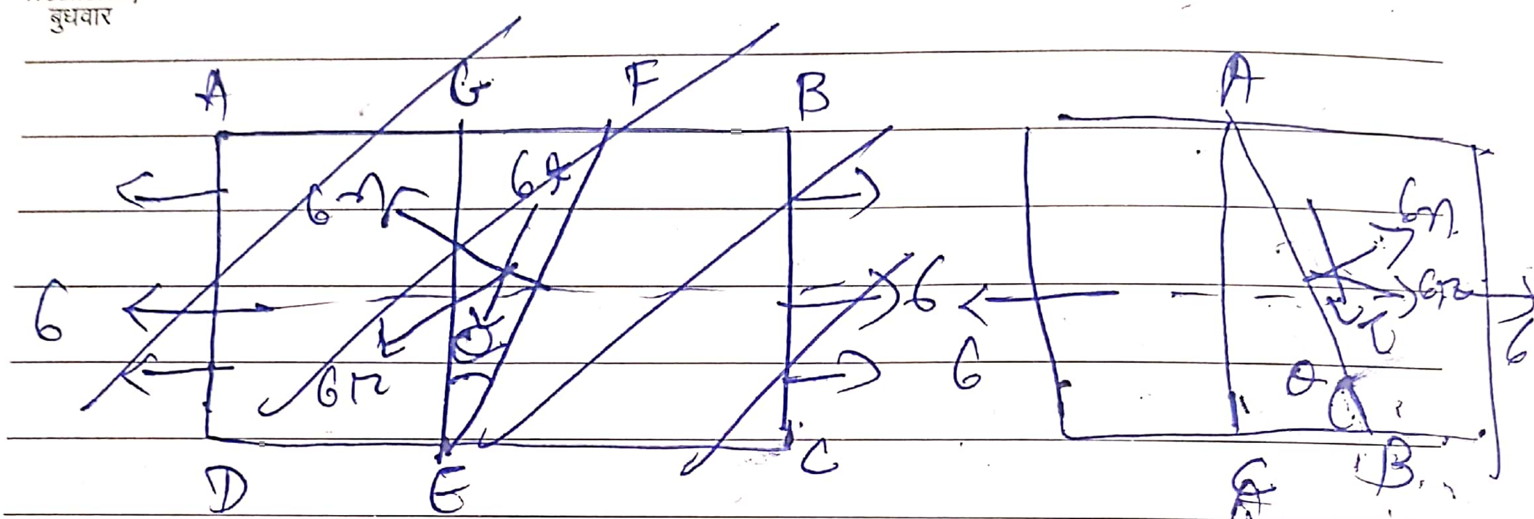
(1) Analytical method for the stresses on an oblique section. (Inclined)

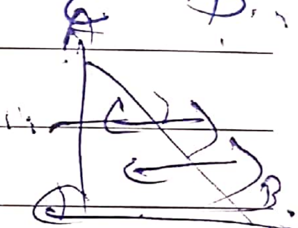
→ A body sub. to a direct stress in one plane.

→ A body sub. to direct stresses in two mutually perpendicular directions

* Normal, tangential and resultant stress on an inclined plane.

(1) when body is sub. to direct stresses in one plane.





σ = Normal stress on body

AB EF = inclined plane

θ = angle of inclined plane with vertical

σ_n = Normal stress on EF plane

σ_t = Shear " " "

σ_R = Resultant stress on EF plane.

A = C/S Area.

Tensile stress on BC,

$\sigma = P/A$.

tensile stress on EF

$= \frac{P}{Area} = \frac{P}{A\sec\theta} = \frac{P\cos\theta}{A} = \sigma\cos\theta$

So, $P = \sigma A$.

Now, Resolving the force ⊥ & ∥ to the section ~~EF~~ AB,

MARCH मार्च 2016

APRIL 2016

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

17 Thursday गुरुवार

& tangential to AB plane is.

$P_t = P\cos\theta = \sigma A_C \cos\theta$.

tensile force perpendicular to AB plane is,

P_n (force) $= P\cancel{\cos\theta}\sin\theta = \cancel{P}\,\sigma A_C \sin\theta$.

Normal stress on ~~EF~~ AB plane,

$$\sigma_n = \frac{P}{AB} = \frac{P\cos\theta}{\cancel{A\sec\theta}} = \frac{P}{\cancel{A}}\cancel{\cos\theta}.\cos\theta$$

$$\frac{\sigma\,\cancel{A_C\sin\theta}}{A_C/\sin\theta}$$

$$\boxed{\sigma_n = \sigma\,\frac{\cancel{\cos^2\theta}}{}\sin^2\theta}$$

$(\cancel{2}\theta)$

$\sigma \cancel{A}\,\cos$

$\frac{\sigma}{2}$

Tangential stress on ~~EF~~ AB plane,

$$\sigma_t = P/AB = \frac{P\sin\theta}{\cancel{A\sec\theta}} = \frac{P}{\cancel{A}}\cancel{\sin\theta\cos\theta}.$$

$$\frac{\sigma A_C\cos\theta}{A_C/\sin\theta}$$

$$\cancel{\sigma_t =}\ \sigma\sin\theta\cos\theta.$$

$2\sin\theta\cos\theta = \sin 2\theta$.

$= \sigma\cos\theta\sin\theta$.

$= \frac{\sigma}{2}\sin 2\theta$.

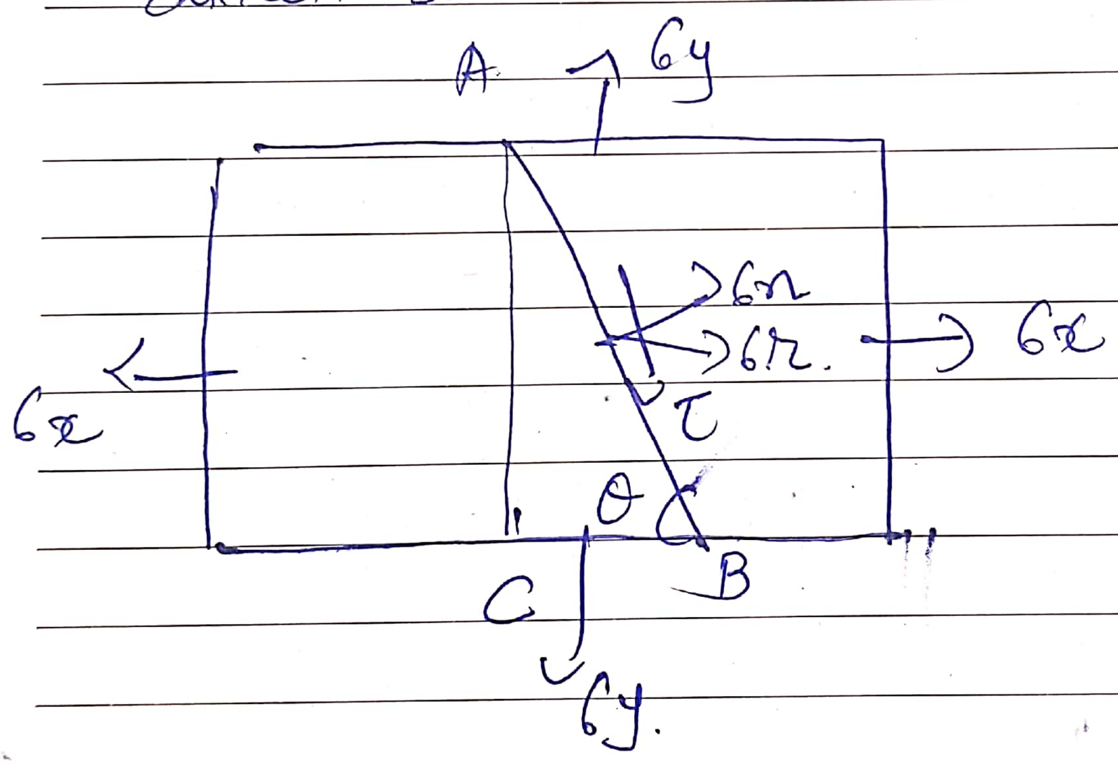
$$\boxed{\sigma_t = \frac{\sigma}{2}\,\cancel{\sin\theta\cos\theta}.(\sin 2\theta)}$$

Resultant stress on EF plane,

$$\boxed{\sigma_r = \sqrt{\sigma_n^2 + \sigma_t^2}}$$

23/7/19

(2). Normal, Tangential and Resultant stress on an inclined plane when body is sub. to direct stresses in two mutually perpendicular directions.



6x = tensile stress along x-x axis (major axis)

6y = tensile stress along y-y axis (minor axis)

Now, horizontal force acting on face Ac (x-x axis).

$$P_x = \sigma_x \, Ac \; (\leftarrow).$$

& vertical force acting on face BC (y-y axis).

$$P_y = \sigma_y \, BC \; (\downarrow).$$

Resolving the forces normal to the section AB,

$$P_n = P_x \sin\theta + P_y \cos\theta.$$
$$= \sigma_x Ac \sin\theta + \sigma_y BC \cos\theta.$$

& resolving forces tangential to the section AB,

$$P_t = P_x \cos\theta - P_y \sin\theta$$
$$= \sigma_x Ac \cos\theta - \sigma_y BC \sin\theta$$

Now, Normal stress across the section AB,

$$\sigma_n = \frac{P_n}{AB} = \frac{\sigma_x . AC\sin\theta + \sigma_y BC\cos\theta}{AB}$$

$$= \frac{\sigma_x AC\sin\theta}{AC/\sin\theta} + \frac{\sigma_y BC\cos\theta}{BC/\cos\theta}$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta$$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} + \frac{\sigma_y}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

& tangential stress across the section AB

$$\tau = \frac{P_t}{AB} = \frac{\sigma_x AC\cos\theta - \sigma_y BC\sin\theta}{AB}$$

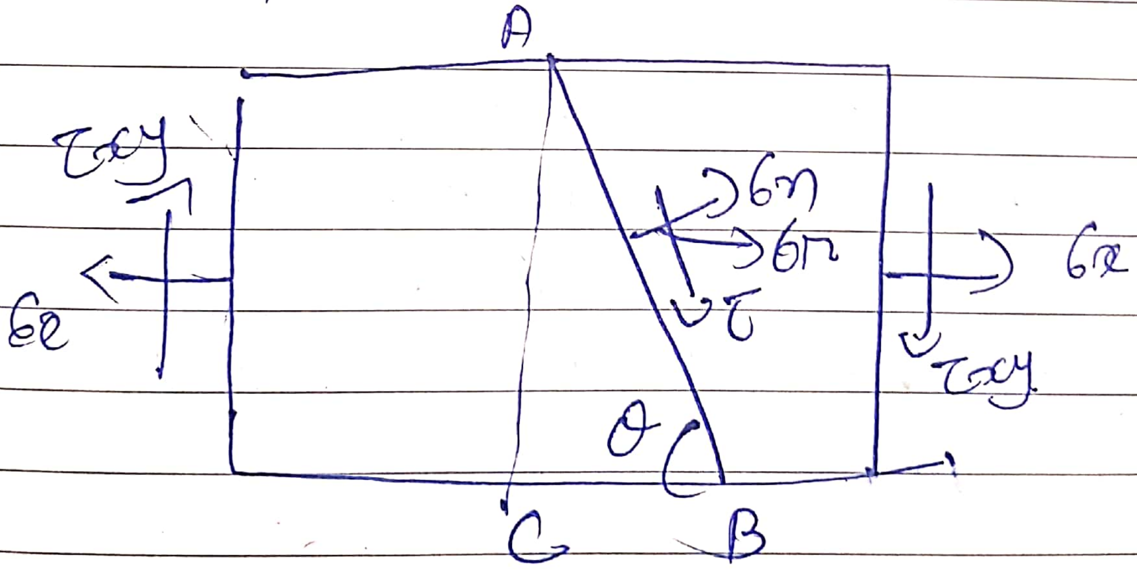
$$= \frac{\sigma_x AC\cos\theta}{AB} - \frac{\sigma_y BC\sin\theta}{AB}$$

$$= \frac{\sigma_x AC \cos\theta}{AC/\sin\theta} - \frac{\sigma_y BC \sin\theta}{BC/\cos\theta}$$

$$= \sigma_x \sin\theta\cos\theta - \sigma_y \sin\theta \cos\theta$$

$$= \frac{\sigma_x - \sigma_y}{2} \sin 2\theta.$$

(3) Stresses on an oblique section of a body sub. to a direct stress in one plane and accompanied by a simple shear stress.



Horizontal force acting on the face AC,

$P_x = \sigma_x . AC \; (\leftarrow)$

vertical force acting on the face AC,

$P_y = \tau_{xy} AC \; (\uparrow)$

& Horizontal force acting on the face BC,

$P = \tau_{xy} BC \; (\rightarrow)$

Resolving the forces perpendicular to section AB,

$P_n = P_x \sin\theta - P_y \sin\theta - P \sin\theta$

$= \sigma_x . AC \sin\theta - \tau_{xy} AC \cos\theta - \tau_{xy} BC \sin\theta$

& forces ~~tangential~~ perpendicular to section AB,

$\sigma_n = \frac{P_n}{AB}$

$= \frac{\sigma_x AC \sin\theta - \tau_{xy} AC \cos\theta - \tau_{xy} BC \sin\theta}{AB}$

$= \frac{\sigma_x AC \sin\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} AC \cos\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} BC \sin\theta}{\frac{BC}{\cos\theta}}$

APRIL | 2016

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

$= \sigma_x \sin^2\theta - \tau_{xy} \sin\theta\cos\theta - \tau_{xy}\sin\theta\cos\theta$

$= \frac{\sigma_x}{2}(1-\cos 2\theta) - 2\tau_{xy}\sin\theta\cos\theta$

$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

& tangential stress across the section AB,

$\tau = \frac{P_t}{AB}$

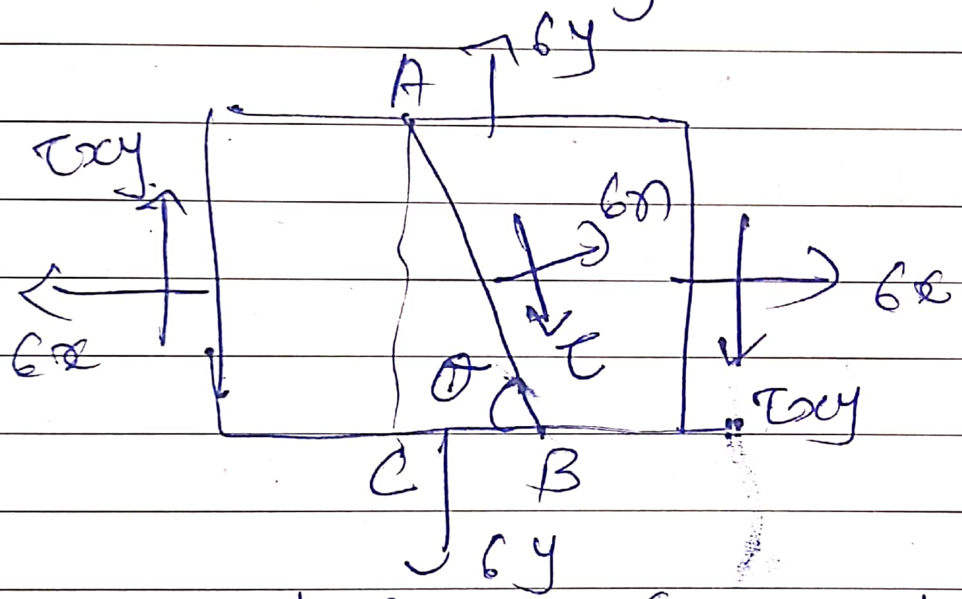
$= \frac{\sigma_x AC\cos\theta}{AB} + \frac{\tau_{xy} AC\sin\theta}{AB} - \frac{\tau_{xy} BC\cos\theta}{AB}$

$= \frac{\sigma_x AC\cos\theta}{\frac{AC}{\sin\theta}} + \frac{\tau_{xy} AC\sin\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} BC\cos\theta}{BC/\cos\theta}$

$= \frac{\sigma_x}{2}\sin 2\theta + \frac{\tau_{xy}}{2}(1-\cos 2\theta) - \frac{\tau_{xy}}{2}(1+\cos 2\theta)$

$= \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$

4) Stresses on an oblique section of a body sub. to direct stress in two mutually perpendicular dirn. Accompanied by a simple shear stress.



Normal Stress (across the AB)

$$\therefore \sigma_n = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

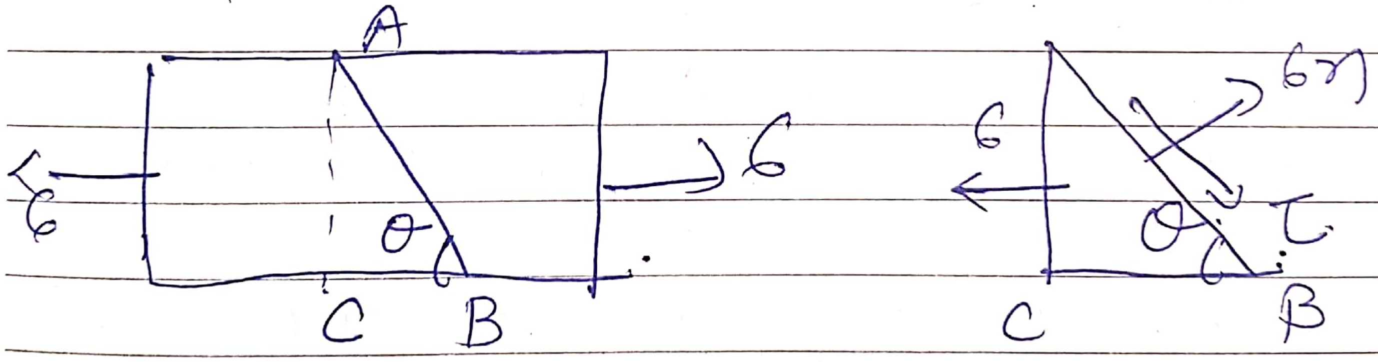
Shear Stress (across the AB)

$$\therefore \tau = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

* Graphical method for the stresses on an oblique section of body.

=) This is done by drawing a Mohr's circle of stresses.

(1). Mohr's circle for stresses on an oblique section of a body sub. to a direct stress in one plane.



(a) (b)

→ Consider a rect. body of uniform c/s area & unit thickness sub. to a direct tensile stress along x-x axis as shown in fig. Now let us consider an oblique section AB inclined with x-x axis, on which we are req^d to find out the stresses as shown in the fig.

σ = tensile stress, in x-x dir^n
θ = Angle which the oblique section AB makes with the x-x axis in clockwise dir^n

Points:-

→ First of all, take some suitable point O & through it draw horizontal line xox.

→ cut off OJ equal to the tensile stress (σ) to some suitable scale & towards right (because σ is tensile). Bisect OJ at C. Now the point O represents the stress system on plane BC & the point J represents the stress system

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

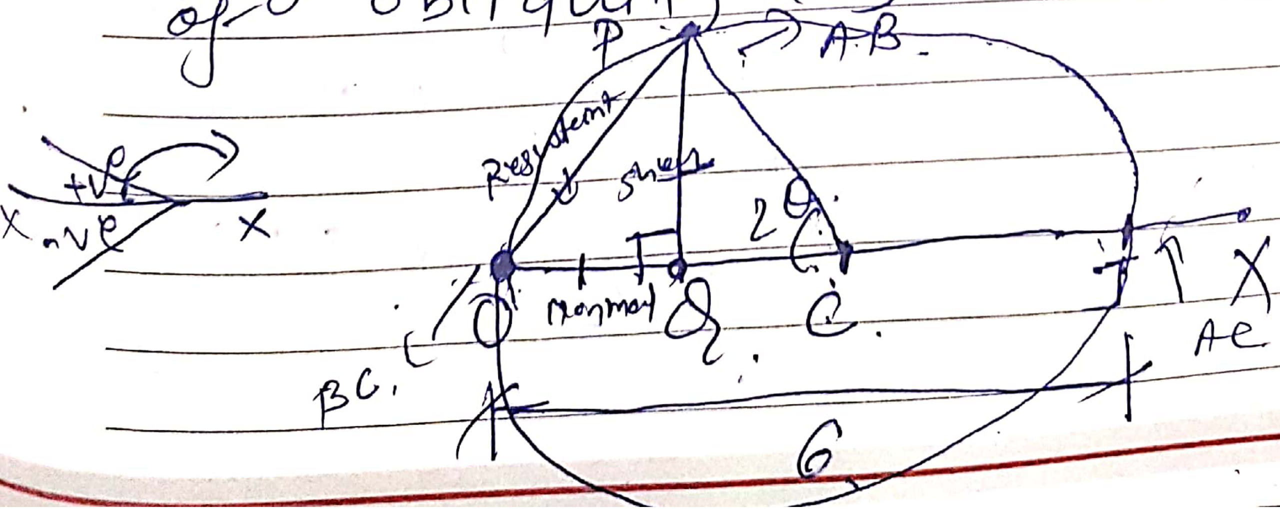
On plane AC.

→ Now with C as centre & radius equal to CO and on CJ draw circle ... Mohr circle.

→ Now through C draw a line CP. making an angle 2θ with CO in the clockwise dir^n meeting the circle at P. The point P. represents the section AB.

→ Through P, draw PQ ⊥ to OX. Join OP.

→ Now OQ, QP & OP will give the normal, shear & resultant stress respectively to the scale. & angle POJ is called ... angle of obliquity (θ).



Proof :- $OC = CQ = CP = \sigma/2$.

∴ Normal stress,

$$\sigma_n = OQ = OC - QC$$
$$= (\sigma/2) - (\sigma/2)\cos 2\theta.$$

& Shear stress,

$$\tau = QP = CP \sin 2\theta$$
$$= \sigma/2 \sin 2\theta.$$

– We find that max. shear stress will be equal to the radius of mohr's circle of stresses. i.e. $\sigma/2$. It will happen when 2θ is equal to 90^0 or 270^0.

$$\Downarrow$$
$$\theta = 45^0 \ \& \ \theta = 135^0$$
$$\Downarrow$$
$$\tau = \sigma/2 \ \& \ \tau = -\sigma/2$$

Ex: point is sub: Tensile stress of 250 MPa in h. dirn & another tensile stress of 25 MPa, such that when it is associated with major tensile stress, it tends to rotate element in the clockwise dirn. magnitude of normal & shear stresses on a element in clockwise dirn. what is magnitude of normal & shear stresses on a section inclined angle of 20° with major tensile stress.

$$\sigma_n = \frac{\overset{\sigma_x}{250} + \overset{\sigma_y}{100}}{2} - \frac{250-100}{2}\cos(2\times 20^\circ) - 25\sin(2\times 20^\circ)$$

$$= 101.48\,\text{MPa}$$