Stresses in beams.

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29					

Bending stresses in beams

* . Introduction

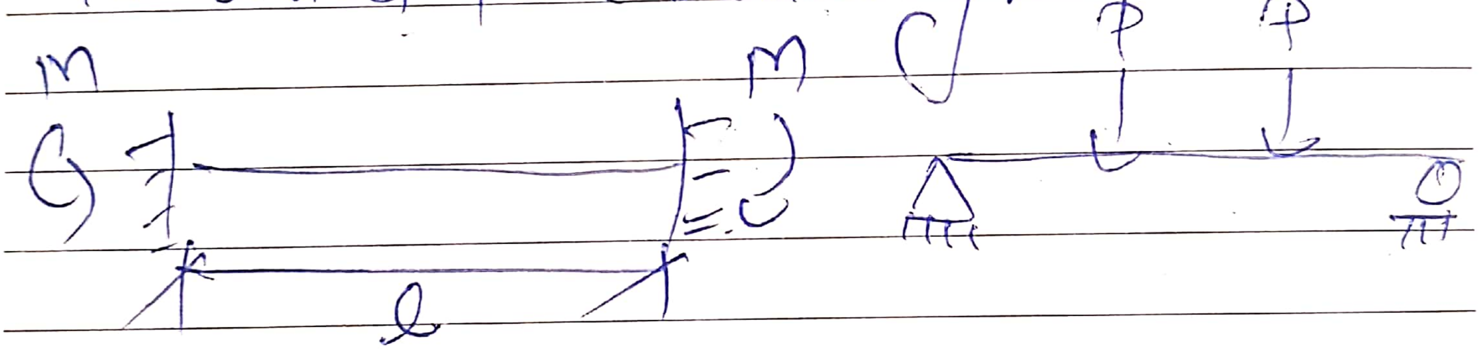
Bending:- effects of a load system on the c/s of beam, viz, SF & BM. These have to be resisted in structural member. In case of beam - has to deform itself. As it bends, resistance to the actions are set up & process of bending will stop when every c/s has set up full resistance to the SF & BM acting on it.

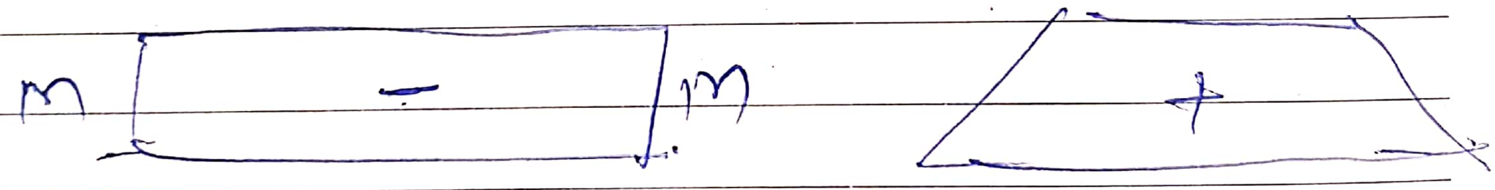
=). stress to resist the bending moment called bending stress or flexural stress.

=) Stress to resist the shear force called shear stress.

Pure bending or simple bending

⇒ beam is loaded in such manner that it is subjected to bending only. such a bending of the beam is called pure bending.





(a) (b)

Stresses setup in the beam due to bending moment are called bending stresses.

* Theory of simple bending:-

Assumptions:-

(1) Transverse sections of the beam which are plane before bending remain plane after bending

(2) The material is homogeneous (same kind of material) and isotropic.
(i.e. it has same elastic properties in all the directions).

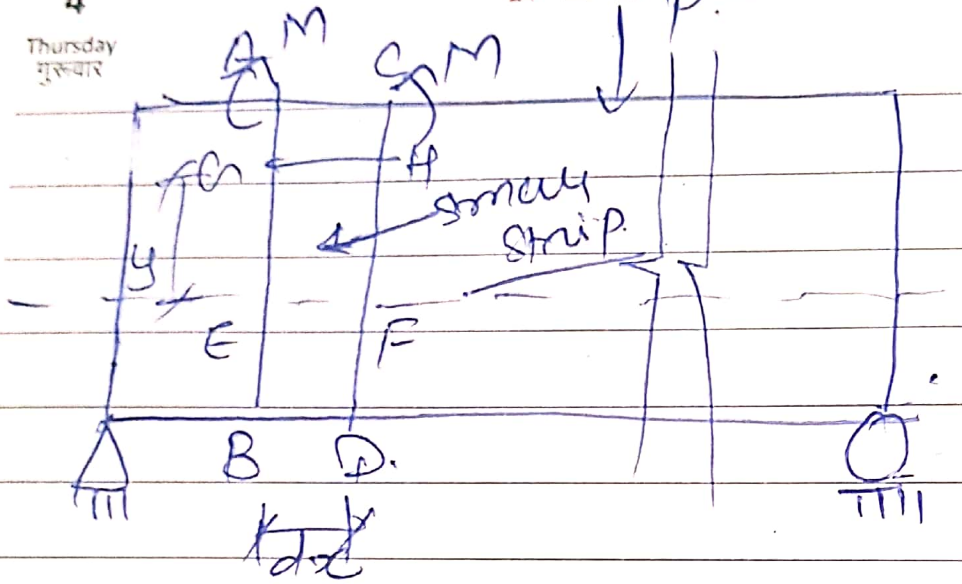
(3) The beam is composed of infinite number of fibres along the longitudinal direction.

(4) The modulus of elasticity are the same for tension & compression.

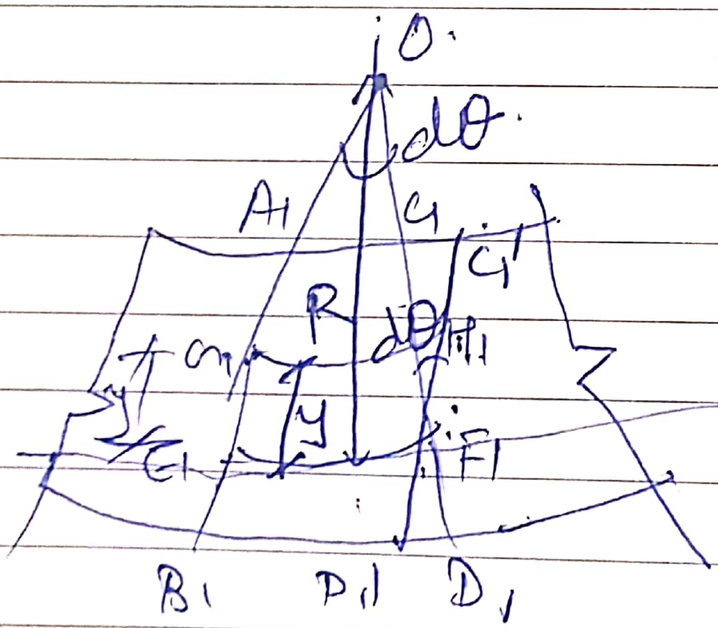
(5) The beam is initially straight and of constant cross-section.

(6) The beam section is symmetrical about the plane of bending

(7) The radius of curvature of the beam is very large as compared to dim. of



Axis of the beam



deformation of fibre say GH located at dist^n of y from neutral axis EF in compression side.

$\delta = H_1H_1' = y\,d\theta$

$\delta l = GH - G_1H_1$

Strain ε is given by,

$\frac{\delta l}{l}$

$\varepsilon = \frac{y\,d\theta}{GH}$ / $\frac{GH - G_1H_1}{GH}$

Now, $GH = EF = R\,d\theta = dx$

M	T	W	T	F	S
	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

From geometry of beam, we find
& sections OG_1H_1 & OG_1F_1

$$\frac{G_1H_1}{E_1F_1} = \frac{R-y}{R}$$

$$1 - \frac{G_1H_1}{E_1F_1} = 1 - \frac{R-y}{R}$$

$$\frac{E_1F_1 - G_1H_1}{E_1F_1} = \frac{y}{R}$$

$$\frac{GH - G_1H_1}{GH} = \frac{y}{R} \qquad E_1F_1 = GH$$

$$\varepsilon = \frac{y\,d\theta}{R\,d\theta} = \frac{y}{R} \qquad \Big| \qquad \varepsilon = \frac{y}{R}$$

We therefore deduce that the strain of any layer is proportional to its distance from the neutral layer. The strain is compressive or tensile according as the layer is above or below the neutral axis, the max^m strain being at the top and bottom layers.

$$\varepsilon_c = \frac{y_c}{R} \quad \& \quad \varepsilon_t = \frac{y_t}{R}$$

Now, variation of flexural stresses

$$\sigma = \varepsilon E = \frac{Ey}{R}$$

$$\therefore \frac{\sigma}{y} = \frac{E}{R} \quad \text{—— (1)}$$

Bending equation: Flexural formula.

→ Final condition of equilibrium, $\Sigma M = 0$, applied bending moment must be balanced by the internal moment of resistance.

optional

$$M - M_R = 0.$$

$$M = M_R = \int y(\sigma_x dA)$$

$$= \frac{E}{R}\int y^2 dA.$$

Now, $\int y^2 dA$ = moment of inertia of the section about the centroidal axis, i.e. neutral axis here.

$$M = \frac{EI}{R}.$$

$$\frac{M}{I} = \frac{E}{R} \quad (2)$$

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\therefore \boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

there are C / T stresses. These stresses form a couple. whose moment must be equal to external moment. The moment of this couple. which resist the external B.M is known as.
Moment of Resistance.

Q2.

we have considered small element of area dA at a dist^n of y from N.A

force on the Area. $dp = \sigma dA$

$$= \frac{E y}{R} dA.$$

moment of force dp, about neutral layer.

$$dM_r = y\, dp.$$

(moment of Resistance)

$$= y \cdot \frac{E y}{R} dA.$$

$$= \frac{E y^2}{R} dA. \quad (\because y^2 dA = \cancel{M.I})$$

$$M_r = \int_{y_c} \frac{E}{R} y^2 dA$$

$$= \frac{E}{R} \int_{-y_t}^{y_c} y^2 dA. \quad (I)$$

$$\boxed{M = \frac{E}{R} \times I}$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$ flexure formula.

M = applied bending moment
I = moment of inertia of section about the neutral axis.
σ = stress in layer under consideration
y = distⁿ of layer from the neutral axis
E = modulus of Elasticity
R = Radius of curvature

Flexure formula is valid for sections which are symmetrical about plane of bending. For rectangular, circular, I section flexure formula can be used for both centroidal axes, but for section like channel, T section. formula is valid about an axis. Centroidal axis about which section is symmetrical.

M.I. to the distⁿ from the N.A. to the point where Bending normal stress is determined

* Section Modulus (Z)

is higher will be the Resistance of bending

The term $\frac{I}{y}$, used is called Z.

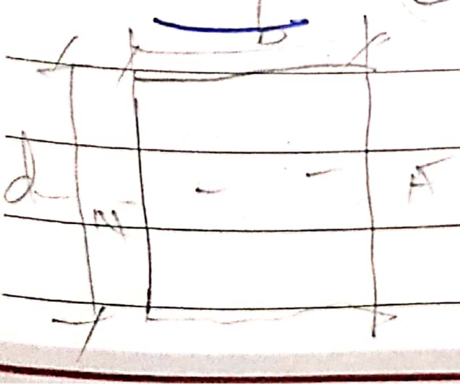
for symmetrical section $y_{max} = \frac{d}{2}$,
d is the depth of section.

⇒ It is used for geometric property of the c/s used for designing beams & flexural members.

⇒ It may be defined as the ratio of total moment resisted by the section to the stress in the extreme fibre which is equal to yield stress.

① Rect! — $I = \frac{bd^3}{12}$, $y_c = y_t = \frac{d}{2}$

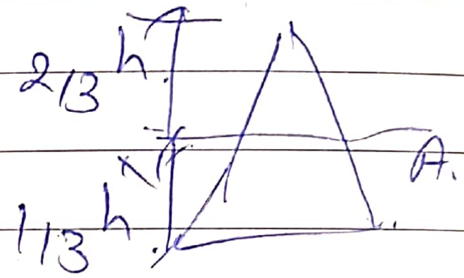
$$Z = \frac{bd^2}{6}$$



(2) Circular $I = \frac{\pi}{64} d^4$, $y_c = y_t = \frac{d}{2}$

$Z = \frac{\pi}{32} d^3$

(3) Triangular $I = \frac{bh^3}{36}$



(top). $y_{max} = \frac{bh^2}{24}$ $\frac{2}{3}h$

$y_{bottom} = \frac{1}{3}h$.

$Z_{top} = \frac{bh^2}{24}$, $Z_{btm} = \frac{bh^2}{12}$

* Modulus of rupture -

$\sigma_{perm} = \frac{M_r}{Z}$,

beam is loaded within elastic limit. when the beam is loaded to failure. failure moment M_F.

unsymmetrical section

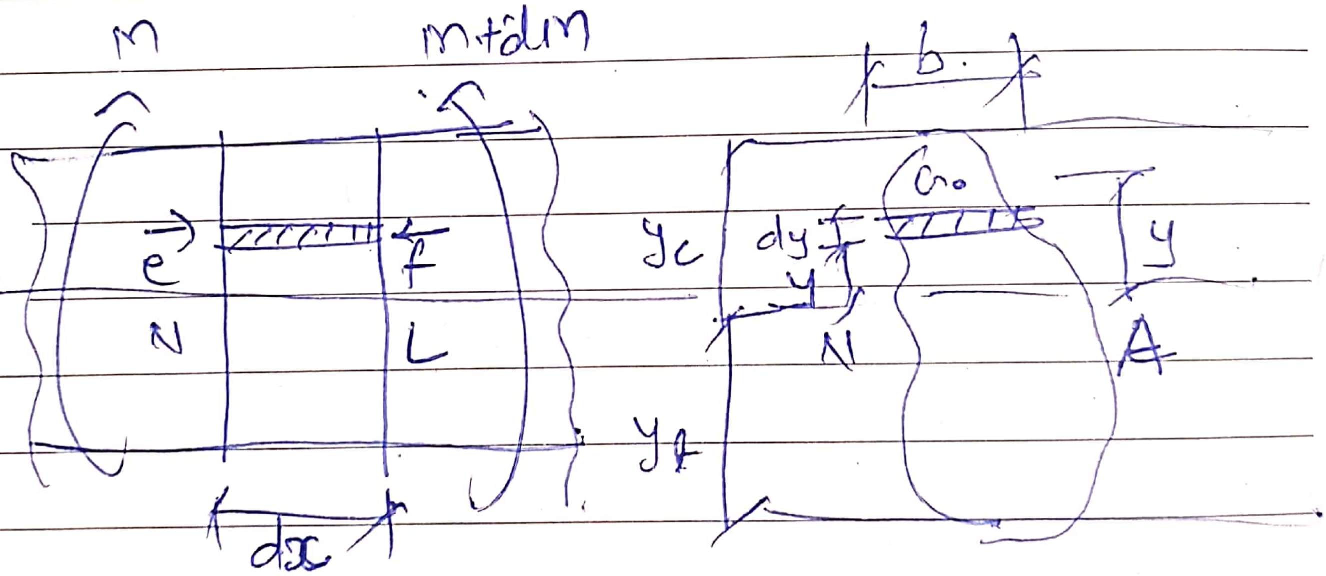
$\sigma_f = \frac{M_r}{Z}$. bending stress at failure, is called modulus of rupture.
It is used to compare the bending strengths of diff^n beams made up of various sizes and materials. estimate the tensile strength of wood & concrete.

17/7/19

Shear stresses in beams

=) About the variation of shear stresses in beam section. As the bending stress section ~~var~~ is not uniform in beam section, it varies linearly. similarly, the shear stress due to shear force is not uniform & varies making a parabolic curve.

Note!
⇒ Shear force in a beam section occurs where there is variation in bending moment. Variation in the bending moment causes unbalanced forces along the length of section



Plane of length dx & breadth b at distance y from neutral layer. beam is sub. to M on left side & M+dM on right side. Consider a layer at dist^n y from neutral layer thickness of layer is dy.

FEBRUARY
फरवरी
2016

MARCH | 2016

S	M	T	W	T	F	S
		1	2	3	4	5
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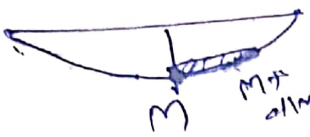
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

15
Monday
सोमवार



$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}.$$

Stress on left side,

$$\sigma = \frac{My}{I}.$$

Stress on right side,

$$\sigma' = \frac{M+dM}{I} \cdot y.$$

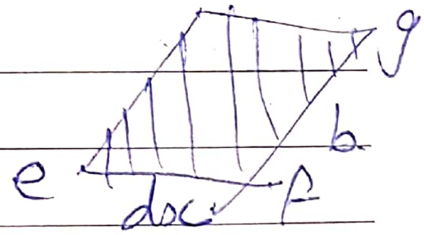
or $\sigma + d\sigma$

forces, $\delta P = \sigma \delta a = \sigma b\, dy$ or $\frac{M}{I} \times y \times a$

$\delta P' = \sigma' \delta a = \sigma' b\, dy$. $\frac{M+dM}{I} \times y \times a$.

Summing up all the unbalanced forces on elemental area, from y to y_c ~~forces on~~ ... Now, Net unbalanced forces on strip.

$$\delta F = \int_y^{y_c} \frac{dM}{I} \cdot y\, b\, dy.$$



$\frac{M+dM \times y \times a}{I}$.

$- \frac{M}{I} \times y \times a$

$= \frac{dM}{I} \times y \times a$.

but $\delta F = \tau b dx$, τ is shear stress developed in beam. efgh,

where side $ef = dx$, $fg = b$.

Now, integrating the above eqn of dM/dx

$$\tau = \frac{1}{bI} \int_y^{y_c} \frac{dM}{dx}\, y\, b\, dy.$$

$\int_0^{d/2} \frac{dM}{I} \times a \times y \times dy$ (cutting).

$\frac{dM}{I} \int_0^{d/2} a\, y\, dy$. dm·ay

$\frac{dM}{dx} = F$, shear force on section

ONGC

JANUARY | 2016

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

16
Tuesday
मंगलवार

$$\tau = \frac{F}{bI}\int_{y}^{y_c} b\,y\,dy.$$

Now, $\tau = \frac{F}{A} = \frac{\frac{dm\,A\bar{y}}{I}}{dx.b}$

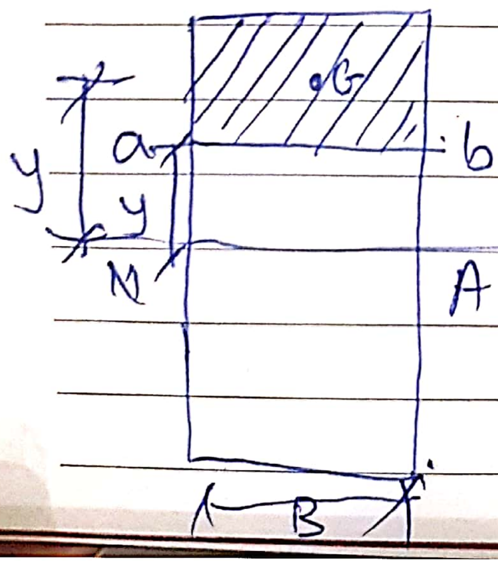
$$\int_{y}^{y_c} b\,y\,dy = \int_{y}^{y} y\,da = A\bar{y}$$

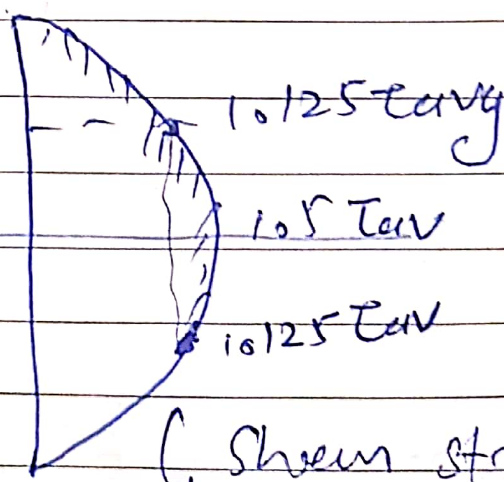
$\frac{dm\,A\bar{y}}{dx.Ib} = \frac{FA\bar{y}}{Ib}$

$$\boxed{\tau = \frac{FA\bar{y}}{Ib}.}$$

F = SF, A = area of CS
I = MOI, b = breadth
$\bar{y}$ = distn of CG of area above the layer

* Shear stress distribution in Rectangular and circular section. / Triangular sections.





(Shear stress distribution)

Scanned with CamScanner

Section of the beam is rectangular with breadth B & depth D is sub. to SF F.
Consider a layer ab of breadth B, at a distⁿ y for neutral axis NA.

Shear stress $\tau = \frac{F A\bar{y}}{Ib}$, $I = \frac{BD^3}{12}$,

$b = B$,

Area above layer ab,

$$A = B\left(D/2 - y\right)$$

$$\bar{y} = y + \frac{1}{2}\left(D/2 - y\right) = D/4 + y - \frac{y}{2} = D/4 + \frac{y}{2}$$

First moment of area $A\bar{y} = B\left(\frac{D}{2} - y\right) \times \left(\frac{D}{4} + \frac{y}{2}\right)$

$$= \frac{B}{2}\left(\frac{D^2}{4} - y^2\right)$$

$$\tau = \frac{F \times B}{2}\left(\frac{D^2}{4} - y^2\right) \times \frac{12}{BD^3} \times \frac{1}{B}$$

$$= \frac{6F}{BD^3}\left(\frac{D^2}{4} - y^2\right)$$

18 Thursday गुरुवार

$\tau = \frac{3}{2}\frac{F}{BD}$, at $y = 0$, $\tau_{max} = 1.5\frac{F}{BD}$ τ_{avg}, Area, $\tau = \frac{F}{A}$

$= \frac{6F}{BD^3}\left(\frac{D^2}{4} - \frac{D^2}{16}\right)$

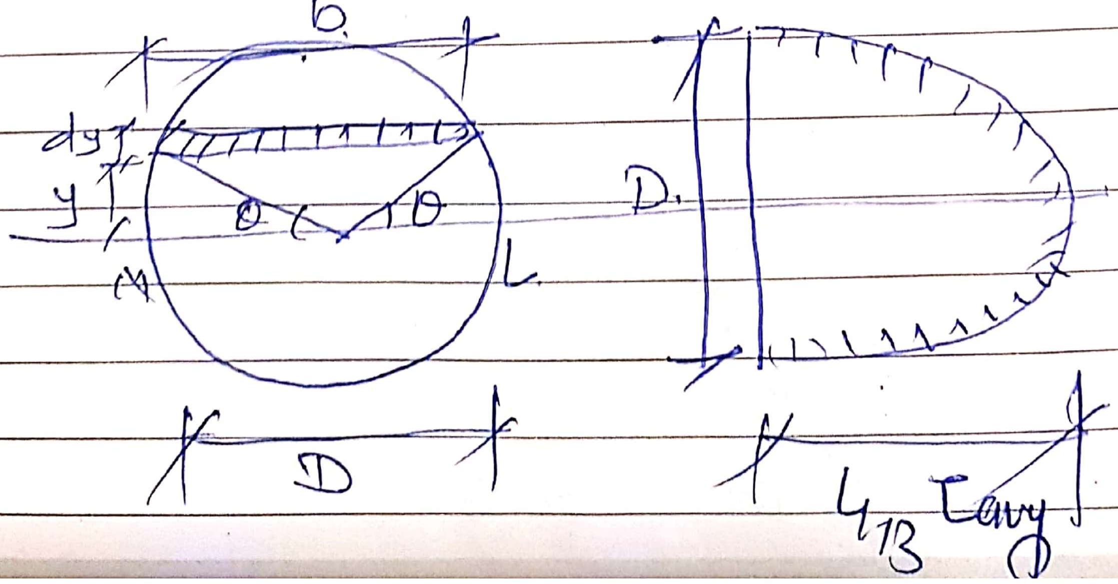
$= \frac{9}{8} \times \frac{F}{BD}$, at $y = D/4$

$= 0$, at $y = D/2$

$\tau_{avg} = F/BD$, $\tau_{max} = \tau_{NL} = 1.5\,\tau_{av}$.

$\tau = 1.125\,\tau_{avg}$ at $y = D/4$

* **circular section**



Consider beam of circular section of dia D.
Ps sub. to SF, F at section under consideration.
SF at any layer also

$$\tau = \frac{FA\bar{y}}{Ib}.$$

Consider layer at distⁿ y from neutral layer, subtending an angle θ at the centre from neutral layer.

$$\sin\theta = y/R \quad \therefore \; y = R\sin\theta.$$

$$dy = R\cos\theta\, d\theta$$

$$b = 2R\cos\theta.$$

$$A\bar{y} = \int_0^R b\,y\,dy = \int_0^{\pi/2} (2R\cos\theta)(R\sin\theta)(R\cos\theta)\,d\theta.$$

$$= 2R^3 \int \left[\cos^3\frac{\pi}{2} - \cos^3\theta\right]$$

$$= -\frac{2R^3}{3}\cos^3\theta, \quad \text{as } \cos\frac{\pi}{2} = 0.$$

$$\tau = \frac{F}{I} \times \frac{2R^3}{3}\cos^3\theta \times \frac{1}{2R\cos\theta}.$$

as $b = 2R\cos\theta$.

JANUARY | 2016

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

FEBRUARY फरवरी 2016

20 Saturday शनिवार

$$= \frac{FR^2\cos^2\theta}{3I}, \quad I = \frac{\pi R^4}{4}$$

$$\tau = \frac{FR^2\cos^2\theta \times 4}{3\times\pi R^4} = \frac{4F\cos^2\theta}{3\pi R^2}$$

$$= \frac{4F}{3\pi R^2}, \text{ at } \theta = 0, \text{ NA.}$$

$$= \frac{F}{\pi R^2}, \text{ at } \theta = 30^\circ.$$

$$= 2/3 \; F/\pi R^2, \; \theta = 45^\circ$$

21 Sunday रविवार

$$= \frac{F}{3\pi R^2}, \text{ at } \theta = 60^\circ.$$

$$= 0 \text{ at } \theta = 90^\circ.$$

$$\tau_{avg} = \frac{F}{\pi R^2}, \text{ avg. shear stress.}$$

$\tau = 4/3\, \tau_{avg}$, at $\theta = 0$

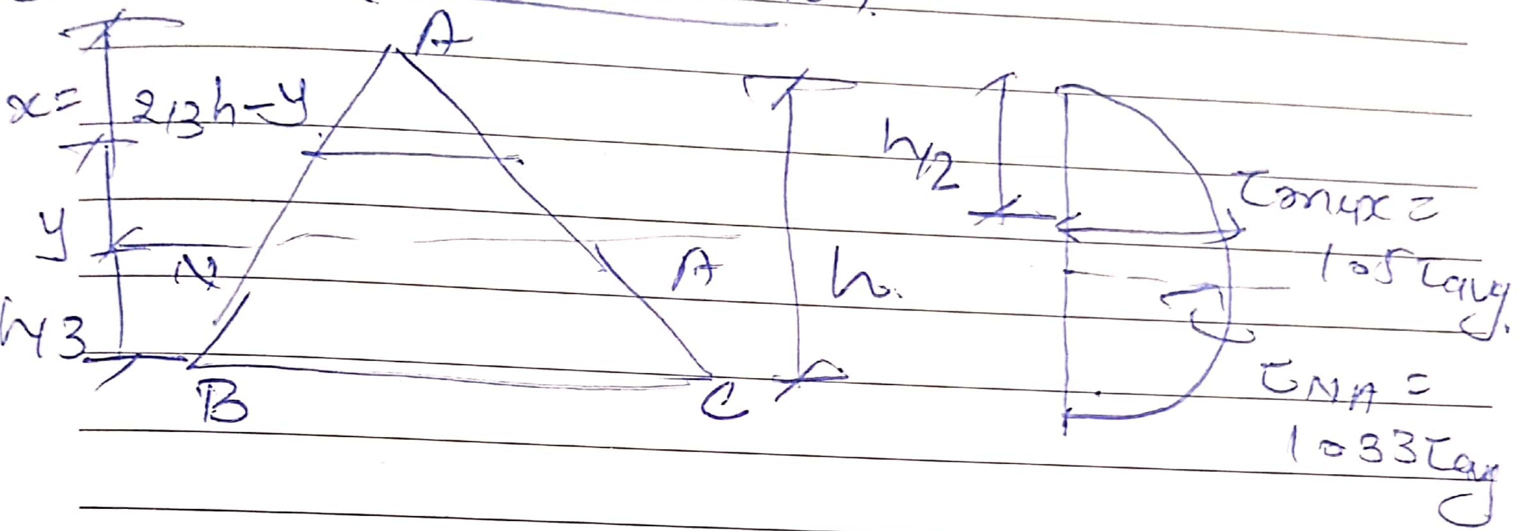
$\tau = \tau_{avg}$ at $\theta = 30^\circ$

$\tau = 2/3\, \tau_{avg}$, $\theta = 45^\circ$

$\tau = 1/3\, \tau_{avg}$, $\theta = 60^\circ$

$\tau = 0$ at $\theta = 90^\circ$

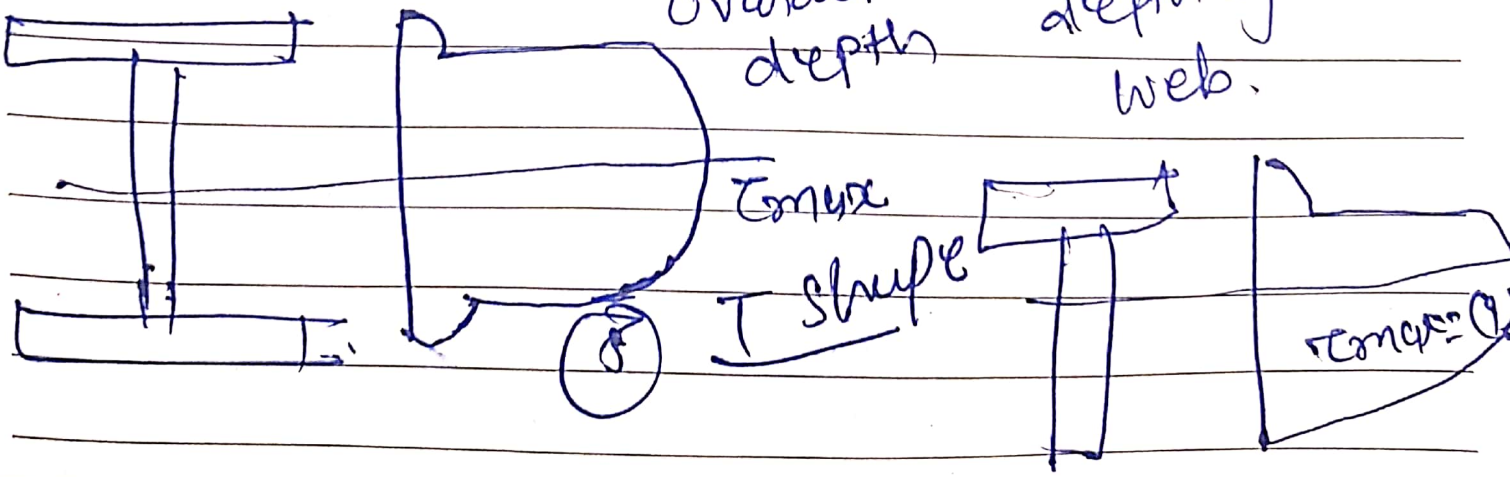
(3) Triangular section



(4) I - Section

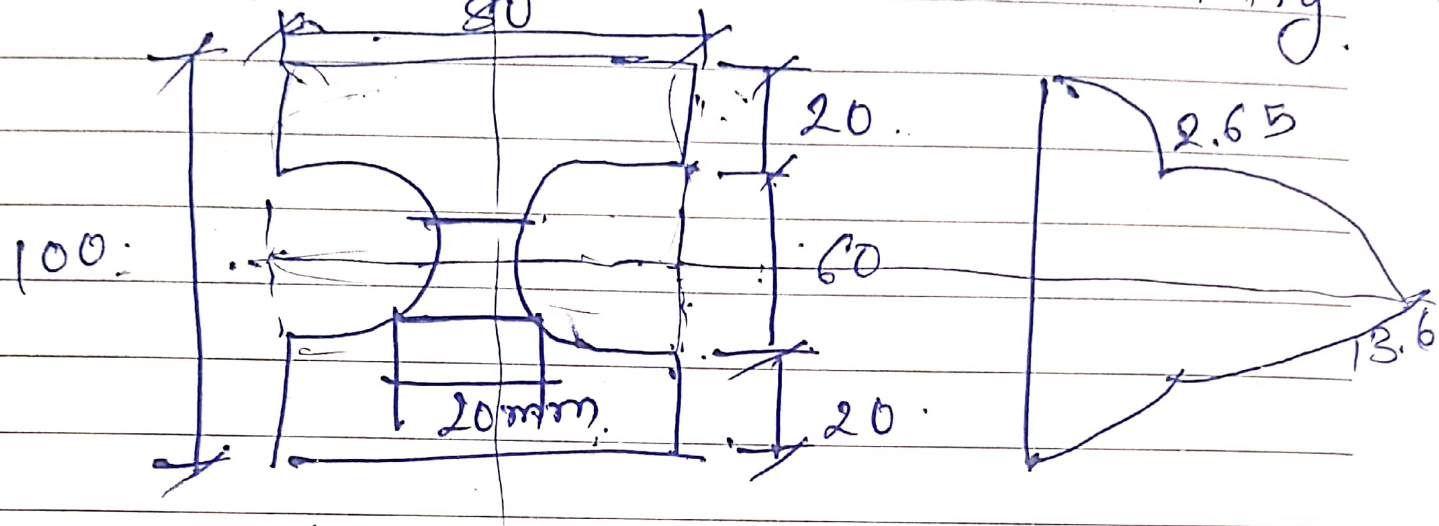
Imp. $\tau_{max} = \frac{F}{Ib}\left[\frac{B}{8}(D^2 - d^2) + \frac{bd^2}{8}\right]$

overall depth — depth of web.



T1

* A steel section shown in fig.



shear force $F = 20\,kN$
$= 20\times10^3\,N$

$$I = \left[\frac{80\times(100)^3}{12}\right] - \left[\frac{\pi}{64}(60)^4\right]$$

$$= 6.03\times10^6\,mm^4$$

upper portion betn A & B.

$A = 80\times20 = 1600\,mm^2$

$\bar{y} = 30 + 20/2 = 40\,mm$

$B = 80$

$$\tau = \frac{20\times10^3\times1600\times40}{6.03\times10^6\times80} = 2.65\,MPa$$

$$\tau = \frac{250-100}{2}\sin(2\times 20) - \overset{\tau_{xy}}{\cancel{25}}\cos(2\times 20)$$

$$= 29.06 \text{ mpa.}$$